Computer algebra independent integration tests

4-Trig-functions/4.3-Tangent/4.3.0-a-trg-^m-b-tan-^n

Nasser M. Abbasi

May 18, 2020 Compiled on May 18, 2020 at 3:29pm

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [387]. This is test number [98].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (387)	% 0. (0)
Mathematica	% 99.74 (386)	% 0.26 (1)
Maple	% 68.48 (265)	% 31.52 (122)
Maxima	% 18.09 (70)	% 81.91 (317)
Fricas	% 35.14 (136)	% 64.86 (251)
Sympy	% 3.36 (13)	% 96.64 (374)
Giac	% 17.57 (68)	% 82.43 (319)

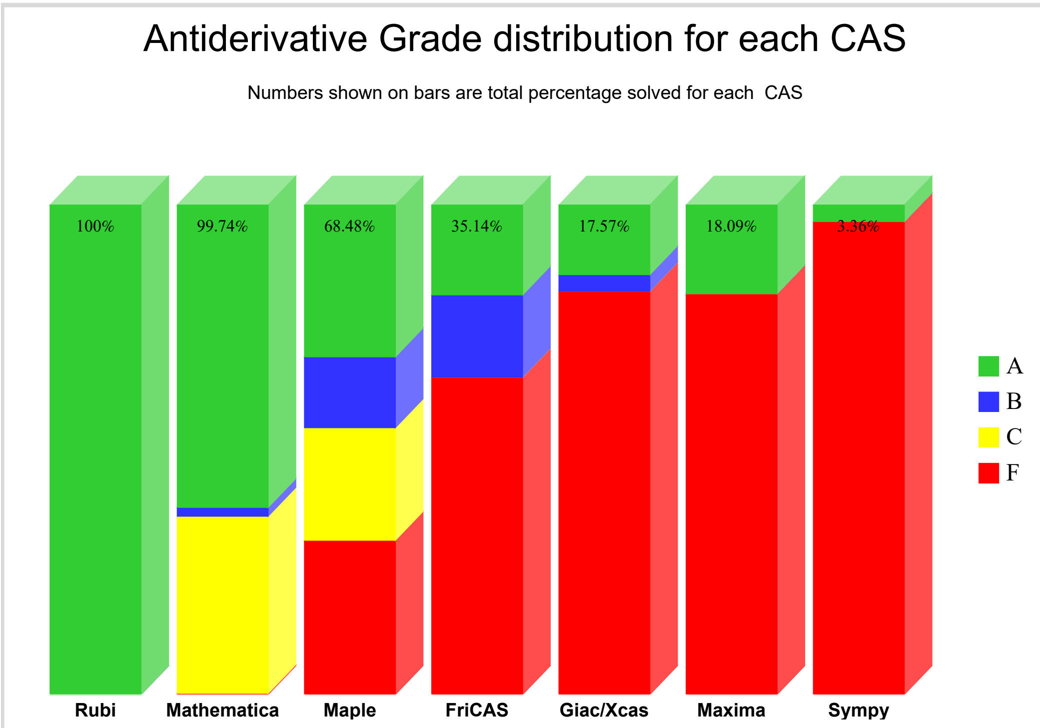
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

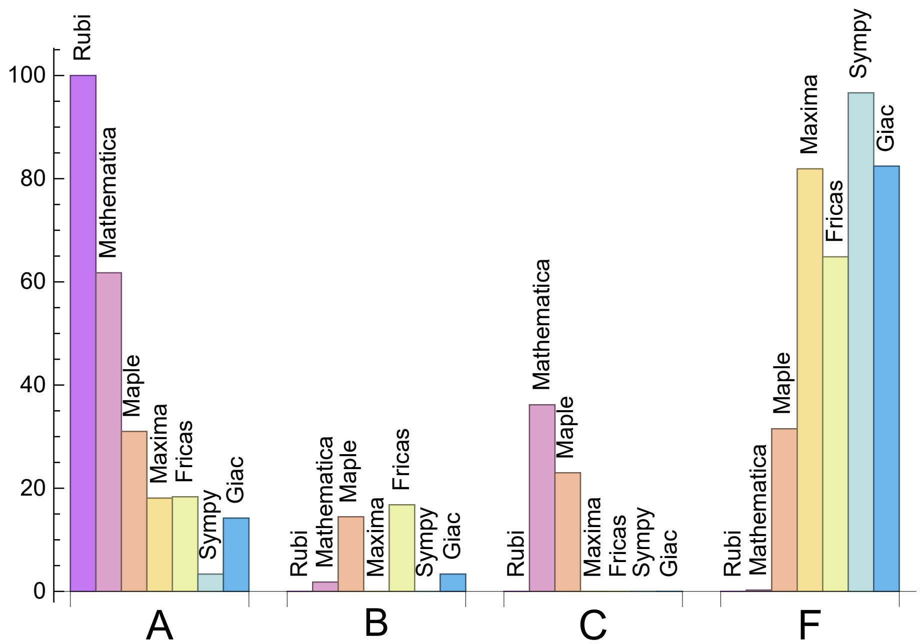
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	61.76	1.81	36.18	0.26
Maple	31.01	14.47	23.	31.52
Maxima	18.09	0.	0.	81.91
Fricas	18.35	16.8	0.	64.86
Sympy	3.36	0.	0.	96.64
Giac	14.21	3.36	0.	82.43

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.1	103.99	1.	78.	1.
Mathematica	1.04	187.28	2.87	69.	0.93
Maple	0.17	688.21	10.25	220.	2.06
Maxima	1.31	71.19	1.16	62.5	1.12
Fricas	2.02	652.29	5.06	185.	4.05
Sympy	7.29	41.54	1.47	44.	1.19
Giac	1.99	299.31	4.01	269.	1.37

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {174, 175, 176, 180, 181, 182, 183, 184, 185, 294, 296, 298, 299, 305, 308, 310, 312, 320, 323, 325, 327, 331, 335, 336, 337, 338, 344, 345, 348, 349, 355, 356, 357, 358, 359, 360, 367, 368, 372, 373, 387}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command AboluteTiming[] was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command Usage was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's time.time() call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is ValueError. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

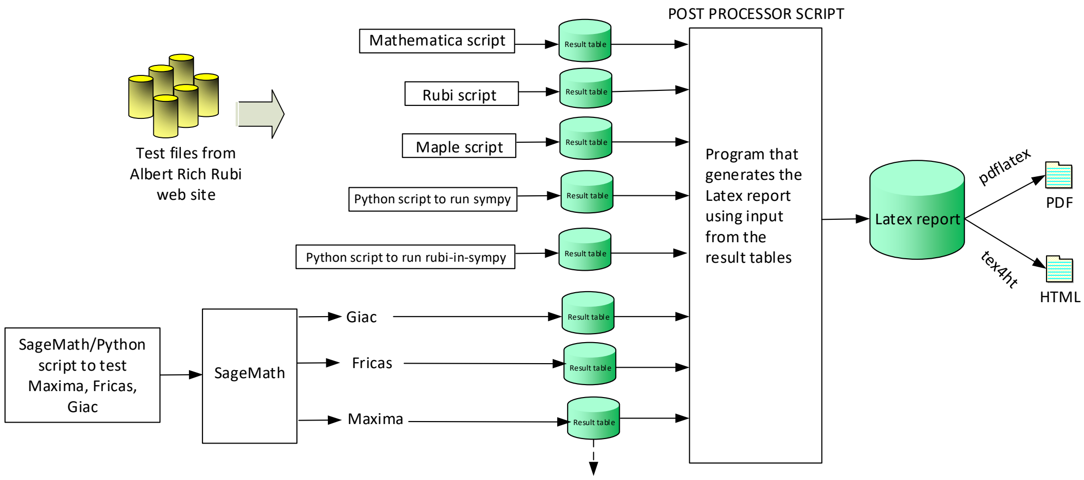
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 20, 23, 24, 25, 26, 27, 28, 29, 30, 32, 36, 37, 38, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 84, 85, 86, 87, 88, 94, 95, 96, 97, 98, 103, 104, 105, 106, 107, 114, 115, 116, 117, 118, 119, 121, 123, 125, 127, 129, 131, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 177, 178, 179, 182, 186, 187, 188, 189, 190, 194, 196, 198, 202, 204, 206, 210, 212, 217, 219, 221, 222, 223, 224, 225, 226, 227, 228, 230, 236, 237, 238, 239, 240, 247, 248, 249, 251, 252, 258, 259, 260, 262, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 293, 295, 297, 300, 306, 307, 309, 311, 313, 315, 317, 319, 321, 322, 324, 326, 328, 330, 332, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 362, 363, 364, 365, 366, 369, 370, 371, 374, 375, 376, 377, 378, 379, 380, 383, 384, 385, 386 }

B grade: { 173, 183, 184, 304, 354, 381, 382 }

C grade: { 10, 12, 14, 15, 16, 17, 18, 19, 21, 22, 31, 33, 34, 35, 39, 40, 41, 59, 60, 61, 62, 63, 69, 70, 71, 72, 78, 79, 80, 81, 82, 83, 89, 90, 91, 92, 93, 99, 100, 101, 102, 108, 109, 110, 111, 112, 113, 120, 122, 124, 126, 128, 130, 132, 174, 175, 176, 180, 181, 185, 191, 192, 193, 195, 197, 199, 200, 201, 203, 205, 207, 208, 209, 211, 213, 214, 215, 216, 218, 220, 229, 231, 232, 233, 234, 235, 241, 242, 243, 244, 245, 246, 250, 253, 254, 255, 256, 257, 261, 263, 264, 265, 266, 267, 268, 269, 270, 292, 294, 296, 298, 299, 301, 302, 303, 305, 308, 310, 312, 314, 316, 318, 320, 323, 325, 327, 329, 331, 333, 355, 356, 357, 358, 359, 360, 367, 368, 372, 373, 387 }

F grade: { 361}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 57, 58, 59, 67, 68, 76, 77, 78, 79, 87, 88, 97, 98, 99, 106, 107, 125, 127, 129, 131, 134, 135, 136, 137, 163, 195, 196, 197, 198, 204, 205, 206, 210, 211, 212, 213, 216, 217, 218, 219, 220, 221, 226, 227, 228, 229, 236, 237, 238, 239, 241, 242, 243, 245, 246, 247, 248, 249, 250, 253, 257, 258, 259, 260, 261, 295, 297, 304, 306, 313, 319, 321, 324, 326, 330, 332, 334, 353, 365, 376 }

B grade: { 56, 60, 61, 62, 63, 66, 69, 70, 71, 72, 75, 80, 81, 82, 83, 86, 89, 90, 91, 92, 93, 96, 100, 101, 102, 105, 108, 109, 110, 111, 112, 113, 114, 116, 118, 121, 123, 133, 138, 231, 232, 233, 234, 235, 244, 254, 255, 256, 263, 264, 265, 266, 267, 268, 269, 270 }

C grade: { 52, 54, 55, 64, 65, 73, 74, 84, 85, 94, 95, 103, 104, 115, 117, 119, 120, 122, 124, 126, 128, 130, 132, 139, 140, 141, 142, 143, 144, 164, 165, 177, 178, 179, 191, 192, 193, 194, 199, 200, 201, 202, 203, 207, 208, 209, 214, 215, 230, 240, 251, 252, 262, 291, 292, 293, 294, 296, 298, 299, 300, 301, 302, 303, 305, 307, 308, 309, 310, 311, 312, 314, 315, 316, 317, 318, 320, 322, 323, 325, 327, 328, 329, 331, 333, 351, 352, 377, 378 }

F grade: { 23, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 222, 223, 224, 225, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 379, 380, 381, 382, 383, 384, 385, 386, 387 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 19, 20, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 56, 57, 58, 66, 67, 68, 75, 76, 77, 86, 87, 88, 96, 97, 98, 105, 106, 107, 163, 164, 165, 177, 178, 179, 226, 227, 228, 236, 237, 238, 247, 248, 249, 258, 259, 260, 351, 352, 353, 376, 377, 378

B grade: { }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 21, 22, 23, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 59, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 78, 79, 80, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 102, 103, 104, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 229, 230, 231, 232, 233, 234, 235, 239, 240, 241, 242, 243, 244, 245, 246, 250, 251, 252, 253, 254, 255, 256, 257, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 379, 380, 381, 382, 383, 384, 385, 386, 387 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 19, 20, 24, 25, 26, 27, 28, 29, 36, 37, 38, 39, 40, 41, 52, 57, 58, 66, 67, 68, 76, 77, 87, 88, 114, 116, 121, 123, 127, 134, 135, 163, 164, 165, 177, 178, 179, 226, 227, 236, 237, 247, 248, 258, 259, 295, 297, 306, 319, 321, 324, 326, 332, 334, 351, 352, 353, 363, 364, 365, 376, 377, 378 }

B grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 21, 22, 56, 75, 86, 96, 97, 98, 105, 106, 107, 118, 125, 129, 131, 133, 136, 137, 138, 191, 192, 193, 194, 199, 200, 201, 202, 203, 207, 208, 209, 214, 215, 228, 229, 238, 239, 249, 250, 260, 261, 291, 293, 300, 302, 304, 307, 309, 311, 313, 315, 317, 322, 328, 330 }

C grade: { }

F grade: { 23, 30, 31, 32, 33, 34, 35, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 54, 55, 59, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 78, 79, 80, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 102, 103, 104, 108, 109, 110, 111, 112, 113, 115, 117, 119, 120, 122, 124, 126, 128, 130, 132, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 195, 196, 197, 198, 204, 205, 206, 210, 211, 212, 213, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 230, 231, 232, 233, 234, 235, 240, 241, 242, 243, 244, 245, 246, 251, 252, 253, 254, 255, 256, 257, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 294, 296, 298, 299, 301, 303, 305, 308, 310, 312, 314, 316, 318, 320, 323, 325, 327, 329, 331, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 354, 355, 356, 357, 358, 359, 360, 361, 362, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 379, 380, 381, 382, 383, 384, 385, 386, 387 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 295, 319, 324, 353, 376 }

B grade: { }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387 }

2.1.7 Giac

A grade: { 1, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 30, 31, 32, 33, 39, 40, 41, 75, 84, 85, 86, 87, 88, 94, 95, 96, 97, 98, 103, 104, 105, 106, 107, 163, 226, 227, 228, 229, 230, 247, 248, 249, 250, 251, 252, 258, 259, 260, 261, 262 }

B grade: { 2, 3, 4, 5, 6, 7, 8, 27, 28, 29, 36, 37, 38 }

C grade: { }

F grade: { 9, 10, 11, 23, 34, 35, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 89, 90, 91, 92, 93, 99, 100, 101, 102, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 253, 254, 255, 256, 257, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	17	15	49	19	18
normalized size	1	1.	1.	1.42	1.25	4.08	1.58	1.5
time (sec)	N/A	0.004	0.008	0.002	2.503	1.729	0.139	1.709

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	23	24	24	34	15	305
normalized size	1	1.	1.64	1.71	1.71	2.43	1.07	21.79
time (sec)	N/A	0.008	0.006	0.003	1.381	1.899	0.169	1.593

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	25	31	42	73	32	332
normalized size	1	1.	0.93	1.15	1.56	2.7	1.19	12.3
time (sec)	N/A	0.011	0.024	0.003	0.872	1.776	0.212	1.93

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	38	35	39	66	27	790
normalized size	1	1.	1.36	1.25	1.39	2.36	0.96	28.21
time (sec)	N/A	0.015	0.011	0.003	2.726	1.8	0.29	2.381

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	37	44	73	101	44	691
normalized size	1	1.	0.86	1.02	1.7	2.35	1.02	16.07
time (sec)	N/A	0.02	0.046	0.003	1.571	1.753	0.441	2.639

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	53	50	55	99	39	1335
normalized size	1	1.	1.2	1.14	1.25	2.25	0.89	30.34
time (sec)	N/A	0.025	0.015	0.003	1.865	1.537	0.637	4.191

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	47	57	100	131	56	1094
normalized size	1	1.	0.82	1.	1.75	2.3	0.98	19.19
time (sec)	N/A	0.027	0.099	0.003	0.945	1.523	0.92	10.313

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	68	61	69	132	51	1945
normalized size	1	1.	1.17	1.05	1.19	2.28	0.88	33.53
time (sec)	N/A	0.03	0.011	0.003	1.408	1.509	1.215	9.255

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	175	200	0	1520	0	0
normalized size	1	1.	0.75	0.86	0.	6.55	0.	0.
time (sec)	N/A	0.196	0.36	0.028	0.	1.747	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	40	182	0	1521	0	0
normalized size	1	1.	0.19	0.86	0.	7.17	0.	0.
time (sec)	N/A	0.145	0.068	0.015	0.	1.718	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	159	176	0	1328	0	0
normalized size	1	1.	0.76	0.84	0.	6.32	0.	0.
time (sec)	N/A	0.151	0.16	0.015	0.	1.696	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	40	160	0	1283	0	251
normalized size	1	1.	0.21	0.83	0.	6.68	0.	1.31
time (sec)	N/A	0.121	0.041	0.018	0.	1.7	0.	1.739

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	131	166	0	1339	0	251
normalized size	1	1.	0.68	0.86	0.	6.97	0.	1.31
time (sec)	N/A	0.121	0.1	0.018	0.	1.675	0.	2.483

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	38	184	0	1693	0	275
normalized size	1	1.	0.18	0.87	0.	7.99	0.	1.3
time (sec)	N/A	0.146	0.065	0.016	0.	1.77	0.	1.577

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	40	184	0	1717	0	288
normalized size	1	1.	0.19	0.86	0.	8.02	0.	1.35
time (sec)	N/A	0.15	0.081	0.017	0.	1.824	0.	1.484

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	40	202	0	1985	0	313
normalized size	1	1.	0.17	0.86	0.	8.48	0.	1.34
time (sec)	N/A	0.181	0.103	0.019	0.	1.779	0.	1.506

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	38	215	0	1481	0	282
normalized size	1	1.	0.16	0.88	0.	6.09	0.	1.16
time (sec)	N/A	0.418	0.027	0.086	0.	1.455	0.	1.411

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	40	202	0	1445	0	282
normalized size	1	1.	0.18	0.9	0.	6.45	0.	1.26
time (sec)	N/A	0.394	0.049	0.051	0.	1.526	0.	1.448

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	40	114	166	358	0	171
normalized size	1	1.	0.31	0.87	1.27	2.73	0.	1.31
time (sec)	N/A	0.104	0.04	0.015	1.445	1.347	0.	1.428

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	100	114	167	898	0	173
normalized size	1	1.	0.76	0.87	1.27	6.85	0.	1.32
time (sec)	N/A	0.1	0.133	0.013	1.424	1.387	0.	1.358

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	38	211	0	1494	0	282
normalized size	1	1.	0.17	0.94	0.	6.67	0.	1.26
time (sec)	N/A	0.326	0.028	0.04	0.	1.603	0.	1.347

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	38	227	0	1852	0	306
normalized size	1	1.	0.16	0.93	0.	7.56	0.	1.25
time (sec)	N/A	0.436	0.059	0.042	0.	1.749	0.	1.588

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	53	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.042	0.438	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	56	58	63	180	0	74
normalized size	1	1.	0.57	0.59	0.64	1.84	0.	0.76
time (sec)	N/A	0.042	0.376	0.033	1.564	1.292	0.	1.589

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	47	48	46	135	0	55
normalized size	1	1.	0.77	0.79	0.75	2.21	0.	0.9
time (sec)	N/A	0.028	0.115	0.019	1.498	1.371	0.	1.512

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	37	26	100	0	35
normalized size	1	1.	1.	1.16	0.81	3.12	0.	1.09
time (sec)	N/A	0.017	0.039	0.026	1.577	1.322	0.	1.443

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	39	45	45	119	0	109
normalized size	1	1.	1.26	1.45	1.45	3.84	0.	3.52
time (sec)	N/A	0.016	0.084	0.025	1.612	1.312	0.	1.62

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	56	63	62	177	0	281
normalized size	1	1.	0.85	0.95	0.94	2.68	0.	4.26
time (sec)	N/A	0.029	0.362	0.022	1.588	1.379	0.	2.577

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	68	74	89	207	0	366
normalized size	1	1.	0.7	0.76	0.92	2.13	0.	3.77
time (sec)	N/A	0.04	0.251	0.021	1.557	1.401	0.	3.499

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	364	199	265	240	0	0	393
normalized size	1	1.	0.55	0.73	0.66	0.	0.	1.08
time (sec)	N/A	0.15	0.81	0.039	1.543	0.	0.	2.167

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	54	235	189	0	0	342
normalized size	1	1.	0.19	0.82	0.66	0.	0.	1.2
time (sec)	N/A	0.126	0.061	0.018	1.43	0.	0.	1.349

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	161	207	180	0	0	263
normalized size	1	1.	0.63	0.81	0.71	0.	0.	1.03
time (sec)	N/A	0.113	0.243	0.024	1.462	0.	0.	1.328

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	43	210	170	0	0	339
normalized size	1	1.	0.17	0.82	0.67	0.	0.	1.33
time (sec)	N/A	0.118	0.032	0.024	1.446	0.	0.	1.332

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	45	235	220	0	0	0
normalized size	1	1.	0.15	0.79	0.74	0.	0.	0.
time (sec)	N/A	0.13	0.067	0.022	1.436	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	364	45	271	232	0	0	0
normalized size	1	1.	0.12	0.74	0.64	0.	0.	0.
time (sec)	N/A	0.151	0.058	0.026	1.443	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	86	84	107	247	0	1296
normalized size	1	1.	0.47	0.46	0.59	1.36	0.	7.12
time (sec)	N/A	0.063	0.767	0.036	1.387	1.558	0.	10.389

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	66	64	72	163	0	1339
normalized size	1	1.	0.6	0.58	0.65	1.48	0.	12.17
time (sec)	N/A	0.043	0.743	0.015	1.411	1.419	0.	6.327

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	41	42	35	88	0	309
normalized size	1	1.	0.82	0.84	0.7	1.76	0.	6.18
time (sec)	N/A	0.021	0.088	0.022	1.5	1.345	0.	1.424

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	43	40	36	93	0	61
normalized size	1	1.	0.84	0.78	0.71	1.82	0.	1.2
time (sec)	N/A	0.022	0.055	0.026	1.405	1.349	0.	1.54

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	45	63	68	162	0	167
normalized size	1	1.	0.38	0.53	0.57	1.36	0.	1.4
time (sec)	N/A	0.043	0.05	0.02	1.407	1.376	0.	1.759

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	45	83	95	225	0	250
normalized size	1	1.	0.25	0.45	0.52	1.23	0.	1.37
time (sec)	N/A	0.064	0.033	0.021	1.407	1.385	0.	3.087

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	57	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.048	12.661	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	49	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.046	0.265	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	55	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.039	0.484	0.	0.	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	53	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.037	0.262	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	62	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.098	180.	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	60	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.065	0.421	0.	0.	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.038	0.121	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	60	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.049	0.126	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	60	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.068	0.129	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	62	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.068	0.117	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	18076	0	59	0	0
normalized size	1	1.	1.	564.88	0.	1.84	0.	0.
time (sec)	N/A	0.019	0.022	3.563	0.	1.096	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.049	5.166	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	122	542	0	0	0	0
normalized size	1	1.	0.47	2.11	0.	0.	0.	0.
time (sec)	N/A	0.196	0.216	0.279	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	104	524	0	0	0	0
normalized size	1	1.	0.46	2.31	0.	0.	0.	0.
time (sec)	N/A	0.161	0.166	0.174	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	38	31	92	0	0
normalized size	1	1.	1.	2.11	1.72	5.11	0.	0.
time (sec)	N/A	0.041	0.073	0.136	2.263	1.639	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	30	50	45	154	0	0
normalized size	1	1.	0.73	1.22	1.1	3.76	0.	0.
time (sec)	N/A	0.045	0.116	0.181	2.991	1.689	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	50	60	65	213	0	0
normalized size	1	1.	0.79	0.95	1.03	3.38	0.	0.
time (sec)	N/A	0.052	0.159	0.187	1.218	1.799	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	139	216	0	0	0	0
normalized size	1	1.	1.32	2.06	0.	0.	0.	0.
time (sec)	N/A	0.133	1.83	0.207	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	57	190	0	0	0	0
normalized size	1	1.	0.76	2.53	0.	0.	0.	0.
time (sec)	N/A	0.09	0.962	0.14	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	73	159	0	0	0	0
normalized size	1	1.	1.55	3.38	0.	0.	0.	0.
time (sec)	N/A	0.068	0.146	0.151	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	115	301	0	0	0	0
normalized size	1	1.	1.49	3.91	0.	0.	0.	0.
time (sec)	N/A	0.106	0.567	0.167	0.	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	124	558	0	0	0	0
normalized size	1	1.	1.18	5.31	0.	0.	0.	0.
time (sec)	N/A	0.143	1.428	0.189	0.	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	123	712	0	0	0	0
normalized size	1	1.	0.44	2.57	0.	0.	0.	0.
time (sec)	N/A	0.196	0.805	0.227	0.	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	113	676	0	0	0	0
normalized size	1	1.	0.46	2.74	0.	0.	0.	0.
time (sec)	N/A	0.176	0.55	0.159	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	58	31	55	0	0
normalized size	1	1.	1.	3.22	1.72	3.06	0.	0.
time (sec)	N/A	0.043	0.05	0.135	1.5	1.596	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	30	50	46	120	0	0
normalized size	1	1.	0.73	1.22	1.12	2.93	0.	0.
time (sec)	N/A	0.048	0.079	0.155	1.695	1.452	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	42	60	78	182	0	0
normalized size	1	1.	0.67	0.95	1.24	2.89	0.	0.
time (sec)	N/A	0.055	0.141	0.192	1.591	1.492	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	90	548	0	0	0	0
normalized size	1	1.	0.82	4.98	0.	0.	0.	0.
time (sec)	N/A	0.141	0.58	0.155	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	58	526	0	0	0	0
normalized size	1	1.	0.76	6.92	0.	0.	0.	0.
time (sec)	N/A	0.087	0.283	0.148	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	61	519	0	0	0	0
normalized size	1	1.	0.8	6.83	0.	0.	0.	0.
time (sec)	N/A	0.094	0.273	0.148	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	71	499	0	0	0	0
normalized size	1	1.	0.7	4.89	0.	0.	0.	0.
time (sec)	N/A	0.143	0.578	0.174	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	142	590	0	0	0	0
normalized size	1	1.	0.51	2.13	0.	0.	0.	0.
time (sec)	N/A	0.194	0.568	0.196	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	126	564	0	0	0	0
normalized size	1	1.	0.51	2.28	0.	0.	0.	0.
time (sec)	N/A	0.175	0.384	0.131	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	38	31	99	0	32
normalized size	1	1.	1.	1.9	1.55	4.95	0.	1.6
time (sec)	N/A	0.042	0.075	0.123	1.396	2.146	0.	1.113

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	32	50	49	134	0	0
normalized size	1	1.	0.78	1.22	1.2	3.27	0.	0.
time (sec)	N/A	0.048	0.113	0.124	1.321	2.207	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	42	60	76	200	0	0
normalized size	1	1.	0.67	0.95	1.21	3.17	0.	0.
time (sec)	N/A	0.054	0.224	0.154	1.519	2.244	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	153	248	0	0	0	0
normalized size	1	1.	1.12	1.81	0.	0.	0.	0.
time (sec)	N/A	0.175	3.246	0.128	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	133	220	0	0	0	0
normalized size	1	1.	1.23	2.04	0.	0.	0.	0.
time (sec)	N/A	0.116	2.191	0.141	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	71	192	0	0	0	0
normalized size	1	1.	0.89	2.4	0.	0.	0.	0.
time (sec)	N/A	0.101	0.376	0.137	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	71	194	0	0	0	0
normalized size	1	1.	0.89	2.42	0.	0.	0.	0.
time (sec)	N/A	0.107	0.341	0.146	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	110	320	0	0	0	0
normalized size	1	1.	1.	2.91	0.	0.	0.	0.
time (sec)	N/A	0.147	0.469	0.158	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	130	571	0	0	0	0
normalized size	1	1.	0.93	4.08	0.	0.	0.	0.
time (sec)	N/A	0.185	1.61	0.187	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	122	698	0	0	0	336
normalized size	1	1.	0.47	2.72	0.	0.	0.	1.31
time (sec)	N/A	0.174	0.706	0.163	0.	0.	0.	1.264

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	109	672	0	0	0	296
normalized size	1	1.	0.48	2.96	0.	0.	0.	1.3
time (sec)	N/A	0.15	0.613	0.142	0.	0.	0.	1.183

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	38	31	109	0	31
normalized size	1	1.	1.	1.9	1.55	5.45	0.	1.55
time (sec)	N/A	0.036	0.1	0.145	1.093	2.142	0.	1.117

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	40	50	47	173	0	61
normalized size	1	1.	0.93	1.16	1.09	4.02	0.	1.42
time (sec)	N/A	0.043	0.127	0.173	1.065	2.312	0.	1.138

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	50	60	65	235	0	78
normalized size	1	1.	0.77	0.92	1.	3.62	0.	1.2
time (sec)	N/A	0.049	0.157	0.194	1.145	2.608	0.	1.108

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	86	558	0	0	0	0
normalized size	1	1.	0.8	5.21	0.	0.	0.	0.
time (sec)	N/A	0.133	0.84	0.177	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	98	545	0	0	0	0
normalized size	1	1.	1.24	6.9	0.	0.	0.	0.
time (sec)	N/A	0.094	0.732	0.173	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	60	531	0	0	0	0
normalized size	1	1.	1.28	11.3	0.	0.	0.	0.
time (sec)	N/A	0.057	0.126	0.147	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	69	490	0	0	0	0
normalized size	1	1.	0.96	6.81	0.	0.	0.	0.
time (sec)	N/A	0.097	0.306	0.164	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	104	972	0	0	0	0
normalized size	1	1.	1.02	9.53	0.	0.	0.	0.
time (sec)	N/A	0.134	0.674	0.186	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	123	558	0	0	0	348
normalized size	1	1.	0.48	2.17	0.	0.	0.	1.35
time (sec)	N/A	0.182	0.327	0.14	0.	0.	0.	1.406

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	105	530	0	0	0	308
normalized size	1	1.	0.46	2.33	0.	0.	0.	1.36
time (sec)	N/A	0.158	0.241	0.154	0.	0.	0.	1.279

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	38	31	135	0	35
normalized size	1	1.	1.	1.9	1.55	6.75	0.	1.75
time (sec)	N/A	0.043	0.12	0.129	1.101	2.196	0.	1.158

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	42	50	47	201	0	61
normalized size	1	1.	0.98	1.16	1.09	4.67	0.	1.42
time (sec)	N/A	0.05	0.095	0.149	1.122	2.132	0.	1.165

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	54	60	65	269	0	78
normalized size	1	1.	0.83	0.92	1.	4.14	0.	1.2
time (sec)	N/A	0.054	0.121	0.2	1.161	2.143	0.	1.16

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	102	222	0	0	0	0
normalized size	1	1.	0.91	1.98	0.	0.	0.	0.
time (sec)	N/A	0.133	0.359	0.15	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	126	199	0	0	0	0
normalized size	1	1.	1.59	2.52	0.	0.	0.	0.
time (sec)	N/A	0.094	0.741	0.125	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	110	306	0	0	0	0
normalized size	1	1.	1.34	3.73	0.	0.	0.	0.
time (sec)	N/A	0.104	0.692	0.135	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	136	566	0	0	0	0
normalized size	1	1.	1.21	5.05	0.	0.	0.	0.
time (sec)	N/A	0.146	1.671	0.169	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	123	698	0	0	0	339
normalized size	1	1.	0.48	2.72	0.	0.	0.	1.32
time (sec)	N/A	0.176	0.741	0.148	0.	0.	0.	1.259

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	113	672	0	0	0	301
normalized size	1	1.	0.5	2.96	0.	0.	0.	1.33
time (sec)	N/A	0.162	0.509	0.129	0.	0.	0.	1.212

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	38	31	150	0	35
normalized size	1	1.	1.	1.9	1.55	7.5	0.	1.75
time (sec)	N/A	0.043	0.16	0.13	1.084	2.373	0.	1.149

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	50	50	47	217	0	61
normalized size	1	1.	1.16	1.16	1.09	5.05	0.	1.42
time (sec)	N/A	0.05	0.173	0.148	1.12	2.803	0.	1.145

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	60	60	65	285	0	78
normalized size	1	1.	0.92	0.92	1.	4.38	0.	1.2
time (sec)	N/A	0.056	0.232	0.172	1.019	3.207	0.	1.143

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	122	571	0	0	0	0
normalized size	1	1.	0.85	3.97	0.	0.	0.	0.
time (sec)	N/A	0.186	1.549	0.187	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	100	550	0	0	0	0
normalized size	1	1.	0.88	4.82	0.	0.	0.	0.
time (sec)	N/A	0.143	1.069	0.138	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	97	544	0	0	0	0
normalized size	1	1.	1.15	6.48	0.	0.	0.	0.
time (sec)	N/A	0.102	0.618	0.167	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	69	511	0	0	0	0
normalized size	1	1.	0.88	6.55	0.	0.	0.	0.
time (sec)	N/A	0.087	0.376	0.138	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	105	980	0	0	0	0
normalized size	1	1.	0.95	8.91	0.	0.	0.	0.
time (sec)	N/A	0.136	1.769	0.164	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	116	1479	0	0	0	0
normalized size	1	1.	0.83	10.56	0.	0.	0.	0.
time (sec)	N/A	0.184	0.801	0.2	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	51	493	0	161	0	0
normalized size	1	1.	0.75	7.25	0.	2.37	0.	0.
time (sec)	N/A	0.09	0.187	0.303	0.	1.636	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	80	131	0	0	0	0
normalized size	1	1.	0.91	1.49	0.	0.	0.	0.
time (sec)	N/A	0.106	0.312	0.199	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	295	0	120	0	0
normalized size	1	1.	1.	9.83	0.	4.	0.	0.
time (sec)	N/A	0.042	0.129	0.197	0.	1.576	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	60	88	0	0	0	0
normalized size	1	1.	1.2	1.76	0.	0.	0.	0.
time (sec)	N/A	0.053	0.115	0.138	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	72	185	0	1035	0	0
normalized size	1	1.	0.67	1.73	0.	9.67	0.	0.
time (sec)	N/A	0.088	0.253	0.167	0.	3.937	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	79	178	0	0	0	0
normalized size	1	1.	0.92	2.07	0.	0.	0.	0.
time (sec)	N/A	0.103	0.243	0.167	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	99	336	0	0	0	0
normalized size	1	1.	0.79	2.67	0.	0.	0.	0.
time (sec)	N/A	0.169	0.325	0.217	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	45	492	0	143	0	0
normalized size	1	1.	0.66	7.24	0.	2.1	0.	0.
time (sec)	N/A	0.105	0.159	0.155	0.	1.652	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	83	328	0	0	0	0
normalized size	1	1.	0.99	3.9	0.	0.	0.	0.
time (sec)	N/A	0.101	0.194	0.172	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	308	0	107	0	0
normalized size	1	1.	1.	10.27	0.	3.57	0.	0.
time (sec)	N/A	0.049	0.066	0.155	0.	1.799	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	92	316	0	0	0	0
normalized size	1	1.	1.02	3.51	0.	0.	0.	0.
time (sec)	N/A	0.111	0.265	0.163	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	104	247	0	1334	0	0
normalized size	1	1.	0.72	1.7	0.	9.2	0.	0.
time (sec)	N/A	0.15	0.346	0.148	0.	5.001	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	100	349	0	0	0	0
normalized size	1	1.	0.81	2.84	0.	0.	0.	0.
time (sec)	N/A	0.162	0.516	0.203	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	52	60	0	170	0	0
normalized size	1	1.	0.76	0.88	0.	2.5	0.	0.
time (sec)	N/A	0.102	0.163	0.148	0.	1.657	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	87	337	0	0	0	0
normalized size	1	1.	0.99	3.83	0.	0.	0.	0.
time (sec)	N/A	0.105	0.233	0.196	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	48	0	131	0	0
normalized size	1	1.	1.	1.5	0.	4.09	0.	0.
time (sec)	N/A	0.049	0.131	0.135	0.	1.632	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	69	327	0	0	0	0
normalized size	1	1.	1.38	6.54	0.	0.	0.	0.
time (sec)	N/A	0.051	0.148	0.166	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	80	177	0	1072	0	0
normalized size	1	1.	0.75	1.67	0.	10.11	0.	0.
time (sec)	N/A	0.083	0.145	0.119	0.	3.725	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	89	315	0	0	0	0
normalized size	1	1.	1.02	3.62	0.	0.	0.	0.
time (sec)	N/A	0.105	0.338	0.171	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	112	319	0	1571	0	0
normalized size	1	1.	0.77	2.18	0.	10.76	0.	0.
time (sec)	N/A	0.142	0.599	0.165	0.	4.465	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	67	70	0	213	0	0
normalized size	1	1.	0.46	0.48	0.	1.46	0.	0.
time (sec)	N/A	0.207	0.424	0.151	0.	1.812	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	57	60	0	173	0	0
normalized size	1	1.	0.52	0.55	0.	1.59	0.	0.
time (sec)	N/A	0.151	0.213	0.115	0.	1.708	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	45	48	0	136	0	0
normalized size	1	1.	1.41	1.5	0.	4.25	0.	0.
time (sec)	N/A	0.055	0.132	0.112	0.	1.6	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	88	237	0	1351	0	0
normalized size	1	1.	0.62	1.68	0.	9.58	0.	0.
time (sec)	N/A	0.143	0.326	0.151	0.	3.895	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	103	320	0	1580	0	0
normalized size	1	1.	0.68	2.12	0.	10.46	0.	0.
time (sec)	N/A	0.155	0.331	0.138	0.	4.496	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	118	181	0	0	0	0
normalized size	1	1.	0.71	1.08	0.	0.	0.	0.
time (sec)	N/A	0.227	0.75	0.221	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	97	161	0	0	0	0
normalized size	1	1.	0.75	1.24	0.	0.	0.	0.
time (sec)	N/A	0.167	0.354	0.179	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	80	137	0	0	0	0
normalized size	1	1.	0.86	1.47	0.	0.	0.	0.
time (sec)	N/A	0.112	0.232	0.15	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	79	185	0	0	0	0
normalized size	1	1.	0.92	2.15	0.	0.	0.	0.
time (sec)	N/A	0.104	0.185	0.16	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	96	337	0	0	0	0
normalized size	1	1.	0.74	2.59	0.	0.	0.	0.
time (sec)	N/A	0.169	0.343	0.172	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	106	487	0	0	0	0
normalized size	1	1.	0.63	2.92	0.	0.	0.	0.
time (sec)	N/A	0.233	0.4	0.218	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	69	0	0	0	0	0
normalized size	1	1.	1.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	0.377	0.327	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	69	0	0	0	0	0
normalized size	1	1.	1.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	0.342	0.175	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	0.313	0.221	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	67	0	0	0	0	0
normalized size	1	1.	1.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	0.315	0.173	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	63	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	0.527	0.13	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	63	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	0.409	0.125	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	63	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	0.437	0.114	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	69	0	0	0	0	0
normalized size	1	1.	1.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	0.478	0.116	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.093	0.423	0.136	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	66	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	0.336	0.211	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	66	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	0.319	0.275	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	64	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	0.39	0.183	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	85	0	0	0	0	0
normalized size	1	1.	1.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	0.577	0.111	0.	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	72	0	0	0	0	0
normalized size	1	1.	1.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	0.476	0.153	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	67	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	0.73	0.101	0.	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	70	0	0	0	0	0
normalized size	1	1.	1.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.093	0.38	0.104	0.	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	53	0	0	0	0	0
normalized size	1	1.	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.061	0.308	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	53	0	0	0	0	0
normalized size	1	1.	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.033	0.546	0.	0.	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	24	35	0	24
normalized size	1	1.	1.	1.06	1.41	2.06	0.	1.41
time (sec)	N/A	0.029	0.009	0.011	0.946	1.582	0.	1.171

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	37	3161	63	132	0	0
normalized size	1	1.	0.8	68.72	1.37	2.87	0.	0.
time (sec)	N/A	0.05	0.056	1.074	0.991	1.595	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	62	7964	96	267	0	0
normalized size	1	1.	0.86	110.61	1.33	3.71	0.	0.
time (sec)	N/A	0.061	0.325	0.691	0.96	1.656	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	71	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.09	0.141	0.211	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	71	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	0.084	0.194	0.	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	66	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	0.08	0.206	0.	0.	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	71	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	0.065	0.238	0.	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	87	0	0	0	0	0
normalized size	1	1.	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	8.104	0.14	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	87	0	0	0	0	0
normalized size	1	1.	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	3.174	0.132	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	87	0	0	0	0	0
normalized size	1	1.	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.103	2.818	0.132	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	225	0	0	0	0	0
normalized size	1	1.	2.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.12	5.139	0.125	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	260	0	0	0	0	0
normalized size	1	1.	3.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.103	1.907	0.915	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	50	50	916	0	0	0	0	0
normalized size	1	1.	18.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	4.745	0.826	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	50	50	450	0	0	0	0	0
normalized size	1	1.	9.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	2.118	0.71	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	22	4284	38	96	0	0
normalized size	1	1.	0.88	171.36	1.52	3.84	0.	0.
time (sec)	N/A	0.042	0.066	1.787	0.964	1.634	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	46	13019	74	204	0	0
normalized size	1	1.	0.87	245.64	1.4	3.85	0.	0.
time (sec)	N/A	0.055	0.147	0.641	0.973	1.622	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	69	26124	109	356	0	0
normalized size	1	1.	0.86	326.55	1.36	4.45	0.	0.
time (sec)	N/A	0.063	0.265	1.119	0.993	1.876	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	456	0	0	0	0	0
normalized size	1	1.	5.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.084	2.856	0.778	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	76	76	252	0	0	0	0	0
normalized size	1	1.	3.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	1.048	0.664	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	64	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	0.201	0.232	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	182	0	0	0	0	0
normalized size	1	1.	2.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	7.399	0.278	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	254	0	0	0	0	0
normalized size	1	1.	3.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	7.633	0.337	0.	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	89	297	0	0	0	0	0
normalized size	1	1.	3.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.121	2.449	0.144	0.	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	91	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	1.573	0.136	0.	0.	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	1.251	0.135	0.	0.	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	90	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.125	1.859	0.13	0.	0.	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	81	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	0.447	0.784	0.	0.	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	66	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.072	0.393	0.	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	45	728	0	1511	0	0
normalized size	1	1.	0.19	3.14	0.	6.51	0.	0.
time (sec)	N/A	0.227	0.079	0.258	0.	2.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	45	540	0	1440	0	0
normalized size	1	1.	0.21	2.52	0.	6.73	0.	0.
time (sec)	N/A	0.182	0.044	0.185	0.	2.055	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	36	650	0	1458	0	0
normalized size	1	1.	0.17	3.1	0.	6.94	0.	0.
time (sec)	N/A	0.179	0.054	0.167	0.	1.911	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	132	287	0	1230	0	0
normalized size	1	1.	0.69	1.49	0.	6.41	0.	0.
time (sec)	N/A	0.138	0.183	0.124	0.	1.88	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	40	160	0	0	0	0
normalized size	1	1.	0.21	0.83	0.	0.	0.	0.
time (sec)	N/A	0.117	0.045	0.025	0.	0.	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	162	172	0	0	0	0
normalized size	1	1.	0.78	0.82	0.	0.	0.	0.
time (sec)	N/A	0.165	0.228	0.02	0.	0.	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	42	178	0	0	0	0
normalized size	1	1.	0.2	0.83	0.	0.	0.	0.
time (sec)	N/A	0.162	0.047	0.027	0.	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	172	190	0	0	0	0
normalized size	1	1.	0.74	0.82	0.	0.	0.	0.
time (sec)	N/A	0.198	0.439	0.029	0.	0.	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	45	718	0	1527	0	0
normalized size	1	1.	0.19	3.07	0.	6.53	0.	0.
time (sec)	N/A	0.214	0.061	0.175	0.	2.219	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	45	540	0	1472	0	0
normalized size	1	1.	0.21	2.52	0.	6.88	0.	0.
time (sec)	N/A	0.179	0.045	0.184	0.	2.153	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	38	650	0	1474	0	0
normalized size	1	1.	0.18	3.07	0.	6.95	0.	0.
time (sec)	N/A	0.174	0.047	0.17	0.	2.121	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	134	287	0	1258	0	0
normalized size	1	1.	0.7	1.49	0.	6.55	0.	0.
time (sec)	N/A	0.147	0.024	0.152	0.	2.257	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	37	318	0	1295	0	0
normalized size	1	1.	0.19	1.66	0.	6.74	0.	0.
time (sec)	N/A	0.138	0.029	0.136	0.	1.892	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	159	176	0	0	0	0
normalized size	1	1.	0.76	0.84	0.	0.	0.	0.
time (sec)	N/A	0.144	0.131	0.013	0.	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	39	181	0	0	0	0
normalized size	1	1.	0.18	0.86	0.	0.	0.	0.
time (sec)	N/A	0.157	0.057	0.012	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	172	194	0	0	0	0
normalized size	1	1.	0.74	0.84	0.	0.	0.	0.
time (sec)	N/A	0.196	0.276	0.019	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	40	718	0	1584	0	0
normalized size	1	1.	0.17	3.11	0.	6.86	0.	0.
time (sec)	N/A	0.207	0.12	0.194	0.	1.805	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	38	540	0	1562	0	0
normalized size	1	1.	0.18	2.55	0.	7.37	0.	0.
time (sec)	N/A	0.171	0.095	0.179	0.	1.816	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	35	642	0	1531	0	0
normalized size	1	1.	0.17	3.07	0.	7.33	0.	0.
time (sec)	N/A	0.164	0.065	0.174	0.	1.745	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	131	166	0	0	0	0
normalized size	1	1.	0.68	0.86	0.	0.	0.	0.
time (sec)	N/A	0.111	0.015	0.023	0.	0.	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	40	157	0	0	0	0
normalized size	1	1.	0.21	0.82	0.	0.	0.	0.
time (sec)	N/A	0.13	0.028	0.021	0.	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	159	184	0	0	0	0
normalized size	1	1.	0.75	0.87	0.	0.	0.	0.
time (sec)	N/A	0.163	0.159	0.027	0.	0.	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	47	175	0	0	0	0
normalized size	1	1.	0.22	0.82	0.	0.	0.	0.
time (sec)	N/A	0.16	0.061	0.028	0.	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	38	718	0	1608	0	0
normalized size	1	1.	0.16	3.09	0.	6.93	0.	0.
time (sec)	N/A	0.209	0.165	0.176	0.	1.784	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	37	532	0	1597	0	0
normalized size	1	1.	0.18	2.52	0.	7.57	0.	0.
time (sec)	N/A	0.174	0.029	0.155	0.	1.767	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	38	184	0	0	0	0
normalized size	1	1.	0.18	0.87	0.	0.	0.	0.
time (sec)	N/A	0.142	0.052	0.017	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	134	166	0	0	0	0
normalized size	1	1.	0.7	0.86	0.	0.	0.	0.
time (sec)	N/A	0.132	0.029	0.014	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	40	166	0	0	0	0
normalized size	1	1.	0.21	0.86	0.	0.	0.	0.
time (sec)	N/A	0.132	0.009	0.02	0.	0.	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	159	184	0	0	0	0
normalized size	1	1.	0.75	0.87	0.	0.	0.	0.
time (sec)	N/A	0.162	0.151	0.02	0.	0.	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	47	184	0	0	0	0
normalized size	1	1.	0.22	0.86	0.	0.	0.	0.
time (sec)	N/A	0.161	0.083	0.017	0.	0.	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	172	202	0	0	0	0
normalized size	1	1.	0.74	0.86	0.	0.	0.	0.
time (sec)	N/A	0.188	0.292	0.016	0.	0.	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	62	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.074	0.444	0.	0.	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	64	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	0.066	0.307	0.	0.	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	0.062	0.372	0.	0.	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	67	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	0.09	0.326	0.	0.	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	52	60	69	159	0	111
normalized size	1	1.	0.78	0.9	1.03	2.37	0.	1.66
time (sec)	N/A	0.054	0.188	0.256	1.111	1.864	0.	1.297

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	34	50	49	128	0	77
normalized size	1	1.	0.76	1.11	1.09	2.84	0.	1.71
time (sec)	N/A	0.045	0.141	0.163	0.973	1.713	0.	1.143

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	24	93	0	31
normalized size	1	1.	1.	0.86	1.09	4.23	0.	1.41
time (sec)	N/A	0.037	0.036	0.03	0.934	1.614	0.	1.181

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	40	160	0	1283	0	259
normalized size	1	1.	0.21	0.83	0.	6.68	0.	1.35
time (sec)	N/A	0.111	0.037	0.019	0.	1.705	0.	1.14

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	102	516	0	0	0	323
normalized size	1	1.	0.45	2.27	0.	0.	0.	1.42
time (sec)	N/A	0.17	0.187	0.183	0.	0.	0.	1.293

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	102	551	0	0	0	0
normalized size	1	1.	0.95	5.15	0.	0.	0.	0.
time (sec)	N/A	0.129	0.423	0.16	0.	0.	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	61	505	0	0	0	0
normalized size	1	1.	0.81	6.73	0.	0.	0.	0.
time (sec)	N/A	0.09	0.266	0.15	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	57	523	0	0	0	0
normalized size	1	1.	1.21	11.13	0.	0.	0.	0.
time (sec)	N/A	0.065	0.094	0.157	0.	0.	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	94	536	0	0	0	0
normalized size	1	1.	1.16	6.62	0.	0.	0.	0.
time (sec)	N/A	0.102	0.431	0.192	0.	0.	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	86	542	0	0	0	0
normalized size	1	1.	0.77	4.88	0.	0.	0.	0.
time (sec)	N/A	0.139	0.75	0.171	0.	0.	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	52	60	69	178	0	0
normalized size	1	1.	0.78	0.9	1.03	2.66	0.	0.
time (sec)	N/A	0.058	0.137	0.189	0.938	2.178	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	42	50	49	143	0	0
normalized size	1	1.	0.93	1.11	1.09	3.18	0.	0.
time (sec)	N/A	0.053	0.133	0.24	0.941	1.894	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	24	111	0	0
normalized size	1	1.	1.	0.86	1.09	5.05	0.	0.
time (sec)	N/A	0.043	0.053	0.02	0.942	1.712	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	159	176	0	1328	0	0
normalized size	1	1.	0.76	0.84	0.	6.32	0.	0.
time (sec)	N/A	0.142	0.26	0.013	0.	1.737	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	110	660	0	0	0	0
normalized size	1	1.	0.49	2.93	0.	0.	0.	0.
time (sec)	N/A	0.163	0.259	0.143	0.	0.	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	90	251	0	0	0	0
normalized size	1	1.	0.66	1.85	0.	0.	0.	0.
time (sec)	N/A	0.183	0.799	0.135	0.	0.	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	80	223	0	0	0	0
normalized size	1	1.	0.74	2.06	0.	0.	0.	0.
time (sec)	N/A	0.145	0.457	0.159	0.	0.	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	69	186	0	0	0	0
normalized size	1	1.	0.86	2.32	0.	0.	0.	0.
time (sec)	N/A	0.085	0.299	0.124	0.	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	58	196	0	0	0	0
normalized size	1	1.	0.74	2.51	0.	0.	0.	0.
time (sec)	N/A	0.095	0.128	0.132	0.	0.	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	96	220	0	0	0	0
normalized size	1	1.	0.89	2.04	0.	0.	0.	0.
time (sec)	N/A	0.134	1.086	0.172	0.	0.	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	131	250	0	0	0	0
normalized size	1	1.	0.96	1.84	0.	0.	0.	0.
time (sec)	N/A	0.174	2.389	0.16	0.	0.	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	52	60	69	211	0	113
normalized size	1	1.	0.78	0.9	1.03	3.15	0.	1.69
time (sec)	N/A	0.058	0.431	0.181	0.948	2.351	0.	1.322

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	42	50	49	171	0	80
normalized size	1	1.	0.93	1.11	1.09	3.8	0.	1.78
time (sec)	N/A	0.05	0.263	0.139	0.978	1.923	0.	1.217

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	24	134	0	38
normalized size	1	1.	1.	0.86	1.09	6.09	0.	1.73
time (sec)	N/A	0.043	0.055	0.02	0.94	1.749	0.	1.336

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	40	182	0	1521	0	277
normalized size	1	1.	0.19	0.86	0.	7.17	0.	1.31
time (sec)	N/A	0.142	0.039	0.011	0.	1.763	0.	1.289

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	107	532	0	0	0	321
normalized size	1	1.	0.48	2.36	0.	0.	0.	1.43
time (sec)	N/A	0.163	0.188	0.158	0.	0.	0.	1.27

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	125	550	0	0	0	366
normalized size	1	1.	0.49	2.17	0.	0.	0.	1.45
time (sec)	N/A	0.18	0.189	0.145	0.	0.	0.	1.333

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	79	222	0	0	0	0
normalized size	1	1.	0.72	2.04	0.	0.	0.	0.
time (sec)	N/A	0.137	0.541	0.196	0.	0.	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	68	194	0	0	0	0
normalized size	1	1.	0.86	2.46	0.	0.	0.	0.
time (sec)	N/A	0.096	0.264	0.171	0.	0.	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	77	165	0	0	0	0
normalized size	1	1.	1.64	3.51	0.	0.	0.	0.
time (sec)	N/A	0.059	0.124	0.122	0.	0.	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	126	196	0	0	0	0
normalized size	1	1.	1.66	2.58	0.	0.	0.	0.
time (sec)	N/A	0.093	0.537	0.145	0.	0.	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	94	224	0	0	0	0
normalized size	1	1.	0.86	2.06	0.	0.	0.	0.
time (sec)	N/A	0.127	0.971	0.174	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	45	60	73	162	0	92
normalized size	1	1.	0.69	0.92	1.12	2.49	0.	1.42
time (sec)	N/A	0.06	0.192	0.163	0.963	1.876	0.	1.305

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	32	50	49	131	0	57
normalized size	1	1.	0.74	1.16	1.14	3.05	0.	1.33
time (sec)	N/A	0.052	0.096	0.138	0.946	1.71	0.	1.342

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	24	97	0	24
normalized size	1	1.	1.	0.95	1.2	4.85	0.	1.2
time (sec)	N/A	0.043	0.058	0.02	0.945	1.663	0.	1.345

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	38	184	0	1693	0	275
normalized size	1	1.	0.18	0.87	0.	7.99	0.	1.3
time (sec)	N/A	0.142	0.034	0.011	0.	1.765	0.	1.248

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	115	982	0	0	0	344
normalized size	1	1.	0.46	3.94	0.	0.	0.	1.38
time (sec)	N/A	0.183	0.282	0.144	0.	0.	0.	1.482

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	104	529	0	0	0	0
normalized size	1	1.	0.75	3.83	0.	0.	0.	0.
time (sec)	N/A	0.174	0.868	0.179	0.	0.	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	93	491	0	0	0	0
normalized size	1	1.	0.89	4.72	0.	0.	0.	0.
time (sec)	N/A	0.134	0.448	0.177	0.	0.	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	69	488	0	0	0	0
normalized size	1	1.	0.88	6.26	0.	0.	0.	0.
time (sec)	N/A	0.092	0.378	0.133	0.	0.	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	66	501	0	0	0	0
normalized size	1	1.	0.85	6.42	0.	0.	0.	0.
time (sec)	N/A	0.098	0.395	0.143	0.	0.	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	77	515	0	0	0	0
normalized size	1	1.	0.69	4.6	0.	0.	0.	0.
time (sec)	N/A	0.149	0.553	0.142	0.	0.	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	89	536	0	0	0	0
normalized size	1	1.	0.63	3.77	0.	0.	0.	0.
time (sec)	N/A	0.187	0.838	0.165	0.	0.	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	113	302	0	0	0	0
normalized size	1	1.	1.38	3.68	0.	0.	0.	0.
time (sec)	N/A	0.087	0.695	0.135	0.	0.	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	103	970	0	0	0	0
normalized size	1	1.	0.94	8.82	0.	0.	0.	0.
time (sec)	N/A	0.137	1.527	0.177	0.	0.	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	77	0	0	0	0	0
normalized size	1	1.	1.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	0.207	0.067	0.	0.	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	56	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	0.083	0.06	0.	0.	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	56	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.085	0.061	0.	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	56	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.08	0.058	0.	0.	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	54	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.071	0.06	0.	0.	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	89	0	0	0	0	0
normalized size	1	1.	1.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	1.1	0.079	0.	0.	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	78	0	0	0	0	0
normalized size	1	1.	1.47	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.859	0.073	0.	0.	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	77	0	0	0	0	0
normalized size	1	1.	1.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.744	0.074	0.	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	78	0	0	0	0	0
normalized size	1	1.	1.47	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.194	0.076	0.	0.	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	77	0	0	0	0	0
normalized size	1	1.	1.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	0.255	0.074	0.	0.	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	80	0	0	0	0	0
normalized size	1	1.	1.4	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.28	0.062	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	80	0	0	0	0	0
normalized size	1	1.	1.4	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.303	0.068	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	80	0	0	0	0	0
normalized size	1	1.	1.4	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.28	0.062	0.	0.	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	80	0	0	0	0	0
normalized size	1	1.	1.4	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.214	0.062	0.	0.	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	79	0	0	0	0	0
normalized size	1	1.	1.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.195	0.062	0.	0.	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	92	0	0	0	0	0
normalized size	1	1.	1.61	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	1.098	0.078	0.	0.	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	69	0	0	0	0	0
normalized size	1	1.	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.155	0.081	0.	0.	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	69	0	0	0	0	0
normalized size	1	1.	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.151	0.077	0.	0.	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	69	0	0	0	0	0
normalized size	1	1.	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.149	0.076	0.	0.	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	67	0	0	0	0	0
normalized size	1	1.	1.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.143	0.076	0.	0.	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	174	600	0	2044	0	0
normalized size	1	1.	0.98	3.37	0.	11.48	0.	0.
time (sec)	N/A	0.162	1.903	0.316	0.	2.905	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	71	572	0	0	0	0
normalized size	1	1.	0.76	6.15	0.	0.	0.	0.
time (sec)	N/A	0.112	1.002	0.277	0.	0.	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	136	306	0	1701	0	0
normalized size	1	1.	1.03	2.32	0.	12.89	0.	0.
time (sec)	N/A	0.098	0.788	0.232	0.	2.801	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	55	55	62	551	0	0	0	0
normalized size	1	1.	1.13	10.02	0.	0.	0.	0.
time (sec)	N/A	0.061	0.514	0.229	0.	0.	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	50	0	127	53	0
normalized size	1	1.	1.	1.47	0.	3.74	1.56	0.
time (sec)	N/A	0.051	0.12	0.18	0.	1.716	42.915	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	95	95	79	573	0	0	0	0
normalized size	1	1.	0.83	6.03	0.	0.	0.	0.
time (sec)	N/A	0.114	0.671	0.242	0.	0.	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	53	62	0	159	0	0
normalized size	1	1.	0.74	0.86	0.	2.21	0.	0.
time (sec)	N/A	0.103	0.17	0.191	0.	1.714	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	132	132	92	585	0	0	0	0
normalized size	1	1.	0.7	4.43	0.	0.	0.	0.
time (sec)	N/A	0.168	0.932	0.216	0.	0.	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	95	239	0	0	0	0
normalized size	1	1.	0.73	1.82	0.	0.	0.	0.
time (sec)	N/A	0.176	0.77	0.249	0.	0.	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	129	759	0	2007	0	0
normalized size	1	1.	0.76	4.49	0.	11.88	0.	0.
time (sec)	N/A	0.174	6.299	0.183	0.	2.999	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	105	211	0	0	0	0
normalized size	1	1.	1.19	2.4	0.	0.	0.	0.
time (sec)	N/A	0.115	0.738	0.226	0.	0.	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	64	719	0	1886	0	0
normalized size	1	1.	0.38	4.31	0.	11.29	0.	0.
time (sec)	N/A	0.125	4.173	0.227	0.	5.356	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	98	207	0	0	0	0
normalized size	1	1.	1.02	2.16	0.	0.	0.	0.
time (sec)	N/A	0.121	0.595	0.202	0.	0.	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	141	50	0	142	0	0
normalized size	1	1.	4.15	1.47	0.	4.18	0.	0.
time (sec)	N/A	0.057	1.338	0.151	0.	2.019	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	105	241	0	0	0	0
normalized size	1	1.	0.8	1.84	0.	0.	0.	0.
time (sec)	N/A	0.18	1.265	0.233	0.	0.	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	158	62	0	174	0	0
normalized size	1	1.	1.53	0.6	0.	1.69	0.	0.
time (sec)	N/A	0.163	3.313	0.162	0.	1.816	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	189	628	0	2182	0	0
normalized size	1	1.	0.91	3.02	0.	10.49	0.	0.
time (sec)	N/A	0.228	3.142	0.229	0.	3.568	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	93	593	0	0	0	0
normalized size	1	1.	0.71	4.53	0.	0.	0.	0.
time (sec)	N/A	0.173	2.339	0.179	0.	0.	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	182	602	0	2047	0	0
normalized size	1	1.	1.08	3.56	0.	12.11	0.	0.
time (sec)	N/A	0.153	1.672	0.196	0.	2.986	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	74	585	0	0	0	0
normalized size	1	1.	0.84	6.65	0.	0.	0.	0.
time (sec)	N/A	0.113	0.71	0.223	0.	0.	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	181	570	0	1952	0	0
normalized size	1	1.	1.08	3.39	0.	11.62	0.	0.
time (sec)	N/A	0.165	1.059	0.211	0.	5.529	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	81	564	0	0	0	0
normalized size	1	1.	0.84	5.88	0.	0.	0.	0.
time (sec)	N/A	0.12	0.739	0.213	0.	0.	0.	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	45	50	0	165	0	0
normalized size	1	1.	1.32	1.47	0.	4.85	0.	0.
time (sec)	N/A	0.055	0.158	0.149	0.	1.713	0.	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	99	586	0	0	0	0
normalized size	1	1.	0.76	4.47	0.	0.	0.	0.
time (sec)	N/A	0.177	0.951	0.186	0.	0.	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	136	758	0	1980	0	0
normalized size	1	1.	0.76	4.26	0.	11.12	0.	0.
time (sec)	N/A	0.168	6.356	0.215	0.	3.324	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	83	208	0	0	0	0
normalized size	1	1.	0.9	2.26	0.	0.	0.	0.
time (sec)	N/A	0.113	2.349	0.21	0.	0.	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	105	344	0	1662	0	0
normalized size	1	1.	0.8	2.63	0.	12.69	0.	0.
time (sec)	N/A	0.106	4.253	0.214	0.	3.012	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	89	175	0	0	0	0
normalized size	1	1.	1.62	3.18	0.	0.	0.	0.
time (sec)	N/A	0.061	0.444	0.207	0.	0.	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	50	0	107	51	0
normalized size	1	1.	1.	1.56	0.	3.34	1.59	0.
time (sec)	N/A	0.045	0.388	0.165	0.	1.671	22.735	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	95	95	91	213	0	0	0	0
normalized size	1	1.	0.96	2.24	0.	0.	0.	0.
time (sec)	N/A	0.115	1.001	0.211	0.	0.	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	112	60	0	140	0	0
normalized size	1	1.	1.56	0.83	0.	1.94	0.	0.
time (sec)	N/A	0.097	1.15	0.17	0.	1.696	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	211	1061	0	2014	0	0
normalized size	1	1.	1.23	6.2	0.	11.78	0.	0.
time (sec)	N/A	0.167	1.037	0.21	0.	3.264	0.	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	70	538	0	0	0	0
normalized size	1	1.	0.72	5.55	0.	0.	0.	0.
time (sec)	N/A	0.123	0.609	0.185	0.	0.	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	50	0	126	53	0
normalized size	1	1.	1.	1.56	0.	3.94	1.66	0.
time (sec)	N/A	0.05	0.124	0.154	0.	1.653	24.034	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	67	556	0	0	0	0
normalized size	1	1.	0.74	6.11	0.	0.	0.	0.
time (sec)	N/A	0.116	0.562	0.206	0.	0.	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	67	52	60	0	161	0	0
normalized size	1	0.93	0.72	0.83	0.	2.24	0.	0.
time (sec)	N/A	0.108	0.176	0.147	0.	1.676	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	81	570	0	0	0	0
normalized size	1	1.	0.62	4.38	0.	0.	0.	0.
time (sec)	N/A	0.186	0.657	0.217	0.	0.	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	144	1367	0	2111	0	0
normalized size	1	1.	0.84	7.95	0.	12.27	0.	0.
time (sec)	N/A	0.18	1.207	0.247	0.	4.123	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	116	315	0	0	0	0
normalized size	1	1.	1.15	3.12	0.	0.	0.	0.
time (sec)	N/A	0.126	0.444	0.205	0.	0.	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	50	0	143	0	0
normalized size	1	1.	1.	1.47	0.	4.21	0.	0.
time (sec)	N/A	0.057	0.156	0.133	0.	1.628	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	95	95	70	322	0	0	0	0
normalized size	1	1.	0.74	3.39	0.	0.	0.	0.
time (sec)	N/A	0.114	0.705	0.214	0.	0.	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	110	62	0	178	0	0
normalized size	1	1.	1.59	0.9	0.	2.58	0.	0.
time (sec)	N/A	0.101	0.851	0.177	0.	1.746	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	112	335	0	0	0	0
normalized size	1	1.	0.85	2.54	0.	0.	0.	0.
time (sec)	N/A	0.184	2.376	0.209	0.	0.	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	159	72	0	213	0	0
normalized size	1	1.	1.5	0.68	0.	2.01	0.	0.
time (sec)	N/A	0.166	3.052	0.156	0.	1.935	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.099	0.173	0.	0.	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.078	0.262	0.	0.	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.093	0.241	0.	0.	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.124	0.207	0.	0.	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.145	0.146	0.	0.	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.133	0.14	0.	0.	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	69	0	0	0	0	0
normalized size	1	1.	1.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.073	0.141	0.	0.	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	71	0	0	0	0	0
normalized size	1	1.	1.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.077	0.186	0.	0.	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	0.142	0.173	0.	0.	0.	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.085	0.247	0.	0.	0.	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.112	0.221	0.	0.	0.	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	62	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.229	0.128	0.	0.	0.	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.146	0.129	0.	0.	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.089	0.128	0.	0.	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.103	0.115	0.	0.	0.	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	64	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	0.203	0.113	0.	0.	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	47	6797	104	188	0	0
normalized size	1	1.	0.7	101.45	1.55	2.81	0.	0.
time (sec)	N/A	0.062	0.352	0.603	1.021	1.729	0.	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	34	2707	69	112	0	0
normalized size	1	1.	0.79	62.95	1.6	2.6	0.	0.
time (sec)	N/A	0.049	0.107	0.175	1.004	1.641	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	27	35	44	0
normalized size	1	1.	1.	1.06	1.59	2.06	2.59	0.
time (sec)	N/A	0.021	0.022	0.01	0.999	1.607	0.538	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	124	0	0	0	0	0
normalized size	1	1.	3.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.804	0.521	0.	0.	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	39	39	3020	0	0	0	0	0
normalized size	1	1.	77.44	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	17.461	0.386	0.	0.	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	40	40	4177	0	0	0	0	0
normalized size	1	1.	104.42	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	21.794	0.191	0.	0.	0.	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	63	63	10908	0	0	0	0	0
normalized size	1	1.	173.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	29.413	0.181	0.	0.	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	63	63	6612	0	0	0	0	0
normalized size	1	1.	104.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	24.92	0.25	0.	0.	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	59	59	4872	0	0	0	0	0
normalized size	1	1.	82.58	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	21.607	0.271	0.	0.	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	63	63	6671	0	0	0	0	0
normalized size	1	1.	105.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	25.186	0.179	0.	0.	0.	0.

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.571	0.192	0.	0.	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	80	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.145	0.748	0.	0.	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	101	0	0	215	0	0
normalized size	1	1.	1.36	0.	0.	2.91	0.	0.
time (sec)	N/A	0.066	2.099	0.196	0.	1.667	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	78	0	0	151	0	0
normalized size	1	1.	1.59	0.	0.	3.08	0.	0.
time (sec)	N/A	0.051	1.146	0.139	0.	1.713	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	25	25	0	96	0	0
normalized size	1	1.	1.04	1.04	0.	4.	0.	0.
time (sec)	N/A	0.039	0.02	0.017	0.	1.652	0.	0.

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	53	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.048	0.347	0.	0.	0.	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	50	50	939	0	0	0	0	0
normalized size	1	1.	18.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	4.376	0.651	0.	0.	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	50	50	1712	0	0	0	0	0
normalized size	1	1.	34.24	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	13.209	180.	0.	0.	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	72	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.107	0.142	0.	0.	0.	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	72	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.095	0.13	0.	0.	0.	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	64	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.067	0.223	0.	0.	0.	0.

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	72	72	452	0	0	0	0	0
normalized size	1	1.	6.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	2.333	0.767	0.	0.	0.	0.

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	1313	0	0	0	0	0
normalized size	1	1.	16.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	6.28	0.826	0.	0.	0.	0.

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	52	0	0	0	0	0
normalized size	1	1.	1.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	0.084	0.287	0.	0.	0.	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	52	0	0	0	0	0
normalized size	1	1.	1.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.049	0.441	0.	0.	0.	0.

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	28	36	56	0
normalized size	1	1.	1.	1.06	1.56	2.	3.11	0.
time (sec)	N/A	0.021	0.017	0.01	0.969	1.602	0.524	0.

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	36	6612	68	134	0	0
normalized size	1	1.	0.84	153.77	1.58	3.12	0.	0.
time (sec)	N/A	0.047	0.082	0.601	0.969	1.663	0.	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	63	16599	105	269	0	0
normalized size	1	1.	0.91	240.57	1.52	3.9	0.	0.
time (sec)	N/A	0.06	0.293	0.553	0.996	1.769	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	79	0	0	0	0	0
normalized size	1	1.	1.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.699	0.221	0.	0.	0.	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	79	0	0	0	0	0
normalized size	1	1.	1.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.561	0.199	0.	0.	0.	0.

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	186	0	0	0	0	0
normalized size	1	1.	2.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	1.056	0.201	0.	0.	0.	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	302	0	0	0	0	0
normalized size	1	1.	4.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	4.854	0.234	0.	0.	0.	0.

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	87	0	0	0	0	0
normalized size	1	1.	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.167	5.681	0.174	0.	0.	0.	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	87	0	0	0	0	0
normalized size	1	1.	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.145	3.107	0.171	0.	0.	0.	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	87	0	0	0	0	0
normalized size	1	1.	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.151	1.225	0.168	0.	0.	0.	0.

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	87	0	0	0	0	0
normalized size	1	1.	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.174	3.542	0.146	0.	0.	0.	0.

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	89	287	0	0	0	0	0
normalized size	1	1.	3.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.15	1.964	0.789	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [9] had the largest ratio of [0.75]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.	6	0.167
2	A	2	2	1.	8	0.25
3	A	2	2	1.	8	0.25
4	A	3	2	1.	8	0.25
5	A	3	2	1.	8	0.25
6	A	4	2	1.	8	0.25
7	A	4	2	1.	8	0.25
8	A	5	2	1.	8	0.25
9	A	13	9	1.	12	0.75
10	A	12	9	1.	12	0.75
11	A	12	9	1.	12	0.75
12	A	11	8	1.	12	0.667
13	A	11	8	1.	12	0.667
14	A	12	9	1.	12	0.75
15	A	12	9	1.	12	0.75

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	13	9	1.	12	0.75
17	A	13	9	1.	12	0.75
18	A	12	8	1.	12	0.667
19	A	9	9	1.	12	0.75
20	A	9	9	1.	12	0.75
21	A	12	8	1.	12	0.667
22	A	13	9	1.	12	0.75
23	A	2	2	1.	10	0.2
24	A	4	3	1.	14	0.214
25	A	3	3	1.	14	0.214
26	A	2	2	1.	14	0.143
27	A	2	2	1.	14	0.143
28	A	3	3	1.	14	0.214
29	A	4	3	1.	14	0.214
30	A	16	10	1.	14	0.714
31	A	14	10	1.	14	0.714
32	A	13	10	1.	14	0.714
33	A	13	10	1.	14	0.714
34	A	14	10	1.	14	0.714
35	A	16	10	1.	14	0.714
36	A	7	3	1.	14	0.214
37	A	5	3	1.	14	0.214
38	A	3	3	1.	14	0.214
39	A	3	3	1.	14	0.214
40	A	5	3	1.	14	0.214
41	A	7	3	1.	14	0.214
42	A	3	3	1.	12	0.25
43	A	3	3	1.	12	0.25
44	A	3	3	1.	12	0.25
45	A	3	3	1.	12	0.25
46	A	3	3	1.	14	0.214
47	A	3	3	1.	14	0.214

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
48	A	3	3	1.	14	0.214
49	A	3	3	1.	14	0.214
50	A	3	3	1.	14	0.214
51	A	3	3	1.	14	0.214
52	A	2	2	1.	14	0.143
53	A	3	3	1.	14	0.214
54	A	13	9	1.	21	0.429
55	A	12	9	1.	21	0.429
56	A	2	2	1.	21	0.095
57	A	3	2	1.	21	0.095
58	A	3	2	1.	21	0.095
59	A	5	4	1.	21	0.19
60	A	4	4	1.	19	0.21
61	A	3	3	1.	19	0.158
62	A	4	4	1.	21	0.19
63	A	5	4	1.	21	0.19
64	A	14	10	1.	21	0.476
65	A	13	10	1.	21	0.476
66	A	2	2	1.	21	0.095
67	A	3	2	1.	21	0.095
68	A	3	2	1.	21	0.095
69	A	5	5	1.	21	0.238
70	A	4	4	1.	19	0.21
71	A	4	4	1.	19	0.21
72	A	5	5	1.	21	0.238
73	A	14	10	1.	21	0.476
74	A	13	10	1.	21	0.476
75	A	2	2	1.	21	0.095
76	A	3	2	1.	21	0.095
77	A	3	2	1.	21	0.095
78	A	6	5	1.	21	0.238
79	A	5	5	1.	19	0.263

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	4	4	1.	19	0.21
81	A	4	4	1.	21	0.19
82	A	5	5	1.	21	0.238
83	A	6	5	1.	21	0.238
84	A	13	9	1.	21	0.429
85	A	12	9	1.	21	0.429
86	A	2	2	1.	21	0.095
87	A	3	2	1.	21	0.095
88	A	3	2	1.	21	0.095
89	A	5	4	1.	21	0.19
90	A	4	4	1.	21	0.19
91	A	3	3	1.	19	0.158
92	A	4	4	1.	19	0.21
93	A	5	5	1.	21	0.238
94	A	13	10	1.	21	0.476
95	A	12	9	1.	21	0.429
96	A	2	2	1.	21	0.095
97	A	3	2	1.	21	0.095
98	A	3	2	1.	21	0.095
99	A	5	5	1.	21	0.238
100	A	4	4	1.	19	0.21
101	A	4	4	1.	19	0.21
102	A	5	5	1.	21	0.238
103	A	13	10	1.	21	0.476
104	A	12	9	1.	21	0.429
105	A	2	2	1.	21	0.095
106	A	3	2	1.	21	0.095
107	A	3	2	1.	21	0.095
108	A	6	5	1.	21	0.238
109	A	5	5	1.	21	0.238
110	A	4	4	1.	21	0.19
111	A	4	4	1.	19	0.21

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
112	A	5	5	1.	19	0.263
113	A	6	6	1.	21	0.286
114	A	2	2	1.	25	0.08
115	A	3	3	1.	25	0.12
116	A	1	1	1.	25	0.04
117	A	2	2	1.	25	0.08
118	A	7	7	1.	25	0.28
119	A	3	3	1.	25	0.12
120	A	4	4	1.	25	0.16
121	A	2	2	1.	25	0.08
122	A	3	3	1.	25	0.12
123	A	1	1	1.	25	0.04
124	A	3	3	1.	25	0.12
125	A	8	8	1.	25	0.32
126	A	4	3	1.	25	0.12
127	A	2	2	1.	25	0.08
128	A	3	3	1.	25	0.12
129	A	1	1	1.	25	0.04
130	A	2	2	1.	25	0.08
131	A	7	7	1.	25	0.28
132	A	3	3	1.	25	0.12
133	A	8	8	1.	25	0.32
134	A	4	3	1.	25	0.12
135	A	3	3	1.	25	0.12
136	A	1	1	1.	25	0.04
137	A	8	8	1.	25	0.32
138	A	8	8	1.	25	0.32
139	A	5	4	1.	25	0.16
140	A	4	4	1.	25	0.16
141	A	3	3	1.	25	0.12
142	A	3	3	1.	25	0.12
143	A	4	4	1.	25	0.16

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
144	A	5	4	1.	25	0.16
145	A	2	2	1.	25	0.08
146	A	2	2	1.	25	0.08
147	A	2	2	1.	25	0.08
148	A	2	2	1.	25	0.08
149	A	2	2	1.	25	0.08
150	A	2	2	1.	25	0.08
151	A	2	2	1.	25	0.08
152	A	2	2	1.	25	0.08
153	A	2	2	1.	25	0.08
154	A	2	2	1.	25	0.08
155	A	2	2	1.	25	0.08
156	A	2	2	1.	25	0.08
157	A	2	2	1.	25	0.08
158	A	2	2	1.	25	0.08
159	A	2	2	1.	25	0.08
160	A	2	2	1.	25	0.08
161	A	2	2	1.	19	0.105
162	A	2	2	1.	17	0.118
163	A	2	2	1.	17	0.118
164	A	3	2	1.	19	0.105
165	A	3	2	1.	19	0.105
166	A	2	2	1.	19	0.105
167	A	2	2	1.	19	0.105
168	A	2	2	1.	19	0.105
169	A	2	2	1.	19	0.105
170	A	2	2	1.	23	0.087
171	A	2	2	1.	23	0.087
172	A	2	2	1.	23	0.087
173	A	2	2	1.	23	0.087
174	A	2	2	1.	21	0.095
175	A	2	2	1.	19	0.105

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
176	A	2	2	1.	19	0.105
177	A	2	2	1.	19	0.105
178	A	3	2	1.	19	0.105
179	A	3	2	1.	19	0.105
180	A	2	2	1.	19	0.105
181	A	2	2	1.	17	0.118
182	A	2	2	1.	17	0.118
183	A	2	2	1.	19	0.105
184	A	2	2	1.	19	0.105
185	A	2	2	1.	23	0.087
186	A	2	2	1.	23	0.087
187	A	2	2	1.	23	0.087
188	A	2	2	1.	23	0.087
189	A	2	2	1.	21	0.095
190	A	3	3	1.	21	0.143
191	A	14	10	1.	21	0.476
192	A	13	10	1.	21	0.476
193	A	13	10	1.	21	0.476
194	A	12	9	1.	19	0.474
195	A	11	8	1.	12	0.667
196	A	13	10	1.	19	0.526
197	A	13	10	1.	21	0.476
198	A	14	10	1.	21	0.476
199	A	14	10	1.	21	0.476
200	A	13	10	1.	21	0.476
201	A	13	10	1.	21	0.476
202	A	12	9	1.	21	0.429
203	A	12	9	1.	19	0.474
204	A	12	9	1.	12	0.75
205	A	13	10	1.	19	0.526
206	A	14	10	1.	21	0.476
207	A	14	10	1.	21	0.476

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
208	A	13	10	1.	21	0.476
209	A	13	10	1.	19	0.526
210	A	11	8	1.	12	0.667
211	A	12	9	1.	19	0.474
212	A	13	10	1.	21	0.476
213	A	13	10	1.	21	0.476
214	A	14	10	1.	21	0.476
215	A	13	10	1.	19	0.526
216	A	12	9	1.	12	0.75
217	A	12	9	1.	19	0.474
218	A	12	9	1.	21	0.429
219	A	13	10	1.	21	0.476
220	A	13	10	1.	21	0.476
221	A	14	10	1.	21	0.476
222	A	3	3	1.	17	0.176
223	A	3	3	1.	19	0.158
224	A	3	3	1.	19	0.158
225	A	3	3	1.	21	0.143
226	A	3	2	1.	21	0.095
227	A	3	2	1.	21	0.095
228	A	2	2	1.	21	0.095
229	A	11	8	1.	12	0.667
230	A	12	9	1.	21	0.429
231	A	5	4	1.	21	0.19
232	A	4	4	1.	19	0.21
233	A	3	3	1.	19	0.158
234	A	4	4	1.	21	0.19
235	A	5	4	1.	21	0.19
236	A	3	2	1.	21	0.095
237	A	3	2	1.	21	0.095
238	A	2	2	1.	21	0.095
239	A	12	9	1.	12	0.75

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
240	A	12	9	1.	21	0.429
241	A	6	5	1.	21	0.238
242	A	5	5	1.	21	0.238
243	A	4	4	1.	19	0.21
244	A	4	4	1.	19	0.21
245	A	5	5	1.	21	0.238
246	A	6	5	1.	21	0.238
247	A	3	2	1.	21	0.095
248	A	3	2	1.	21	0.095
249	A	2	2	1.	21	0.095
250	A	12	9	1.	12	0.75
251	A	12	9	1.	21	0.429
252	A	13	10	1.	21	0.476
253	A	5	4	1.	21	0.19
254	A	4	4	1.	21	0.19
255	A	3	3	1.	19	0.158
256	A	4	4	1.	19	0.21
257	A	5	4	1.	21	0.19
258	A	3	2	1.	21	0.095
259	A	3	2	1.	21	0.095
260	A	2	2	1.	21	0.095
261	A	12	9	1.	12	0.75
262	A	13	10	1.	21	0.476
263	A	6	5	1.	21	0.238
264	A	5	5	1.	21	0.238
265	A	4	4	1.	19	0.21
266	A	4	4	1.	19	0.21
267	A	5	5	1.	21	0.238
268	A	6	5	1.	21	0.238
269	A	4	4	1.	19	0.21
270	A	5	4	1.	21	0.19
271	A	2	2	1.	19	0.105

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
272	A	2	2	1.	19	0.105
273	A	2	2	1.	19	0.105
274	A	2	2	1.	19	0.105
275	A	2	2	1.	19	0.105
276	A	2	2	1.	19	0.105
277	A	2	2	1.	19	0.105
278	A	2	2	1.	19	0.105
279	A	2	2	1.	19	0.105
280	A	2	2	1.	19	0.105
281	A	1	1	1.	21	0.048
282	A	1	1	1.	21	0.048
283	A	1	1	1.	21	0.048
284	A	1	1	1.	21	0.048
285	A	1	1	1.	21	0.048
286	A	1	1	1.	21	0.048
287	A	1	1	1.	21	0.048
288	A	1	1	1.	21	0.048
289	A	1	1	1.	21	0.048
290	A	1	1	1.	21	0.048
291	A	7	7	1.	25	0.28
292	A	4	4	1.	25	0.16
293	A	6	6	1.	25	0.24
294	A	3	3	1.	25	0.12
295	A	1	1	1.	25	0.04
296	A	4	4	1.	25	0.16
297	A	2	2	1.	25	0.08
298	A	5	4	1.	25	0.16
299	A	5	5	1.	25	0.2
300	A	7	7	1.	25	0.28
301	A	4	4	1.	25	0.16
302	A	7	7	1.	25	0.28
303	A	4	4	1.	25	0.16

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
304	A	1	1	1.	25	0.04
305	A	5	5	1.	25	0.2
306	A	3	3	1.	25	0.12
307	A	8	8	1.	25	0.32
308	A	5	5	1.	25	0.2
309	A	7	7	1.	25	0.28
310	A	4	4	1.	25	0.16
311	A	7	7	1.	25	0.28
312	A	4	4	1.	25	0.16
313	A	1	1	1.	25	0.04
314	A	5	5	1.	25	0.2
315	A	7	7	1.	25	0.28
316	A	4	4	1.	25	0.16
317	A	6	6	1.	25	0.24
318	A	3	3	1.	25	0.12
319	A	1	1	1.	25	0.04
320	A	4	4	1.	25	0.16
321	A	2	2	1.	25	0.08
322	A	7	7	1.	25	0.28
323	A	4	4	1.	25	0.16
324	A	1	1	1.	25	0.04
325	A	4	4	1.	25	0.16
326	A	2	2	0.93	25	0.08
327	A	5	5	1.	25	0.2
328	A	7	7	1.	25	0.28
329	A	4	4	1.	25	0.16
330	A	1	1	1.	25	0.04
331	A	4	4	1.	25	0.16
332	A	2	2	1.	25	0.08
333	A	5	5	1.	25	0.2
334	A	3	3	1.	25	0.12
335	A	1	1	1.	25	0.04

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
336	A	1	1	1.	25	0.04
337	A	1	1	1.	25	0.04
338	A	1	1	1.	25	0.04
339	A	1	1	1.	25	0.04
340	A	1	1	1.	25	0.04
341	A	1	1	1.	25	0.04
342	A	1	1	1.	25	0.04
343	A	1	1	1.	25	0.04
344	A	1	1	1.	25	0.04
345	A	1	1	1.	25	0.04
346	A	1	1	1.	25	0.04
347	A	1	1	1.	25	0.04
348	A	1	1	1.	25	0.04
349	A	1	1	1.	25	0.04
350	A	1	1	1.	25	0.04
351	A	3	2	1.	19	0.105
352	A	3	2	1.	19	0.105
353	A	2	2	1.	17	0.118
354	A	2	2	1.	17	0.118
355	A	2	2	1.	19	0.105
356	A	2	2	1.	19	0.105
357	A	1	1	1.	19	0.053
358	A	1	1	1.	19	0.053
359	A	1	1	1.	19	0.053
360	A	1	1	1.	19	0.053
361	A	1	1	1.	19	0.053
362	A	1	1	1.	21	0.048
363	A	3	2	1.	19	0.105
364	A	3	2	1.	19	0.105
365	A	2	2	1.	19	0.105
366	A	2	2	1.	10	0.2
367	A	2	2	1.	19	0.105

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
368	A	2	2	1.	19	0.105
369	A	1	1	1.	19	0.053
370	A	1	1	1.	19	0.053
371	A	1	1	1.	17	0.059
372	A	1	1	1.	17	0.059
373	A	1	1	1.	19	0.053
374	A	2	2	1.	19	0.105
375	A	2	2	1.	17	0.118
376	A	2	2	1.	17	0.118
377	A	3	2	1.	19	0.105
378	A	3	2	1.	19	0.105
379	A	1	1	1.	19	0.053
380	A	1	1	1.	19	0.053
381	A	1	1	1.	19	0.053
382	A	1	1	1.	19	0.053
383	A	3	3	1.	23	0.13
384	A	3	3	1.	23	0.13
385	A	3	3	1.	23	0.13
386	A	3	3	1.	23	0.13
387	A	3	3	1.	21	0.143

Chapter 3

Listing of integrals

3.1 $\int \tan(c + dx)\, dx$

Optimal. Leaf size=12

$$-\frac{\log(\cos(c + dx))}{d}$$

```
[Out] -(Log[Cos[c + d*x]]/d)
```

Rubi [A] time = 0.004157, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3475}

$$-\frac{\log(\cos(c + dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x],x]
```

```
[Out] -(Log[Cos[c + d*x]]/d)
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \tan(c+dx)\,dx = -\frac{\log(\cos(c+dx))}{d}$$

Mathematica [A] time = 0.007772, size = 12, normalized size = 1.

$$-\frac{\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x],x]
```

```
[Out] -(Log[Cos[c + d*x]]/d)
```

Maple [A] time = 0.002, size = 17, normalized size = 1.4

$$\frac{\ln\left(1+(\tan(dx+c))^2\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c),x)
```

```
[Out] 1/2/d*ln(1+tan(d*x+c)^2)
```

Maxima [A] time = 2.50265, size = 15, normalized size = 1.25

$$\frac{\log(\sec(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c),x, algorithm="maxima")
```

```
[Out] log(sec(d*x + c))/d
```

Fricas [A] time = 1.72891, size = 49, normalized size = 4.08

$$-\frac{\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*log(1/(tan(d*x + c)^2 + 1))/d
```

Sympy [A] time = 0.139407, size = 19, normalized size = 1.58

$$\begin{cases} \frac{\log\left(\tan^2(c+dx)+1\right)}{2d} & \text{for } d \neq 0 \\ x\tan(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c),x)
```

```
[Out] Piecewise((log(tan(c + d*x)**2 + 1)/(2*d), Ne(d, 0)), (x*tan(c), True))
```

Giac [A] time = 1.70937, size = 18, normalized size = 1.5

$$-\frac{\log\left(|\cos\left(dx+c\right)|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c),x, algorithm="giac")
```

```
[Out] -log(abs(cos(d*x + c)))/d
```

3.2 $\int \tan^2(c+dx)\,dx$

Optimal. Leaf size=14

$$\frac{\tan(c+dx)}{d}-x$$

```
[Out] -x + Tan[c + d*x]/d
```

Rubi [A] time = 0.0076875, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 8}

$$\frac{\tan(c+dx)}{d}-x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2,x]
```

```
[Out] -x + Tan[c + d*x]/d
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \tan^2(c+dx)\,dx &= \frac{\tan(c+dx)}{d}-\int 1\,dx\\ &= -x+\frac{\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0063154, size = 23, normalized size = 1.64

$$\frac{\tan(c+dx)}{d}-\frac{\tan^{-1}(\tan(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2,x]
```

```
[Out] -(ArcTan[Tan[c + d*x]]/d) + Tan[c + d*x]/d
```

Maple [A] time = 0.003, size = 24, normalized size = 1.7

$$\frac{\tan\left(dx+c\right)}{d}-\frac{\arctan\left(\tan\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2,x)
```

```
[Out] tan(d*x+c)/d-1/d*arctan(tan(d*x+c))
```

Maxima [A] time = 1.38102, size = 24, normalized size = 1.71

$$-\frac{dx+c-\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -(d*x + c - tan(d*x + c))/d
```

Fricas [A] time = 1.89881, size = 34, normalized size = 2.43

$$-\frac{dx-\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^2,x, algorithm="fricas")

[Out] -(d*x - tan(d*x + c))/d

Sympy [A] time = 0.169093, size = 15, normalized size = 1.07

$$\begin{cases} -x + \frac{\tan(c+dx)}{d} & \text{for}\ d \neq 0 \\ x\tan^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)**2,x)

[Out] Piecewise((-x + tan(c + d*x)/d, Ne(d, 0)), (x*tan(c)**2, True))

Giac [B] time = 1.59345, size = 305, normalized size = 21.79

$$\frac{\pi - 4\,dx\tan(dx)\tan(c) - \pi\mathrm{sgn}\left(2\,\tan(dx)^2\tan(c) + 2\,\tan(dx)\tan(c)^2 - 2\,\tan(dx) - 2\,\tan(c)\right)\tan(dx)\tan(c) - \pi}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^2,x, algorithm="giac")

[Out] 1/4*(pi - 4*d*x*tan(d*x)*tan(c) - pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*t
an(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)*tan(c) - pi*tan(d*x)*tan(c) + 2*a
rctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d*x)*tan(c) + 2*arctan
((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)*tan(c) + 4*d*x + pi*sg
n(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c)) - 2*ar
ctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c))) - 2*arctan((tan(d*x) + tan(
c))/(tan(d*x)*tan(c) - 1)) - 4*tan(d*x) - 4*tan(c))/(d*tan(d*x)*tan(c) - d)

3.3 $\int \tan^3(c+dx)\,dx$

Optimal. Leaf size=27

$$\frac{\tan^2(c+dx)}{2d}+\frac{\log(\cos(c+dx))}{d}$$

```
[Out] Log[Cos[c + d*x]]/d + Tan[c + d*x]^2/(2*d)
```

Rubi [A] time = 0.0114313, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 3475}

$$\frac{\tan^2(c+dx)}{2d}+\frac{\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3,x]
```

```
[Out] Log[Cos[c + d*x]]/d + Tan[c + d*x]^2/(2*d)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \tan^3(c+dx)\,dx &= \frac{\tan^2(c+dx)}{2d}-\int \tan(c+dx)\,dx\\ &= \frac{\log(\cos(c+dx))}{d}+\frac{\tan^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0240356, size = 25, normalized size = 0.93

$$\frac{\tan^2(c+dx)+2\log(\cos(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3,x]
```

```
[Out] (2*Log[Cos[c + d*x]] + Tan[c + d*x]^2)/(2*d)
```

Maple [A] time = 0.003, size = 31, normalized size = 1.2

$$\frac{(\tan(dx+c))^2}{2d}-\frac{\ln\left(1+(\tan(dx+c))^2\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3,x)
```

```
[Out] 1/2*tan(d*x+c)^2/d-1/2/d*ln(1+tan(d*x+c)^2)
```

Maxima [A] time = 0.872092, size = 42, normalized size = 1.56

$$-\frac{\frac{1}{\sin(dx+c)^2-1}-\log\left(\sin(dx+c)^2-1\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1))/d
```

Fricas [A] time = 1.77591, size = 73, normalized size = 2.7

$$\frac{\tan(dx+c)^2+\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(tan(d*x + c)^2 + log(1/(tan(d*x + c)^2 + 1)))/d
```

Sympy [A] time = 0.211822, size = 32, normalized size = 1.19

$$\begin{cases} -\frac{\log\left(\tan^{2}(c+dx)+1\right)}{2d} + \frac{\tan^{2}(c+dx)}{2d} & \text{for } d \neq 0 \\ x\tan^{3}(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3,x)
```

```
[Out] Piecewise((-log(tan(c + d*x)**2 + 1)/(2*d) + tan(c + d*x)**2/(2*d), Ne(d, 0
)), (x*tan(c)**3, True))
```

Giac [B] time = 1.92996, size = 332, normalized size = 12.3

$$\log\left(\frac{4\left(\tan(c)^{2}+1\right)}{\tan(dx)^{4}\tan(c)^{2}-2\tan(dx)^{3}\tan(c)+\tan(dx)^{2}\tan(c)^{2}+\tan(dx)^{2}-2\tan(dx)\tan(c)+1}\right)\tan(dx)^{2}\tan(c)^{2}+\tan(dx)^{2}\tan(c)^{2}-2\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(
d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2
+ tan(d*x)^2*tan(c)^2 - 2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan
(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))
*tan(d*x)*tan(c) + tan(d*x)^2 + tan(c)^2 + log(4*(tan(c)^2 + 1)/(tan(d*x)^4
*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(
d*x)*tan(c) + 1)) + 1)/(d*tan(d*x)^2*tan(c)^2 - 2*d*tan(d*x)*tan(c) + d)
```

3.4 $\int \tan^4(c+dx)\,dx$

Optimal. Leaf size=28

$$\frac{\tan^3(c+dx)}{3d}-\frac{\tan(c+dx)}{d}+x$$

```
[Out] x - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d)
```

Rubi [A] time = 0.015247, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 8}

$$\frac{\tan^3(c+dx)}{3d}-\frac{\tan(c+dx)}{d}+x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4,x]
```

```
[Out] x - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \tan^4(c+dx)\,dx &= \frac{\tan^3(c+dx)}{3d}-\int \tan^2(c+dx)\,dx\\ &= -\frac{\tan(c+dx)}{d}+\frac{\tan^3(c+dx)}{3d}+\int 1\,dx\\ &= x-\frac{\tan(c+dx)}{d}+\frac{\tan^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0109749, size = 38, normalized size = 1.36

$$\frac{\tan^3(c+dx)}{3d}+\frac{\tan^{-1}(\tan(c+dx))}{d}-\frac{\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4,x]
```

```
[Out] ArcTan[Tan[c + d*x]]/d - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d)
```

Maple [A] time = 0.003, size = 35, normalized size = 1.3

$$\frac{(\tan(dx+c))^3}{3d}-\frac{\tan(dx+c)}{d}+\frac{dx+c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4,x)
```

```
[Out] 1/3*tan(d*x+c)^3/d-tan(d*x+c)/d+1/d*(d*x+c)
```

Maxima [A] time = 2.72585, size = 39, normalized size = 1.39

$$\frac{\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/3*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))/d
```

Fricas [A] time = 1.80023, size = 66, normalized size = 2.36

$$\frac{\tan(dx+c)^3+3\,dx-3\,\tan(dx+c)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(tan(d*x + c)^3 + 3*d*x - 3*tan(d*x + c))/d
```

Sympy [A] time = 0.290155, size = 27, normalized size = 0.96

$$\begin{cases} x + \frac{\tan^3(c+dx)}{3d} - \frac{\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x\tan^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4,x)
```

```
[Out] Piecewise((x + tan(c + d*x)**3/(3*d) - tan(c + d*x)/d, Ne(d, 0)), (x*tan(c)
**4, True))
```

Giac [B] time = 2.38089, size = 790, normalized size = 28.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/12*(3*pi + 12*d*x*tan(d*x)^3*tan(c)^3 - 3*pi*sgn(2*tan(d*x)^2*tan(c) + 2*
tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^3*tan(c)^3 - 3*pi*tan(d
*x)^3*tan(c)^3 + 6*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d*
x)^3*tan(c)^3 + 6*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x
)^3*tan(c)^3 - 36*d*x*tan(d*x)^2*tan(c)^2 + 9*pi*sgn(2*tan(d*x)^2*tan(c) +
2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^2 + 9*pi*tan
(d*x)^2*tan(c)^2 - 18*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan
(d*x)^2*tan(c)^2 - 18*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan
(d*x)^2*tan(c)^2 + 12*tan(d*x)^3*tan(c)^2 + 12*tan(d*x)^2*tan(c)^3 + 36*d*x
*tan(d*x)*tan(c) - 9*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*t
an(d*x) - 2*tan(c))*tan(d*x)*tan(c) - 4*tan(d*x)^3 - 9*pi*tan(d*x)*tan(c) +
 18*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d*x)*tan(c) + 18*
```

```
arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)*tan(c) - 36*tan(
d*x)^2*tan(c) - 36*tan(d*x)*tan(c)^2 - 4*tan(c)^3 - 12*d*x + 3*pi*sgn(2*tan
(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c)) - 6*arctan((t
an(d*x)*tan(c) - 1)/(tan(d*x) + tan(c))) - 6*arctan((tan(d*x) + tan(c))/(ta
n(d*x)*tan(c) - 1)) + 12*tan(d*x) + 12*tan(c))/(d*tan(d*x)^3*tan(c)^3 - 3*d
*tan(d*x)^2*tan(c)^2 + 3*d*tan(d*x)*tan(c) - d)
```

3.5 $\int \tan^5(c+dx)\,dx$

Optimal. Leaf size=43

$$\frac{\tan^4(c+dx)}{4d}-\frac{\tan^2(c+dx)}{2d}-\frac{\log(\cos(c+dx))}{d}$$

```
[Out] -(Log[Cos[c + d*x]]/d) - Tan[c + d*x]^2/(2*d) + Tan[c + d*x]^4/(4*d)
```

Rubi [A] time = 0.0197208, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 3475}

$$\frac{\tan^4(c+dx)}{4d}-\frac{\tan^2(c+dx)}{2d}-\frac{\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5,x]
```

```
[Out] -(Log[Cos[c + d*x]]/d) - Tan[c + d*x]^2/(2*d) + Tan[c + d*x]^4/(4*d)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \tan^5(c+dx)\,dx &= \frac{\tan^4(c+dx)}{4d}-\int \tan^3(c+dx)\,dx\\
&= -\frac{\tan^2(c+dx)}{2d}+\frac{\tan^4(c+dx)}{4d}+\int \tan(c+dx)\,dx\\
&= -\frac{\log(\cos(c+dx))}{d}-\frac{\tan^2(c+dx)}{2d}+\frac{\tan^4(c+dx)}{4d}
\end{aligned}$$

Mathematica [A] time = 0.0457365, size = 37, normalized size = 0.86

$$-\frac{-\tan^4(c+dx)+2\tan^2(c+dx)+4\log(\cos(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5,x]
```

```
[Out] -(4*Log[Cos[c + d*x]] + 2*Tan[c + d*x]^2 - Tan[c + d*x]^4)/(4*d)
```

Maple [A] time = 0.003, size = 44, normalized size = 1.

$$\frac{(\tan(dx+c))^4}{4d}-\frac{(\tan(dx+c))^2}{2d}+\frac{\ln\left(1+(\tan(dx+c))^2\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5,x)
```

```
[Out] 1/4*tan(d*x+c)^4/d-1/2*tan(d*x+c)^2/d+1/2/d*ln(1+tan(d*x+c)^2)
```

Maxima [A] time = 1.57118, size = 73, normalized size = 1.7

$$\frac{\frac{4\sin(dx+c)^2-3}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-2\log\left(\sin(dx+c)^2-1\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/4*((4*sin(d*x + c)^2 - 3)/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 2*log
(sin(d*x + c)^2 - 1))/d
```

Fricas [A] time = 1.753, size = 101, normalized size = 2.35

$$\frac{\tan(dx+c)^4-2\tan(dx+c)^2-2\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/4*(tan(d*x + c)^4 - 2*tan(d*x + c)^2 - 2*log(1/(tan(d*x + c)^2 + 1)))/d
```

Sympy [A] time = 0.441391, size = 44, normalized size = 1.02

$$\begin{cases} \frac{\log\left(\tan^2(c+dx)+1\right)}{2d} + \frac{\tan^4(c+dx)}{4d} - \frac{\tan^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x\tan^5(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5,x)
```

```
[Out] Piecewise((log(tan(c + d*x)**2 + 1)/(2*d) + tan(c + d*x)**4/(4*d) - tan(c +
 d*x)**2/(2*d), Ne(d, 0)), (x*tan(c)**5, True))
```

Giac [B] time = 2.63925, size = 691, normalized size = 16.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] -1/4*(2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + t
an(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)
^4 + 3*tan(d*x)^4*tan(c)^4 - 8*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 -
2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c)
+ 1))*tan(d*x)^3*tan(c)^3 + 2*tan(d*x)^4*tan(c)^2 - 8*tan(d*x)^3*tan(c)^3 +
 2*tan(d*x)^2*tan(c)^4 + 12*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*t
an(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1
))*tan(d*x)^2*tan(c)^2 - tan(d*x)^4 - 8*tan(d*x)^3*tan(c) + 4*tan(d*x)^2*ta
n(c)^2 - 8*tan(d*x)*tan(c)^3 - tan(c)^4 - 8*log(4*(tan(c)^2 + 1)/(tan(d*x)^
4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan
(d*x)*tan(c) + 1))*tan(d*x)*tan(c) + 2*tan(d*x)^2 - 8*tan(d*x)*tan(c) + 2*t
an(c)^2 + 2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c)
```

```
 + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)) + 3)/(d*tan(d
*x)^4*tan(c)^4 - 4*d*tan(d*x)^3*tan(c)^3 + 6*d*tan(d*x)^2*tan(c)^2 - 4*d*ta
n(d*x)*tan(c) + d)
```

3.6 $\int \tan^6(c+dx)\,dx$

Optimal. Leaf size=44

$$\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan(c+dx)}{d}-x$$

```
[Out] -x + Tan[c + d*x]/d - Tan[c + d*x]^3/(3*d) + Tan[c + d*x]^5/(5*d)
```

Rubi [A] time = 0.0246057, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 8}

$$\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan(c+dx)}{d}-x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6,x]
```

```
[Out] -x + Tan[c + d*x]/d - Tan[c + d*x]^3/(3*d) + Tan[c + d*x]^5/(5*d)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \tan^6(c+dx)\,dx &= \frac{\tan^5(c+dx)}{5d} - \int \tan^4(c+dx)\,dx \\ &= -\frac{\tan^3(c+dx)}{3d} + \frac{\tan^5(c+dx)}{5d} + \int \tan^2(c+dx)\,dx \\ &= \frac{\tan(c+dx)}{d} - \frac{\tan^3(c+dx)}{3d} + \frac{\tan^5(c+dx)}{5d} - \int 1\,dx \\ &= -x + \frac{\tan(c+dx)}{d} - \frac{\tan^3(c+dx)}{3d} + \frac{\tan^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.0150459, size = 53, normalized size = 1.2

$$\frac{\tan^5(c+dx)}{5d} - \frac{\tan^3(c+dx)}{3d} - \frac{\tan^{-1}(\tan(c+dx))}{d} + \frac{\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^6,x]
```

```
[Out] -(ArcTan[Tan[c + d*x]]/d) + Tan[c + d*x]/d - Tan[c + d*x]^3/(3*d) + Tan[c +
 d*x]^5/(5*d)
```

Maple [A] time = 0.003, size = 50, normalized size = 1.1

$$\frac{(\tan(dx+c))^5}{5\,d} - \frac{(\tan(dx+c))^3}{3\,d} + \frac{\tan(dx+c)}{d} - \frac{\arctan(\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6,x)
```

```
[Out] 1/5*tan(d*x+c)^5/d-1/3*tan(d*x+c)^3/d+tan(d*x+c)/d-1/d*arctan(tan(d*x+c))
```

Maxima [A] time = 1.86456, size = 55, normalized size = 1.25

$$\frac{3\,\tan(dx+c)^5 - 5\,\tan(dx+c)^3 - 15\,dx - 15\,c + 15\,\tan(dx+c)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/15*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x + c)
)/d
```

Fricas [A] time = 1.53716, size = 99, normalized size = 2.25

$$\frac{3\,\tan\left(dx+c\right)^{5}-5\,\tan\left(dx+c\right)^{3}-15\,dx+15\,\tan\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/15*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x + 15*tan(d*x + c))/d
```

Sympy [A] time = 0.636812, size = 39, normalized size = 0.89

$$\begin{cases} -x+\frac{\tan^{5}(c+dx)}{5d}-\frac{\tan^{3}(c+dx)}{3d}+\frac{\tan(c+dx)}{d} & \text{for } d\neq 0\\ x\tan^{6}(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6,x)
```

```
[Out] Piecewise((-x + tan(c + d*x)**5/(5*d) - tan(c + d*x)**3/(3*d) + tan(c + d*x
)/d, Ne(d, 0)), (x*tan(c)**6, True))
```

Giac [B] time = 4.1907, size = 1335, normalized size = 30.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/60*(15*pi - 60*d*x*tan(d*x)^5*tan(c)^5 - 15*pi*sgn(2*tan(d*x)^2*tan(c) +
2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*tan(c)^5 - 15*pi*ta
n(d*x)^5*tan(c)^5 + 30*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*ta
n(d*x)^5*tan(c)^5 + 30*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*ta
n(d*x)^5*tan(c)^5 + 300*d*x*tan(d*x)^4*tan(c)^4 + 75*pi*sgn(2*tan(d*x)^2*ta
n(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 7
5*pi*tan(d*x)^4*tan(c)^4 - 150*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan
(c)))*tan(d*x)^4*tan(c)^4 - 150*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c)
 - 1))*tan(d*x)^4*tan(c)^4 - 60*tan(d*x)^5*tan(c)^4 - 60*tan(d*x)^4*tan(c)^
5 - 600*d*x*tan(d*x)^3*tan(c)^3 - 150*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*
x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^3*tan(c)^3 + 20*tan(d*x)^5*ta
n(c)^2 - 150*pi*tan(d*x)^3*tan(c)^3 + 300*arctan((tan(d*x)*tan(c) - 1)/(tan
(d*x) + tan(c)))*tan(d*x)^3*tan(c)^3 + 300*arctan((tan(d*x) + tan(c))/(tan(
d*x)*tan(c) - 1))*tan(d*x)^3*tan(c)^3 + 300*tan(d*x)^4*tan(c)^3 + 300*tan(d
*x)^3*tan(c)^4 + 20*tan(d*x)^2*tan(c)^5 + 600*d*x*tan(d*x)^2*tan(c)^2 + 150
*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*
tan(d*x)^2*tan(c)^2 - 12*tan(d*x)^5 - 100*tan(d*x)^4*tan(c) + 150*pi*tan(d*
x)^2*tan(c)^2 - 300*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d
*x)^2*tan(c)^2 - 300*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(
d*x)^2*tan(c)^2 - 600*tan(d*x)^3*tan(c)^2 - 600*tan(d*x)^2*tan(c)^3 - 100*t
an(d*x)*tan(c)^4 - 12*tan(c)^5 - 300*d*x*tan(d*x)*tan(c) - 75*pi*sgn(2*tan(
d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)*tan(c
) + 20*tan(d*x)^3 - 75*pi*tan(d*x)*tan(c) + 150*arctan((tan(d*x)*tan(c) - 1
)/(tan(d*x) + tan(c)))*tan(d*x)*tan(c) + 150*arctan((tan(d*x) + tan(c))/(ta
n(d*x)*tan(c) - 1))*tan(d*x)*tan(c) + 300*tan(d*x)^2*tan(c) + 300*tan(d*x)*
tan(c)^2 + 20*tan(c)^3 + 60*d*x + 15*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x
)*tan(c)^2 - 2*tan(d*x) - 2*tan(c)) - 30*arctan((tan(d*x)*tan(c) - 1)/(tan(
d*x) + tan(c))) - 30*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1)) - 60
*tan(d*x) - 60*tan(c))/(d*tan(d*x)^5*tan(c)^5 - 5*d*tan(d*x)^4*tan(c)^4 + 1
0*d*tan(d*x)^3*tan(c)^3 - 10*d*tan(d*x)^2*tan(c)^2 + 5*d*tan(d*x)*tan(c) -
d)
```

3.7 $\int \tan^7(c+dx)\,dx$

Optimal. Leaf size=57

$$\frac{\tan^6(c+dx)}{6d}-\frac{\tan^4(c+dx)}{4d}+\frac{\tan^2(c+dx)}{2d}+\frac{\log(\cos(c+dx))}{d}$$

```
[Out] Log[Cos[c + d*x]]/d + Tan[c + d*x]^2/(2*d) - Tan[c + d*x]^4/(4*d) + Tan[c +
 d*x]^6/(6*d)
```

Rubi [A] time = 0.0267256, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 3475}

$$\frac{\tan^6(c+dx)}{6d}-\frac{\tan^4(c+dx)}{4d}+\frac{\tan^2(c+dx)}{2d}+\frac{\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^7,x]
```

```
[Out] Log[Cos[c + d*x]]/d + Tan[c + d*x]^2/(2*d) - Tan[c + d*x]^4/(4*d) + Tan[c +
 d*x]^6/(6*d)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \tan^7(c+dx)\,dx &= \frac{\tan^6(c+dx)}{6d} - \int \tan^5(c+dx)\,dx\\ &= -\frac{\tan^4(c+dx)}{4d} + \frac{\tan^6(c+dx)}{6d} + \int \tan^3(c+dx)\,dx\\ &= \frac{\tan^2(c+dx)}{2d} - \frac{\tan^4(c+dx)}{4d} + \frac{\tan^6(c+dx)}{6d} - \int \tan(c+dx)\,dx\\ &= \frac{\log(\cos(c+dx))}{d} + \frac{\tan^2(c+dx)}{2d} - \frac{\tan^4(c+dx)}{4d} + \frac{\tan^6(c+dx)}{6d}\end{aligned}$$

Mathematica [A] time = 0.0989485, size = 47, normalized size = 0.82

$$\frac{2\tan^6(c+dx) - 3\tan^4(c+dx) + 6\tan^2(c+dx) + 12\log(\cos(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^7,x]
```

```
[Out] (12*Log[Cos[c + d*x]] + 6*Tan[c + d*x]^2 - 3*Tan[c + d*x]^4 + 2*Tan[c + d*x
]^6)/(12*d)
```

Maple [A] time = 0.003, size = 57, normalized size = 1.

$$\frac{(\tan(dx+c))^6}{6\,d} - \frac{(\tan(dx+c))^4}{4\,d} + \frac{(\tan(dx+c))^2}{2\,d} - \frac{\ln\left(1+(\tan(dx+c))^2\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^7,x)
```

```
[Out] 1/6*tan(d*x+c)^6/d-1/4*tan(d*x+c)^4/d+1/2*tan(d*x+c)^2/d-1/2/d*ln(1+tan(d*x
+c)^2)
```

Maxima [A] time = 0.945125, size = 100, normalized size = 1.75

$$-\frac{\frac{18\sin(dx+c)^4 - 27\sin(dx+c)^2 + 11}{\sin(dx+c)^6 - 3\sin(dx+c)^4 + 3\sin(dx+c)^2 - 1} - 6\log\left(\sin(dx+c)^2 - 1\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] -1/12*((18*sin(d*x + c)^4 - 27*sin(d*x + c)^2 + 11)/(sin(d*x + c)^6 - 3*sin
(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 6*log(sin(d*x + c)^2 - 1))/d
```

Fricas [A] time = 1.52297, size = 131, normalized size = 2.3

$$\frac{2\,\tan\left(dx+c\right)^{6}-3\,\tan\left(dx+c\right)^{4}+6\,\tan\left(dx+c\right)^{2}+6\,\log\left(\frac{1}{\tan\left(dx+c\right)^{2}+1}\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/12*(2*tan(d*x + c)^6 - 3*tan(d*x + c)^4 + 6*tan(d*x + c)^2 + 6*log(1/(tan
(d*x + c)^2 + 1)))/d
```

Sympy [A] time = 0.920456, size = 56, normalized size = 0.98

$$\begin{cases} -\frac{\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}+\frac{\tan^{6}\left(c+dx\right)}{6d}-\frac{\tan^{4}\left(c+dx\right)}{4d}+\frac{\tan^{2}\left(c+dx\right)}{2d} & \text{for}\ d\neq 0\\ x\tan^{7}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**7,x)
```

```
[Out] Piecewise((-log(tan(c + d*x)**2 + 1)/(2*d) + tan(c + d*x)**6/(6*d) - tan(c
+ d*x)**4/(4*d) + tan(c + d*x)**2/(2*d), Ne(d, 0)), (x*tan(c)**7, True))
```

Giac [B] time = 10.3129, size = 1094, normalized size = 19.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/12*(6*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + t
an(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^6*tan(c)
^6 + 11*tan(d*x)^6*tan(c)^6 - 36*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2
- 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c
) + 1))*tan(d*x)^5*tan(c)^5 + 6*tan(d*x)^6*tan(c)^4 - 54*tan(d*x)^5*tan(c)^
5 + 6*tan(d*x)^4*tan(c)^6 + 90*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 -
2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c)
+ 1))*tan(d*x)^4*tan(c)^4 - 3*tan(d*x)^6*tan(c)^2 - 36*tan(d*x)^5*tan(c)^3
+ 99*tan(d*x)^4*tan(c)^4 - 36*tan(d*x)^3*tan(c)^5 - 3*tan(d*x)^2*tan(c)^6 -
 120*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(
d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c)^3
+ 2*tan(d*x)^6 + 18*tan(d*x)^5*tan(c) + 90*tan(d*x)^4*tan(c)^2 - 72*tan(d*x
)^3*tan(c)^3 + 90*tan(d*x)^2*tan(c)^4 + 18*tan(d*x)*tan(c)^5 + 2*tan(c)^6 +
 90*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d
*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2 -
 3*tan(d*x)^4 - 36*tan(d*x)^3*tan(c) + 99*tan(d*x)^2*tan(c)^2 - 36*tan(d*x)
*tan(c)^3 - 3*tan(c)^4 - 36*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*t
an(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1
))*tan(d*x)*tan(c) + 6*tan(d*x)^2 - 54*tan(d*x)*tan(c) + 6*tan(c)^2 + 6*log
(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*t
an(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)) + 11)/(d*tan(d*x)^6*tan(c)^6
 - 6*d*tan(d*x)^5*tan(c)^5 + 15*d*tan(d*x)^4*tan(c)^4 - 20*d*tan(d*x)^3*tan
(c)^3 + 15*d*tan(d*x)^2*tan(c)^2 - 6*d*tan(d*x)*tan(c) + d)
```

3.8 $\int \tan^8(c + dx)\, dx$

Optimal. Leaf size=58

$$\frac{\tan^7(c+dx)}{7d} - \frac{\tan^5(c+dx)}{5d} + \frac{\tan^3(c+dx)}{3d} - \frac{\tan(c+dx)}{d} + x$$

```
[Out] x - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d) - Tan[c + d*x]^5/(5*d) + Tan[c +
d*x]^7/(7*d)
```

Rubi [A] time = 0.0299271, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 8}

$$\frac{\tan^7(c+dx)}{7d} - \frac{\tan^5(c+dx)}{5d} + \frac{\tan^3(c+dx)}{3d} - \frac{\tan(c+dx)}{d} + x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^8,x]
```

```
[Out] x - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d) - Tan[c + d*x]^5/(5*d) + Tan[c +
d*x]^7/(7*d)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \tan^8(c+dx)\,dx &= \frac{\tan^7(c+dx)}{7d} - \int \tan^6(c+dx)\,dx \\
&= -\frac{\tan^5(c+dx)}{5d} + \frac{\tan^7(c+dx)}{7d} + \int \tan^4(c+dx)\,dx \\
&= \frac{\tan^3(c+dx)}{3d} - \frac{\tan^5(c+dx)}{5d} + \frac{\tan^7(c+dx)}{7d} - \int \tan^2(c+dx)\,dx \\
&= -\frac{\tan(c+dx)}{d} + \frac{\tan^3(c+dx)}{3d} - \frac{\tan^5(c+dx)}{5d} + \frac{\tan^7(c+dx)}{7d} + \int 1\,dx \\
&= x - \frac{\tan(c+dx)}{d} + \frac{\tan^3(c+dx)}{3d} - \frac{\tan^5(c+dx)}{5d} + \frac{\tan^7(c+dx)}{7d}
\end{aligned}$$

Mathematica [A] time = 0.0112421, size = 68, normalized size = 1.17

$$\frac{\tan^7(c+dx)}{7d} - \frac{\tan^5(c+dx)}{5d} + \frac{\tan^3(c+dx)}{3d} + \frac{\tan^{-1}(\tan(c+dx))}{d} - \frac{\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^8,x]
```

```
[Out] ArcTan[Tan[c + d*x]]/d - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d) - Tan[c + d*
x]^5/(5*d) + Tan[c + d*x]^7/(7*d)
```

Maple [A] time = 0.003, size = 61, normalized size = 1.1

$$\frac{(\tan(dx+c))^7}{7d} - \frac{(\tan(dx+c))^5}{5d} + \frac{(\tan(dx+c))^3}{3d} - \frac{\tan(dx+c)}{d} + \frac{dx+c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^8,x)
```

```
[Out] 1/7*tan(d*x+c)^7/d-1/5*tan(d*x+c)^5/d+1/3*tan(d*x+c)^3/d-tan(d*x+c)/d+1/d*(
d*x+c)
```

Maxima [A] time = 1.40812, size = 69, normalized size = 1.19

$$\frac{15\,\tan\left(dx+c\right)^{7}-21\,\tan\left(dx+c\right)^{5}+35\,\tan\left(dx+c\right)^{3}+105\,dx+105\,c-105\,\tan\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] 1/105*(15*tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 105*d*x
+ 105*c - 105*tan(d*x + c))/d
```

Fricas [A] time = 1.50851, size = 132, normalized size = 2.28

$$\frac{15\,\tan\left(dx+c\right)^{7}-21\,\tan\left(dx+c\right)^{5}+35\,\tan\left(dx+c\right)^{3}+105\,dx-105\,\tan\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/105*(15*tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 105*d*x
- 105*tan(d*x + c))/d
```

Sympy [A] time = 1.21523, size = 51, normalized size = 0.88

$$\begin{cases} x+\frac{\tan^{7}(c+dx)}{7d}-\frac{\tan^{5}(c+dx)}{5d}+\frac{\tan^{3}(c+dx)}{3d}-\frac{\tan(c+dx)}{d} & \text{for } d\neq 0 \\ x\tan^{8}(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**8,x)
```

```
[Out] Piecewise((x + tan(c + d*x)**7/(7*d) - tan(c + d*x)**5/(5*d) + tan(c + d*x)
**3/(3*d) - tan(c + d*x)/d, Ne(d, 0)), (x*tan(c)**8, True))
```

Giac [B] time = 9.25495, size = 1945, normalized size = 33.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8,x, algorithm="giac")
```

```
[Out] 1/420*(105*pi + 420*d*x*tan(d*x)^7*tan(c)^7 - 105*pi*sgn(2*tan(d*x)^2*tan(c
) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^7*tan(c)^7 - 105*
pi*tan(d*x)^7*tan(c)^7 + 210*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c
)))*tan(d*x)^7*tan(c)^7 + 210*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) -
 1))*tan(d*x)^7*tan(c)^7 - 2940*d*x*tan(d*x)^6*tan(c)^6 + 735*pi*sgn(2*tan(
d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^6*tan
(c)^6 + 735*pi*tan(d*x)^6*tan(c)^6 - 1470*arctan((tan(d*x)*tan(c) - 1)/(tan
(d*x) + tan(c)))*tan(d*x)^6*tan(c)^6 - 1470*arctan((tan(d*x) + tan(c))/(tan
(d*x)*tan(c) - 1))*tan(d*x)^6*tan(c)^6 + 420*tan(d*x)^7*tan(c)^6 + 420*tan(
d*x)^6*tan(c)^7 + 8820*d*x*tan(d*x)^5*tan(c)^5 - 2205*pi*sgn(2*tan(d*x)^2*t
an(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*tan(c)^5 -
140*tan(d*x)^7*tan(c)^4 - 2205*pi*tan(d*x)^5*tan(c)^5 + 4410*arctan((tan(d*
x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d*x)^5*tan(c)^5 + 4410*arctan((tan(
d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^5*tan(c)^5 - 2940*tan(d*x)^6
*tan(c)^5 - 2940*tan(d*x)^5*tan(c)^6 - 140*tan(d*x)^4*tan(c)^7 - 14700*d*x*
tan(d*x)^4*tan(c)^4 + 3675*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2
 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 84*tan(d*x)^7*tan(c)^2 + 98
0*tan(d*x)^6*tan(c)^3 + 3675*pi*tan(d*x)^4*tan(c)^4 - 7350*arctan((tan(d*x)
*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d*x)^4*tan(c)^4 - 7350*arctan((tan(d*
x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4*tan(c)^4 + 8820*tan(d*x)^5*t
an(c)^4 + 8820*tan(d*x)^4*tan(c)^5 + 980*tan(d*x)^3*tan(c)^6 + 84*tan(d*x)^
2*tan(c)^7 + 14700*d*x*tan(d*x)^3*tan(c)^3 - 3675*pi*sgn(2*tan(d*x)^2*tan(c
) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^3*tan(c)^3 - 60*t
an(d*x)^7 - 588*tan(d*x)^6*tan(c) - 2940*tan(d*x)^5*tan(c)^2 - 3675*pi*tan(
d*x)^3*tan(c)^3 + 7350*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*ta
n(d*x)^3*tan(c)^3 + 7350*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*
tan(d*x)^3*tan(c)^3 - 14700*tan(d*x)^4*tan(c)^3 - 14700*tan(d*x)^3*tan(c)^4
 - 2940*tan(d*x)^2*tan(c)^5 - 588*tan(d*x)*tan(c)^6 - 60*tan(c)^7 - 8820*d*
x*tan(d*x)^2*tan(c)^2 + 2205*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)
^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^2 + 84*tan(d*x)^5 + 980*tan(d
*x)^4*tan(c) + 2205*pi*tan(d*x)^2*tan(c)^2 - 4410*arctan((tan(d*x)*tan(c) -
 1)/(tan(d*x) + tan(c)))*tan(d*x)^2*tan(c)^2 - 4410*arctan((tan(d*x) + tan(
c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^2 + 8820*tan(d*x)^3*tan(c)^2 +
 8820*tan(d*x)^2*tan(c)^3 + 980*tan(d*x)*tan(c)^4 + 84*tan(c)^5 + 2940*d*x*
tan(d*x)*tan(c) - 735*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*
tan(d*x) - 2*tan(c))*tan(d*x)*tan(c) - 140*tan(d*x)^3 - 735*pi*tan(d*x)*tan
```

```
(c) + 1470*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d*x)*tan(c
) + 1470*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)*tan(c)
- 2940*tan(d*x)^2*tan(c) - 2940*tan(d*x)*tan(c)^2 - 140*tan(c)^3 - 420*d*x
+ 105*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan
(c)) - 210*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c))) - 210*arctan((
tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1)) + 420*tan(d*x) + 420*tan(c))/(d*t
an(d*x)^7*tan(c)^7 - 7*d*tan(d*x)^6*tan(c)^6 + 21*d*tan(d*x)^5*tan(c)^5 - 3
5*d*tan(d*x)^4*tan(c)^4 + 35*d*tan(d*x)^3*tan(c)^3 - 21*d*tan(d*x)^2*tan(c)
^2 + 7*d*tan(d*x)*tan(c) - d)
```

3.9 $\int (b\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=232

$$-\frac{b^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}d}-\frac{2b^3\sqrt{b\tan(c+dx)}}{d}-\frac{b^{7/2}\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\right.}{2\sqrt{2}d}$$

```
[Out] -((b^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d))
 + (b^(7/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d)
 - (b^(7/2)*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x
]]])/(2*Sqrt[2]*d) + (b^(7/2)*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*
Sqrt[b*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (2*b^3*Sqrt[b*Tan[c + d*x]])/d + (2*
b*(b*Tan[c + d*x])^(5/2))/(5*d)
```

Rubi [A] time = 0.196008, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3473, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{b^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}d}-\frac{2b^3\sqrt{b\tan(c+dx)}}{d}-\frac{b^{7/2}\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\right.}{2\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x])^(7/2),x]
```


```
[Out] -((b^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d))
 + (b^(7/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d)
 - (b^(7/2)*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x
]]])/(2*Sqrt[2]*d) + (b^(7/2)*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*
Sqrt[b*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (2*b^3*Sqrt[b*Tan[c + d*x]])/d + (2*
b*(b*Tan[c + d*x])^(5/2))/(5*d)
```


Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (b\tan(c+dx))^{7/2}\,dx = \frac{2b(b\tan(c+dx))^{5/2}}{5d} - b^2\int (b\tan(c+dx))^{3/2}\,dx$$
$$= -\frac{2b^3\sqrt{b\tan(c+dx)}}{d} + \frac{2b(b\tan(c+dx))^{5/2}}{5d} + b^4\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx$$
$$= -\frac{2b^3\sqrt{b\tan(c+dx)}}{d} + \frac{2b(b\tan(c+dx))^{5/2}}{5d} + \frac{b^5\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(b^2+x^2\right)}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= -\frac{2b^3\sqrt{b\tan(c+dx)}}{d} + \frac{2b(b\tan(c+dx))^{5/2}}{5d} + \frac{\left(2b^5\right)\text{Subst}\left(\int \frac{1}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{d}$$
$$= -\frac{2b^3\sqrt{b\tan(c+dx)}}{d} + \frac{2b(b\tan(c+dx))^{5/2}}{5d} + \frac{b^4\,\text{Subst}\left(\int \frac{b-x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{d} + \frac{b^4\,\text{S}}{}$$
$$= -\frac{2b^3\sqrt{b\tan(c+dx)}}{d} + \frac{2b(b\tan(c+dx))^{5/2}}{5d} - \frac{b^{7/2}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}+2x}{-b-\sqrt{2}\sqrt{b}x-x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right.}{2\sqrt{2}d}$$
$$= -\frac{b^{7/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d} + \frac{b^{7/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\right.}{2\sqrt{2}d}$$
$$= -\frac{b^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} + \frac{b^{7/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} - \frac{b^{7/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+\right.}{2\sqrt{}}$$

Mathematica [A] time = 0.359906, size = 175, normalized size = 0.75

$$\frac{b^3\sqrt{b\tan(c+dx)}\left(8\tan^{\frac{5}{2}}(c+dx)-10\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)+10\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)-40\sqrt{\tan}\right.}{20d\sqrt{\tan(c+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x])^(7/2),x]
```

```
[Out] (b^3*Sqrt[b*Tan[c + d*x]]*(-10*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]
]] + 10*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] - 5*Sqrt[2]*Log[1 -
Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + 5*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt
[Tan[c + d*x]] + Tan[c + d*x]] - 40*Sqrt[Tan[c + d*x]] + 8*Tan[c + d*x]^(5/
2)))/(20*d*Sqrt[Tan[c + d*x]])
```

Maple [A] time = 0.028, size = 200, normalized size = 0.9

$$\frac{2b}{5d}(b\tan(dx+c))^{\frac{5}{2}}-2\frac{b^3\sqrt{b\tan(dx+c)}}{d}+\frac{b^3\sqrt{2}}{2d}\sqrt[4]{b^2}\arctan\left(\sqrt{2}\sqrt{b\tan(dx+c)}\frac{1}{\sqrt[4]{b^2}}+1\right)-\frac{b^3\sqrt{2}}{2d}\sqrt[4]{b^2}\arctan\left(-\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c))^(7/2),x)
```

```
[Out] 2/5*b*(b*tan(d*x+c))^(5/2)/d-2*b^3*(b*tan(d*x+c))^(1/2)/d+1/2/d*b^3*(b^2)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)-1/2/d*b^3*(
b^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)+1/4/
d*b^3*(b^2)^(1/4)*2^(1/2)*ln((b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)
*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)-(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2
)+(b^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.74677, size = 1520, normalized size = 6.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/20*(20*sqrt(2)*(b^14/d^4)^(1/4)*d*arctan(-(b^14 + sqrt(2)*(b^14/d^4)^(3/
4)*b^3*d^3*sqrt(b*sin(d*x + c)/cos(d*x + c)) - sqrt(2)*(b^14/d^4)^(3/4)*d^3
*sqrt((b^7*sin(d*x + c) + sqrt(2)*(b^14/d^4)^(1/4)*b^3*d*sqrt(b*sin(d*x + c
)/cos(d*x + c))*cos(d*x + c) + sqrt(b^14/d^4)*d^2*cos(d*x + c))/cos(d*x + c
)))/b^14)*cos(d*x + c)^2 + 20*sqrt(2)*(b^14/d^4)^(1/4)*d*arctan((b^14 - sqr
t(2)*(b^14/d^4)^(3/4)*b^3*d^3*sqrt(b*sin(d*x + c)/cos(d*x + c)) + sqrt(2)*(
b^14/d^4)^(3/4)*d^3*sqrt((b^7*sin(d*x + c) - sqrt(2)*(b^14/d^4)^(1/4)*b^3*d
*sqrt(b*sin(d*x + c)/cos(d*x + c))*cos(d*x + c) + sqrt(b^14/d^4)*d^2*cos(d*
x + c))/cos(d*x + c)))/b^14)*cos(d*x + c)^2 - 5*sqrt(2)*(b^14/d^4)^(1/4)*d*
cos(d*x + c)^2*log((b^7*sin(d*x + c) + sqrt(2)*(b^14/d^4)^(1/4)*b^3*d*sqrt(
b*sin(d*x + c)/cos(d*x + c))*cos(d*x + c) + sqrt(b^14/d^4)*d^2*cos(d*x + c)
)/cos(d*x + c)) + 5*sqrt(2)*(b^14/d^4)^(1/4)*d*cos(d*x + c)^2*log((b^7*sin(
d*x + c) - sqrt(2)*(b^14/d^4)^(1/4)*b^3*d*sqrt(b*sin(d*x + c)/cos(d*x + c))
*cos(d*x + c) + sqrt(b^14/d^4)*d^2*cos(d*x + c))/cos(d*x + c)) + 8*(6*b^3*c
os(d*x + c)^2 - b^3)*sqrt(b*sin(d*x + c)/cos(d*x + c)))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tan(c+dx))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))**(7/2),x)
```

```
[Out] Integral((b*tan(c + d*x))**(7/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.10 $\int (b\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=212

$$\frac{b^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}d}-\frac{b^{5/2}\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}d}+$$

```
[Out] (b^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d) -
(b^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d) -
(b^(5/2)*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x]]]
)/(2*Sqrt[2]*d) + (b^(5/2)*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*Sqr
t[b*Tan[c + d*x]]])/(2*Sqrt[2]*d) + (2*b*(b*Tan[c + d*x])^(3/2))/(3*d)
```

Rubi [A] time = 0.144697, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3473, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{b^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}d}-\frac{b^{5/2}\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}d}+$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (b^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d) -
(b^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d) -
(b^(5/2)*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x]]]
)/(2*Sqrt[2]*d) + (b^(5/2)*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*Sqr
t[b*Tan[c + d*x]]])/(2*Sqrt[2]*d) + (2*b*(b*Tan[c + d*x])^(3/2))/(3*d)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int (b\tan(c+dx))^{5/2}\,dx = \frac{2b(b\tan(c+dx))^{3/2}}{3d} - b^2\int\sqrt{b\tan(c+dx)}\,dx$$

$$= \frac{2b(b\tan(c+dx))^{3/2}}{3d} - \frac{b^3\,\text{Subst}\left(\int\frac{\sqrt{x}}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{2b(b\tan(c+dx))^{3/2}}{3d} - \frac{\left(2b^3\right)\text{Subst}\left(\int\frac{x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{d}$$

$$= \frac{2b(b\tan(c+dx))^{3/2}}{3d} + \frac{b^3\,\text{Subst}\left(\int\frac{b-x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{d} - \frac{b^3\,\text{Subst}\left(\int\frac{b+x^2}{b^2+x^4}\,dx, x, \sqrt{b\,\text{t}\right.}{d}$$

$$= \frac{2b(b\tan(c+dx))^{3/2}}{3d} - \frac{b^{5/2}\,\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{b}+2x}{-b-\sqrt{2}\sqrt{b}x-x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{b^{5/2}\,\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{}}{-b+\sqrt{2}}\right.}{}$$

$$= -\frac{b^{5/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d} + \frac{b^{5/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b}\right.}{2\sqrt{2}d}$$

$$= \frac{b^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} - \frac{b^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} - \frac{b^{5/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx\right.}{2\sqrt{2}d}$$

Mathematica [C] time = 0.0683589, size = 40, normalized size = 0.19

$$-\frac{2b(b\tan(c+dx))^{3/2}\left({}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(c+dx)\right)-1\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (-2*b*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d*x]^2])*(b*Tan[c + d*x
])^(3/2))/(3*d)
```

Maple [A] time = 0.015, size = 182, normalized size = 0.9

$$\frac{2b}{3d}(b\tan(dx+c))^{\frac{3}{2}}-\frac{b^3\sqrt{2}}{4d}\ln\left(\left(b\tan(dx+c)-\sqrt[4]{b^2}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}\right)\left(b\tan(dx+c)+\sqrt[4]{b^2}\sqrt{b\tan(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c))^(5/2),x)
```

```
[Out] 2/3*b*(b*tan(d*x+c))^(3/2)/d-1/4/d*b^3/(b^2)^(1/4)*2^(1/2)*ln((b*tan(d*x+c)
-(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)+(b^2)^
(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))-1/2/d*b^3/(b^2)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)+1/2/d*b^3/(b^2)^(1/4
)*2^(1/2)*arctan(-2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.71795, size = 1521, normalized size = 7.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(12*sqrt(2)*(b^10/d^4)^(1/4)*d*arctan(-(b^10 + sqrt(2)*(b^10/d^4)^(1/4
)*b^7*d*sqrt(b*sin(d*x + c)/cos(d*x + c)) - sqrt(2)*(b^10/d^4)^(1/4)*d*sqrt
((b^15*sin(d*x + c) + sqrt(b^10/d^4)*b^10*d^2*cos(d*x + c) + sqrt(2)*(b^10/
d^4)^(3/4)*b^7*d^3*sqrt(b*sin(d*x + c)/cos(d*x + c))*cos(d*x + c))/cos(d*x
+ c)))/b^10)*cos(d*x + c) + 12*sqrt(2)*(b^10/d^4)^(1/4)*d*arctan((b^10 - sq
rt(2)*(b^10/d^4)^(1/4)*b^7*d*sqrt(b*sin(d*x + c)/cos(d*x + c)) + sqrt(2)*(b
```

```
^10/d^4)^(1/4)*d*sqrt((b^15*sin(d*x + c) + sqrt(b^10/d^4)*b^10*d^2*cos(d*x
+ c) - sqrt(2)*(b^10/d^4)^(3/4)*b^7*d^3*sqrt(b*sin(d*x + c)/cos(d*x + c))*c
os(d*x + c))/cos(d*x + c)))/b^10)*cos(d*x + c) + 3*sqrt(2)*(b^10/d^4)^(1/4)
*d*cos(d*x + c)*log((b^15*sin(d*x + c) + sqrt(b^10/d^4)*b^10*d^2*cos(d*x +
c) + sqrt(2)*(b^10/d^4)^(3/4)*b^7*d^3*sqrt(b*sin(d*x + c)/cos(d*x + c))*cos
(d*x + c))/cos(d*x + c)) - 3*sqrt(2)*(b^10/d^4)^(1/4)*d*cos(d*x + c)*log((b
^15*sin(d*x + c) + sqrt(b^10/d^4)*b^10*d^2*cos(d*x + c) - sqrt(2)*(b^10/d^4
)^(3/4)*b^7*d^3*sqrt(b*sin(d*x + c)/cos(d*x + c))*cos(d*x + c))/cos(d*x + c
)) + 8*b^2*sqrt(b*sin(d*x + c)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tan{(c+dx)})^{\frac{5}{2}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*tan(c + d*x))**(5/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.11 $\int (b\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=210

$$\frac{b^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}d}+\frac{b^{3/2}\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}d}-$$

```
[Out] (b^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d) -
(b^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d) +
(b^(3/2)*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x]]]
)/(2*Sqrt[2]*d) - (b^(3/2)*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*Sqr
t[b*Tan[c + d*x]]])/(2*Sqrt[2]*d) + (2*b*Sqrt[b*Tan[c + d*x]])/d
```

Rubi [A] time = 0.151164, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3473, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}d}+\frac{b^{3/2}\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}d}-$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x])^(3/2),x]
```


```
[Out] (b^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d) -
(b^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d) +
(b^(3/2)*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x]]]
)/(2*Sqrt[2]*d) - (b^(3/2)*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*Sqr
t[b*Tan[c + d*x]]])/(2*Sqrt[2]*d) + (2*b*Sqrt[b*Tan[c + d*x]])/d
```


Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (b\tan(c+dx))^{3/2}\,dx = \frac{2b\sqrt{b\tan(c+dx)}}{d} - b^2\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx$$

$$= \frac{2b\sqrt{b\tan(c+dx)}}{d} - \frac{b^3\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(b^2+x^2)}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{2b\sqrt{b\tan(c+dx)}}{d} - \frac{(2b^3)\,\text{Subst}\left(\int \frac{1}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{d}$$

$$= \frac{2b\sqrt{b\tan(c+dx)}}{d} - \frac{b^2\,\text{Subst}\left(\int \frac{b-x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{d} - \frac{b^2\,\text{Subst}\left(\int \frac{b+x^2}{b^2+x^4}\,dx, x, \sqrt{b\,\text{ta}}\right.}{d}$$

$$= \frac{2b\sqrt{b\tan(c+dx)}}{d} + \frac{b^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}+2x}{-b-\sqrt{2}\sqrt{b}x-x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d} + \frac{b^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}}{-b+\sqrt{2}\sqrt{}}\right.}{}$$

$$= \frac{b^{3/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{b^{3/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b}\right.}{2\sqrt{2}d}$$

$$= \frac{b^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} - \frac{b^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} + \frac{b^{3/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+d\right.}{2\sqrt{2}}$$

Mathematica [A] time = 0.159505, size = 159, normalized size = 0.76

$$\frac{(b\tan(c+dx))^{3/2}\left(2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-2\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)+8\sqrt{\tan(c+dx)}+\sqrt{2}\log\left(\text{ta}\right.\right.}{4d\tan^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 2*Sqrt[2]*ArcTan[1 + S
qrt[2]*Sqrt[Tan[c + d*x]]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + T
an[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] +
```

```
8*Sqrt[Tan[c + d*x]])*(b*Tan[c + d*x])^(3/2))/(4*d*Tan[c + d*x]^(3/2))
```

Maple [A] time = 0.015, size = 176, normalized size = 0.8

$$2\,\frac{b\sqrt{b\tan\left(dx+c\right)}}{d}-\frac{b\sqrt{2}}{4\,d}\sqrt[4]{b^{2}}\ln\left(\left(b\tan\left(dx+c\right)+\sqrt[4]{b^{2}}\sqrt{b\tan\left(dx+c\right)}\sqrt{2}+\sqrt{b^{2}}\right)\left(b\tan\left(dx+c\right)-\sqrt[4]{b^{2}}\sqrt{b\tan\left(dx+c\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c))^(3/2),x)
```

```
[Out] 2*b*(b*tan(d*x+c))^(1/2)/d-1/4/d*b*(b^2)^(1/4)*2^(1/2)*ln((b*tan(d*x+c)+(b^
2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)-(b^2)^(1/4
)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))-1/2/d*b*(b^2)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)+1/2/d*b*(b^2)^(1/4)*2^(1/2
)*arctan(-2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.696, size = 1328, normalized size = 6.32

$$4\,\sqrt{2}\left(\frac{b^{6}}{d^{4}}\right)^{\frac{1}{4}}d\arctan\left(-\frac{b^{6}+\sqrt{2}\left(\frac{b^{6}}{d^{4}}\right)^{\frac{3}{4}}bd^{3}\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}-\sqrt{2}\left(\frac{b^{6}}{d^{4}}\right)^{\frac{3}{4}}d^{3}\sqrt{\frac{\sqrt{2}\left(\frac{b^{6}}{d^{4}}\right)^{\frac{1}{4}}bd\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\cos(dx+c)+b^{3}\sin(dx+c)+\sqrt{\frac{b^{6}}{d^{4}}}d^{2}\cos(dx+c)}{\cos(dx+c)}}}{b^{6}}\right)+4\,\sqrt{2}\left(\frac{b^{6}}{d^{4}}\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*(b^6/d^4)^(1/4)*d*arctan(-(b^6 + sqrt(2)*(b^6/d^4)^(3/4)*b*d
^3*sqrt(b*sin(d*x + c)/cos(d*x + c)) - sqrt(2)*(b^6/d^4)^(3/4)*d^3*sqrt((sq
rt(2)*(b^6/d^4)^(1/4)*b*d*sqrt(b*sin(d*x + c)/cos(d*x + c))*cos(d*x + c) +
b^3*sin(d*x + c) + sqrt(b^6/d^4)*d^2*cos(d*x + c))/cos(d*x + c)))/b^6) + 4*
sqrt(2)*(b^6/d^4)^(1/4)*d*arctan((b^6 - sqrt(2)*(b^6/d^4)^(3/4)*b*d^3*sqrt(
b*sin(d*x + c)/cos(d*x + c)) + sqrt(2)*(b^6/d^4)^(3/4)*d^3*sqrt(-(sqrt(2)*(
b^6/d^4)^(1/4)*b*d*sqrt(b*sin(d*x + c)/cos(d*x + c))*cos(d*x + c) - b^3*sin
(d*x + c) - sqrt(b^6/d^4)*d^2*cos(d*x + c))/cos(d*x + c)))/b^6) - sqrt(2)*(
b^6/d^4)^(1/4)*d*log((sqrt(2)*(b^6/d^4)^(1/4)*b*d*sqrt(b*sin(d*x + c)/cos(d
*x + c))*cos(d*x + c) + b^3*sin(d*x + c) + sqrt(b^6/d^4)*d^2*cos(d*x + c))/
cos(d*x + c)) + sqrt(2)*(b^6/d^4)^(1/4)*d*log(-(sqrt(2)*(b^6/d^4)^(1/4)*b*d
*sqrt(b*sin(d*x + c)/cos(d*x + c))*cos(d*x + c) - b^3*sin(d*x + c) - sqrt(b
^6/d^4)*d^2*cos(d*x + c))/cos(d*x + c)) + 8*b*sqrt(b*sin(d*x + c)/cos(d*x +
 c)))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tan (c+dx))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*tan(c + d*x))**(3/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.12 $\int \sqrt{b\tan(c+dx)}\,dx$

Optimal. Leaf size=192

$$-\frac{\sqrt{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}d}+\frac{\sqrt{b}\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}d}-$$

```
[Out] -((Sqrt[b]*ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d))
 + (Sqrt[b]*ArcTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d)
 + (Sqrt[b]*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x
]]])/(2*Sqrt[2]*d) - (Sqrt[b]*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*
Sqrt[b*Tan[c + d*x]]])/(2*Sqrt[2]*d)
```

Rubi [A] time = 0.120601, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3476, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}d}+\frac{\sqrt{b}\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}d}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[c + d*x]],x]
```


```
[Out] -((Sqrt[b]*ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d))
 + (Sqrt[b]*ArcTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d)
 + (Sqrt[b]*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x
]]])/(2*Sqrt[2]*d) - (Sqrt[b]*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*
Sqrt[b*Tan[c + d*x]]])/(2*Sqrt[2]*d)
```


Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \sqrt{b\tan(c+dx)}\,dx = \frac{b\,\text{Subst}\left(\int \frac{\sqrt{x}}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= \frac{(2b)\,\text{Subst}\left(\int \frac{x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{d}$$
$$= -\frac{b\,\text{Subst}\left(\int \frac{b-x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{d} + \frac{b\,\text{Subst}\left(\int \frac{b+x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{d}$$
$$= \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}+2x}{-b-\sqrt{2}\sqrt{b}x-x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d} + \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}-2x}{-b+\sqrt{2}\sqrt{b}x-x^2}\,dx, x, \sqrt{b\tan(c+dx}\right.}{2\sqrt{2}d}$$
$$= \frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan}\right.}{2\sqrt{2}d}$$
$$= -\frac{\sqrt{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} + \frac{\sqrt{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} + \frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\right.}{2\sqrt{2}d}$$

Mathematica [C] time = 0.0409806, size = 40, normalized size = 0.21

$$\frac{2(b\tan(c+dx))^{3/2}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(c+dx)\right)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[c + d*x]],x]
```

```
[Out] (2*Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d*x]^2]*(b*Tan[c + d*x])^(3/2))/
(3*b*d)
```

Maple [A] time = 0.018, size = 160, normalized size = 0.8

$$\frac{b\sqrt{2}}{4\,d}\ln\left(\left(b\tan(dx+c)-\sqrt[4]{b^2}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}\right)\left(b\tan(dx+c)+\sqrt[4]{b^2}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}\right)^{-1}\right)\frac{1}{\sqrt[4]{b^2}}+\frac{b\sqrt{2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c))^(1/2),x)
```

```
[Out] 1/4/d*b/(b^2)^(1/4)*2^(1/2)*ln((b*tan(d*x+c)-(b^2)^(1/4)*(b*tan(d*x+c))^(1/
2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1
/2)+(b^2)^(1/2)))+1/2/d*b/(b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b^2)^(1/4)*(b
*tan(d*x+c))^(1/2)+1)-1/2/d*b/(b^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(b^2)^(1/
4)*(b*tan(d*x+c))^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.69988, size = 1283, normalized size = 6.68

$$-\sqrt{2}\left(\frac{b^2}{d^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}bd\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\left(\frac{b^2}{d^4}\right)^{\frac{1}{4}}-\sqrt{2}d\sqrt{\frac{\sqrt{2}bd^3\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\left(\frac{b^2}{d^4}\right)^{\frac{3}{4}}\cos(dx+c)+b^2d^2\sqrt{\frac{b^2}{d^4}}\cos(dx+c)+b^3\sin(dx+c)}{\cos(dx+c)}}\left(\frac{b^2}{d^4}\right)^{\frac{1}{4}}+}{b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*(b^2/d^4)^(1/4)*arctan(-(sqrt(2)*b*d*sqrt(b*sin(d*x + c)/cos(d*x +
 c))*(b^2/d^4)^(1/4) - sqrt(2)*d*sqrt((sqrt(2)*b*d^3*sqrt(b*sin(d*x + c)/co
s(d*x + c))*(b^2/d^4)^(3/4)*cos(d*x + c) + b^2*d^2*sqrt(b^2/d^4)*cos(d*x +
c) + b^3*sin(d*x + c))/cos(d*x + c))*(b^2/d^4)^(1/4) + b^2)/b^2) - sqrt(2)*
(b^2/d^4)^(1/4)*arctan(-(sqrt(2)*b*d*sqrt(b*sin(d*x + c)/cos(d*x + c))*(b^2
/d^4)^(1/4) - sqrt(2)*d*sqrt(-(sqrt(2)*b*d^3*sqrt(b*sin(d*x + c)/cos(d*x +
c))*(b^2/d^4)^(3/4)*cos(d*x + c) - b^2*d^2*sqrt(b^2/d^4)*cos(d*x + c) - b^3
*sin(d*x + c))/cos(d*x + c))*(b^2/d^4)^(1/4) - b^2)/b^2) - 1/4*sqrt(2)*(b^2
```

```
/d^4)^(1/4)*log((sqrt(2)*b*d^3*sqrt(b*sin(d*x + c)/cos(d*x + c))*(b^2/d^4)^
(3/4)*cos(d*x + c) + b^2*d^2*sqrt(b^2/d^4)*cos(d*x + c) + b^3*sin(d*x + c))
/cos(d*x + c)) + 1/4*sqrt(2)*(b^2/d^4)^(1/4)*log(-(sqrt(2)*b*d^3*sqrt(b*sin
(d*x + c)/cos(d*x + c))*(b^2/d^4)^(3/4)*cos(d*x + c) - b^2*d^2*sqrt(b^2/d^4
)*cos(d*x + c) - b^3*sin(d*x + c))/cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*tan(c + d*x)), x)
```

Giac [A] time = 1.73897, size = 251, normalized size = 1.31

$$\frac{1}{4}\,b\left(\frac{2\,\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\,\sqrt{b\tan(dx+c)}\right)}{2\,\sqrt{|b|}}\right)}{b^2d}+\frac{2\,\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\,\sqrt{b\tan(dx+c)}\right)}{2\,\sqrt{|b|}}\right)}{b^2d}-\frac{\sqrt{2}|b|^{\frac{3}{2}}\log\left(b\tan\left(dx+c\right)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*b*(2*sqrt(2)*abs(b)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) + 2*
sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/(b^2*d) + 2*sqrt(2)*abs(b)^(3/2)*arctan
(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))
/(b^2*d) - sqrt(2)*abs(b)^(3/2)*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x
 + c))*sqrt(abs(b)) + abs(b))/(b^2*d) + sqrt(2)*abs(b)^(3/2)*log(b*tan(d*x
+ c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(b^2*d))
```

3.13 $\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx$

Optimal. Leaf size=192

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{b}d}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}\sqrt{b}d}-\frac{\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}\sqrt{b}d}+\frac{\log\left(\sqrt{b}\tan\right.}{}$$

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*Sqrt[b]*d)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*Sqrt[b]*d) - L
og[Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x]]]/(2*Sqrt[2
]*Sqrt[b]*d) + Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*Sqrt[b*Tan[c +
d*x]]]/(2*Sqrt[2]*Sqrt[b]*d)
```

Rubi [A] time = 0.121426, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3476, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{b}d}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}\sqrt{b}d}-\frac{\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}\sqrt{b}d}+\frac{\log\left(\sqrt{b}\tan\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Tan[c + d*x]],x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*Sqrt[b]*d)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*Sqrt[b]*d) - L
og[Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x]]]/(2*Sqrt[2
]*Sqrt[b]*d) + Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*Sqrt[b*Tan[c +
d*x]]]/(2*Sqrt[2]*Sqrt[b]*d)
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx = \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(b^2+x^2\right)}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{(2b)\,\text{Subst}\left(\int \frac{1}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{b-x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{d} + \frac{\text{Subst}\left(\int \frac{b+x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{b-\sqrt{2}\sqrt{b}x+x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{2d} + \frac{\text{Subst}\left(\int \frac{1}{b+\sqrt{2}\sqrt{b}x+x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{2d} - \cdots$$

$$= -\frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}\sqrt{b}d} + \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c \cdots}{2\sqrt{2}\sqrt{b}d}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{b}d} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{b}d} - \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b \cdots}{2\sqrt{2}\sqrt{b}d}$$

Mathematica [A] time = 0.0998046, size = 131, normalized size = 0.68

$$\frac{\sqrt{\tan(c+dx)}\left(-2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)+2\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)-\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)} \cdots\right.\right.}{2\sqrt{2}d\sqrt{b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Tan[c + d*x]],x]
```

```
[Out] ((-2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] + 2*ArcTan[1 + Sqrt[2]*Sqrt[Tan
[c + d*x]]] - Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + Log[1 +
Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])*Sqrt[Tan[c + d*x]])/(2*Sqrt[2]*
d*Sqrt[b*Tan[c + d*x]])
```

Maple [A] time = 0.018, size = 166, normalized size = 0.9

$$\frac{\sqrt{2}}{4\,bd}\sqrt[4]{b^2}\ln\left(\left(b\tan(dx+c)+\sqrt[4]{b^2}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}\right)\left(b\tan(dx+c)-\sqrt[4]{b^2}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}\right)^{-1}\right)+\frac{\sqrt{2}}{2\,b \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c))^(1/2),x)
```

```
[Out] 1/4/d/b*(b^2)^(1/4)*2^(1/2)*ln((b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/
2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)-(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1
/2)+(b^2)^(1/2)))+1/2/d/b*(b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b^2)^(1/4)*(b
*tan(d*x+c))^(1/2)+1)-1/2/d/b*(b^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(b^2)^(1/
4)*(b*tan(d*x+c))^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.67544, size = 1339, normalized size = 6.97

$$-\sqrt{2}\left(\frac{1}{b^2d^4}\right)^{\frac{1}{4}}\arctan\left(-\sqrt{2}bd^3\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\left(\frac{1}{b^2d^4}\right)^{\frac{3}{4}}+\sqrt{2}bd^3\sqrt{\frac{b^2d^2\sqrt{\frac{1}{b^2d^4}}\cos(dx+c)+\sqrt{2}bd\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\left(\frac{1}{b^2d^4}\right)^{\frac{1}{4}}\mathrm{co}}{\cos(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*(1/(b^2*d^4))^(1/4)*arctan(-sqrt(2)*b*d^3*sqrt(b*sin(d*x + c)/cos(
d*x + c))*(1/(b^2*d^4))^(3/4) + sqrt(2)*b*d^3*sqrt((b^2*d^2*sqrt(1/(b^2*d^4
))*cos(d*x + c) + sqrt(2)*b*d*sqrt(b*sin(d*x + c)/cos(d*x + c))*(1/(b^2*d^4
))^(1/4)*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*(1/(b^2*d^4))^(3/4) -
 1) - sqrt(2)*(1/(b^2*d^4))^(1/4)*arctan(-sqrt(2)*b*d^3*sqrt(b*sin(d*x + c)
/cos(d*x + c))*(1/(b^2*d^4))^(3/4) + sqrt(2)*b*d^3*sqrt((b^2*d^2*sqrt(1/(b^
2*d^4))*cos(d*x + c) - sqrt(2)*b*d*sqrt(b*sin(d*x + c)/cos(d*x + c))*(1/(b^
```

```
2*d^4))^(1/4)*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*(1/(b^2*d^4))^(3
/4) + 1) + 1/4*sqrt(2)*(1/(b^2*d^4))^(1/4)*log((b^2*d^2*sqrt(1/(b^2*d^4))*c
os(d*x + c) + sqrt(2)*b*d*sqrt(b*sin(d*x + c)/cos(d*x + c))*(1/(b^2*d^4))^(
1/4)*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c)) - 1/4*sqrt(2)*(1/(b^2*d^4
))^(1/4)*log((b^2*d^2*sqrt(1/(b^2*d^4))*cos(d*x + c) - sqrt(2)*b*d*sqrt(b*s
in(d*x + c)/cos(d*x + c))*(1/(b^2*d^4))^(1/4)*cos(d*x + c) + b*sin(d*x + c)
)/cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*tan(c + d*x)), x)
```

Giac [A] time = 2.483, size = 251, normalized size = 1.31

$$\frac{1}{4}b\left(\frac{2\sqrt{2}\sqrt{|b|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{b^2d}+\frac{2\sqrt{2}\sqrt{|b|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{b^2d}+\frac{\sqrt{2}\sqrt{|b|}\log\left(b\tan(dx+\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*b*(2*sqrt(2)*sqrt(abs(b))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) + 2*
sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/(b^2*d) + 2*sqrt(2)*sqrt(abs(b))*arctan
(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))
/(b^2*d) + sqrt(2)*sqrt(abs(b))*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x
 + c))*sqrt(abs(b)) + abs(b))/(b^2*d) - sqrt(2)*sqrt(abs(b))*log(b*tan(d*x
+ c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(b^2*d))
```

3.14 $\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=212

$$\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{3/2}d}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}b^{3/2}d}-\frac{\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}b^{3/2}d}+\frac{\log\left(\sqrt{b}\tan\right.}{}$$

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*b^(3/2)*d) - Ar
cTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*b^(3/2)*d) - Log[
Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x]]]/(2*Sqrt[2]*b
^(3/2)*d) + Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*Sqrt[b*Tan[c + d*x
]]]/(2*Sqrt[2]*b^(3/2)*d) - 2/(b*d*Sqrt[b*Tan[c + d*x]])
```

Rubi [A] time = 0.146247, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3474, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{3/2}d}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}b^{3/2}d}-\frac{\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}b^{3/2}d}+\frac{\log\left(\sqrt{b}\tan\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x])^(-3/2),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*b^(3/2)*d) - Ar
cTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*b^(3/2)*d) - Log[
Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x]]]/(2*Sqrt[2]*b
^(3/2)*d) + Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*Sqrt[b*Tan[c + d*x
]]]/(2*Sqrt[2]*b^(3/2)*d) - 2/(b*d*Sqrt[b*Tan[c + d*x]])
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx = -\frac{2}{bd\sqrt{b\tan(c+dx)}} - \frac{\int \sqrt{b\tan(c+dx)}\,dx}{b^2}$$

$$= -\frac{2}{bd\sqrt{b\tan(c+dx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= -\frac{2}{bd\sqrt{b\tan(c+dx)}} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{bd}$$

$$= -\frac{2}{bd\sqrt{b\tan(c+dx)}} + \frac{\text{Subst}\left(\int \frac{b-x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{bd} - \frac{\text{Subst}\left(\int \frac{b+x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c}{bd}$$

$$= -\frac{2}{bd\sqrt{b\tan(c+dx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}+2x}{-b-\sqrt{2}\sqrt{b}x-x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{3/2}d} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}-2x}{-b+\sqrt{2}\sqrt{b}x-x^2}\,d}{2\sqrt{2}b^3}$$

$$= -\frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{3/2}d} + \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c}{2\sqrt{2}b^{3/2}d}$$

$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{3/2}d} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{3/2}d} - \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\,\text{ta}}{2\sqrt{2}b^{3/2}d}$$

Mathematica [C] time = 0.0646589, size = 38, normalized size = 0.18

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\tan^2(c+dx)\right)}{bd\sqrt{b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x])^(-3/2),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[c + d*x]^2])/(b*d*Sqrt[b*Tan[c + d
*x]])
```

Maple [A] time = 0.016, size = 184, normalized size = 0.9

$$-\frac{\sqrt{2}}{4\,bd}\ln\left(\left(b\tan\left(dx+c\right)-\sqrt[4]{b^2}\sqrt{b\tan\left(dx+c\right)}\sqrt{2}+\sqrt{b^2}\right)\left(b\tan\left(dx+c\right)+\sqrt[4]{b^2}\sqrt{b\tan\left(dx+c\right)}\sqrt{2}+\sqrt{b^2}\right)^{-1}\right)\frac{1}{\sqrt[4]{b^2}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d/b/(b^2)^(1/4)*2^(1/2)*ln((b*tan(d*x+c)-(b^2)^(1/4)*(b*tan(d*x+c))^(1
/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(
1/2)+(b^2)^(1/2)))-1/2/d/b/(b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b^2)^(1/4)*(
b*tan(d*x+c))^(1/2)+1)+1/2/d/b/(b^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(b^2)^(1
/4)*(b*tan(d*x+c))^(1/2)+1)-2/b/d/(b*tan(d*x+c))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.7696, size = 1693, normalized size = 7.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(8*sqrt(b*sin(d*x + c)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 4*(sqr
t(2)*b^2*d*cos(d*x + c)^2 - sqrt(2)*b^2*d)*(1/(b^6*d^4))^(1/4)*arctan(-sqrt
(2)*b*d*sqrt(b*sin(d*x + c)/cos(d*x + c))*(1/(b^6*d^4))^(1/4) + sqrt(2)*b*d
*sqrt((sqrt(2)*b^5*d^3*sqrt(b*sin(d*x + c)/cos(d*x + c))*(1/(b^6*d^4))^(3/4
```

```
)*cos(d*x + c) + b^4*d^2*sqrt(1/(b^6*d^4))*cos(d*x + c) + b*sin(d*x + c))/c
os(d*x + c))*(1/(b^6*d^4))^(1/4) - 1) + 4*(sqrt(2)*b^2*d*cos(d*x + c)^2 - s
qrt(2)*b^2*d)*(1/(b^6*d^4))^(1/4)*arctan(-sqrt(2)*b*d*sqrt(b*sin(d*x + c)/c
os(d*x + c))*(1/(b^6*d^4))^(1/4) + sqrt(2)*b*d*sqrt(-(sqrt(2)*b^5*d^3*sqrt(
b*sin(d*x + c)/cos(d*x + c))*(1/(b^6*d^4))^(3/4)*cos(d*x + c) - b^4*d^2*sqr
t(1/(b^6*d^4))*cos(d*x + c) - b*sin(d*x + c))/cos(d*x + c))*(1/(b^6*d^4))^(
1/4) + 1) + (sqrt(2)*b^2*d*cos(d*x + c)^2 - sqrt(2)*b^2*d)*(1/(b^6*d^4))^(1
/4)*log((sqrt(2)*b^5*d^3*sqrt(b*sin(d*x + c)/cos(d*x + c))*(1/(b^6*d^4))^(3
/4)*cos(d*x + c) + b^4*d^2*sqrt(1/(b^6*d^4))*cos(d*x + c) + b*sin(d*x + c))
/cos(d*x + c)) - (sqrt(2)*b^2*d*cos(d*x + c)^2 - sqrt(2)*b^2*d)*(1/(b^6*d^4
))^(1/4)*log(-(sqrt(2)*b^5*d^3*sqrt(b*sin(d*x + c)/cos(d*x + c))*(1/(b^6*d^
4))^(3/4)*cos(d*x + c) - b^4*d^2*sqrt(1/(b^6*d^4))*cos(d*x + c) - b*sin(d*x
 + c))/cos(d*x + c)))/(b^2*d*cos(d*x + c)^2 - b^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\tan(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*tan(c + d*x))**(-3/2), x)
```

Giac [A] time = 1.57727, size = 275, normalized size = 1.3

$$-\frac{1}{4}b\left(\frac{2\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{b^4d}+\frac{2\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{b^4d}-\frac{\sqrt{2}|b|^{\frac{3}{2}}\log\left(b\tan(dx+c\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*b*(2*sqrt(2)*abs(b)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) + 2
*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/(b^4*d) + 2*sqrt(2)*abs(b)^(3/2)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b))
```

```
)/(b^4*d) - sqrt(2)*abs(b)^(3/2)*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*
x + c))*sqrt(abs(b)) + abs(b))/(b^4*d) + sqrt(2)*abs(b)^(3/2)*log(b*tan(d*x
 + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(b^4*d) + 8/(sq
rt(b*tan(d*x + c))*b^2*d))
```

3.15 $\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=214

$$\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{5/2}d}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}b^{5/2}d}+\frac{\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}b^{5/2}d}-\frac{\log\left(\sqrt{b}\tan\cdots\right.}{}$$

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*b^(5/2)*d) - Ar
cTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*b^(5/2)*d) + Log[
Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x]]]/(2*Sqrt[2]*b
^(5/2)*d) - Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*Sqrt[b*Tan[c + d*x
]]]/(2*Sqrt[2]*b^(5/2)*d) - 2/(3*b*d*(b*Tan[c + d*x])^(3/2))
```

Rubi [A] time = 0.150489, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3474, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{5/2}d}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}b^{5/2}d}+\frac{\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}b^{5/2}d}-\frac{\log\left(\sqrt{b}\tan\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x])^(-5/2),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*b^(5/2)*d) - Ar
cTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*b^(5/2)*d) + Log[
Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x]]]/(2*Sqrt[2]*b
^(5/2)*d) - Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*Sqrt[b*Tan[c + d*x
]]]/(2*Sqrt[2]*b^(5/2)*d) - 2/(3*b*d*(b*Tan[c + d*x])^(3/2))
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx = -\frac{2}{3bd(b\tan(c+dx))^{3/2}} - \frac{\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx}{b^2}$$

$$= -\frac{2}{3bd(b\tan(c+dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}(b^2+x^2)}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= -\frac{2}{3bd(b\tan(c+dx))^{3/2}} - \frac{2\,\text{Subst}\left(\int \frac{1}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{bd}$$

$$= -\frac{2}{3bd(b\tan(c+dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{b-x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{b^2d} - \frac{\text{Subst}\left(\int \frac{b+x^2}{b^2+x^4}\,dx, x, \sqrt{b\,\text{ta}}\right.}{b^2d}$$

$$= -\frac{2}{3bd(b\tan(c+dx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}+2x}{-b-\sqrt{2}\sqrt{b}x-x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{5/2}d} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}-2x}{-b+\sqrt{2}\sqrt{b}x-}\right.}{2\sqrt{}}$$

$$= \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{5/2}d} - \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+}\right.}{2\sqrt{2}b^{5/2}d}$$

$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{5/2}d} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{5/2}d} + \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\,\text{t}}\right.}{2\sqrt{2}b^{5/2}d}$$

Mathematica [C] time = 0.0805243, size = 40, normalized size = 0.19

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\tan^2(c+dx)\right)}{3bd(b\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x])^(-5/2),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/4, 1, 1/4, -Tan[c + d*x]^2])/(3*b*d*(b*Tan[c + d*x
])^(3/2))
```

Maple [A] time = 0.017, size = 184, normalized size = 0.9

$$-\frac{\sqrt{2}}{4\,d b^{3}}\sqrt[4]{b^{2}}\ln\left(\left(b\tan\left(dx+c\right)+\sqrt[4]{b^{2}}\sqrt{b\tan\left(dx+c\right)}\sqrt{2}+\sqrt{b^{2}}\right)\left(b\tan\left(dx+c\right)-\sqrt[4]{b^{2}}\sqrt{b\tan\left(dx+c\right)}\sqrt{2}+\sqrt{b^{2}}\right)^{-1}\right)-\frac{}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c))^(5/2),x)
```

```
[Out] -1/4/d/b^3*(b^2)^(1/4)*2^(1/2)*ln((b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^
(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)-(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2
^(1/2)+(b^2)^(1/2)))-1/2/d/b^3*(b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b^2)^(1/
4)*(b*tan(d*x+c))^(1/2)+1)+1/2/d/b^3*(b^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(b
^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)-2/3/b/d/(b*tan(d*x+c))^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.82421, size = 1717, normalized size = 8.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(8*sqrt(b*sin(d*x + c)/cos(d*x + c))*cos(d*x + c)^2 + 12*(sqrt(2)*b^3*
d*cos(d*x + c)^2 - sqrt(2)*b^3*d)*(1/(b^10*d^4))^(1/4)*arctan(-sqrt(2)*b^7*
d^3*sqrt(b*sin(d*x + c)/cos(d*x + c))*(1/(b^10*d^4))^(3/4) + sqrt(2)*b^7*d^
3*sqrt((b^6*d^2*sqrt(1/(b^10*d^4))*cos(d*x + c) + sqrt(2)*b^3*d*sqrt(b*sin(
```

```
d*x + c)/cos(d*x + c))*(1/(b^10*d^4))^(1/4)*cos(d*x + c) + b*sin(d*x + c))/
cos(d*x + c))*(1/(b^10*d^4))^(3/4) - 1) + 12*(sqrt(2)*b^3*d*cos(d*x + c)^2
- sqrt(2)*b^3*d)*(1/(b^10*d^4))^(1/4)*arctan(-sqrt(2)*b^7*d^3*sqrt(b*sin(d*
x + c)/cos(d*x + c))*(1/(b^10*d^4))^(3/4) + sqrt(2)*b^7*d^3*sqrt((b^6*d^2*s
qrt(1/(b^10*d^4))*cos(d*x + c) - sqrt(2)*b^3*d*sqrt(b*sin(d*x + c)/cos(d*x
+ c))*(1/(b^10*d^4))^(1/4)*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*(1/
(b^10*d^4))^(3/4) + 1) - 3*(sqrt(2)*b^3*d*cos(d*x + c)^2 - sqrt(2)*b^3*d)*(
1/(b^10*d^4))^(1/4)*log((b^6*d^2*sqrt(1/(b^10*d^4))*cos(d*x + c) + sqrt(2)*
b^3*d*sqrt(b*sin(d*x + c)/cos(d*x + c))*(1/(b^10*d^4))^(1/4)*cos(d*x + c) +
 b*sin(d*x + c))/cos(d*x + c)) + 3*(sqrt(2)*b^3*d*cos(d*x + c)^2 - sqrt(2)*
b^3*d)*(1/(b^10*d^4))^(1/4)*log((b^6*d^2*sqrt(1/(b^10*d^4))*cos(d*x + c) -
sqrt(2)*b^3*d*sqrt(b*sin(d*x + c)/cos(d*x + c))*(1/(b^10*d^4))^(1/4)*cos(d*
x + c) + b*sin(d*x + c))/cos(d*x + c)))/(b^3*d*cos(d*x + c)^2 - b^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\tan{(c+dx)})^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*tan(c + d*x))**(-5/2), x)
```

Giac [A] time = 1.48354, size = 288, normalized size = 1.35

$$-\frac{1}{12}b\left(\frac{6\sqrt{2}\sqrt{|b|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{b^4d}+\frac{6\sqrt{2}\sqrt{|b|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{b^4d}+\frac{3\sqrt{2}\sqrt{|b|}\log\left(b\tan(d\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/12*b*(6*sqrt(2)*sqrt(abs(b))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) +
2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/(b^4*d) + 6*sqrt(2)*sqrt(abs(b))*arct
an(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)
```

```
))/(b^4*d) + 3*sqrt(2)*sqrt(abs(b))*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan
(d*x + c))*sqrt(abs(b)) + abs(b))/(b^4*d) - 3*sqrt(2)*sqrt(abs(b))*log(b*ta
n(d*x + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(b^4*d) +
8/(sqrt(b*tan(d*x + c))*b^3*d*tan(d*x + c)))
```

3.16 $\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=234

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{7/2}d}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}b^{7/2}d}+\frac{2}{b^3d\sqrt{b\tan(c+dx)}}+\frac{\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right.}{2\sqrt{2}b^{7/2}d}$$

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*b^(7/2)*d)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*b^(7/2)*d) + L
og[Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x]]]/(2*Sqrt[2
]*b^(7/2)*d) - Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*Sqrt[b*Tan[c +
d*x]]]/(2*Sqrt[2]*b^(7/2)*d) - 2/(5*b*d*(b*Tan[c + d*x])^(5/2)) + 2/(b^3*d*
Sqrt[b*Tan[c + d*x]])
```

Rubi [A] time = 0.180953, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3474, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{7/2}d}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}b^{7/2}d}+\frac{2}{b^3d\sqrt{b\tan(c+dx)}}+\frac{\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right.}{2\sqrt{2}b^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x])^(-7/2),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*b^(7/2)*d)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*b^(7/2)*d) + L
og[Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x]]]/(2*Sqrt[2
]*b^(7/2)*d) - Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*Sqrt[b*Tan[c +
d*x]]]/(2*Sqrt[2]*b^(7/2)*d) - 2/(5*b*d*(b*Tan[c + d*x])^(5/2)) + 2/(b^3*d*
Sqrt[b*Tan[c + d*x]])
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx = -\frac{2}{5bd(b\tan(c+dx))^{5/2}} - \frac{\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx}{b^2}$$

$$= -\frac{2}{5bd(b\tan(c+dx))^{5/2}} + \frac{2}{b^3d\sqrt{b\tan(c+dx)}} + \frac{\int \sqrt{b\tan(c+dx)}\,dx}{b^4}$$

$$= -\frac{2}{5bd(b\tan(c+dx))^{5/2}} + \frac{2}{b^3d\sqrt{b\tan(c+dx)}} + \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{b^3d}$$

$$= -\frac{2}{5bd(b\tan(c+dx))^{5/2}} + \frac{2}{b^3d\sqrt{b\tan(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{b^3d}$$

$$= -\frac{2}{5bd(b\tan(c+dx))^{5/2}} + \frac{2}{b^3d\sqrt{b\tan(c+dx)}} - \frac{\text{Subst}\left(\int \frac{b-x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{b^3d} + \frac{\text{Subs}}{}$$

$$= -\frac{2}{5bd(b\tan(c+dx))^{5/2}} + \frac{2}{b^3d\sqrt{b\tan(c+dx)}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}+2x}{-b-\sqrt{2}\sqrt{b}x-x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{7/2}d} +$$

$$= \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{7/2}d} - \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+\right.}}{2\sqrt{2}b^{7/2}d}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{7/2}d} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{7/2}d} + \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\right.}}{2\sqrt{2}b^{7/2}d}$$

Mathematica [C] time = 0.102988, size = 40, normalized size = 0.17

$$-\frac{2\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\tan^2(c+dx)\right)}{5bd(b\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x])^(-7/2),x]
```

```
[Out] (-2*Hypergeometric2F1[-5/4, 1, -1/4, -Tan[c + d*x]^2])/(5*b*d*(b*Tan[c + d*
x])^(5/2))
```

Maple [A] time = 0.019, size = 202, normalized size = 0.9

$$\frac{\sqrt{2}}{4\,db^3}\ln\left(\left(b\tan(dx+c)-\sqrt[4]{b^2}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}\right)\left(b\tan(dx+c)+\sqrt[4]{b^2}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}\right)^{-1}\right)\frac{1}{\sqrt[4]{b^2}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c))^(7/2),x)
```

```
[Out] 1/4/d/b^3/(b^2)^(1/4)*2^(1/2)*ln((b*tan(d*x+c)-(b^2)^(1/4)*(b*tan(d*x+c))^(
1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^
(1/2)+(b^2)^(1/2)))+1/2/d/b^3/(b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b^2)^(1/4
)*(b*tan(d*x+c))^(1/2)+1)-1/2/d/b^3/(b^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(b^
2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)-2/5/b/d/(b*tan(d*x+c))^(5/2)+2/b^3/d/(b*ta
n(d*x+c))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.77854, size = 1985, normalized size = 8.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/20*(8*(6*cos(d*x + c)^3 - 5*cos(d*x + c))*sqrt(b*sin(d*x + c)/cos(d*x +
c))*sin(d*x + c) + 20*(sqrt(2)*b^4*d*cos(d*x + c)^4 - 2*sqrt(2)*b^4*d*cos(d
*x + c)^2 + sqrt(2)*b^4*d)*(1/(b^14*d^4))^(1/4)*arctan(-sqrt(2)*b^3*d*sqrt(
b*sin(d*x + c)/cos(d*x + c))*(1/(b^14*d^4))^(1/4) + sqrt(2)*b^3*d*sqrt((sqr
t(2)*b^11*d^3*sqrt(b*sin(d*x + c)/cos(d*x + c))*(1/(b^14*d^4))^(3/4)*cos(d*
x + c) + b^8*d^2*sqrt(1/(b^14*d^4))*cos(d*x + c) + b*sin(d*x + c))/cos(d*x
+ c))*(1/(b^14*d^4))^(1/4) - 1) + 20*(sqrt(2)*b^4*d*cos(d*x + c)^4 - 2*sqrt
(2)*b^4*d*cos(d*x + c)^2 + sqrt(2)*b^4*d)*(1/(b^14*d^4))^(1/4)*arctan(-sqrt
(2)*b^3*d*sqrt(b*sin(d*x + c)/cos(d*x + c))*(1/(b^14*d^4))^(1/4) + sqrt(2)*
b^3*d*sqrt(-(sqrt(2)*b^11*d^3*sqrt(b*sin(d*x + c)/cos(d*x + c))*(1/(b^14*d^
4))^(3/4)*cos(d*x + c) - b^8*d^2*sqrt(1/(b^14*d^4))*cos(d*x + c) - b*sin(d*
x + c))/cos(d*x + c))*(1/(b^14*d^4))^(1/4) + 1) + 5*(sqrt(2)*b^4*d*cos(d*x
+ c)^4 - 2*sqrt(2)*b^4*d*cos(d*x + c)^2 + sqrt(2)*b^4*d)*(1/(b^14*d^4))^(1/
4)*log((sqrt(2)*b^11*d^3*sqrt(b*sin(d*x + c)/cos(d*x + c))*(1/(b^14*d^4))^(
3/4)*cos(d*x + c) + b^8*d^2*sqrt(1/(b^14*d^4))*cos(d*x + c) + b*sin(d*x + c
))/cos(d*x + c)) - 5*(sqrt(2)*b^4*d*cos(d*x + c)^4 - 2*sqrt(2)*b^4*d*cos(d*
x + c)^2 + sqrt(2)*b^4*d)*(1/(b^14*d^4))^(1/4)*log(-(sqrt(2)*b^11*d^3*sqrt(
b*sin(d*x + c)/cos(d*x + c))*(1/(b^14*d^4))^(3/4)*cos(d*x + c) - b^8*d^2*sq
rt(1/(b^14*d^4))*cos(d*x + c) - b*sin(d*x + c))/cos(d*x + c)))/(b^4*d*cos(d
*x + c)^4 - 2*b^4*d*cos(d*x + c)^2 + b^4*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\tan(c+dx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))**(7/2),x)
```

```
[Out] Integral((b*tan(c + d*x))**(-7/2), x)
```

Giac [A] time = 1.50618, size = 313, normalized size = 1.34

$$\frac{1}{20}b\left(\frac{10\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{b^6d}+\frac{10\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{b^6d}-\frac{5\sqrt{2}|b|^{\frac{3}{2}}\log\left(b\tan(dx\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] 1/20*b*(10*sqrt(2)*abs(b)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) +
2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/(b^6*d) + 10*sqrt(2)*abs(b)^(3/2)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b
)))/(b^6*d) - 5*sqrt(2)*abs(b)^(3/2)*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*ta
n(d*x + c))*sqrt(abs(b)) + abs(b))/(b^6*d) + 5*sqrt(2)*abs(b)^(3/2)*log(b*t
an(d*x + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(b^6*d) +
 8*(5*b^2*tan(d*x + c)^2 - b^2)/(sqrt(b*tan(d*x + c))*b^6*d*tan(d*x + c)^2)
)
```

3.17 $\int (b\tan(c+dx))^{4/3}\,dx$

Optimal. Leaf size=243

$$-\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d}+\frac{b^{4/3}\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2d}-\frac{b^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}+\sqrt{3}\right)}{2d}+\frac{\sqrt{3}b^{4/3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\right.}{}$$

```
[Out] -((b^(4/3)*ArcTan[(b*Tan[c + d*x])^(1/3)/b^(1/3)])/d) + (b^(4/3)*ArcTan[Sqr
t[3] - (2*(b*Tan[c + d*x])^(1/3))/b^(1/3)])/(2*d) - (b^(4/3)*ArcTan[Sqrt[3]
 + (2*(b*Tan[c + d*x])^(1/3))/b^(1/3)])/(2*d) + (Sqrt[3]*b^(4/3)*Log[b^(2/3
) - Sqrt[3]*b^(1/3)*(b*Tan[c + d*x])^(1/3) + (b*Tan[c + d*x])^(2/3)])/(4*d)
 - (Sqrt[3]*b^(4/3)*Log[b^(2/3) + Sqrt[3]*b^(1/3)*(b*Tan[c + d*x])^(1/3) +
(b*Tan[c + d*x])^(2/3)])/(4*d) + (3*b*(b*Tan[c + d*x])^(1/3))/d
```

Rubi [A] time = 0.417682, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3473, 3476, 329, 209, 634, 618, 204, 628, 203}

$$-\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d}+\frac{b^{4/3}\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2d}-\frac{b^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}+\sqrt{3}\right)}{2d}+\frac{\sqrt{3}b^{4/3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x])^(4/3),x]
```

```
[Out] -((b^(4/3)*ArcTan[(b*Tan[c + d*x])^(1/3)/b^(1/3)])/d) + (b^(4/3)*ArcTan[Sqr
t[3] - (2*(b*Tan[c + d*x])^(1/3))/b^(1/3)])/(2*d) - (b^(4/3)*ArcTan[Sqrt[3]
 + (2*(b*Tan[c + d*x])^(1/3))/b^(1/3)])/(2*d) + (Sqrt[3]*b^(4/3)*Log[b^(2/3
) - Sqrt[3]*b^(1/3)*(b*Tan[c + d*x])^(1/3) + (b*Tan[c + d*x])^(2/3)])/(4*d)
 - (Sqrt[3]*b^(4/3)*Log[b^(2/3) + Sqrt[3]*b^(1/3)*(b*Tan[c + d*x])^(1/3) +
(b*Tan[c + d*x])^(2/3)])/(4*d) + (3*b*(b*Tan[c + d*x])^(1/3))/d
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (b\tan(c+dx))^{4/3}\,dx = \frac{3b\sqrt[3]{b\tan(c+dx)}}{d} - b^2\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx$$

$$= \frac{3b\sqrt[3]{b\tan(c+dx)}}{d} - \frac{b^3\,\text{Subst}\left(\int \frac{1}{x^{2/3}(b^2+x^2)}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{3b\sqrt[3]{b\tan(c+dx)}}{d} - \frac{(3b^3)\,\text{Subst}\left(\int \frac{1}{b^2+x^6}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{d}$$

$$= \frac{3b\sqrt[3]{b\tan(c+dx)}}{d} - \frac{b^{4/3}\,\text{Subst}\left(\int \frac{\sqrt[3]{b}-\frac{\sqrt{3}x}{2}}{b^{2/3}-\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{d} - \frac{b^{4/3}\,\text{Subst}\left(\int \frac{\sqrt[3]{b}+}{b^{2/3}+\sqrt{3}}\right.}{}$$

$$= -\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d} + \frac{3b\sqrt[3]{b\tan(c+dx)}}{d} + \frac{\left(\sqrt{3}b^{4/3}\right)\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[3]{b}+2x}{b^{2/3}-\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan\right.}{4d}$$

$$= -\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d} + \frac{\sqrt{3}b^{4/3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4d} - \sqrt{3}$$

$$= -\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d} + \frac{b^{4/3}\tan^{-1}\left(\frac{1}{3}\left(3\sqrt{3}-\frac{6\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)\right)}{2d} - \frac{b^{4/3}\tan^{-1}\left(\frac{1}{3}\left(3\sqrt{3}+\frac{6\sqrt[3]{b\tan}}{\sqrt[3]{}}\right.\right.}{2d}$$

Mathematica [C] time = 0.0269183, size = 38, normalized size = 0.16

$$-\frac{3b\sqrt[3]{b\tan(c+dx)}\left({}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\tan^2(c+dx)\right)-1\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x])^(4/3),x]
```

```
[Out] (-3*b*(-1 + Hypergeometric2F1[1/6, 1, 7/6, -Tan[c + d*x]^2])*(b*Tan[c + d*x
])^(1/3))/d
```

Maple [A] time = 0.086, size = 215, normalized size = 0.9

$$3\,{\frac {b\sqrt [3]{b\tan \left(dx+c \right) }}{d}}-{\frac {b\sqrt {3}}{4\,d}\sqrt [6]{{b}^{2}}\ln \left(\left(b\tan \left(dx+c \right) \right) ^{{\frac {2}{3}}}+\sqrt {3}\sqrt [6]{{b}^{2}}\sqrt [3]{b\tan \left(dx+c \right) }+\sqrt [3]{{b}^{2}} \right) }-{\frac {b}{2\,d}\sqrt [6]{{b}^{2}}\arctan \left(2\,{\frac {\sqrt [3]{b\tan \left(dx+ \right.}}{\sqrt [6]{{b}^{2}}}} \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c))^(4/3),x)
```

```
[Out] 3*b*(b*tan(d*x+c))^(1/3)/d-1/4/d*b*3^(1/2)*(b^2)^(1/6)*ln((b*tan(d*x+c))^(2
/3)+3^(1/2)*(b^2)^(1/6)*(b*tan(d*x+c))^(1/3)+(b^2)^(1/3))-1/2/d*b*(b^2)^(1/
6)*arctan(2*(b*tan(d*x+c))^(1/3)/(b^2)^(1/6)+3^(1/2))+1/4/d*b*3^(1/2)*(b^2)
^(1/6)*ln((b*tan(d*x+c))^(2/3)-3^(1/2)*(b^2)^(1/6)*(b*tan(d*x+c))^(1/3)+(b^
2)^(1/3))-1/2/d*b*(b^2)^(1/6)*arctan(2*(b*tan(d*x+c))^(1/3)/(b^2)^(1/6)-3^(
1/2))-1/d*b*(b^2)^(1/6)*arctan((b*tan(d*x+c))^(1/3)/(b^2)^(1/6))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.45459, size = 1481, normalized size = 6.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(3)*(b^8/d^6)^(1/6)*d*log(sqrt(3)*(b^8/d^6)^(1/6)*b*d*(b*sin(d*x
+ c)/cos(d*x + c))^(1/3) + b^2*(b*sin(d*x + c)/cos(d*x + c))^(2/3) + (b^8/d
^6)^(1/3)*d^2) - sqrt(3)*(b^8/d^6)^(1/6)*d*log(-sqrt(3)*(b^8/d^6)^(1/6)*b*d
*(b*sin(d*x + c)/cos(d*x + c))^(1/3) + b^2*(b*sin(d*x + c)/cos(d*x + c))^(2
/3) + (b^8/d^6)^(1/3)*d^2) - 4*(b^8/d^6)^(1/6)*d*arctan(-(sqrt(3)*b^8 + 2*(
b^8/d^6)^(5/6)*b*d^5*(b*sin(d*x + c)/cos(d*x + c))^(1/3) - 2*sqrt(sqrt(3)*(
b^8/d^6)^(1/6)*b*d*(b*sin(d*x + c)/cos(d*x + c))^(1/3) + b^2*(b*sin(d*x + c
)/cos(d*x + c))^(2/3) + (b^8/d^6)^(1/3)*d^2)*(b^8/d^6)^(5/6)*d^5)/b^8) - 4*
(b^8/d^6)^(1/6)*d*arctan((sqrt(3)*b^8 - 2*(b^8/d^6)^(5/6)*b*d^5*(b*sin(d*x
+ c)/cos(d*x + c))^(1/3) + 2*sqrt(-sqrt(3)*(b^8/d^6)^(1/6)*b*d*(b*sin(d*x +
 c)/cos(d*x + c))^(1/3) + b^2*(b*sin(d*x + c)/cos(d*x + c))^(2/3) + (b^8/d^
6)^(1/3)*d^2)*(b^8/d^6)^(5/6)*d^5)/b^8) - 8*(b^8/d^6)^(1/6)*d*arctan(-((b^8
/d^6)^(5/6)*b*d^5*(b*sin(d*x + c)/cos(d*x + c))^(1/3) - sqrt(b^2*(b*sin(d*x
 + c)/cos(d*x + c))^(2/3) + (b^8/d^6)^(1/3)*d^2)*(b^8/d^6)^(5/6)*d^5)/b^8)
- 12*b*(b*sin(d*x + c)/cos(d*x + c))^(1/3))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tan{(c+dx)})^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))**(4/3),x)
```

```
[Out] Integral((b*tan(c + d*x))**(4/3), x)
```

Giac [A] time = 1.41056, size = 282, normalized size = 1.16

$$-\frac{1}{4}\,b\left(\frac{\sqrt{3}|b|^{\frac{1}{3}}\log\left(\sqrt{3}\,(b\tan{(dx+c)})^{\frac{1}{3}}\,|b|^{\frac{1}{3}}+(b\tan{(dx+c)})^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{d}-\frac{\sqrt{3}|b|^{\frac{1}{3}}\log\left(-\sqrt{3}\,(b\tan{(dx+c)})^{\frac{1}{3}}\,|b|^{\frac{1}{3}}+(b\tan{(\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] -1/4*b*(sqrt(3)*abs(b)^(1/3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3
) + (b*tan(d*x + c))^(2/3) + abs(b)^(2/3))/d - sqrt(3)*abs(b)^(1/3)*log(-sq
```

```
rt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3) + (b*tan(d*x + c))^(2/3) + abs(b)
^(2/3))/d + 2*abs(b)^(1/3)*arctan((sqrt(3)*abs(b)^(1/3) + 2*(b*tan(d*x + c)
)^(1/3))/abs(b)^(1/3))/d + 2*abs(b)^(1/3)*arctan(-(sqrt(3)*abs(b)^(1/3) - 2
*(b*tan(d*x + c))^(1/3))/abs(b)^(1/3))/d + 4*abs(b)^(1/3)*arctan((b*tan(d*x
 + c))^(1/3)/abs(b)^(1/3))/d - 12*(b*tan(d*x + c))^(1/3)/d)
```

3.18 $\int (b\tan(c+dx))^{2/3}\,dx$

Optimal. Leaf size=224

$$\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d}-\frac{b^{2/3}\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2d}+\frac{b^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}+\sqrt{3}\right)}{2d}+\frac{\sqrt{3}b^{2/3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{\cdots}\right.}{}$$

```
[Out] (b^(2/3)*ArcTan[(b*Tan[c + d*x])^(1/3)/b^(1/3)])/d - (b^(2/3)*ArcTan[Sqrt[3
] - (2*(b*Tan[c + d*x])^(1/3))/b^(1/3)]])/(2*d) + (b^(2/3)*ArcTan[Sqrt[3] +
(2*(b*Tan[c + d*x])^(1/3))/b^(1/3)]])/(2*d) + (Sqrt[3]*b^(2/3)*Log[b^(2/3) -
 Sqrt[3]*b^(1/3)*(b*Tan[c + d*x])^(1/3) + (b*Tan[c + d*x])^(2/3)])/(4*d) -
(Sqrt[3]*b^(2/3)*Log[b^(2/3) + Sqrt[3]*b^(1/3)*(b*Tan[c + d*x])^(1/3) + (b*
Tan[c + d*x])^(2/3)])/(4*d)
```

Rubi [A] time = 0.393851, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3476, 329, 295, 634, 618, 204, 628, 203}

$$\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d}-\frac{b^{2/3}\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2d}+\frac{b^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}+\sqrt{3}\right)}{2d}+\frac{\sqrt{3}b^{2/3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{\cdots}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x])^(2/3),x]
```

```
[Out] (b^(2/3)*ArcTan[(b*Tan[c + d*x])^(1/3)/b^(1/3)])/d - (b^(2/3)*ArcTan[Sqrt[3
] - (2*(b*Tan[c + d*x])^(1/3))/b^(1/3)]])/(2*d) + (b^(2/3)*ArcTan[Sqrt[3] +
(2*(b*Tan[c + d*x])^(1/3))/b^(1/3)]])/(2*d) + (Sqrt[3]*b^(2/3)*Log[b^(2/3) -
 Sqrt[3]*b^(1/3)*(b*Tan[c + d*x])^(1/3) + (b*Tan[c + d*x])^(2/3)])/(4*d) -
(Sqrt[3]*b^(2/3)*Log[b^(2/3) + Sqrt[3]*b^(1/3)*(b*Tan[c + d*x])^(1/3) + (b*
Tan[c + d*x])^(2/3)])/(4*d)
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (b\tan(c+dx))^{2/3}\,dx = \frac{b\,\text{Subst}\left(\int \frac{x^{2/3}}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= \frac{(3b)\,\text{Subst}\left(\int \frac{x^4}{b^2+x^6}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{d}$$
$$= \frac{b^{2/3}\,\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{b}}{2}+\frac{\sqrt{3}x}{2}}{b^{2/3}-\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{d} + \frac{b^{2/3}\,\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{b}}{2}-\frac{\sqrt{3}x}{2}}{b^{2/3}+\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(\right.}{d}$$
$$= \frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d} + \frac{\left(\sqrt{3}b^{2/3}\right)\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[3]{b}+2x}{b^{2/3}-\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{4d} - \frac{\left(\sqrt{3}b^{2/3}\right)\text{S}}{}$$
$$= \frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d} + \frac{\sqrt{3}b^{2/3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4d} - \frac{\sqrt{3}b}{}$$
$$= \frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d} - \frac{b^{2/3}\tan^{-1}\left(\frac{1}{3}\left(3\sqrt{3}-\frac{6\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)\right)}{2d} + \frac{b^{2/3}\tan^{-1}\left(\frac{1}{3}\left(3\sqrt{3}+\frac{6\sqrt[3]{b\tan(}}{\sqrt[3]{b}}\right.\right.}{2d}$$

Mathematica [C] time = 0.0491014, size = 40, normalized size = 0.18

$$\frac{3(b\tan(c+dx))^{5/3}\,{}_2F_1\left(\frac{5}{6},1;\frac{11}{6};-\tan^2(c+dx)\right)}{5bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x])^(2/3),x]
```

```
[Out] (3*Hypergeometric2F1[5/6, 1, 11/6, -Tan[c + d*x]^2]*(b*Tan[c + d*x])^(5/3))
/(5*b*d)
```

Maple [A] time = 0.051, size = 202, normalized size = 0.9

$$-\frac{\sqrt{3}}{4\,bd}\left(b^2\right)^{\frac{5}{6}}\ln\left((b\tan(dx+c))^{\frac{2}{3}}+\sqrt{3}\sqrt[6]{b^2}\sqrt[3]{b\tan(dx+c)}+\sqrt[3]{b^2}\right)+\frac{b}{2\,d}\arctan\left(2\,\frac{\sqrt[3]{b\tan(dx+c)}}{\sqrt[6]{b^2}}+\sqrt{3}\right)\frac{1}{\sqrt[6]{b^2}}+\frac{\sqrt{3}}{4\,bd}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c))^(2/3),x)
```

```
[Out] -1/4/d/b*3^(1/2)*(b^2)^(5/6)*ln((b*tan(d*x+c))^(2/3)+3^(1/2)*(b^2)^(1/6)*(b
*tan(d*x+c))^(1/3)+(b^2)^(1/3))+1/2/d*b/(b^2)^(1/6)*arctan(2*(b*tan(d*x+c))
^(1/3)/(b^2)^(1/6)+3^(1/2))+1/4/d/b*3^(1/2)*(b^2)^(5/6)*ln((b*tan(d*x+c))^(
2/3)-3^(1/2)*(b^2)^(1/6)*(b*tan(d*x+c))^(1/3)+(b^2)^(1/3))+1/2/d*b/(b^2)^(1
/6)*arctan(2*(b*tan(d*x+c))^(1/3)/(b^2)^(1/6)-3^(1/2))+1/d*b/(b^2)^(1/6)*ar
ctan((b*tan(d*x+c))^(1/3)/(b^2)^(1/6))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.52602, size = 1445, normalized size = 6.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(3)*(b^4/d^6)^(1/6)*log(sqrt(3)*b^3*d^5*(b*sin(d*x + c)/cos(d*x +
c))^(1/3)*(b^4/d^6)^(5/6) + b^4*d^4*(b^4/d^6)^(2/3) + b^6*(b*sin(d*x + c)/c
os(d*x + c))^(2/3)) + 1/4*sqrt(3)*(b^4/d^6)^(1/6)*log(-sqrt(3)*b^3*d^5*(b*s
in(d*x + c)/cos(d*x + c))^(1/3)*(b^4/d^6)^(5/6) + b^4*d^4*(b^4/d^6)^(2/3) +
 b^6*(b*sin(d*x + c)/cos(d*x + c))^(2/3)) - (b^4/d^6)^(1/6)*arctan(-(sqrt(3
)*b^4 + 2*b^3*d*(b*sin(d*x + c)/cos(d*x + c))^(1/3)*(b^4/d^6)^(1/6) - 2*sqr
t(sqrt(3)*b^3*d^5*(b*sin(d*x + c)/cos(d*x + c))^(1/3)*(b^4/d^6)^(5/6) + b^4
*d^4*(b^4/d^6)^(2/3) + b^6*(b*sin(d*x + c)/cos(d*x + c))^(2/3))*d*(b^4/d^6)
^(1/6))/b^4) - (b^4/d^6)^(1/6)*arctan((sqrt(3)*b^4 - 2*b^3*d*(b*sin(d*x + c
)/cos(d*x + c))^(1/3)*(b^4/d^6)^(1/6) + 2*sqrt(-sqrt(3)*b^3*d^5*(b*sin(d*x
```

```
+ c)/cos(d*x + c))^(1/3)*(b^4/d^6)^(5/6) + b^4*d^4*(b^4/d^6)^(2/3) + b^6*(b
*sin(d*x + c)/cos(d*x + c))^(2/3))*d*(b^4/d^6)^(1/6))/b^4) - 2*(b^4/d^6)^(1
/6)*arctan(-(b^3*d*(b*sin(d*x + c)/cos(d*x + c))^(1/3)*(b^4/d^6)^(1/6) - sq
rt(b^4*d^4*(b^4/d^6)^(2/3) + b^6*(b*sin(d*x + c)/cos(d*x + c))^(2/3))*d*(b^
4/d^6)^(1/6))/b^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tan(c+dx))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))**(2/3),x)
```

```
[Out] Integral((b*tan(c + d*x))**(2/3), x)
```

Giac [A] time = 1.4479, size = 282, normalized size = 1.26

$$-\frac{1}{4}b\left(\frac{\sqrt{3}|b|^{\frac{5}{3}}\log\left(\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}|b|^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{b^2d}-\frac{\sqrt{3}|b|^{\frac{5}{3}}\log\left(-\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}|b|^{\frac{1}{3}}+(b\tan(\right.}{b^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] -1/4*b*(sqrt(3)*abs(b)^(5/3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3
) + (b*tan(d*x + c))^(2/3) + abs(b)^(2/3))/(b^2*d) - sqrt(3)*abs(b)^(5/3)*l
og(-sqrt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3) + (b*tan(d*x + c))^(2/3) +
abs(b)^(2/3))/(b^2*d) - 2*abs(b)^(5/3)*arctan((sqrt(3)*abs(b)^(1/3) + 2*(b*
tan(d*x + c))^(1/3))/abs(b)^(1/3))/(b^2*d) - 2*abs(b)^(5/3)*arctan(-(sqrt(3
)*abs(b)^(1/3) - 2*(b*tan(d*x + c))^(1/3))/abs(b)^(1/3))/(b^2*d) - 4*abs(b)
^(5/3)*arctan((b*tan(d*x + c))^(1/3)/abs(b)^(1/3))/(b^2*d))
```

3.19 $\int \sqrt[3]{b\tan(c+dx)}\,dx$

Optimal. Leaf size=131

$$-\frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{b^{2/3}-2(b\tan(c+dx))^{2/3}}{\sqrt{3}b^{2/3}}\right)}{2d}-\frac{\sqrt[3]{b}\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2d}+\frac{\sqrt[3]{b}\log\left(-b^{2/3}(b\tan(c+dx))^{2/3}+b^{4/3}+(b\tan\right.}{4d}$$

```
[Out] -(Sqrt[3]*b^(1/3)*ArcTan[(b^(2/3) - 2*(b*Tan[c + d*x])^(2/3))/(Sqrt[3]*b^(2
/3))])/(2*d) - (b^(1/3)*Log[b^(2/3) + (b*Tan[c + d*x])^(2/3)])/(2*d) + (b^(
1/3)*Log[b^(4/3) - b^(2/3)*(b*Tan[c + d*x])^(2/3) + (b*Tan[c + d*x])^(4/3)]
)/(4*d)
```

Rubi [A] time = 0.103936, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3476, 329, 275, 292, 31, 634, 617, 204, 628}

$$-\frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{b^{2/3}-2(b\tan(c+dx))^{2/3}}{\sqrt{3}b^{2/3}}\right)}{2d}-\frac{\sqrt[3]{b}\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2d}+\frac{\sqrt[3]{b}\log\left(-b^{2/3}(b\tan(c+dx))^{2/3}+b^{4/3}+(b\tan\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x])^(1/3),x]
```

```
[Out] -(Sqrt[3]*b^(1/3)*ArcTan[(b^(2/3) - 2*(b*Tan[c + d*x])^(2/3))/(Sqrt[3]*b^(2
/3))])/(2*d) - (b^(1/3)*Log[b^(2/3) + (b*Tan[c + d*x])^(2/3)])/(2*d) + (b^(
1/3)*Log[b^(4/3) - b^(2/3)*(b*Tan[c + d*x])^(2/3) + (b*Tan[c + d*x])^(4/3)]
)/(4*d)
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \sqrt[3]{b\tan(c+dx)}\,dx = \frac{b\,\text{Subst}\left(\int \frac{\sqrt[3]{x}}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= \frac{(3b)\,\text{Subst}\left(\int \frac{x^3}{b^2+x^6}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{d}$$
$$= \frac{(3b)\,\text{Subst}\left(\int \frac{x}{b^2+x^3}\,dx, x, (b\tan(c+dx))^{2/3}\right)}{2d}$$
$$= -\frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{1}{b^{2/3}+x}\,dx, x, (b\tan(c+dx))^{2/3}\right)}{2d} + \frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{b^{2/3}+x}{b^{4/3}-b^{2/3}x+x^2}\,dx, x, (b\tan(c+dx))^{2/3}\right)}{2d}$$
$$= -\frac{\sqrt[3]{b}\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2d} + \frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{-b^{2/3}+2x}{b^{4/3}-b^{2/3}x+x^2}\,dx, x, (b\tan(c+dx))^{2/3}\right)}{4d} + \frac{(3b)}{}$$
$$= -\frac{\sqrt[3]{b}\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2d} + \frac{\sqrt[3]{b}\log\left(b^{4/3}-b^{2/3}(b\tan(c+dx))^{2/3}+(b\tan(c+dx))^{4/3}\right)}{4d}$$
$$= -\frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2(b\tan(c+dx))^{2/3}}{b^{2/3}}}{\sqrt{3}}\right)}{2d} - \frac{\sqrt[3]{b}\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2d} + \frac{\sqrt[3]{b}\log\left(b^{4/3}-b^{2/3}(b\tan\right.}{}$$

Mathematica [C] time = 0.0397503, size = 40, normalized size = 0.31

$$\frac{3(b\tan(c+dx))^{4/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\tan^2(c+dx)\right)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x])^(1/3),x]
```

```
[Out] (3*Hypergeometric2F1[2/3, 1, 5/3, -Tan[c + d*x]^2]*(b*Tan[c + d*x])^(4/3))/
(4*b*d)
```

Maple [A] time = 0.015, size = 114, normalized size = 0.9

$$-\frac{b}{2d}\ln\left((b\tan(dx+c))^{\frac{2}{3}}+\sqrt[3]{b^2}\right)\frac{1}{\sqrt[3]{b^2}}+\frac{b}{4d}\ln\left((b\tan(dx+c))^{\frac{4}{3}}-\sqrt[3]{b^2}\,(b\tan(dx+c))^{\frac{2}{3}}+\left(b^2\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{b^2}}+\frac{b\sqrt{3}}{2d}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c))^(1/3),x)
```

```
[Out] -1/2/d*b/(b^2)^(1/3)*ln((b*tan(d*x+c))^(2/3)+(b^2)^(1/3))+1/4/d*b/(b^2)^(1/
3)*ln((b*tan(d*x+c))^(4/3)-(b^2)^(1/3)*(b*tan(d*x+c))^(2/3)+(b^2)^(2/3))+1/
2/d*b*3^(1/2)/(b^2)^(1/3)*arctan(1/3*3^(1/2)*(2/(b^2)^(1/3)*(b*tan(d*x+c))^
(2/3)-1))
```

Maxima [A] time = 1.44522, size = 166, normalized size = 1.27

$$\frac{\frac{2\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\,(b\tan(dx+c))^{\frac{2}{3}}-\left(b^2\right)^{\frac{1}{3}}\right)}{3\left(b^2\right)^{\frac{1}{3}}}\right)}{\left(b^2\right)^{\frac{1}{3}}}+\frac{b^2\log\left((b\tan(dx+c))^{\frac{4}{3}}-\left(b^2\right)^{\frac{1}{3}}(b\tan(dx+c))^{\frac{2}{3}}+\left(b^2\right)^{\frac{2}{3}}\right)}{\left(b^2\right)^{\frac{1}{3}}}-\frac{2\,b^2\log\left((b\tan(dx+c))^{\frac{2}{3}}+\left(b^2\right)^{\frac{1}{3}}\right)}{\left(b^2\right)^{\frac{1}{3}}}}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*tan(d*x + c))^(2/3) - (b^2)^(1/
3))/(b^2)^(1/3))/(b^2)^(1/3) + b^2*log((b*tan(d*x + c))^(4/3) - (b^2)^(1/3)
*(b*tan(d*x + c))^(2/3) + (b^2)^(2/3))/(b^2)^(1/3) - 2*b^2*log((b*tan(d*x +
 c))^(2/3) + (b^2)^(1/3))/(b^2)^(1/3))/(b*d)
```

Fricas [A] time = 1.34673, size = 358, normalized size = 2.73

$$\frac{2\sqrt{3}\,(-b)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(b\tan(dx+c))^{\frac{2}{3}}(-b)^{\frac{1}{3}}+\sqrt{3}b}{3\,b}\right)-(-b)^{\frac{1}{3}}\log\left((b\tan(dx+c))^{\frac{1}{3}}\,b\tan(dx+c)-(b\tan(dx+c))^{\frac{2}{3}}\,(-b)^{\frac{2}{3}}-(-\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*(-b)^(1/3)*arctan(1/3*(2*sqrt(3)*(b*tan(d*x + c))^(2/3)*(-b)
^(1/3) + sqrt(3)*b)/b) - (-b)^(1/3)*log((b*tan(d*x + c))^(1/3)*b*tan(d*x +
```

```
c) - (b*tan(d*x + c))^(2/3)*(-b)^(2/3) - (-b)^(1/3)*b) + 2*(-b)^(1/3)*log((
b*tan(d*x + c))^(2/3) + (-b)^(2/3)))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))**(1/3),x)
```

```
[Out] Integral((b*tan(c + d*x))**(1/3), x)
```

Giac [A] time = 1.42799, size = 171, normalized size = 1.31

$$\frac{1}{4}\,b\left(\frac{2\,\sqrt{3}|b|^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(b\tan(dx+c))^{\frac{2}{3}}-|b|^{\frac{2}{3}}\right)}{3\,|b|^{\frac{2}{3}}}\right)}{b^2d}+\frac{|b|^{\frac{4}{3}}\log\left((b\tan(dx+c))^{\frac{1}{3}}\,b\tan(dx+c)-(b\tan(dx+c))^{\frac{2}{3}}\,|b|^{\frac{2}{3}}+|b|^{\frac{4}{3}}\right)}{b^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] 1/4*b*(2*sqrt(3)*abs(b)^(4/3)*arctan(1/3*sqrt(3)*(2*(b*tan(d*x + c))^(2/3)
- abs(b)^(2/3))/abs(b)^(2/3))/(b^2*d) + abs(b)^(4/3)*log((b*tan(d*x + c))^(
1/3)*b*tan(d*x + c) - (b*tan(d*x + c))^(2/3)*abs(b)^(2/3) + abs(b)^(4/3))/(
b^2*d) - 2*abs(b)^(4/3)*log((b*tan(d*x + c))^(2/3) + abs(b)^(2/3))/(b^2*d))
```

3.20 $\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx$

Optimal. Leaf size=131

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{b^{2/3}-2(b\tan(c+dx))^{2/3}}{\sqrt{3}b^{2/3}}\right)}{2\sqrt[3]{b}d}+\frac{\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2\sqrt[3]{b}d}-\frac{\log\left(-b^{2/3}(b\tan(c+dx))^{2/3}+b^{4/3}+(b\tan(c+dx))^{4/3}\right)}{4\sqrt[3]{b}d}$$

```
[Out] -(Sqrt[3]*ArcTan[(b^(2/3) - 2*(b*Tan[c + d*x])^(2/3))/(Sqrt[3]*b^(2/3))])/(
2*b^(1/3)*d) + Log[b^(2/3) + (b*Tan[c + d*x])^(2/3)]/(2*b^(1/3)*d) - Log[b^
(4/3) - b^(2/3)*(b*Tan[c + d*x])^(2/3) + (b*Tan[c + d*x])^(4/3)]/(4*b^(1/3)
*d)
```

Rubi [A] time = 0.0996872, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3476, 329, 275, 200, 31, 634, 617, 204, 628}

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{b^{2/3}-2(b\tan(c+dx))^{2/3}}{\sqrt{3}b^{2/3}}\right)}{2\sqrt[3]{b}d}+\frac{\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2\sqrt[3]{b}d}-\frac{\log\left(-b^{2/3}(b\tan(c+dx))^{2/3}+b^{4/3}+(b\tan(c+dx))^{4/3}\right)}{4\sqrt[3]{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x])^(-1/3),x]
```


```
[Out] -(Sqrt[3]*ArcTan[(b^(2/3) - 2*(b*Tan[c + d*x])^(2/3))/(Sqrt[3]*b^(2/3))])/(
2*b^(1/3)*d) + Log[b^(2/3) + (b*Tan[c + d*x])^(2/3)]/(2*b^(1/3)*d) - Log[b^
(4/3) - b^(2/3)*(b*Tan[c + d*x])^(2/3) + (b*Tan[c + d*x])^(4/3)]/(4*b^(1/3)
*d)
```


Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx = \frac{b\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{x}\left(b^2+x^2\right)}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= \frac{(3b)\,\mathrm{Subst}\left(\int \frac{x}{b^2+x^6}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{d}$$
$$= \frac{(3b)\,\mathrm{Subst}\left(\int \frac{1}{b^2+x^3}\,dx, x, (b\tan(c+dx))^{2/3}\right)}{2d}$$
$$= \frac{\mathrm{Subst}\left(\int \frac{1}{b^{2/3}+x}\,dx, x, (b\tan(c+dx))^{2/3}\right)}{2\sqrt[3]{b}d} + \frac{\mathrm{Subst}\left(\int \frac{2b^{2/3}-x}{b^{4/3}-b^{2/3}x+x^2}\,dx, x, (b\tan(c+dx))^{2/3}\right)}{2\sqrt[3]{b}d}$$
$$= \frac{\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2\sqrt[3]{b}d} - \frac{\mathrm{Subst}\left(\int \frac{-b^{2/3}+2x}{b^{4/3}-b^{2/3}x+x^2}\,dx, x, (b\tan(c+dx))^{2/3}\right)}{4\sqrt[3]{b}d} + \frac{\left(3\sqrt[3]{b}\right)\mathrm{Subst}\left(\right.}{}$$
$$= \frac{\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2\sqrt[3]{b}d} - \frac{\log\left(b^{4/3}-b^{2/3}(b\tan(c+dx))^{2/3}+(b\tan(c+dx))^{4/3}\right)}{4\sqrt[3]{b}d} + \frac{3\,\mathrm{Subs}}{}$$
$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2(b\tan(c+dx))^{2/3}}{b^{2/3}}}{\sqrt{3}}\right)}{2\sqrt[3]{b}d} + \frac{\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2\sqrt[3]{b}d} - \frac{\log\left(b^{4/3}-b^{2/3}(b\tan(c+dx))^{2/3}\right.}{4\sqrt[3]{b}d}$$

Mathematica [A] time = 0.133476, size = 100, normalized size = 0.76

$$\frac{\sqrt[3]{\tan(c+dx)}\left(2\sqrt{3}\tan^{-1}\left(\frac{2\tan^{\frac{2}{3}}(c+dx)-1}{\sqrt{3}}\right)+2\log\left(\tan^{\frac{2}{3}}(c+dx)+1\right)-\log\left(\tan^{\frac{4}{3}}(c+dx)-\tan^{\frac{2}{3}}(c+dx)+1\right)\right)}{4d\sqrt[3]{b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x])^(-1/3),x]
```

```
[Out] ((2*Sqrt[3]*ArcTan[(-1 + 2*Tan[c + d*x]^(2/3))/Sqrt[3]] + 2*Log[1 + Tan[c +
 d*x]^(2/3)] - Log[1 - Tan[c + d*x]^(2/3) + Tan[c + d*x]^(4/3)])*Tan[c + d*
x]^(1/3))/(4*d*(b*Tan[c + d*x])^(1/3))
```

Maple [A] time = 0.013, size = 114, normalized size = 0.9

$$\frac{b}{2d}\ln\left(\left(b\tan\left(dx+c\right)\right)^{\frac{2}{3}}+\sqrt[3]{b^{2}}\right)\left(b^{2}\right)^{-\frac{2}{3}}-\frac{b}{4d}\ln\left(\left(b\tan\left(dx+c\right)\right)^{\frac{4}{3}}-\sqrt[3]{b^{2}}\left(b\tan\left(dx+c\right)\right)^{\frac{2}{3}}+\left(b^{2}\right)^{\frac{2}{3}}\right)\left(b^{2}\right)^{-\frac{2}{3}}+\frac{b\sqrt{3}}{2d}\mathrm{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c))^(1/3),x)
```

```
[Out] 1/2/d*b/(b^2)^(2/3)*ln((b*tan(d*x+c))^(2/3)+(b^2)^(1/3))-1/4/d*b/(b^2)^(2/3
)*ln((b*tan(d*x+c))^(4/3)-(b^2)^(1/3)*(b*tan(d*x+c))^(2/3)+(b^2)^(2/3))+1/2
/d*b/(b^2)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(b^2)^(1/3)*(b*tan(d*x+c))^(
2/3)-1))
```

Maxima [A] time = 1.4241, size = 167, normalized size = 1.27

$$\frac{\frac{2\sqrt{3}b^{2}\arctan\left(\frac{\sqrt{3}\left(2\left(b\tan(dx+c)\right)^{\frac{2}{3}}-\left(b^{2}\right)^{\frac{1}{3}}\right)}{3\left(b^{2}\right)^{\frac{1}{3}}}\right)}{\left(b^{2}\right)^{\frac{2}{3}}}-\frac{b^{2}\log\left(\left(b\tan(dx+c)\right)^{\frac{4}{3}}-\left(b^{2}\right)^{\frac{1}{3}}\left(b\tan(dx+c)\right)^{\frac{2}{3}}+\left(b^{2}\right)^{\frac{2}{3}}\right)}{\left(b^{2}\right)^{\frac{2}{3}}}+\frac{2b^{2}\log\left(\left(b\tan(dx+c)\right)^{\frac{2}{3}}+\left(b^{2}\right)^{\frac{1}{3}}\right)}{\left(b^{2}\right)^{\frac{2}{3}}}}{4bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*tan(d*x + c))^(2/3) - (b^2)^(1/
3))/(b^2)^(1/3))/(b^2)^(2/3) - b^2*log((b*tan(d*x + c))^(4/3) - (b^2)^(1/3)
*(b*tan(d*x + c))^(2/3) + (b^2)^(2/3))/(b^2)^(2/3) + 2*b^2*log((b*tan(d*x +
 c))^(2/3) + (b^2)^(1/3))/(b^2)^(2/3))/(b*d)
```

Fricas [A] time = 1.38658, size = 898, normalized size = 6.85

$$\left[\frac{\sqrt{3}b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{2\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\tan(dx+c)+2b\tan(dx+c)^2-\sqrt{3}b^{\frac{4}{3}}\sqrt{-\frac{1}{b^{\frac{2}{3}}}}+(b\tan(dx+c))^{\frac{2}{3}}\left(\sqrt{3}b^{\frac{2}{3}}\sqrt{-\frac{1}{b^{\frac{2}{3}}}}-3b^{\frac{1}{3}}\right)-b}{\tan(dx+c)^2+1}\right)-b^{\frac{2}{3}}\log\left((b\tan(\ldots\right.}{4bd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(3)*b*sqrt(-1/b^(2/3))*log((2*sqrt(3)*(b*tan(d*x + c))^(1/3)*b*sq
rt(-1/b^(2/3))*tan(d*x + c) + 2*b*tan(d*x + c)^2 - sqrt(3)*b^(4/3)*sqrt(-1/
b^(2/3)) + (b*tan(d*x + c))^(2/3)*(sqrt(3)*b^(2/3)*sqrt(-1/b^(2/3)) - 3*b^(
1/3)) - b)/(tan(d*x + c)^2 + 1)) - b^(2/3)*log((b*tan(d*x + c))^(1/3)*b*tan
(d*x + c) - (b*tan(d*x + c))^(2/3)*b^(2/3) + b^(4/3)) + 2*b^(2/3)*log((b*ta
n(d*x + c))^(2/3) + b^(2/3)))/(b*d), 1/4*(2*sqrt(3)*b^(2/3)*arctan(1/3*sqrt
(3)*(2*(b*tan(d*x + c))^(2/3)*b^(2/3) - b^(4/3))/b^(4/3)) - b^(2/3)*log((b*
tan(d*x + c))^(1/3)*b*tan(d*x + c) - (b*tan(d*x + c))^(2/3)*b^(2/3) + b^(4/
3)) + 2*b^(2/3)*log((b*tan(d*x + c))^(2/3) + b^(2/3)))/(b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))**(1/3),x)
```

```
[Out] Integral((b*tan(c + d*x))**(-1/3), x)
```

Giac [A] time = 1.35768, size = 173, normalized size = 1.32

$$\frac{1}{4}\,b\left(\frac{2\,\sqrt{3}|b|^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(b\tan(dx+c))^{\frac{2}{3}}-|b|^{\frac{2}{3}}\right)}{3\,|b|^{\frac{2}{3}}}\right)}{b^2d}-\frac{|b|^{\frac{2}{3}}\log\left((b\tan\left(dx+c\right))^{\frac{1}{3}}\,b\tan\left(dx+c\right)-(b\tan\left(dx+c\right))^{\frac{2}{3}}\,|b|^{\frac{2}{3}}+|b|^{\frac{4}{3}}\right)}{b^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] 1/4*b*(2*sqrt(3)*abs(b)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*tan(d*x + c))^(2/3)
- abs(b)^(2/3))/abs(b)^(2/3))/(b^2*d) - abs(b)^(2/3)*log((b*tan(d*x + c))^(
1/3)*b*tan(d*x + c) - (b*tan(d*x + c))^(2/3)*abs(b)^(2/3) + abs(b)^(4/3))/(
b^2*d) + 2*abs(b)^(2/3)*log((b*tan(d*x + c))^(2/3) + abs(b)^(2/3))/(b^2*d))
```

3.21 $\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=224

$$\frac{\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d}-\frac{\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2b^{2/3}d}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}+\sqrt{3}\right)}{2b^{2/3}d}-\frac{\sqrt{3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+\right.}{4b^{2/3}d}$$

```
[Out] ArcTan[(b*Tan[c + d*x])^(1/3)/b^(1/3)]/(b^(2/3)*d) - ArcTan[Sqrt[3] - (2*(b
*Tan[c + d*x])^(1/3))/b^(1/3)]/(2*b^(2/3)*d) + ArcTan[Sqrt[3] + (2*(b*Tan[c
 + d*x])^(1/3))/b^(1/3)]/(2*b^(2/3)*d) - (Sqrt[3]*Log[b^(2/3) - Sqrt[3]*b^(
1/3)*(b*Tan[c + d*x])^(1/3) + (b*Tan[c + d*x])^(2/3)])/(4*b^(2/3)*d) + (Sqr
t[3]*Log[b^(2/3) + Sqrt[3]*b^(1/3)*(b*Tan[c + d*x])^(1/3) + (b*Tan[c + d*x]
)^(2/3)])/(4*b^(2/3)*d)
```

Rubi [A] time = 0.326431, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3476, 329, 209, 634, 618, 204, 628, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d}-\frac{\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2b^{2/3}d}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}+\sqrt{3}\right)}{2b^{2/3}d}-\frac{\sqrt{3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+\right.}{4b^{2/3}d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x])^(-2/3),x]
```

```
[Out] ArcTan[(b*Tan[c + d*x])^(1/3)/b^(1/3)]/(b^(2/3)*d) - ArcTan[Sqrt[3] - (2*(b
*Tan[c + d*x])^(1/3))/b^(1/3)]/(2*b^(2/3)*d) + ArcTan[Sqrt[3] + (2*(b*Tan[c
 + d*x])^(1/3))/b^(1/3)]/(2*b^(2/3)*d) - (Sqrt[3]*Log[b^(2/3) - Sqrt[3]*b^(
1/3)*(b*Tan[c + d*x])^(1/3) + (b*Tan[c + d*x])^(2/3)])/(4*b^(2/3)*d) + (Sqr
t[3]*Log[b^(2/3) + Sqrt[3]*b^(1/3)*(b*Tan[c + d*x])^(1/3) + (b*Tan[c + d*x]
)^(2/3)])/(4*b^(2/3)*d)
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx = \frac{b\,\text{Subst}\left(\int \frac{1}{x^{2/3}(b^2+x^2)}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{(3b)\,\text{Subst}\left(\int \frac{1}{b^2+x^6}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}-\frac{\sqrt{3}x}{2}}{b^{2/3}-\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{b^{2/3}d} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}+\frac{\sqrt{3}x}{2}}{b^{2/3}+\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{b^{2/3}d}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d} - \frac{\sqrt{3}\,\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[3]{b}+2x}{b^{2/3}-\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{4b^{2/3}d} + \frac{\sqrt{3}\,\text{Subst}\left(\int \frac{\sqrt{3}}{b^{2/3}+}\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d} - \frac{\sqrt{3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4b^{2/3}d} + \frac{\sqrt{3}\log\left(b^{2}\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d} - \frac{\tan^{-1}\left(\frac{1}{3}\left(3\sqrt{3}-\frac{6\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)\right)}{2b^{2/3}d} + \frac{\tan^{-1}\left(\frac{1}{3}\left(3\sqrt{3}+\frac{6\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)\right)}{2b^{2/3}d} - \cdots$$

Mathematica [C] time = 0.0281756, size = 38, normalized size = 0.17

$$\frac{3\sqrt[3]{b\tan(c+dx)}\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\tan^2(c+dx)\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x])^(-2/3),x]
```

```
[Out] (3*Hypergeometric2F1[1/6, 1, 7/6, -Tan[c + d*x]^2]*(b*Tan[c + d*x])^(1/3))/
(b*d)
```

Maple [A] time = 0.04, size = 211, normalized size = 0.9

$$\frac{\sqrt{3}}{4\,bd}\sqrt[6]{b^2}\ln\left((b\tan(dx+c))^{\frac{2}{3}}+\sqrt{3}\sqrt[6]{b^2}\sqrt[3]{b\tan(dx+c)}+\sqrt[3]{b^2}\right)+\frac{1}{2\,bd}\sqrt[6]{b^2}\arctan\left(2\frac{\sqrt[3]{b\tan(dx+c)}}{\sqrt[6]{b^2}}+\sqrt{3}\right)-\frac{\sqrt{3}}{4\,bd}\sqrt[6]{b^2}\,\mathrm{l}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c))^(2/3),x)
```

```
[Out] 1/4/d/b*3^(1/2)*(b^2)^(1/6)*ln((b*tan(d*x+c))^(2/3)+3^(1/2)*(b^2)^(1/6)*(b*
tan(d*x+c))^(1/3)+(b^2)^(1/3))+1/2/d/b*(b^2)^(1/6)*arctan(2*(b*tan(d*x+c))^
(1/3)/(b^2)^(1/6)+3^(1/2))-1/4/d/b*3^(1/2)*(b^2)^(1/6)*ln(3^(1/2)*(b^2)^(1/
6)*(b*tan(d*x+c))^(1/3)-(b*tan(d*x+c))^(2/3)-(b^2)^(1/3))+1/2/d/b*(b^2)^(1/
6)*arctan(2*(b*tan(d*x+c))^(1/3)/(b^2)^(1/6)-3^(1/2))+1/d/b*(b^2)^(1/6)*arc
tan((b*tan(d*x+c))^(1/3)/(b^2)^(1/6))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.60274, size = 1494, normalized size = 6.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(3)*(1/(b^4*d^6))^(1/6)*log(b^2*d^2*(1/(b^4*d^6))^(1/3) + sqrt(3)*b
*d*(b*sin(d*x + c)/cos(d*x + c))^(1/3)*(1/(b^4*d^6))^(1/6) + (b*sin(d*x + c
)/cos(d*x + c))^(2/3)) - 1/4*sqrt(3)*(1/(b^4*d^6))^(1/6)*log(b^2*d^2*(1/(b^
4*d^6))^(1/3) - sqrt(3)*b*d*(b*sin(d*x + c)/cos(d*x + c))^(1/3)*(1/(b^4*d^6
))^(1/6) + (b*sin(d*x + c)/cos(d*x + c))^(2/3)) - (1/(b^4*d^6))^(1/6)*arcta
n(2*sqrt(b^2*d^2*(1/(b^4*d^6))^(1/3) + sqrt(3)*b*d*(b*sin(d*x + c)/cos(d*x
+ c))^(1/3)*(1/(b^4*d^6))^(1/6) + (b*sin(d*x + c)/cos(d*x + c))^(2/3))*b^3*
d^5*(1/(b^4*d^6))^(5/6) - 2*b^3*d^5*(b*sin(d*x + c)/cos(d*x + c))^(1/3)*(1/
(b^4*d^6))^(5/6) - sqrt(3)) - (1/(b^4*d^6))^(1/6)*arctan(2*sqrt(b^2*d^2*(1/
(b^4*d^6))^(1/3) - sqrt(3)*b*d*(b*sin(d*x + c)/cos(d*x + c))^(1/3)*(1/(b^4*
```

```
d^6))^(1/6) + (b*sin(d*x + c)/cos(d*x + c))^(2/3))*b^3*d^5*(1/(b^4*d^6))^(5
/6) - 2*b^3*d^5*(b*sin(d*x + c)/cos(d*x + c))^(1/3)*(1/(b^4*d^6))^(5/6) + s
qrt(3)) - 2*(1/(b^4*d^6))^(1/6)*arctan(sqrt(b^2*d^2*(1/(b^4*d^6))^(1/3) + (
b*sin(d*x + c)/cos(d*x + c))^(2/3))*b^3*d^5*(1/(b^4*d^6))^(5/6) - b^3*d^5*(
b*sin(d*x + c)/cos(d*x + c))^(1/3)*(1/(b^4*d^6))^(5/6))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tan\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))**(2/3),x)
```

```
[Out] Integral((b*tan(c + d*x))**(-2/3), x)
```

Giac [A] time = 1.34661, size = 282, normalized size = 1.26

$$\frac{1}{4}\,b\left(\frac{\sqrt{3}|b|^{\frac{1}{3}}\log\left(\sqrt{3}\left(b\tan\left(dx+c\right)\right)^{\frac{1}{3}}|b|^{\frac{1}{3}}+\left(b\tan\left(dx+c\right)\right)^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{b^{2}d}-\frac{\sqrt{3}|b|^{\frac{1}{3}}\log\left(-\sqrt{3}\left(b\tan\left(dx+c\right)\right)^{\frac{1}{3}}|b|^{\frac{1}{3}}+\left(b\tan\left(dx\right.\right.\right.}{b^{2}d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] 1/4*b*(sqrt(3)*abs(b)^(1/3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3)
 + (b*tan(d*x + c))^(2/3) + abs(b)^(2/3))/(b^2*d) - sqrt(3)*abs(b)^(1/3)*lo
g(-sqrt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3) + (b*tan(d*x + c))^(2/3) + a
bs(b)^(2/3))/(b^2*d) + 2*abs(b)^(1/3)*arctan((sqrt(3)*abs(b)^(1/3) + 2*(b*t
an(d*x + c))^(1/3))/abs(b)^(1/3))/(b^2*d) + 2*abs(b)^(1/3)*arctan(-(sqrt(3)
*abs(b)^(1/3) - 2*(b*tan(d*x + c))^(1/3))/abs(b)^(1/3))/(b^2*d) + 4*abs(b)^
(1/3)*arctan((b*tan(d*x + c))^(1/3)/abs(b)^(1/3))/(b^2*d))
```

3.22 $\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=245

$$-\frac{\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d}+\frac{\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2b^{4/3}d}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}+\sqrt{3}\right)}{2b^{4/3}d}-\frac{\sqrt{3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx}\right.}{4b^{4/3}d}$$

```
[Out] -(ArcTan[(b*Tan[c + d*x])^(1/3)/b^(1/3)]/(b^(4/3)*d)) + ArcTan[Sqrt[3] - (2
*(b*Tan[c + d*x])^(1/3))/b^(1/3)]/(2*b^(4/3)*d) - ArcTan[Sqrt[3] + (2*(b*Ta
n[c + d*x])^(1/3))/b^(1/3)]/(2*b^(4/3)*d) - (Sqrt[3]*Log[b^(2/3) - Sqrt[3]*
b^(1/3)*(b*Tan[c + d*x])^(1/3) + (b*Tan[c + d*x])^(2/3)])/(4*b^(4/3)*d) + (
Sqrt[3]*Log[b^(2/3) + Sqrt[3]*b^(1/3)*(b*Tan[c + d*x])^(1/3) + (b*Tan[c + d
*x])^(2/3)])/(4*b^(4/3)*d) - 3/(b*d*(b*Tan[c + d*x])^(1/3))
```

Rubi [A] time = 0.43572, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3474, 3476, 329, 295, 634, 618, 204, 628, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d}+\frac{\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2b^{4/3}d}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}+\sqrt{3}\right)}{2b^{4/3}d}-\frac{\sqrt{3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx}\right.}{4b^{4/3}d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x])^(-4/3),x]
```

```
[Out] -(ArcTan[(b*Tan[c + d*x])^(1/3)/b^(1/3)]/(b^(4/3)*d)) + ArcTan[Sqrt[3] - (2
*(b*Tan[c + d*x])^(1/3))/b^(1/3)]/(2*b^(4/3)*d) - ArcTan[Sqrt[3] + (2*(b*Ta
n[c + d*x])^(1/3))/b^(1/3)]/(2*b^(4/3)*d) - (Sqrt[3]*Log[b^(2/3) - Sqrt[3]*
b^(1/3)*(b*Tan[c + d*x])^(1/3) + (b*Tan[c + d*x])^(2/3)])/(4*b^(4/3)*d) + (
Sqrt[3]*Log[b^(2/3) + Sqrt[3]*b^(1/3)*(b*Tan[c + d*x])^(1/3) + (b*Tan[c + d
*x])^(2/3)])/(4*b^(4/3)*d) - 3/(b*d*(b*Tan[c + d*x])^(1/3))
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx &= -\frac{3}{bd\sqrt[3]{b\tan(c+dx)}}-\frac{\int (b\tan(c+dx))^{2/3}\,dx}{b^2}\\
&= -\frac{3}{bd\sqrt[3]{b\tan(c+dx)}}-\frac{\text{Subst}\left(\int \frac{x^{2/3}}{b^2+x^2}\,dx,x,b\tan(c+dx)\right)}{bd}\\
&= -\frac{3}{bd\sqrt[3]{b\tan(c+dx)}}-\frac{3\,\text{Subst}\left(\int \frac{x^4}{b^2+x^6}\,dx,x,\sqrt[3]{b\tan(c+dx)}\right)}{bd}\\
&= -\frac{3}{bd\sqrt[3]{b\tan(c+dx)}}-\frac{\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{b}}{2}+\frac{\sqrt{3}x}{2}}{b^{2/3}-\sqrt{3}\sqrt[3]{b}x+x^2}\,dx,x,\sqrt[3]{b\tan(c+dx)}\right)}{b^{4/3}d}-\frac{\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{b}}{2}-\frac{\sqrt{3}x}{2}}{b^{2/3}+\sqrt{3}\sqrt[3]{b}x+\ldots}\right.}{b\ldots}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d}-\frac{3}{bd\sqrt[3]{b\tan(c+dx)}}-\frac{\sqrt{3}\,\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[3]{b}+2x}{b^{2/3}-\sqrt{3}\sqrt[3]{b}x+x^2}\,dx,x,\sqrt[3]{b\tan(c+dx\ldots}\right.}{4b^{4/3}d}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d}-\frac{\sqrt{3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4b^{4/3}d}+\frac{\sqrt{3}\log\ldots}{\ldots}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d}+\frac{\tan^{-1}\left(\frac{1}{3}\left(3\sqrt{3}-\frac{6\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)\right)}{2b^{4/3}d}-\frac{\tan^{-1}\left(\frac{1}{3}\left(3\sqrt{3}+\frac{6\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)\right)}{2b^{4/3}d}\ldots
\end{aligned}$$

Mathematica [C] time = 0.0590962, size = 38, normalized size = 0.16

$$-\frac{3\,{}_2F_1\left(-\frac{1}{6},1;\frac{5}{6};-\tan^2(c+dx)\right)}{bd\sqrt[3]{b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x])^(-4/3),x]
```

```
[Out] (-3*Hypergeometric2F1[-1/6, 1, 5/6, -Tan[c + d*x]^2])/(b*d*(b*Tan[c + d*x])
^(1/3))
```

Maple [A] time = 0.042, size = 227, normalized size = 0.9

$$\frac{\sqrt{3}}{4\,d b^3}\left(b^2\right)^{\frac{5}{6}}\ln\left(\left(b\tan\left(dx+c\right)\right)^{\frac{2}{3}}+\sqrt{3}\sqrt[6]{b^2}\sqrt[3]{b\tan\left(dx+c\right)}+\sqrt[3]{b^2}\right)-\frac{1}{2\,bd}\arctan\left(2\,\frac{\sqrt[3]{b\tan\left(dx+c\right)}}{\sqrt[6]{b^2}}+\sqrt{3}\right)\frac{1}{\sqrt[6]{b^2}}-\frac{\sqrt{3}}{4\,d b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c))^(4/3),x)
```

```
[Out] 1/4/d/b^3*3^(1/2)*(b^2)^(5/6)*ln((b*tan(d*x+c))^(2/3)+3^(1/2)*(b^2)^(1/6)*(
b*tan(d*x+c))^(1/3)+(b^2)^(1/3))-1/2/d/b/(b^2)^(1/6)*arctan(2*(b*tan(d*x+c)
)^(1/3)/(b^2)^(1/6)+3^(1/2))-1/4/d/b^3*3^(1/2)*(b^2)^(5/6)*ln((b*tan(d*x+c)
)^(2/3)-3^(1/2)*(b^2)^(1/6)*(b*tan(d*x+c))^(1/3)+(b^2)^(1/3))-1/2/d/b/(b^2)
^(1/6)*arctan(2*(b*tan(d*x+c))^(1/3)/(b^2)^(1/6)-3^(1/2))-1/d/b/(b^2)^(1/6)
*arctan((b*tan(d*x+c))^(1/3)/(b^2)^(1/6))-3/b/d/(b*tan(d*x+c))^(1/3)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.74943, size = 1852, normalized size = 7.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] 1/4*(12*(b*sin(d*x + c)/cos(d*x + c))^(2/3)*cos(d*x + c)*sin(d*x + c) + 4*(
b^2*d*cos(d*x + c)^2 - b^2*d)*(1/(b^8*d^6))^(1/6)*arctan(2*sqrt(sqrt(3)*b^7
*d^5*(b*sin(d*x + c)/cos(d*x + c))^(1/3)*(1/(b^8*d^6))^(5/6) + b^6*d^4*(1/(
b^8*d^6))^(2/3) + (b*sin(d*x + c)/cos(d*x + c))^(2/3))*b*d*(1/(b^8*d^6))^(1
/6) - 2*b*d*(b*sin(d*x + c)/cos(d*x + c))^(1/3)*(1/(b^8*d^6))^(1/6) - sqrt(
3)) + 4*(b^2*d*cos(d*x + c)^2 - b^2*d)*(1/(b^8*d^6))^(1/6)*arctan(2*sqrt(-s
qrt(3)*b^7*d^5*(b*sin(d*x + c)/cos(d*x + c))^(1/3)*(1/(b^8*d^6))^(5/6) + b^
6*d^4*(1/(b^8*d^6))^(2/3) + (b*sin(d*x + c)/cos(d*x + c))^(2/3))*b*d*(1/(b^
8*d^6))^(1/6) - 2*b*d*(b*sin(d*x + c)/cos(d*x + c))^(1/3)*(1/(b^8*d^6))^(1/
6) + sqrt(3)) + 8*(b^2*d*cos(d*x + c)^2 - b^2*d)*(1/(b^8*d^6))^(1/6)*arctan
(sqrt(b^6*d^4*(1/(b^8*d^6))^(2/3) + (b*sin(d*x + c)/cos(d*x + c))^(2/3))*b*
d*(1/(b^8*d^6))^(1/6) - b*d*(b*sin(d*x + c)/cos(d*x + c))^(1/3)*(1/(b^8*d^6
))^(1/6)) + (sqrt(3)*b^2*d*cos(d*x + c)^2 - sqrt(3)*b^2*d)*(1/(b^8*d^6))^(1
/6)*log(sqrt(3)*b^7*d^5*(b*sin(d*x + c)/cos(d*x + c))^(1/3)*(1/(b^8*d^6))^(
5/6) + b^6*d^4*(1/(b^8*d^6))^(2/3) + (b*sin(d*x + c)/cos(d*x + c))^(2/3)) -
 (sqrt(3)*b^2*d*cos(d*x + c)^2 - sqrt(3)*b^2*d)*(1/(b^8*d^6))^(1/6)*log(-sq
rt(3)*b^7*d^5*(b*sin(d*x + c)/cos(d*x + c))^(1/3)*(1/(b^8*d^6))^(5/6) + b^6
*d^4*(1/(b^8*d^6))^(2/3) + (b*sin(d*x + c)/cos(d*x + c))^(2/3)))/(b^2*d*cos
(d*x + c)^2 - b^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\tan(c+dx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))**(4/3),x)
```

```
[Out] Integral((b*tan(c + d*x))**(-4/3), x)
```

Giac [A] time = 1.58753, size = 306, normalized size = 1.25

$$\frac{1}{4}b\left(\frac{\sqrt{3}|b|^{\frac{5}{3}}\log\left(\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}|b|^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{b^4 d}-\frac{\sqrt{3}|b|^{\frac{5}{3}}\log\left(-\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}|b|^{\frac{1}{3}}+(b\tan(\right.}{b^4 d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] 1/4*b*(sqrt(3)*abs(b)^(5/3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3)
 + (b*tan(d*x + c))^(2/3) + abs(b)^(2/3))/(b^4*d) - sqrt(3)*abs(b)^(5/3)*lo
g(-sqrt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3) + (b*tan(d*x + c))^(2/3) + a
bs(b)^(2/3))/(b^4*d) - 2*abs(b)^(5/3)*arctan((sqrt(3)*abs(b)^(1/3) + 2*(b*t
an(d*x + c))^(1/3))/abs(b)^(1/3))/(b^4*d) - 2*abs(b)^(5/3)*arctan(-(sqrt(3)
*abs(b)^(1/3) - 2*(b*tan(d*x + c))^(1/3))/abs(b)^(1/3))/(b^4*d) - 4*abs(b)^
(5/3)*arctan((b*tan(d*x + c))^(1/3)/abs(b)^(1/3))/(b^4*d) - 12/((b*tan(d*x
+ c))^(1/3)*b^2*d))
```

3.23 $\int (b\tan(c+dx))^n\,dx$

Optimal. Leaf size=50

$$\frac{(b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(c+dx)\right)}{bd(n+1)}$$

```
[Out] (Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[c + d*x]^2]*(b*Tan[c + d*x
])^(1 + n))/(b*d*(1 + n))
```

Rubi [A] time = 0.028416, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3476, 364}

$$\frac{(b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(c+dx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[c + d*x]^2]*(b*Tan[c + d*x
])^(1 + n))/(b*d*(1 + n))
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (b\tan(c+dx))^n\,dx = \frac{b\,\mathrm{Subst}\left(\int \frac{x^n}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= \frac{{}_2F_1\left(1, \frac{1+n}{2}; \frac{3+n}{2}; -\tan^2(c+dx)\right)(b\tan(c+dx))^{1+n}}{bd(1+n)}$$

Mathematica [A] time = 0.0418573, size = 53, normalized size = 1.06

$$\frac{\tan(c+dx)(b\tan(c+dx))^n\,{}_2F_1\left(1, \frac{n+1}{2}; \frac{n+1}{2}+1; -\tan^2(c+dx)\right)}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n)/2, 1 + (1 + n)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]*(b*Tan[c + d*x])^n)/(d*(1 + n))
```

Maple [F] time = 0.438, size = 0, normalized size = 0.

$$\int (b\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c))^n,x)
```

```
[Out] int((b*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left((b\tan(dx+c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tan(c+dx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))**n,x)
```

```
[Out] Integral((b*tan(c + d*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c))^n, x)
```

3.24 $\int \left(b\tan^2(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=98

$$\frac{b^2\tan^3(c+dx)\sqrt{b\tan^2(c+dx)}}{4d}-\frac{b^2\tan(c+dx)\sqrt{b\tan^2(c+dx)}}{2d}-\frac{b^2\cot(c+dx)\sqrt{b\tan^2(c+dx)}\log(\cos(c+dx))}{d}$$

```
[Out] -((b^2*Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[b*Tan[c + d*x]^2])/d) - (b^2*Tan
[c + d*x]*Sqrt[b*Tan[c + d*x]^2])/(2*d) + (b^2*Tan[c + d*x]^3*Sqrt[b*Tan[c
+ d*x]^2])/(4*d)
```

Rubi [A] time = 0.0416749, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 3475}

$$\frac{b^2\tan^3(c+dx)\sqrt{b\tan^2(c+dx)}}{4d}-\frac{b^2\tan(c+dx)\sqrt{b\tan^2(c+dx)}}{2d}-\frac{b^2\cot(c+dx)\sqrt{b\tan^2(c+dx)}\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^2)^(5/2),x]
```

```
[Out] -((b^2*Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[b*Tan[c + d*x]^2])/d) - (b^2*Tan
[c + d*x]*Sqrt[b*Tan[c + d*x]^2])/(2*d) + (b^2*Tan[c + d*x]^3*Sqrt[b*Tan[c
+ d*x]^2])/(4*d)
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(b\tan^2(c+dx)\right)^{5/2}\,dx = \left(b^2\cot(c+dx)\sqrt{b\tan^2(c+dx)}\right)\int \tan^5(c+dx)\,dx$$

$$= \frac{b^2\tan^3(c+dx)\sqrt{b\tan^2(c+dx)}}{4d} - \left(b^2\cot(c+dx)\sqrt{b\tan^2(c+dx)}\right)\int \tan^3(c+dx)\,dx$$

$$= -\frac{b^2\tan(c+dx)\sqrt{b\tan^2(c+dx)}}{2d} + \frac{b^2\tan^3(c+dx)\sqrt{b\tan^2(c+dx)}}{4d} + \left(b^2\cot(c+dx)\sqrt{b\,\mathrm{ta}\right.$$

$$= -\frac{b^2\cot(c+dx)\log(\cos(c+dx))\sqrt{b\tan^2(c+dx)}}{d} - \frac{b^2\tan(c+dx)\sqrt{b\tan^2(c+dx)}}{2d} + \frac{b^2\,\mathrm{ta}}{}$$

Mathematica [A] time = 0.376226, size = 56, normalized size = 0.57

$$-\frac{\cot(c+dx)\left(b\tan^2(c+dx)\right)^{5/2}\left(2\cot^2(c+dx)+4\cot^4(c+dx)\log(\cos(c+dx))-1\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^2)^(5/2),x]
```

```
[Out] -(Cot[c + d*x]*(-1 + 2*Cot[c + d*x]^2 + 4*Cot[c + d*x]^4*Log[Cos[c + d*x]])
*(b*Tan[c + d*x]^2)^(5/2))/(4*d)
```

Maple [A] time = 0.033, size = 58, normalized size = 0.6

$$\frac{(\tan(dx+c))^4 - 2\,(\tan(dx+c))^2 + 2\,\ln\left(1+(\tan(dx+c))^2\right)}{4\,d\,(\tan(dx+c))^5}\left(b\,(\tan(dx+c))^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^2)^(5/2),x)
```

```
[Out] 1/4/d*(b*tan(d*x+c)^2)^(5/2)*(tan(d*x+c)^4-2*tan(d*x+c)^2+2*ln(1+tan(d*x+c)
^2))/tan(d*x+c)^5
```

Maxima [A] time = 1.56432, size = 63, normalized size = 0.64

$$\frac{b^{\frac{5}{2}} \tan \left(dx+c\right)^{4}-2 \, b^{\frac{5}{2}} \tan \left(dx+c\right)^{2}+2 \, b^{\frac{5}{2}} \log \left(\tan \left(dx+c\right)^{2}+1\right)}{4 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(b^(5/2)*tan(d*x + c)^4 - 2*b^(5/2)*tan(d*x + c)^2 + 2*b^(5/2)*log(tan(
d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.29161, size = 180, normalized size = 1.84

$$\frac{\left(b^{2} \tan \left(dx+c\right)^{4}-2 \, b^{2} \tan \left(dx+c\right)^{2}-2 \, b^{2} \log \left(\frac{1}{\tan \left(dx+c\right)^{2}+1}\right)-3 \, b^{2}\right) \sqrt{b \tan \left(dx+c\right)^{2}}}{4 \, d \tan \left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*tan(d*x + c)^4 - 2*b^2*tan(d*x + c)^2 - 2*b^2*log(1/(tan(d*x + c)^
2 + 1)) - 3*b^2)*sqrt(b*tan(d*x + c)^2)/(d*tan(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \tan^{2}{\left (c+dx \right)}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)**2)**(5/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**2)**(5/2), x)
```

Giac [A] time = 1.58912, size = 74, normalized size = 0.76

$$\frac{1}{4}\,b^{\frac{5}{2}}\left(\frac{2\,\log\left(\tan\left(dx+c\right)^{2}+1\right)}{d}+\frac{d\tan\left(dx+c\right)^{4}-2\,d\tan\left(dx+c\right)^{2}}{d^{2}}\right)\operatorname{sgn}\left(\tan\left(dx+c\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/4*b^(5/2)*(2*log(tan(d*x + c)^2 + 1)/d + (d*tan(d*x + c)^4 - 2*d*tan(d*x
+ c)^2)/d^2)*sgn(tan(d*x + c))
```

3.25 $\int \left(b \tan^2(c+dx)\right)^{3/2} dx$

Optimal. Leaf size=61

$$\frac{b \tan(c+dx)\sqrt{b \tan^2(c+dx)}}{2d}+\frac{b \cot(c+dx)\sqrt{b \tan^2(c+dx)}\log(\cos(c+dx))}{d}$$

```
[Out] (b*Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[b*Tan[c + d*x]^2])/d + (b*Tan[c + d*
x]*Sqrt[b*Tan[c + d*x]^2])/(2*d)
```

Rubi [A] time = 0.027738, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 3475}

$$\frac{b \tan(c+dx)\sqrt{b \tan^2(c+dx)}}{2d}+\frac{b \cot(c+dx)\sqrt{b \tan^2(c+dx)}\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^2)^(3/2),x]
```

```
[Out] (b*Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[b*Tan[c + d*x]^2])/d + (b*Tan[c + d*
x]*Sqrt[b*Tan[c + d*x]^2])/(2*d)
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(b\tan^2(c+dx)\right)^{3/2}\,dx = \left(b\cot(c+dx)\sqrt{b\tan^2(c+dx)}\right)\int \tan^3(c+dx)\,dx$$

$$= \frac{b\tan(c+dx)\sqrt{b\tan^2(c+dx)}}{2d} - \left(b\cot(c+dx)\sqrt{b\tan^2(c+dx)}\right)\int \tan(c+dx)\,dx$$

$$= \frac{b\cot(c+dx)\log(\cos(c+dx))\sqrt{b\tan^2(c+dx)}}{d} + \frac{b\tan(c+dx)\sqrt{b\tan^2(c+dx)}}{2d}$$

Mathematica [A] time = 0.115158, size = 47, normalized size = 0.77

$$\frac{\cot^3(c+dx)\left(b\tan^2(c+dx)\right)^{3/2}\left(\tan^2(c+dx)+2\log(\cos(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^2)^(3/2),x]
```

```
[Out] (Cot[c + d*x]^3*(b*Tan[c + d*x]^2)^(3/2)*(2*Log[Cos[c + d*x]] + Tan[c + d*x
]^2))/(2*d)
```

Maple [A] time = 0.019, size = 48, normalized size = 0.8

$$-\frac{-\left(\tan\left(dx+c\right)\right)^2+\ln\left(1+\left(\tan\left(dx+c\right)\right)^2\right)}{2\,d\left(\tan\left(dx+c\right)\right)^3}\left(b\left(\tan\left(dx+c\right)\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^2)^(3/2),x)
```

```
[Out] -1/2/d*(b*tan(d*x+c)^2)^(3/2)*(-tan(d*x+c)^2+ln(1+tan(d*x+c)^2))/tan(d*x+c)
^3
```

Maxima [A] time = 1.49755, size = 46, normalized size = 0.75

$$\frac{b^{\frac{3}{2}}\tan\left(dx+c\right)^{2}-b^{\frac{3}{2}}\log\left(\tan\left(dx+c\right)^{2}+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(b^(3/2)*tan(d*x + c)^2 - b^(3/2)*log(tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.37052, size = 135, normalized size = 2.21

$$\frac{\left(b\tan\left(dx+c\right)^{2}+b\log\left(\frac{1}{\tan\left(dx+c\right)^{2}+1}\right)+b\right)\sqrt{b\tan\left(dx+c\right)^{2}}}{2\,d\tan\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*tan(d*x + c)^2 + b*log(1/(tan(d*x + c)^2 + 1)) + b)*sqrt(b*tan(d*x +
 c)^2)/(d*tan(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan^{2}\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)**2)**(3/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**2)**(3/2), x)
```

Giac [A] time = 1.51155, size = 55, normalized size = 0.9

$$\frac{1}{2}b^{\frac{3}{2}}\left(\frac{\tan(dx+c)^2}{d}-\frac{\log\left(\tan(dx+c)^2+1\right)}{d}\right)\mathrm{sgn}\left(\tan(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*b^(3/2)*(tan(d*x + c)^2/d - log(tan(d*x + c)^2 + 1)/d)*sgn(tan(d*x + c)
)
```

3.26 $\int \sqrt{b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=32

$$-\frac{\cot(c+dx)\sqrt{b\tan^2(c+dx)}\log(\cos(c+dx))}{d}$$

```
[Out] -((Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[b*Tan[c + d*x]^2])/d)
```

Rubi [A] time = 0.0166386, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3658, 3475}

$$-\frac{\cot(c+dx)\sqrt{b\tan^2(c+dx)}\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[c + d*x]^2],x]
```

```
[Out] -((Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[b*Tan[c + d*x]^2])/d)
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{b\tan^2(c+dx)}\,dx = \left(\cot(c+dx)\sqrt{b\tan^2(c+dx)}\right)\int \tan(c+dx)\,dx$$
$$= -\frac{\cot(c+dx)\log(\cos(c+dx))\sqrt{b\tan^2(c+dx)}}{d}$$

Mathematica [A] time = 0.0389083, size = 32, normalized size = 1.

$$-\frac{\cot(c+dx)\sqrt{b\tan^2(c+dx)}\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[c + d*x]^2],x]
```

```
[Out] -((Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[b*Tan[c + d*x]^2])/d)
```

Maple [A] time = 0.026, size = 37, normalized size = 1.2

$$\frac{\ln\left(1+(\tan(dx+c))^2\right)}{2d\tan(dx+c)}\sqrt{b(\tan(dx+c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^2)^(1/2),x)
```

```
[Out] 1/2/d*(b*tan(d*x+c)^2)^(1/2)/tan(d*x+c)*ln(1+tan(d*x+c)^2)
```

Maxima [A] time = 1.57723, size = 26, normalized size = 0.81

$$\frac{\sqrt{b}\log\left(\tan(dx+c)^2+1\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b)*log(tan(d*x + c)^2 + 1)/d
```

Fricas [A] time = 1.32182, size = 100, normalized size = 3.12

$$-\frac{\sqrt{b\tan\left(dx+c\right)^2}\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d\tan\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(b*tan(d*x + c)^2)*log(1/(tan(d*x + c)^2 + 1))/(d*tan(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\tan^2\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(b*tan(c + d*x)**2), x)
```

Giac [A] time = 1.44285, size = 35, normalized size = 1.09

$$\frac{\sqrt{b}\log\left(\tan\left(dx+c\right)^2+1\right)\operatorname{sgn}\left(\tan\left(dx+c\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b)*log(tan(d*x + c)^2 + 1)*sgn(tan(d*x + c))/d
```

3.27 $\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx$

Optimal. Leaf size=31

$$\frac{\tan(c+dx)\log(\sin(c+dx))}{d\sqrt{b\tan^2(c+dx)}}$$

```
[Out] (Log[Sin[c + d*x]]*Tan[c + d*x])/(d*Sqrt[b*Tan[c + d*x]^2])
```

Rubi [A] time = 0.0160449, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3658, 3475}

$$\frac{\tan(c+dx)\log(\sin(c+dx))}{d\sqrt{b\tan^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Tan[c + d*x]^2],x]
```

```
[Out] (Log[Sin[c + d*x]]*Tan[c + d*x])/(d*Sqrt[b*Tan[c + d*x]^2])
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx = \frac{\tan(c+dx)\int\cot(c+dx)\,dx}{\sqrt{b\tan^2(c+dx)}}$$
$$= \frac{\log(\sin(c+dx))\tan(c+dx)}{d\sqrt{b\tan^2(c+dx)}}$$

Mathematica [A] time = 0.0835878, size = 39, normalized size = 1.26

$$\frac{\tan(c+dx)(\log(\tan(c+dx))+\log(\cos(c+dx)))}{d\sqrt{b\tan^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Tan[c + d*x]^2],x]
```

```
[Out] ((Log[Cos[c + d*x]] + Log[Tan[c + d*x]])*Tan[c + d*x])/(d*Sqrt[b*Tan[c + d*
x]^2])
```

Maple [A] time = 0.025, size = 45, normalized size = 1.5

$$-\frac{\tan(dx+c)\left(\ln\left(1+(\tan(dx+c))^2\right)-2\ln(\tan(dx+c))\right)}{2d}\frac{1}{\sqrt{b(\tan(dx+c))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c)^2)^(1/2),x)
```

```
[Out] -1/2/d*tan(d*x+c)*(ln(1+tan(d*x+c)^2)-2*ln(tan(d*x+c)))/(b*tan(d*x+c)^2)^(1
/2)
```

Maxima [A] time = 1.61233, size = 45, normalized size = 1.45

$$-\frac{\frac{\log\left(\tan(dx+c)^2+1\right)}{\sqrt{b}}-\frac{2\log(\tan(dx+c))}{\sqrt{b}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(log(tan(d*x + c)^2 + 1)/sqrt(b) - 2*log(tan(d*x + c))/sqrt(b))/d
```

Fricas [A] time = 1.31158, size = 119, normalized size = 3.84

$$\frac{\sqrt{b\tan\left(dx+c\right)^2}\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)}{2\,bd\tan\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(b*tan(d*x + c)^2)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))/(b*d*ta
n(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{b\tan^2\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*tan(c + d*x)**2), x)
```

Giac [B] time = 1.61994, size = 109, normalized size = 3.52

$$\frac{\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{\sqrt{b}\mathrm{sgn}(\tan(dx+c))}-\frac{2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{\sqrt{b}\mathrm{sgn}(\tan(dx+c))}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(sqrt(b)*sgn(tan(d*x
 + c))) - 2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/(sqrt(b)*s
gn(tan(d*x + c))))/d
```

3.28 $\int \frac{1}{\left(b\tan^2(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{\cot(c+dx)}{2bd\sqrt{b\tan^2(c+dx)}}-\frac{\tan(c+dx)\log(\sin(c+dx))}{bd\sqrt{b\tan^2(c+dx)}}$$

```
[Out] -Cot[c + d*x]/(2*b*d*Sqrt[b*Tan[c + d*x]^2]) - (Log[Sin[c + d*x]]*Tan[c + d
*x])/(b*d*Sqrt[b*Tan[c + d*x]^2])
```

Rubi [A] time = 0.0291696, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 3475}

$$-\frac{\cot(c+dx)}{2bd\sqrt{b\tan^2(c+dx)}}-\frac{\tan(c+dx)\log(\sin(c+dx))}{bd\sqrt{b\tan^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^2)^(-3/2),x]
```

```
[Out] -Cot[c + d*x]/(2*b*d*Sqrt[b*Tan[c + d*x]^2]) - (Log[Sin[c + d*x]]*Tan[c + d
*x])/(b*d*Sqrt[b*Tan[c + d*x]^2])
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(b\tan^2(c+dx)\right)^{3/2}}\,dx &= \frac{\tan(c+dx)\int \cot^3(c+dx)\,dx}{b\sqrt{b\tan^2(c+dx)}}\\ &= -\frac{\cot(c+dx)}{2bd\sqrt{b\tan^2(c+dx)}}-\frac{\tan(c+dx)\int \cot(c+dx)\,dx}{b\sqrt{b\tan^2(c+dx)}}\\ &= -\frac{\cot(c+dx)}{2bd\sqrt{b\tan^2(c+dx)}}-\frac{\log(\sin(c+dx))\tan(c+dx)}{bd\sqrt{b\tan^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.361502, size = 56, normalized size = 0.85

$$-\frac{\tan^3(c+dx)\left(\cot^2(c+dx)+2\log(\tan(c+dx))+2\log(\cos(c+dx))\right)}{2d\left(b\tan^2(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^2)^(-3/2),x]
```

```
[Out] -((Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[Tan[c + d*x]])*Tan[c + d*x]
^3)/(2*d*(b*Tan[c + d*x]^2)^(3/2))
```

Maple [A] time = 0.022, size = 63, normalized size = 1.

$$\frac{\tan\left(dx+c\right)\left(\ln\left(1+\left(\tan\left(dx+c\right)\right)^2\right)\left(\tan\left(dx+c\right)\right)^2-2\,\ln\left(\tan\left(dx+c\right)\right)\left(\tan\left(dx+c\right)\right)^2-1\right)}{2\,d}\left(b\left(\tan\left(dx+c\right)\right)^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c)^2)^(3/2),x)
```

```
[Out] 1/2/d*tan(d*x+c)*(ln(1+tan(d*x+c)^2)*tan(d*x+c)^2-2*ln(tan(d*x+c))*tan(d*x+
c)^2-1)/(b*tan(d*x+c)^2)^(3/2)
```

Maxima [A] time = 1.58767, size = 62, normalized size = 0.94

$$\frac{\frac{\log\left(\tan(dx+c)^2+1\right)}{b^{\frac{3}{2}}} - \frac{2\,\log(\tan(dx+c))}{b^{\frac{3}{2}}} - \frac{1}{b^{\frac{3}{2}}\,\tan(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(log(tan(d*x + c)^2 + 1)/b^(3/2) - 2*log(tan(d*x + c))/b^(3/2) - 1/(b^(
3/2)*tan(d*x + c)^2))/d
```

Fricas [A] time = 1.37851, size = 177, normalized size = 2.68

$$-\frac{\sqrt{b\tan\left(dx+c\right)^2}\left(\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^2+\tan\left(dx+c\right)^2+1\right)}{2\,b^2d\tan\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(b*tan(d*x + c)^2)*(log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d
*x + c)^2 + tan(d*x + c)^2 + 1)/(b^2*d*tan(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tan^2\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)**2)**(3/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**2)**(-3/2), x)
```

Giac [B] time = 2.57734, size = 281, normalized size = 4.26

$$-\frac{\frac{\operatorname{sgn}\left(-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{8\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)\operatorname{sgn}\left(-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}+\frac{4\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\right)\operatorname{sgn}\left(-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{8\,b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*(sgn(-tan(1/2*d*x + 1/2*c)^2 + 1)*tan(1/2*d*x + 1/2*c)^2/sgn(tan(1/2*d
*x + 1/2*c)) - 8*log(tan(1/2*d*x + 1/2*c)^2 + 1)*sgn(-tan(1/2*d*x + 1/2*c)^
2 + 1)/sgn(tan(1/2*d*x + 1/2*c)) + 4*log(tan(1/2*d*x + 1/2*c)^2)*sgn(-tan(1
/2*d*x + 1/2*c)^2 + 1)/sgn(tan(1/2*d*x + 1/2*c)) - (4*sgn(-tan(1/2*d*x + 1/
2*c)^2 + 1)*tan(1/2*d*x + 1/2*c)^2 - sgn(-tan(1/2*d*x + 1/2*c)^2 + 1))/(sgn
(tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c)^2))/(b^(3/2)*d)
```

3.29 $\int \frac{1}{\left(b\tan^2(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{\cot^3(c+dx)}{4b^2d\sqrt{b\tan^2(c+dx)}}+\frac{\cot(c+dx)}{2b^2d\sqrt{b\tan^2(c+dx)}}+\frac{\tan(c+dx)\log(\sin(c+dx))}{b^2d\sqrt{b\tan^2(c+dx)}}$$

```
[Out] Cot[c + d*x]/(2*b^2*d*Sqrt[b*Tan[c + d*x]^2]) - Cot[c + d*x]^3/(4*b^2*d*Sqr
t[b*Tan[c + d*x]^2]) + (Log[Sin[c + d*x]]*Tan[c + d*x])/(b^2*d*Sqrt[b*Tan[c
 + d*x]^2])
```

Rubi [A] time = 0.0398231, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 3475}

$$-\frac{\cot^3(c+dx)}{4b^2d\sqrt{b\tan^2(c+dx)}}+\frac{\cot(c+dx)}{2b^2d\sqrt{b\tan^2(c+dx)}}+\frac{\tan(c+dx)\log(\sin(c+dx))}{b^2d\sqrt{b\tan^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^2)^(-5/2),x]
```

```
[Out] Cot[c + d*x]/(2*b^2*d*Sqrt[b*Tan[c + d*x]^2]) - Cot[c + d*x]^3/(4*b^2*d*Sqr
t[b*Tan[c + d*x]^2]) + (Log[Sin[c + d*x]]*Tan[c + d*x])/(b^2*d*Sqrt[b*Tan[c
 + d*x]^2])
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(b\tan^2(c+dx)\right)^{5/2}}\,dx &= \frac{\tan(c+dx)\int \cot^5(c+dx)\,dx}{b^2\sqrt{b\tan^2(c+dx)}}\\ &= -\frac{\cot^3(c+dx)}{4b^2d\sqrt{b\tan^2(c+dx)}}-\frac{\tan(c+dx)\int \cot^3(c+dx)\,dx}{b^2\sqrt{b\tan^2(c+dx)}}\\ &= \frac{\cot(c+dx)}{2b^2d\sqrt{b\tan^2(c+dx)}}-\frac{\cot^3(c+dx)}{4b^2d\sqrt{b\tan^2(c+dx)}}+\frac{\tan(c+dx)\int \cot(c+dx)\,dx}{b^2\sqrt{b\tan^2(c+dx)}}\\ &= \frac{\cot(c+dx)}{2b^2d\sqrt{b\tan^2(c+dx)}}-\frac{\cot^3(c+dx)}{4b^2d\sqrt{b\tan^2(c+dx)}}+\frac{\log(\sin(c+dx))\tan(c+dx)}{b^2d\sqrt{b\tan^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.251275, size = 68, normalized size = 0.7

$$\frac{\tan^5(c+dx)\left(-\cot^4(c+dx)+2\cot^2(c+dx)+4\log(\tan(c+dx))+4\log(\cos(c+dx))\right)}{4d\left(b\tan^2(c+dx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^2)^(-5/2),x]
```

```
[Out] ((2*Cot[c + d*x]^2 - Cot[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Log[Tan[c + d
*x]])*Tan[c + d*x]^5)/(4*d*(b*Tan[c + d*x]^2)^(5/2))
```

Maple [A] time = 0.021, size = 74, normalized size = 0.8

$$-\frac{\tan(dx+c)\left(2\ln\left(1+(\tan(dx+c))^2\right)(\tan(dx+c))^4-4\ln(\tan(dx+c))(\tan(dx+c))^4-2(\tan(dx+c))^2+1\right)}{4d}\left(b(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c)^2)^(5/2),x)
```

```
[Out] -1/4/d*tan(d*x+c)*(2*ln(1+tan(d*x+c)^2)*tan(d*x+c)^4-4*ln(tan(d*x+c))*tan(d
*x+c)^4-2*tan(d*x+c)^2+1)/(b*tan(d*x+c)^2)^(5/2)
```

Maxima [A] time = 1.55698, size = 89, normalized size = 0.92

$$-\frac{\frac{2\,\log\left(\tan\left(dx+c\right)^{2}+1\right)}{b^{\frac{5}{2}}}-\frac{4\,\log\left(\tan\left(dx+c\right)\right)}{b^{\frac{5}{2}}}-\frac{2\,\sqrt{b}\tan\left(dx+c\right)^{2}-\sqrt{b}}{b^{3}\tan\left(dx+c\right)^{4}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*log(tan(d*x + c)^2 + 1)/b^(5/2) - 4*log(tan(d*x + c))/b^(5/2) - (2*
sqrt(b)*tan(d*x + c)^2 - sqrt(b))/(b^3*tan(d*x + c)^4))/d
```

Fricas [A] time = 1.40071, size = 207, normalized size = 2.13

$$\frac{\left(2\,\log\left(\frac{\tan\left(dx+c\right)^{2}}{\tan\left(dx+c\right)^{2}+1}\right)\tan\left(dx+c\right)^{4}+3\,\tan\left(dx+c\right)^{4}+2\,\tan\left(dx+c\right)^{2}-1\right)\sqrt{b\tan\left(dx+c\right)^{2}}}{4\,b^{3}d\tan\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^4 + 3*tan(d*x
+ c)^4 + 2*tan(d*x + c)^2 - 1)*sqrt(b*tan(d*x + c)^2)/(b^3*d*tan(d*x + c)^5
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(b\tan^{2}\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)**2)**(5/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**2)**(-5/2), x)
```

Giac [B] time = 3.49898, size = 366, normalized size = 3.77

$$-\frac{\operatorname{sgn}\left(-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-12\operatorname{sgn}\left(-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/64*(sgn(-tan(1/2*d*x + 1/2*c)^2 + 1)*sgn(tan(1/2*d*x + 1/2*c))*tan(1/2*d
*x + 1/2*c)^4 - 12*sgn(-tan(1/2*d*x + 1/2*c)^2 + 1)*sgn(tan(1/2*d*x + 1/2*c
))*tan(1/2*d*x + 1/2*c)^2 + 64*log(tan(1/2*d*x + 1/2*c)^2 + 1)*sgn(-tan(1/2
*d*x + 1/2*c)^2 + 1)/sgn(tan(1/2*d*x + 1/2*c)) - 32*log(tan(1/2*d*x + 1/2*c
)^2)*sgn(-tan(1/2*d*x + 1/2*c)^2 + 1)/sgn(tan(1/2*d*x + 1/2*c)) + (48*sgn(-
tan(1/2*d*x + 1/2*c)^2 + 1)*tan(1/2*d*x + 1/2*c)^4 - 12*sgn(-tan(1/2*d*x +
1/2*c)^2 + 1)*tan(1/2*d*x + 1/2*c)^2 + sgn(-tan(1/2*d*x + 1/2*c)^2 + 1))/(s
gn(tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c)^4))/(b^(5/2)*d)
```

3.30 $\int \left(b\tan^3(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=364

$$\frac{2b^2\tan^5(c+dx)\sqrt{b\tan^3(c+dx)}}{13d}-\frac{2b^2\tan^3(c+dx)\sqrt{b\tan^3(c+dx)}}{9d}+\frac{2b^2\tan(c+dx)\sqrt{b\tan^3(c+dx)}}{5d}-\frac{b^2\tan^{-1}\left(1\right.}{}$$

```
[Out] (-2*b^2*Cot[c + d*x]*Sqrt[b*Tan[c + d*x]^3])/d - (b^2*ArcTan[1 - Sqrt[2]*Sq
rt[Tan[c + d*x]]]*Sqrt[b*Tan[c + d*x]^3])/(Sqrt[2]*d*Tan[c + d*x]^(3/2)) +
(b^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]*Sqrt[b*Tan[c + d*x]^3])/(Sqrt[2
]*d*Tan[c + d*x]^(3/2)) - (b^2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
 d*x]]*Sqrt[b*Tan[c + d*x]^3])/(2*Sqrt[2]*d*Tan[c + d*x]^(3/2)) + (b^2*Log[
1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sqrt[b*Tan[c + d*x]^3])/(2*S
qrt[2]*d*Tan[c + d*x]^(3/2)) + (2*b^2*Tan[c + d*x]*Sqrt[b*Tan[c + d*x]^3])/
(5*d) - (2*b^2*Tan[c + d*x]^3*Sqrt[b*Tan[c + d*x]^3])/(9*d) + (2*b^2*Tan[c
+ d*x]^5*Sqrt[b*Tan[c + d*x]^3])/(13*d)
```

Rubi [A] time = 0.150337, antiderivative size = 364, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3473, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{2b^2\tan^5(c+dx)\sqrt{b\tan^3(c+dx)}}{13d}-\frac{2b^2\tan^3(c+dx)\sqrt{b\tan^3(c+dx)}}{9d}+\frac{2b^2\tan(c+dx)\sqrt{b\tan^3(c+dx)}}{5d}-\frac{b^2\tan^{-1}\left(1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^3)^(5/2),x]
```



```
[Out] (-2*b^2*Cot[c + d*x]*Sqrt[b*Tan[c + d*x]^3])/d - (b^2*ArcTan[1 - Sqrt[2]*Sq
rt[Tan[c + d*x]]]*Sqrt[b*Tan[c + d*x]^3])/(Sqrt[2]*d*Tan[c + d*x]^(3/2)) +
(b^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]*Sqrt[b*Tan[c + d*x]^3])/(Sqrt[2
]*d*Tan[c + d*x]^(3/2)) - (b^2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
 d*x]]*Sqrt[b*Tan[c + d*x]^3])/(2*Sqrt[2]*d*Tan[c + d*x]^(3/2)) + (b^2*Log[
1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sqrt[b*Tan[c + d*x]^3])/(2*S
qrt[2]*d*Tan[c + d*x]^(3/2)) + (2*b^2*Tan[c + d*x]*Sqrt[b*Tan[c + d*x]^3])/
(5*d) - (2*b^2*Tan[c + d*x]^3*Sqrt[b*Tan[c + d*x]^3])/(9*d) + (2*b^2*Tan[c
+ d*x]^5*Sqrt[b*Tan[c + d*x]^3])/(13*d)
```


Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(b\tan^3(c+dx)\right)^{5/2}\,dx = \frac{\left(b^2\sqrt{b\tan^3(c+dx)}\right)\int \tan^{\frac{15}{2}}(c+dx)\,dx}{\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2b^2\tan^5(c+dx)\sqrt{b\tan^3(c+dx)}}{13d} - \frac{\left(b^2\sqrt{b\tan^3(c+dx)}\right)\int \tan^{\frac{11}{2}}(c+dx)\,dx}{\tan^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b^2\tan^3(c+dx)\sqrt{b\tan^3(c+dx)}}{9d} + \frac{2b^2\tan^5(c+dx)\sqrt{b\tan^3(c+dx)}}{13d} + \frac{\left(b^2\sqrt{b\tan^3(c+dx\right.}}{\tan^{\frac{3}{2}}}$$

$$= \frac{2b^2\tan(c+dx)\sqrt{b\tan^3(c+dx)}}{5d} - \frac{2b^2\tan^3(c+dx)\sqrt{b\tan^3(c+dx)}}{9d} + \frac{2b^2\tan^5(c+dx)\sqrt{b\,t}}{13d}$$

$$= -\frac{2b^2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} + \frac{2b^2\tan(c+dx)\sqrt{b\tan^3(c+dx)}}{5d} - \frac{2b^2\tan^3(c+dx)\sqrt{b\,t}}{9d}$$

$$= -\frac{2b^2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} + \frac{2b^2\tan(c+dx)\sqrt{b\tan^3(c+dx)}}{5d} - \frac{2b^2\tan^3(c+dx)\sqrt{b\,t}}{9d}$$

$$= -\frac{2b^2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} + \frac{2b^2\tan(c+dx)\sqrt{b\tan^3(c+dx)}}{5d} - \frac{2b^2\tan^3(c+dx)\sqrt{b\,t}}{9d}$$

$$= -\frac{2b^2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} + \frac{2b^2\tan(c+dx)\sqrt{b\tan^3(c+dx)}}{5d} - \frac{2b^2\tan^3(c+dx)\sqrt{b\,t}}{9d}$$

$$= -\frac{2b^2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} + \frac{2b^2\tan(c+dx)\sqrt{b\tan^3(c+dx)}}{5d} - \frac{2b^2\tan^3(c+dx)\sqrt{b\,t}}{9d}$$

$$= -\frac{2b^2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} - \frac{b^2\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{b\tan^3(c+}}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b^2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} - \frac{b^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} + \frac{b^2\,\mathrm{ta}}{}$$

Mathematica [A] time = 0.810332, size = 199, normalized size = 0.55

$$b\left(b\tan^3(c+dx)\right)^{3/2}\left(360\tan^{\frac{13}{2}}(c+dx)-520\tan^{\frac{9}{2}}(c+dx)+936\tan^{\frac{5}{2}}(c+dx)-1170\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^3)^(5/2),x]
```

```
[Out] (b*(b*Tan[c + d*x]^3)^(3/2)*(-1170*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c +
d*x]]] + 1170*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] - 585*Sqrt[2]*
Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + 585*Sqrt[2]*Log[1 + Sq
rt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - 4680*Sqrt[Tan[c + d*x]] + 936*Ta
n[c + d*x]^(5/2) - 520*Tan[c + d*x]^(9/2) + 360*Tan[c + d*x]^(13/2)))/(2340
*d*Tan[c + d*x]^(9/2))
```

Maple [A] time = 0.039, size = 265, normalized size = 0.7

$$\frac{1}{2340\,d\left(\tan(dx+c)\right)^5 b^4}\left(b\left(\tan(dx+c)\right)^3\right)^{\frac{5}{2}}\left(360\,\left(b\tan(dx+c)\right)^{13/2}-520\,b^2\left(b\tan(dx+c)\right)^{9/2}+585\,b^6\sqrt[4]{b^2}\sqrt{2}\ln\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^3)^(5/2),x)
```

```
[Out] 1/2340/d*(b*tan(d*x+c)^3)^(5/2)*(360*(b*tan(d*x+c))^(13/2)-520*b^2*(b*tan(d
*x+c))^(9/2)+585*b^6*(b^2)^(1/4)*2^(1/2)*ln(-(b*tan(d*x+c)+(b^2)^(1/4)*(b*t
an(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/((b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(
1/2)-b*tan(d*x+c)-(b^2)^(1/2)))+1170*b^6*(b^2)^(1/4)*2^(1/2)*arctan((2^(1/2
)*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))-1170*b^6*(b^2)^(1/4)*2^(1/
2)*arctan((-2^(1/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))+936*b^4*
(b*tan(d*x+c))^(5/2)-4680*b^6*(b*tan(d*x+c))^(1/2))/tan(d*x+c)^5/(b*tan(d*x
+c))^(5/2)/b^4
```

Maxima [A] time = 1.5432, size = 240, normalized size = 0.66

$$360\,b^{\frac{5}{2}}\tan(dx+c)^{\frac{13}{2}}-520\,b^{\frac{5}{2}}\tan(dx+c)^{\frac{9}{2}}+936\,b^{\frac{5}{2}}\tan(dx+c)^{\frac{5}{2}}+585\left(2\sqrt{2}\sqrt{b}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2340*(360*b^(5/2)*tan(d*x + c)^(13/2) - 520*b^(5/2)*tan(d*x + c)^(9/2) +
936*b^(5/2)*tan(d*x + c)^(5/2) + 585*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*
(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*sqrt(b)*arctan(-1/2*sqrt(2)*(
sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*sqrt(b)*log(sqrt(2)*sqrt(tan(d*x
 + c)) + tan(d*x + c) + 1) - sqrt(2)*sqrt(b)*log(-sqrt(2)*sqrt(tan(d*x + c)
) + tan(d*x + c) + 1))*b^2 - 4680*b^(5/2)*sqrt(tan(d*x + c)))/d
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan^3\left(c+dx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)**3)**(5/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**3)**(5/2), x)
```

Giac [A] time = 2.1667, size = 393, normalized size = 1.08

$$\frac{1}{2340}\left(\frac{1170\sqrt{2}b\sqrt{|b|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{d}+\frac{1170\sqrt{2}b\sqrt{|b|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{d}+\frac{585\sqrt{2}b\sqrt{|b|}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^3)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2340*(1170*sqrt(2)*b*sqrt(abs(b))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)
) + 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/d + 1170*sqrt(2)*b*sqrt(abs(b))*a
rctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d*x + c)))/sqrt(abs
(b)))/d + 585*sqrt(2)*b*sqrt(abs(b))*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*ta
n(d*x + c))*sqrt(abs(b)) + abs(b))/d - 585*sqrt(2)*b*sqrt(abs(b))*log(b*tan
(d*x + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/d + 8*(45*s
qrt(b*tan(d*x + c))*b^66*d^12*tan(d*x + c)^6 - 65*sqrt(b*tan(d*x + c))*b^66
*d^12*tan(d*x + c)^4 + 117*sqrt(b*tan(d*x + c))*b^66*d^12*tan(d*x + c)^2 -
585*sqrt(b*tan(d*x + c))*b^66*d^12)/(b^65*d^13))*b*sgn(tan(d*x + c))
```

3.31 $\int \left(b\tan^3(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=286

$$\frac{2b\tan^2(c+dx)\sqrt{b\tan^3(c+dx)}}{7d}-\frac{2b\sqrt{b\tan^3(c+dx)}}{3d}-\frac{b\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)}+\frac{b\tan^{-1}\left(\sqrt{2}\cdots\right.}{\cdots}$$

```
[Out] (-2*b*Sqrt[b*Tan[c + d*x]^3])/(3*d) - (b*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*
x]]]*Sqrt[b*Tan[c + d*x]^3])/(Sqrt[2]*d*Tan[c + d*x]^(3/2)) + (b*ArcTan[1 +
 Sqrt[2]*Sqrt[Tan[c + d*x]]]*Sqrt[b*Tan[c + d*x]^3])/(Sqrt[2]*d*Tan[c + d*x
]^(3/2)) + (b*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sqrt[b*Tan
[c + d*x]^3])/(2*Sqrt[2]*d*Tan[c + d*x]^(3/2)) - (b*Log[1 + Sqrt[2]*Sqrt[Ta
n[c + d*x]] + Tan[c + d*x]]*Sqrt[b*Tan[c + d*x]^3])/(2*Sqrt[2]*d*Tan[c + d*
x]^(3/2)) + (2*b*Tan[c + d*x]^2*Sqrt[b*Tan[c + d*x]^3])/(7*d)
```

Rubi [A] time = 0.126066, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3473, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{2b\tan^2(c+dx)\sqrt{b\tan^3(c+dx)}}{7d}-\frac{2b\sqrt{b\tan^3(c+dx)}}{3d}-\frac{b\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)}+\frac{b\tan^{-1}\left(\sqrt{2}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^3)^(3/2),x]
```

```
[Out] (-2*b*Sqrt[b*Tan[c + d*x]^3])/(3*d) - (b*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*
x]]]*Sqrt[b*Tan[c + d*x]^3])/(Sqrt[2]*d*Tan[c + d*x]^(3/2)) + (b*ArcTan[1 +
 Sqrt[2]*Sqrt[Tan[c + d*x]]]*Sqrt[b*Tan[c + d*x]^3])/(Sqrt[2]*d*Tan[c + d*x
]^(3/2)) + (b*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sqrt[b*Tan
[c + d*x]^3])/(2*Sqrt[2]*d*Tan[c + d*x]^(3/2)) - (b*Log[1 + Sqrt[2]*Sqrt[Ta
n[c + d*x]] + Tan[c + d*x]]*Sqrt[b*Tan[c + d*x]^3])/(2*Sqrt[2]*d*Tan[c + d*
x]^(3/2)) + (2*b*Tan[c + d*x]^2*Sqrt[b*Tan[c + d*x]^3])/(7*d)
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
```

```
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \left(b\tan^3(c+dx)\right)^{3/2}\,dx = \frac{\left(b\sqrt{b\tan^3(c+dx)}\right)\int \tan^{\frac{9}{2}}(c+dx)\,dx}{\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2b\tan^2(c+dx)\sqrt{b\tan^3(c+dx)}}{7d} - \frac{\left(b\sqrt{b\tan^3(c+dx)}\right)\int \tan^{\frac{5}{2}}(c+dx)\,dx}{\tan^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b\sqrt{b\tan^3(c+dx)}}{3d} + \frac{2b\tan^2(c+dx)\sqrt{b\tan^3(c+dx)}}{7d} + \frac{\left(b\sqrt{b\tan^3(c+dx)}\right)\int \sqrt{\tan(c+ \cdots}}{\tan^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b\sqrt{b\tan^3(c+dx)}}{3d} + \frac{2b\tan^2(c+dx)\sqrt{b\tan^3(c+dx)}}{7d} + \frac{\left(b\sqrt{b\tan^3(c+dx)}\right)\text{Subst}\left(\int \cdots\right.}{d\tan^{\frac{3}{2}}(c+d \cdots}$$

$$= -\frac{2b\sqrt{b\tan^3(c+dx)}}{3d} + \frac{2b\tan^2(c+dx)\sqrt{b\tan^3(c+dx)}}{7d} + \frac{\left(2b\sqrt{b\tan^3(c+dx)}\right)\text{Subst}\left(\int \cdots\right.}{d\tan^{\frac{3}{2}}(c+ \cdots}$$

$$= -\frac{2b\sqrt{b\tan^3(c+dx)}}{3d} + \frac{2b\tan^2(c+dx)\sqrt{b\tan^3(c+dx)}}{7d} - \frac{\left(b\sqrt{b\tan^3(c+dx)}\right)\text{Subst}\left(\int \cdots\right.}{d\tan^{\frac{3}{2}}(c+ \cdots}$$

$$= -\frac{2b\sqrt{b\tan^3(c+dx)}}{3d} + \frac{2b\tan^2(c+dx)\sqrt{b\tan^3(c+dx)}}{7d} + \frac{\left(b\sqrt{b\tan^3(c+dx)}\right)\text{Subst}\left(\int \cdots\right.}{2d\tan^{\frac{3}{2}}(c \cdots}$$

$$= -\frac{2b\sqrt{b\tan^3(c+dx)}}{3d} + \frac{b\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{b\tan^3(c+dx)}}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} - \frac{b\log \cdots}{\cdots}$$

$$= -\frac{2b\sqrt{b\tan^3(c+dx)}}{3d} - \frac{b\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} + \frac{b\tan^{-1}\left(1+\sqrt{2} \cdots\right.}{\sqrt{\cdots}}$$

Mathematica [C] time = 0.0606522, size = 54, normalized size = 0.19

$$\frac{2b\sqrt{b\tan^3(c+dx)}\left(7\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(c+dx)\right)+3\tan^2(c+dx)-7\right)}{21d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^3)^(3/2),x]
```

```
[Out] (2*b*Sqrt[b*Tan[c + d*x]^3]*(-7 + 7*Hypergeometric2F1[3/4, 1, 7/4, -Tan[c +
 d*x]^2] + 3*Tan[c + d*x]^2))/(21*d)
```

Maple [A] time = 0.018, size = 235, normalized size = 0.8

$$\frac{1}{84\,d\,(\tan(dx+c))^3\,b^2}\left(b\,(\tan(dx+c))^3\right)^{\frac{3}{2}}\left(24\,\left(b\tan(dx+c)\right)^{7/2}\sqrt[4]{b^2}+21\,b^4\sqrt{2}\ln\left(-\frac{\sqrt[4]{b^2}\sqrt{b\tan(dx+c)}\sqrt{2}-b\tan(dx}{b\tan(dx+c)+\sqrt[4]{b^2}\sqrt{b\tan(dx+}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^3)^(3/2),x)
```

```
[Out] 1/84/d*(b*tan(d*x+c)^3)^(3/2)*(24*(b*tan(d*x+c))^(7/2)*(b^2)^(1/4)+21*b^4*2
^(1/2)*ln(-((b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)-b*tan(d*x+c)-(b^2)^(1/
2))/(b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+42
*b^4*2^(1/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))
-42*b^4*2^(1/2)*arctan((-2^(1/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4))/(b^2)^(1
/4))-56*(b*tan(d*x+c))^(3/2)*b^2*(b^2)^(1/4))/tan(d*x+c)^3/(b*tan(d*x+c))^(
3/2)/b^2/(b^2)^(1/4)
```

Maxima [A] time = 1.43031, size = 189, normalized size = 0.66

$$\frac{24\,b^{\frac{3}{2}}\tan(dx+c)^{\frac{7}{2}}-56\,b^{\frac{3}{2}}\tan(dx+c)^{\frac{3}{2}}+21\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\right.\right.\right.}{84}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/84*(24*b^(3/2)*tan(d*x + c)^(7/2) - 56*b^(3/2)*tan(d*x + c)^(3/2) + 21*(2
*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*log(sqrt(2)*
sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*log(-sqrt(2)*sqrt(tan(d*x
+ c)) + tan(d*x + c) + 1))*b^(3/2))/d
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan^3\left(c+dx\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)**3)**(3/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**3)**(3/2), x)
```

Giac [A] time = 1.34921, size = 342, normalized size = 1.2

$$\frac{1}{84}b\left(\frac{42\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{bd}+\frac{42\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{bd}-\frac{21\sqrt{2}|b|^{\frac{3}{2}}\log\left(b\tan\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/84*b*(42*sqrt(2)*abs(b)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) +
2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/(b*d) + 42*sqrt(2)*abs(b)^(3/2)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b))
)/(b*d) - 21*sqrt(2)*abs(b)^(3/2)*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d
*x + c))*sqrt(abs(b)) + abs(b))/(b*d) + 21*sqrt(2)*abs(b)^(3/2)*log(b*tan(d
*x + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(b*d) + 8*(3*
sqrt(b*tan(d*x + c))*b^21*d^6*tan(d*x + c)^3 - 7*sqrt(b*tan(d*x + c))*b^21*
d^6*tan(d*x + c))/(b^21*d^7))*sgn(tan(d*x + c))
```

3.32 $\int \sqrt{b\tan^3(c+dx)}\,dx$

Optimal. Leaf size=255

$$\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)}+\frac{\sqrt{b\tan^3(c+dx)}\log\left(t\right.}{2\sqrt{}}$$

```
[Out] (2*Cot[c + d*x]*Sqrt[b*Tan[c + d*x]^3])/d + (ArcTan[1 - Sqrt[2]*Sqrt[Tan[c
+ d*x]]]*Sqrt[b*Tan[c + d*x]^3])/(Sqrt[2]*d*Tan[c + d*x]^(3/2)) - (ArcTan[1
 + Sqrt[2]*Sqrt[Tan[c + d*x]]]*Sqrt[b*Tan[c + d*x]^3])/(Sqrt[2]*d*Tan[c + d
*x]^(3/2)) + (Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sqrt[b*Tan
[c + d*x]^3])/(2*Sqrt[2]*d*Tan[c + d*x]^(3/2)) - (Log[1 + Sqrt[2]*Sqrt[Tan[
c + d*x]] + Tan[c + d*x]]*Sqrt[b*Tan[c + d*x]^3])/(2*Sqrt[2]*d*Tan[c + d*x]
^(3/2))
```

Rubi [A] time = 0.113415, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3473, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)}+\frac{\sqrt{b\tan^3(c+dx)}\log\left(t\right.}{2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[c + d*x]^3],x]
```


```
[Out] (2*Cot[c + d*x]*Sqrt[b*Tan[c + d*x]^3])/d + (ArcTan[1 - Sqrt[2]*Sqrt[Tan[c
+ d*x]]]*Sqrt[b*Tan[c + d*x]^3])/(Sqrt[2]*d*Tan[c + d*x]^(3/2)) - (ArcTan[1
 + Sqrt[2]*Sqrt[Tan[c + d*x]]]*Sqrt[b*Tan[c + d*x]^3])/(Sqrt[2]*d*Tan[c + d
*x]^(3/2)) + (Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sqrt[b*Tan
[c + d*x]^3])/(2*Sqrt[2]*d*Tan[c + d*x]^(3/2)) - (Log[1 + Sqrt[2]*Sqrt[Tan[
c + d*x]] + Tan[c + d*x]]*Sqrt[b*Tan[c + d*x]^3])/(2*Sqrt[2]*d*Tan[c + d*x]
^(3/2))
```


Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
```

```
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{b\tan^3(c+dx)}\,dx = \frac{\sqrt{b\tan^3(c+dx)}\int \tan^{\frac{3}{2}}(c+dx)\,dx}{\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} - \frac{\sqrt{b\tan^3(c+dx)}\int \frac{1}{\sqrt{\tan(c+dx)}}\,dx}{\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} - \frac{\sqrt{b\tan^3(c+dx)}\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} - \frac{\left(2\sqrt{b\tan^3(c+dx)}\right)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} - \frac{\sqrt{b\tan^3(c+dx)}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d\tan^{\frac{3}{2}}(c+dx)} - \frac{\sqrt{b\,\text{ta}}}{}$$

$$= \frac{2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} - \frac{\sqrt{b\tan^3(c+dx)}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2d\tan^{\frac{3}{2}}(c+dx)} - $$

$$= \frac{2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{b\tan^3(c+dx)}}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} - $$

$$= \frac{2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} + \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} - \frac{\tan^{-1}\left(1+\right.}{}$$

Mathematica [A] time = 0.242687, size = 161, normalized size = 0.63

$$\frac{\sqrt{b\tan^3(c+dx)}\left(2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-2\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)+8\sqrt{\tan(c+dx)}+\sqrt{2}\log\left(\text{ta}\right.\right.}{4d\tan^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[c + d*x]^3],x]
```

```
[Out] ((2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 2*Sqrt[2]*ArcTan[1 + S
qrt[2]*Sqrt[Tan[c + d*x]]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + T
an[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] +
```

```
8*Sqrt[Tan[c + d*x]])*Sqrt[b*Tan[c + d*x]^3])/(4*d*Tan[c + d*x]^(3/2))
```

Maple [A] time = 0.024, size = 207, normalized size = 0.8

$$-\frac{1}{4\,d\tan(dx+c)}\sqrt{b(\tan(dx+c))^3}\left(\sqrt[4]{b^2}\sqrt{2}\ln\left(-\left(b\tan(dx+c)+\sqrt[4]{b^2}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}\right)\left(\sqrt[4]{b^2}\sqrt{b\tan(dx+c)}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^3)^(1/2),x)
```

```
[Out] -1/4/d*(b*tan(d*x+c)^3)^(1/2)*((b^2)^(1/4)*2^(1/2)*ln(-(b*tan(d*x+c)+(b^2)^
(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/((b^2)^(1/4)*(b*tan(d*x+c))
^(1/2)*2^(1/2)-b*tan(d*x+c)-(b^2)^(1/2)))+2*(b^2)^(1/4)*2^(1/2)*arctan((2^(
1/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))-2*(b^2)^(1/4)*2^(1/2)*a
rctan((-2^(1/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))-8*(b*tan(d*x
+c))^(1/2))/tan(d*x+c)/(b*tan(d*x+c))^(1/2)
```

Maxima [A] time = 1.46182, size = 180, normalized size = 0.71

$$-\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\sqrt{b}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)+\sqrt{2}\sqrt{b}\log\left(\sqrt{2}\sqrt{\tan\cdots}\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))
) + 2*sqrt(2)*sqrt(b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c))))
 + sqrt(2)*sqrt(b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqr
t(2)*sqrt(b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - 8*sqrt(b
)*sqrt(tan(d*x + c)))/d
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(b*tan(c + d*x)**3), x)
```

Giac [A] time = 1.3278, size = 263, normalized size = 1.03

$$-\frac{1}{4}\left(\frac{2\sqrt{2}\sqrt{|b|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{d}+\frac{2\sqrt{2}\sqrt{|b|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{d}+\frac{\sqrt{2}\sqrt{|b|}\log\left(b\tan(dx+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(2*sqrt(2)*sqrt(abs(b))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) + 2*s
qrt(b*tan(d*x + c)))/sqrt(abs(b)))/d + 2*sqrt(2)*sqrt(abs(b))*arctan(-1/2*s
qrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/d + sq
rt(2)*sqrt(abs(b))*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(a
bs(b)) + abs(b))/d - sqrt(2)*sqrt(abs(b))*log(b*tan(d*x + c) - sqrt(2)*sqrt
(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/d - 8*sqrt(b*tan(d*x + c))/d)*sgn(t
an(d*x + c))
```

3.33 $\int \frac{1}{\sqrt{b \tan^3(c+dx)}}\,dx$

Optimal. Leaf size=255

$$\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}d\sqrt{b\tan^3(c+dx)}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}d\sqrt{b\tan^3(c+dx)}}-\frac{2\tan(c+dx)}{d\sqrt{b\tan^3(c+dx)}}-\frac{\tan^{\frac{3}{2}}(c+}{}$$

```
[Out] (-2*Tan[c + d*x])/(d*Sqrt[b*Tan[c + d*x]^3]) + (ArcTan[1 - Sqrt[2]*Sqrt[Tan
[c + d*x]]]*Tan[c + d*x]^(3/2))/(Sqrt[2]*d*Sqrt[b*Tan[c + d*x]^3]) - (ArcTa
n[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]*Tan[c + d*x]^(3/2))/(Sqrt[2]*d*Sqrt[b*Tan
[c + d*x]^3]) - (Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c +
 d*x]^(3/2))/(2*Sqrt[2]*d*Sqrt[b*Tan[c + d*x]^3]) + (Log[1 + Sqrt[2]*Sqrt[T
an[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*d*Sqrt[b*Tan[c
+ d*x]^3])
```

Rubi [A] time = 0.117849, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3474, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}d\sqrt{b\tan^3(c+dx)}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}d\sqrt{b\tan^3(c+dx)}}-\frac{2\tan(c+dx)}{d\sqrt{b\tan^3(c+dx)}}-\frac{\tan^{\frac{3}{2}}(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Tan[c + d*x]^3],x]
```

```
[Out] (-2*Tan[c + d*x])/(d*Sqrt[b*Tan[c + d*x]^3]) + (ArcTan[1 - Sqrt[2]*Sqrt[Tan
[c + d*x]]]*Tan[c + d*x]^(3/2))/(Sqrt[2]*d*Sqrt[b*Tan[c + d*x]^3]) - (ArcTa
n[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]*Tan[c + d*x]^(3/2))/(Sqrt[2]*d*Sqrt[b*Tan
[c + d*x]^3]) - (Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c +
 d*x]^(3/2))/(2*Sqrt[2]*d*Sqrt[b*Tan[c + d*x]^3]) + (Log[1 + Sqrt[2]*Sqrt[T
an[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*d*Sqrt[b*Tan[c
+ d*x]^3])
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
```

```
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx = \frac{\tan^{\frac{3}{2}}(c+dx)\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)}\,dx}{\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\tan(c+dx)}{d\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{\frac{3}{2}}(c+dx)\int \sqrt{\tan(c+dx)}\,dx}{\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\tan(c+dx)}{d\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{\frac{3}{2}}(c+dx)\,\mathrm{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\tan(c+dx)}{d\sqrt{b\tan^3(c+dx)}} - \frac{\left(2\tan^{\frac{3}{2}}(c+dx)\right)\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\tan(c+dx)}{d\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{\frac{3}{2}}(c+dx)\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{\frac{3}{2}}(c+dx)\,\mathrm{Subs}}{d\sqrt{}}$$

$$= -\frac{2\tan(c+dx)}{d\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{\frac{3}{2}}(c+dx)\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2d\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{\frac{3}{2}}(c+dx)\,\mathrm{S}}{}$$

$$= -\frac{2\tan(c+dx)}{d\sqrt{b\tan^3(c+dx)}} - \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}d\sqrt{b\tan^3(c+dx)}} + \frac{\log\left(1+\sqrt{2}\sqrt{}\right.}{}$$

$$= -\frac{2\tan(c+dx)}{d\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}d\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{\tan(c+}\right.}{\sqrt{2}d\sqrt{b\tan^3(}}$$

Mathematica [C] time = 0.0321529, size = 43, normalized size = 0.17

$$-\frac{2\tan(c+dx)\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\tan^2(c+dx)\right)}{d\sqrt{b\tan^3(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Tan[c + d*x]^3],x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[c + d*x]^2]*Tan[c + d*x])/(d*Sqrt[
b*Tan[c + d*x]^3])
```

Maple [A] time = 0.024, size = 210, normalized size = 0.8

$$-\frac{\tan(dx+c)}{4d}\left(\sqrt{2}\sqrt{b\tan(dx+c)}\ln\left(-\left(\sqrt[4]{b^2}\sqrt{b\tan(dx+c)}\sqrt{2}-b\tan(dx+c)-\sqrt{b^2}\right)\left(b\tan(dx+c)+\sqrt[4]{b^2}\sqrt{b\tan(dx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c)^3)^(1/2),x)
```

```
[Out] -1/4/d*tan(d*x+c)*(2^(1/2)*(b*tan(d*x+c))^(1/2)*ln(-((b^2)^(1/4)*(b*tan(d*x
+c))^(1/2)*2^(1/2)-b*tan(d*x+c)-(b^2)^(1/2))/(b*tan(d*x+c)+(b^2)^(1/4)*(b*t
an(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+2*2^(1/2)*(b*tan(d*x+c))^(1/2)*arcta
n((2^(1/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))-2*2^(1/2)*(b*tan(
d*x+c))^(1/2)*arctan((-2^(1/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4
))+8*(b^2)^(1/4))/(b*tan(d*x+c)^3)^(1/2)/(b^2)^(1/4)
```

Maxima [A] time = 1.4457, size = 170, normalized size = 0.67

$$-\frac{\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(d\right.}{\sqrt{b}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*log(
sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*log(-sqrt(2)*sqrt(
tan(d*x + c)) + tan(d*x + c) + 1))/sqrt(b) + 8/(sqrt(b)*sqrt(tan(d*x + c)))
)/d
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*tan(c + d*x)**3), x)
```

Giac [A] time = 1.33162, size = 339, normalized size = 1.33

$$-\frac{1}{4}b^2\left(\frac{2\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{b^4 d\,\mathrm{sgn}(\tan(dx+c))}+\frac{2\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{b^4 d\,\mathrm{sgn}(\tan(dx+c))}-\frac{\sqrt{2}|b|^{\frac{3}{2}}\log\left(b\tan(dx\right.}{b^4 ds}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*b^2*(2*sqrt(2)*abs(b)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) +
 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/(b^4*d*sgn(tan(d*x + c))) + 2*sqrt(2
)*abs(b)^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d*x
 + c)))/sqrt(abs(b)))/(b^4*d*sgn(tan(d*x + c))) - sqrt(2)*abs(b)^(3/2)*log(
b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(b^4*d
*sgn(tan(d*x + c))) + sqrt(2)*abs(b)^(3/2)*log(b*tan(d*x + c) - sqrt(2)*sqr
t(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(b^4*d*sgn(tan(d*x + c))) + 8/(sqr
t(b*tan(d*x + c))*b^2*d*sgn(tan(d*x + c))))
```

3.34 $\int \frac{1}{\left(b\tan^3(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=298

$$-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}bd\sqrt{b\tan^3(c+dx)}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}bd\sqrt{b\tan^3(c+dx)}}+\frac{2}{3bd\sqrt{b\tan^3(c+dx)}}-\frac{\tan^{\frac{3}{2}}}{}$$

```
[Out] 2/(3*b*d*Sqrt[b*Tan[c + d*x]^3]) - (2*Cot[c + d*x]^2)/(7*b*d*Sqrt[b*Tan[c +
 d*x]^3]) - (ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*Tan[c + d*x]^(3/2))/(Sq
rt[2]*b*d*Sqrt[b*Tan[c + d*x]^3]) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]
*Tan[c + d*x]^(3/2))/(Sqrt[2]*b*d*Sqrt[b*Tan[c + d*x]^3]) - (Log[1 - Sqrt[2
]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*b*d*Sqr
t[b*Tan[c + d*x]^3]) + (Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*
Tan[c + d*x]^(3/2))/(2*Sqrt[2]*b*d*Sqrt[b*Tan[c + d*x]^3])
```

Rubi [A] time = 0.130004, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3474, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}bd\sqrt{b\tan^3(c+dx)}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}bd\sqrt{b\tan^3(c+dx)}}+\frac{2}{3bd\sqrt{b\tan^3(c+dx)}}-\frac{\tan^{\frac{3}{2}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^3)^(-3/2),x]
```


```
[Out] 2/(3*b*d*Sqrt[b*Tan[c + d*x]^3]) - (2*Cot[c + d*x]^2)/(7*b*d*Sqrt[b*Tan[c +
 d*x]^3]) - (ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*Tan[c + d*x]^(3/2))/(Sq
rt[2]*b*d*Sqrt[b*Tan[c + d*x]^3]) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]
*Tan[c + d*x]^(3/2))/(Sqrt[2]*b*d*Sqrt[b*Tan[c + d*x]^3]) - (Log[1 - Sqrt[2
]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*b*d*Sqr
t[b*Tan[c + d*x]^3]) + (Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*
Tan[c + d*x]^(3/2))/(2*Sqrt[2]*b*d*Sqrt[b*Tan[c + d*x]^3])
```


Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
```

```
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(b\tan^3(c+dx)\right)^{3/2}}\,dx = \frac{\tan^{\frac{3}{2}}(c+dx)\int \frac{1}{\tan^{\frac{9}{2}}(c+dx)}\,dx}{b\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\cot^2(c+dx)}{7bd\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{\frac{3}{2}}(c+dx)\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)}\,dx}{b\sqrt{b\tan^3(c+dx)}}$$

$$= \frac{2}{3bd\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^2(c+dx)}{7bd\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{\frac{3}{2}}(c+dx)\int \frac{1}{\sqrt{\tan(c+dx)}}\,dx}{b\sqrt{b\tan^3(c+dx)}}$$

$$= \frac{2}{3bd\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^2(c+dx)}{7bd\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{\frac{3}{2}}(c+dx)\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x^2)}\,dx, x, \tan(c\right.}{bd\sqrt{b\tan^3(c+dx)}}$$

$$= \frac{2}{3bd\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^2(c+dx)}{7bd\sqrt{b\tan^3(c+dx)}} + \frac{\left(2\tan^{\frac{3}{2}}(c+dx)\right)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{\tan(\right.}{bd\sqrt{b\tan^3(c+dx)}}$$

$$= \frac{2}{3bd\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^2(c+dx)}{7bd\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{\frac{3}{2}}(c+dx)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+\right.}{bd\sqrt{b\tan^3(c+dx)}}$$

$$= \frac{2}{3bd\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^2(c+dx)}{7bd\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{\frac{3}{2}}(c+dx)\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan\right.}{2bd\sqrt{b\tan^3(c+dx)}}$$

$$= \frac{2}{3bd\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^2(c+dx)}{7bd\sqrt{b\tan^3(c+dx)}} - \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan}{2\sqrt{2}bd\sqrt{b\tan^3(c+dx)}}$$

$$= \frac{2}{3bd\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^2(c+dx)}{7bd\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}bd\sqrt{b\tan^3(c+dx)}} +$$

Mathematica [C] time = 0.0667278, size = 45, normalized size = 0.15

$$-\frac{2\tan(c+dx)\,{}_2F_1\left(-\frac{7}{4},1;-\frac{3}{4};-\tan^2(c+dx)\right)}{7d\left(b\tan^3(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^3)^(-3/2),x]
```

```
[Out] (-2*Hypergeometric2F1[-7/4, 1, -3/4, -Tan[c + d*x]^2]*Tan[c + d*x])/(7*d*(b
*Tan[c + d*x]^3)^(3/2))
```

Maple [A] time = 0.022, size = 235, normalized size = 0.8

$$\frac{\tan(dx+c)}{84\,db^4}\left(21\,\sqrt[4]{b^2}\sqrt{2}\,(b\tan(dx+c))^{7/2}\ln\left(-\frac{b\tan(dx+c)+\sqrt[4]{b^2}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{\sqrt[4]{b^2}\sqrt{b\tan(dx+c)}\sqrt{2}-b\tan(dx+c)-\sqrt{b^2}}\right)+42\,\sqrt[4]{b^2}\sqrt{2}\,(b\tan(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c)^3)^(3/2),x)
```

```
[Out] 1/84/d*tan(d*x+c)/b^4*(21*(b^2)^(1/4)*2^(1/2)*(b*tan(d*x+c))^(7/2)*ln(-(b*t
an(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/((b^2)^(1/4
)*(b*tan(d*x+c))^(1/2)*2^(1/2)-b*tan(d*x+c)-(b^2)^(1/2)))+42*(b^2)^(1/4)*2^
(1/2)*(b*tan(d*x+c))^(7/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4)
)/(b^2)^(1/4))-42*(b^2)^(1/4)*2^(1/2)*(b*tan(d*x+c))^(7/2)*arctan((-2^(1/2)
*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))+56*b^4*tan(d*x+c)^2-24*b^4)
/(b*tan(d*x+c)^3)^(3/2)
```

Maxima [A] time = 1.4358, size = 220, normalized size = 0.74

$$\frac{\frac{21\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)-\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(\right.\right.}{b^{\frac{3}{2}}}}{84\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/84*(21*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) +
2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*l
og(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*log(-sqrt(2)*sq
rt(tan(d*x + c)) + tan(d*x + c) + 1))/b^(3/2) + 8*(21*sqrt(tan(d*x + c)) +
7/tan(d*x + c)^(3/2) - 3/tan(d*x + c)^(7/2))/b^(3/2) - 168*sqrt(tan(d*x + c
```

```
))/b^(3/2))/d
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tan^{3}\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)**3)**(3/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**3)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tan\left(dx+c\right)^{3}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c)^3)^(-3/2), x)
```

3.35 $\int \frac{1}{\left(b\tan^3(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=364

$$-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}b^2d\sqrt{b\tan^3(c+dx)}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}b^2d\sqrt{b\tan^3(c+dx)}}+\frac{2\tan(c+dx)}{b^2d\sqrt{b\tan^3(c+dx)}}+\frac{\tan^{\frac{3}{2}}}{}$$

```
[Out] (-2*Cot[c + d*x])/(5*b^2*d*Sqrt[b*Tan[c + d*x]^3]) + (2*Cot[c + d*x]^3)/(9*
b^2*d*Sqrt[b*Tan[c + d*x]^3]) - (2*Cot[c + d*x]^5)/(13*b^2*d*Sqrt[b*Tan[c +
 d*x]^3]) + (2*Tan[c + d*x])/(b^2*d*Sqrt[b*Tan[c + d*x]^3]) - (ArcTan[1 - S
qrt[2]*Sqrt[Tan[c + d*x]]]*Tan[c + d*x]^(3/2))/(Sqrt[2]*b^2*d*Sqrt[b*Tan[c
+ d*x]^3]) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]*Tan[c + d*x]^(3/2))/(S
qrt[2]*b^2*d*Sqrt[b*Tan[c + d*x]^3]) + (Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
+ Tan[c + d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*b^2*d*Sqrt[b*Tan[c + d*x]^3]
) - (Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(3/2))
/(2*Sqrt[2]*b^2*d*Sqrt[b*Tan[c + d*x]^3])
```

Rubi [A] time = 0.151434, antiderivative size = 364, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3474, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}b^2d\sqrt{b\tan^3(c+dx)}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}b^2d\sqrt{b\tan^3(c+dx)}}+\frac{2\tan(c+dx)}{b^2d\sqrt{b\tan^3(c+dx)}}+\frac{\tan^{\frac{3}{2}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^3)^(-5/2),x]
```

```
[Out] (-2*Cot[c + d*x])/(5*b^2*d*Sqrt[b*Tan[c + d*x]^3]) + (2*Cot[c + d*x]^3)/(9*
b^2*d*Sqrt[b*Tan[c + d*x]^3]) - (2*Cot[c + d*x]^5)/(13*b^2*d*Sqrt[b*Tan[c +
 d*x]^3]) + (2*Tan[c + d*x])/(b^2*d*Sqrt[b*Tan[c + d*x]^3]) - (ArcTan[1 - S
qrt[2]*Sqrt[Tan[c + d*x]]]*Tan[c + d*x]^(3/2))/(Sqrt[2]*b^2*d*Sqrt[b*Tan[c
+ d*x]^3]) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]*Tan[c + d*x]^(3/2))/(S
qrt[2]*b^2*d*Sqrt[b*Tan[c + d*x]^3]) + (Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
+ Tan[c + d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*b^2*d*Sqrt[b*Tan[c + d*x]^3]
) - (Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(3/2))
/(2*Sqrt[2]*b^2*d*Sqrt[b*Tan[c + d*x]^3])
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(b\tan^3(c+dx)\right)^{5/2}}\,dx = \frac{\tan^{\frac{3}{2}}(c+dx)\int \frac{1}{\tan^{\frac{15}{2}}(c+dx)}\,dx}{b^2\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{\frac{3}{2}}(c+dx)\int \frac{1}{\tan^{\frac{11}{2}}(c+dx)}\,dx}{b^2\sqrt{b\tan^3(c+dx)}}$$

$$= \frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{\frac{3}{2}}(c+dx)\int \frac{1}{\tan^{\frac{7}{2}}(c+dx)}\,dx}{b^2\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\cot(c+dx)}{5b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{\frac{3}{2}}(c+dx)\int}{b^2\sqrt{b\tan^3}}$$

$$= -\frac{2\cot(c+dx)}{5b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\tan(c+dx}{b^2d\sqrt{b\tan^3(c}}$$

$$= -\frac{2\cot(c+dx)}{5b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\tan(c+dx}{b^2d\sqrt{b\tan^3(c}}$$

$$= -\frac{2\cot(c+dx)}{5b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\tan(c+dx}{b^2d\sqrt{b\tan^3(c}}$$

$$= -\frac{2\cot(c+dx)}{5b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\tan(c+dx}{b^2d\sqrt{b\tan^3(c}}$$

$$= -\frac{2\cot(c+dx)}{5b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\tan(c+dx}{b^2d\sqrt{b\tan^3(c}}$$

$$= -\frac{2\cot(c+dx)}{5b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\tan(c+dx}{b^2d\sqrt{b\tan^3(c}}$$

$$= -\frac{2\cot(c+dx)}{5b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\tan(c+dx}{b^2d\sqrt{b\tan^3(c}}$$

Mathematica [C] time = 0.05845, size = 45, normalized size = 0.12

$$-\frac{2\tan(c+dx)\,{}_2F_1\left(-\frac{13}{4},1;-\frac{9}{4};-\tan^2(c+dx)\right)}{13d\left(b\tan^3(c+dx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^3)^(-5/2),x]
```

```
[Out] (-2*Hypergeometric2F1[-13/4, 1, -9/4, -Tan[c + d*x]^2]*Tan[c + d*x])/(13*d*
(b*Tan[c + d*x]^3)^(5/2))
```

Maple [A] time = 0.026, size = 271, normalized size = 0.7

$$\frac{\tan(dx+c)}{2340\,db^6}\left(585\sqrt{2}\,(b\tan(dx+c))^{13/2}\ln\left(-\frac{\sqrt[4]{b^2}\sqrt{b\tan(dx+c)}\sqrt{2}-b\tan(dx+c)-\sqrt{b^2}}{b\tan(dx+c)+\sqrt[4]{b^2}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+1170\sqrt{2}\,(b\tan(dx+c))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c)^3)^(5/2),x)
```

```
[Out] 1/2340/d*tan(d*x+c)/b^6*(585*2^(1/2)*(b*tan(d*x+c))^(13/2)*ln(-((b^2)^(1/4)
*(b*tan(d*x+c))^(1/2)*2^(1/2)-b*tan(d*x+c)-(b^2)^(1/2))/(b*tan(d*x+c)+(b^2)
^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+1170*2^(1/2)*(b*tan(d*x+c
))^(13/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))-11
70*2^(1/2)*(b*tan(d*x+c))^(13/2)*arctan((-2^(1/2)*(b*tan(d*x+c))^(1/2)+(b^2
)^(1/4))/(b^2)^(1/4))+4680*(b^2)^(1/4)*tan(d*x+c)^6*b^6-936*b^6*(b^2)^(1/4)
*tan(d*x+c)^4+520*b^6*(b^2)^(1/4)*tan(d*x+c)^2-360*b^6*(b^2)^(1/4))/(b*tan(
d*x+c)^3)^(5/2)/(b^2)^(1/4)
```

Maxima [A] time = 1.44309, size = 232, normalized size = 0.64

$$\frac{\frac{585\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan\right.}{b^{\frac{5}{2}}}}{2340\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2340*(585*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c))))
 + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2
)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*log(-sqrt(2)
*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/b^(5/2) + 8*(585*sqrt(b)/sqrt(tan(
d*x + c)) - 117*sqrt(b)/tan(d*x + c)^(5/2) + 65*sqrt(b)/tan(d*x + c)^(9/2)
- 45*sqrt(b)/tan(d*x + c)^(13/2))/b^3)/d
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tan^3\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)**3)**(5/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**3)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tan\left(dx+c\right)^3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^3)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c)^3)^(-5/2), x)
```

3.36 $\int \left(b\tan^4(c+dx)\right)^{5/2} dx$

Optimal. Leaf size=182

$$\frac{b^2\tan^7(c+dx)\sqrt{b\tan^4(c+dx)}}{9d} - \frac{b^2\tan^5(c+dx)\sqrt{b\tan^4(c+dx)}}{7d} + \frac{b^2\tan^3(c+dx)\sqrt{b\tan^4(c+dx)}}{5d} - \frac{b^2\tan(c+dx)}{}$$

```
[Out] (b^2*Cot[c + d*x]*Sqrt[b*Tan[c + d*x]^4])/d - b^2*x*Cot[c + d*x]^2*Sqrt[b*T
an[c + d*x]^4] - (b^2*Tan[c + d*x]*Sqrt[b*Tan[c + d*x]^4])/(3*d) + (b^2*Tan
[c + d*x]^3*Sqrt[b*Tan[c + d*x]^4])/(5*d) - (b^2*Tan[c + d*x]^5*Sqrt[b*Tan[
c + d*x]^4])/(7*d) + (b^2*Tan[c + d*x]^7*Sqrt[b*Tan[c + d*x]^4])/(9*d)
```

Rubi [A] time = 0.0628983, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$\frac{b^2\tan^7(c+dx)\sqrt{b\tan^4(c+dx)}}{9d} - \frac{b^2\tan^5(c+dx)\sqrt{b\tan^4(c+dx)}}{7d} + \frac{b^2\tan^3(c+dx)\sqrt{b\tan^4(c+dx)}}{5d} - \frac{b^2\tan(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^4)^(5/2),x]
```

```
[Out] (b^2*Cot[c + d*x]*Sqrt[b*Tan[c + d*x]^4])/d - b^2*x*Cot[c + d*x]^2*Sqrt[b*T
an[c + d*x]^4] - (b^2*Tan[c + d*x]*Sqrt[b*Tan[c + d*x]^4])/(3*d) + (b^2*Tan
[c + d*x]^3*Sqrt[b*Tan[c + d*x]^4])/(5*d) - (b^2*Tan[c + d*x]^5*Sqrt[b*Tan[
c + d*x]^4])/(7*d) + (b^2*Tan[c + d*x]^7*Sqrt[b*Tan[c + d*x]^4])/(9*d)
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(b\tan^4(c+dx)\right)^{5/2}\,dx = \left(b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\right)\int \tan^{10}(c+dx)\,dx$$
$$= \frac{b^2\tan^7(c+dx)\sqrt{b\tan^4(c+dx)}}{9d} - \left(b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\right)\int \tan^8(c+dx)\,dx$$
$$= -\frac{b^2\tan^5(c+dx)\sqrt{b\tan^4(c+dx)}}{7d} + \frac{b^2\tan^7(c+dx)\sqrt{b\tan^4(c+dx)}}{9d} + \left(b^2\cot^2(c+dx)\sqrt{b\,\mathrm{ta}}\right.$$
$$= \frac{b^2\tan^3(c+dx)\sqrt{b\tan^4(c+dx)}}{5d} - \frac{b^2\tan^5(c+dx)\sqrt{b\tan^4(c+dx)}}{7d} + \frac{b^2\tan^7(c+dx)\sqrt{b\,\mathrm{tan}}}{9d}$$
$$= -\frac{b^2\tan(c+dx)\sqrt{b\tan^4(c+dx)}}{3d} + \frac{b^2\tan^3(c+dx)\sqrt{b\tan^4(c+dx)}}{5d} - \frac{b^2\tan^5(c+dx)\sqrt{b\,\mathrm{tan}}}{7d}$$
$$= \frac{b^2\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d} - \frac{b^2\tan(c+dx)\sqrt{b\tan^4(c+dx)}}{3d} + \frac{b^2\tan^3(c+dx)\sqrt{b\tan^4(}}{5d}$$
$$= \frac{b^2\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d} - b^2x\cot^2(c+dx)\sqrt{b\tan^4(c+dx)} - \frac{b^2\tan(c+dx)\sqrt{b\tan^4(}}{3d}$$

Mathematica [A] time = 0.766847, size = 86, normalized size = 0.47

$$\frac{\cot(c+dx)\left(b\tan^4(c+dx)\right)^{5/2}\left(315\cot^8(c+dx)-105\cot^6(c+dx)+63\cot^4(c+dx)-45\cot^2(c+dx)-315\tan^{-1}(\tan\right.}{315d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^4)^(5/2),x]
```

```
[Out] (Cot[c + d*x]*(35 - 45*Cot[c + d*x]^2 + 63*Cot[c + d*x]^4 - 105*Cot[c + d*x
]^6 + 315*Cot[c + d*x]^8 - 315*ArcTan[Tan[c + d*x]]*Cot[c + d*x]^9)*(b*Tan[
c + d*x]^4)^(5/2))/(315*d)
```

Maple [A] time = 0.036, size = 84, normalized size = 0.5

$$-\frac{-35\,(\tan(dx+c))^9+45\,(\tan(dx+c))^7-63\,(\tan(dx+c))^5+105\,(\tan(dx+c))^3+315\arctan(\tan(dx+c))-315}{315\,d\,(\tan(dx+c))^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^4)^(5/2),x)
```

```
[Out] -1/315/d*(b*tan(d*x+c)^4)^(5/2)*(-35*tan(d*x+c)^9+45*tan(d*x+c)^7-63*tan(d*
x+c)^5+105*tan(d*x+c)^3+315*arctan(tan(d*x+c))-315*tan(d*x+c))/tan(d*x+c)^1
0
```

Maxima [A] time = 1.38655, size = 107, normalized size = 0.59

$$\frac{35\,b^{\frac{5}{2}}\tan(dx+c)^9-45\,b^{\frac{5}{2}}\tan(dx+c)^7+63\,b^{\frac{5}{2}}\tan(dx+c)^5-105\,b^{\frac{5}{2}}\tan(dx+c)^3-315\,(dx+c)b^{\frac{5}{2}}+315\,b^{\frac{5}{2}}\tan(dx}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)^4*b)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/315*(35*b^(5/2)*tan(d*x + c)^9 - 45*b^(5/2)*tan(d*x + c)^7 + 63*b^(5/2)*t
an(d*x + c)^5 - 105*b^(5/2)*tan(d*x + c)^3 - 315*(d*x + c)*b^(5/2) + 315*b^
(5/2)*tan(d*x + c))/d
```

Fricas [A] time = 1.55799, size = 247, normalized size = 1.36

$$\frac{\left(35\,b^2\tan(dx+c)^9-45\,b^2\tan(dx+c)^7+63\,b^2\tan(dx+c)^5-105\,b^2\tan(dx+c)^3-315\,b^2dx+315\,b^2\tan(dx+c)\right)}{315\,d\tan(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)^4*b)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^2*tan(d*x + c)^9 - 45*b^2*tan(d*x + c)^7 + 63*b^2*tan(d*x + c)^
5 - 105*b^2*tan(d*x + c)^3 - 315*b^2*d*x + 315*b^2*tan(d*x + c))*sqrt(b*tan
```

```
(d*x + c)^4)/(d*tan(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan^4\left(c+dx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)**4*b)**(5/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**4)**(5/2), x)
```

Giac [B] time = 10.3889, size = 1296, normalized size = 7.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)^4*b)^(5/2),x, algorithm="giac")
```

```
[Out] -1/315*(315*b^2*d*x*tan(d*x)^9*tan(c)^9 - 2835*b^2*d*x*tan(d*x)^8*tan(c)^8
+ 315*b^2*tan(d*x)^9*tan(c)^8 + 315*b^2*tan(d*x)^8*tan(c)^9 + 11340*b^2*d*x
*tan(d*x)^7*tan(c)^7 - 105*b^2*tan(d*x)^9*tan(c)^6 - 2835*b^2*tan(d*x)^8*ta
n(c)^7 - 2835*b^2*tan(d*x)^7*tan(c)^8 - 105*b^2*tan(d*x)^6*tan(c)^9 - 26460
*b^2*d*x*tan(d*x)^6*tan(c)^6 + 63*b^2*tan(d*x)^9*tan(c)^4 + 945*b^2*tan(d*x
)^8*tan(c)^5 + 11340*b^2*tan(d*x)^7*tan(c)^6 + 11340*b^2*tan(d*x)^6*tan(c)^
7 + 945*b^2*tan(d*x)^5*tan(c)^8 + 63*b^2*tan(d*x)^4*tan(c)^9 + 39690*b^2*d*
x*tan(d*x)^5*tan(c)^5 - 45*b^2*tan(d*x)^9*tan(c)^2 - 567*b^2*tan(d*x)^8*tan
(c)^3 - 3780*b^2*tan(d*x)^7*tan(c)^4 - 26460*b^2*tan(d*x)^6*tan(c)^5 - 2646
0*b^2*tan(d*x)^5*tan(c)^6 - 3780*b^2*tan(d*x)^4*tan(c)^7 - 567*b^2*tan(d*x)
^3*tan(c)^8 - 45*b^2*tan(d*x)^2*tan(c)^9 - 39690*b^2*d*x*tan(d*x)^4*tan(c)^
4 + 35*b^2*tan(d*x)^9 + 405*b^2*tan(d*x)^8*tan(c) + 2268*b^2*tan(d*x)^7*tan
(c)^2 + 8820*b^2*tan(d*x)^6*tan(c)^3 + 39690*b^2*tan(d*x)^5*tan(c)^4 + 3969
0*b^2*tan(d*x)^4*tan(c)^5 + 8820*b^2*tan(d*x)^3*tan(c)^6 + 2268*b^2*tan(d*x
)^2*tan(c)^7 + 405*b^2*tan(d*x)*tan(c)^8 + 35*b^2*tan(c)^9 + 26460*b^2*d*x*
tan(d*x)^3*tan(c)^3 - 45*b^2*tan(d*x)^7 - 567*b^2*tan(d*x)^6*tan(c) - 3780*
b^2*tan(d*x)^5*tan(c)^2 - 26460*b^2*tan(d*x)^4*tan(c)^3 - 26460*b^2*tan(d*x
)^3*tan(c)^4 - 3780*b^2*tan(d*x)^2*tan(c)^5 - 567*b^2*tan(d*x)*tan(c)^6 - 4
5*b^2*tan(c)^7 - 11340*b^2*d*x*tan(d*x)^2*tan(c)^2 + 63*b^2*tan(d*x)^5 + 94
```

```
5*b^2*tan(d*x)^4*tan(c) + 11340*b^2*tan(d*x)^3*tan(c)^2 + 11340*b^2*tan(d*x
)^2*tan(c)^3 + 945*b^2*tan(d*x)*tan(c)^4 + 63*b^2*tan(c)^5 + 2835*b^2*d*x*t
an(d*x)*tan(c) - 105*b^2*tan(d*x)^3 - 2835*b^2*tan(d*x)^2*tan(c) - 2835*b^2
*tan(d*x)*tan(c)^2 - 105*b^2*tan(c)^3 - 315*b^2*d*x + 315*b^2*tan(d*x) + 31
5*b^2*tan(c))*sqrt(b)/(d*tan(d*x)^9*tan(c)^9 - 9*d*tan(d*x)^8*tan(c)^8 + 36
*d*tan(d*x)^7*tan(c)^7 - 84*d*tan(d*x)^6*tan(c)^6 + 126*d*tan(d*x)^5*tan(c)
^5 - 126*d*tan(d*x)^4*tan(c)^4 + 84*d*tan(d*x)^3*tan(c)^3 - 36*d*tan(d*x)^2
*tan(c)^2 + 9*d*tan(d*x)*tan(c) - d)
```

3.37 $\int \left(b\tan^4(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=110

$$\frac{b\tan^3(c+dx)\sqrt{b\tan^4(c+dx)}}{5d}-\frac{b\tan(c+dx)\sqrt{b\tan^4(c+dx)}}{3d}-bx\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}+\frac{b\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d}$$

```
[Out] (b*Cot[c + d*x]*Sqrt[b*Tan[c + d*x]^4])/d - b*x*Cot[c + d*x]^2*Sqrt[b*Tan[c
 + d*x]^4] - (b*Tan[c + d*x]*Sqrt[b*Tan[c + d*x]^4])/(3*d) + (b*Tan[c + d*x
]^3*Sqrt[b*Tan[c + d*x]^4])/(5*d)
```

Rubi [A] time = 0.0427885, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$\frac{b\tan^3(c+dx)\sqrt{b\tan^4(c+dx)}}{5d}-\frac{b\tan(c+dx)\sqrt{b\tan^4(c+dx)}}{3d}-bx\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}+\frac{b\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^4)^(3/2),x]
```

```
[Out] (b*Cot[c + d*x]*Sqrt[b*Tan[c + d*x]^4])/d - b*x*Cot[c + d*x]^2*Sqrt[b*Tan[c
 + d*x]^4] - (b*Tan[c + d*x]*Sqrt[b*Tan[c + d*x]^4])/(3*d) + (b*Tan[c + d*x
]^3*Sqrt[b*Tan[c + d*x]^4])/(5*d)
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(b\tan^4(c+dx)\right)^{3/2}\,dx = \left(b\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\right)\int \tan^6(c+dx)\,dx$$

$$= \frac{b\tan^3(c+dx)\sqrt{b\tan^4(c+dx)}}{5d} - \left(b\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\right)\int \tan^4(c+dx)\,dx$$

$$= -\frac{b\tan(c+dx)\sqrt{b\tan^4(c+dx)}}{3d} + \frac{b\tan^3(c+dx)\sqrt{b\tan^4(c+dx)}}{5d} + \left(b\cot^2(c+dx)\sqrt{b\tan$$

$$= \frac{b\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d} - \frac{b\tan(c+dx)\sqrt{b\tan^4(c+dx)}}{3d} + \frac{b\tan^3(c+dx)\sqrt{b\tan^4(c}{5d}$$

$$= \frac{b\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d} - bx\cot^2(c+dx)\sqrt{b\tan^4(c+dx)} - \frac{b\tan(c+dx)\sqrt{b\tan^4(c}{3d}$$

Mathematica [A] time = 0.743299, size = 66, normalized size = 0.6

$$\frac{\cot(c+dx)\left(b\tan^4(c+dx)\right)^{3/2}\left(15\cot^4(c+dx)-5\cot^2(c+dx)-15\tan^{-1}(\tan(c+dx))\cot^5(c+dx)+3\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^4)^(3/2),x]
```

```
[Out] (Cot[c + d*x]*(3 - 5*Cot[c + d*x]^2 + 15*Cot[c + d*x]^4 - 15*ArcTan[Tan[c +
 d*x]]*Cot[c + d*x]^5)*(b*Tan[c + d*x]^4)^(3/2))/(15*d)
```

Maple [A] time = 0.015, size = 64, normalized size = 0.6

$$-\frac{-3\,(\tan(dx+c))^5+5\,(\tan(dx+c))^3+15\,\arctan(\tan(dx+c))-15\,\tan(dx+c)}{15\,d\,(\tan(dx+c))^6}\left(b\,(\tan(dx+c))^4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^4)^(3/2),x)
```

```
[Out] -1/15/d*(b*tan(d*x+c)^4)^(3/2)*(-3*tan(d*x+c)^5+5*tan(d*x+c)^3+15*arctan(ta
n(d*x+c))-15*tan(d*x+c))/tan(d*x+c)^6
```

Maxima [A] time = 1.41117, size = 72, normalized size = 0.65

$$\frac{3\,b^{\frac{3}{2}}\tan\left(dx+c\right)^5-5\,b^{\frac{3}{2}}\tan\left(dx+c\right)^3-15\,(dx+c)b^{\frac{3}{2}}+15\,b^{\frac{3}{2}}\tan\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)^4*b)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*b^(3/2)*tan(d*x + c)^5 - 5*b^(3/2)*tan(d*x + c)^3 - 15*(d*x + c)*b^
(3/2) + 15*b^(3/2)*tan(d*x + c))/d
```

Fricas [A] time = 1.41854, size = 163, normalized size = 1.48

$$\frac{\left(3\,b\tan\left(dx+c\right)^5-5\,b\tan\left(dx+c\right)^3-15\,bdx+15\,b\tan\left(dx+c\right)\right)\sqrt{b\tan\left(dx+c\right)^4}}{15\,d\tan\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)^4*b)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b*tan(d*x + c)^5 - 5*b*tan(d*x + c)^3 - 15*b*d*x + 15*b*tan(d*x + c
))*sqrt(b*tan(d*x + c)^4)/(d*tan(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan^4\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)**4*b)**(3/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**4)**(3/2), x)
```

Giac [B] time = 6.32711, size = 1339, normalized size = 12.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)^4*b)^(3/2),x, algorithm="giac")
```

```
[Out] 1/60*(15*pi - 60*d*x*tan(d*x)^5*tan(c)^5 - 15*pi*sgn(2*tan(d*x)^2*tan(c) +
2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*tan(c)^5 - 15*pi*ta
n(d*x)^5*tan(c)^5 + 30*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*ta
n(d*x)^5*tan(c)^5 + 30*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*ta
n(d*x)^5*tan(c)^5 + 300*d*x*tan(d*x)^4*tan(c)^4 + 75*pi*sgn(2*tan(d*x)^2*ta
n(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 7
5*pi*tan(d*x)^4*tan(c)^4 - 150*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan
(c)))*tan(d*x)^4*tan(c)^4 - 150*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c)
 - 1))*tan(d*x)^4*tan(c)^4 - 60*tan(d*x)^5*tan(c)^4 - 60*tan(d*x)^4*tan(c)^
5 - 600*d*x*tan(d*x)^3*tan(c)^3 - 150*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*
x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^3*tan(c)^3 + 20*tan(d*x)^5*ta
n(c)^2 - 150*pi*tan(d*x)^3*tan(c)^3 + 300*arctan((tan(d*x)*tan(c) - 1)/(tan
(d*x) + tan(c)))*tan(d*x)^3*tan(c)^3 + 300*arctan((tan(d*x) + tan(c))/(tan(
d*x)*tan(c) - 1))*tan(d*x)^3*tan(c)^3 + 300*tan(d*x)^4*tan(c)^3 + 300*tan(d
*x)^3*tan(c)^4 + 20*tan(d*x)^2*tan(c)^5 + 600*d*x*tan(d*x)^2*tan(c)^2 + 150
*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*
tan(d*x)^2*tan(c)^2 - 12*tan(d*x)^5 - 100*tan(d*x)^4*tan(c) + 150*pi*tan(d*
x)^2*tan(c)^2 - 300*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d
*x)^2*tan(c)^2 - 300*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(
d*x)^2*tan(c)^2 - 600*tan(d*x)^3*tan(c)^2 - 600*tan(d*x)^2*tan(c)^3 - 100*t
an(d*x)*tan(c)^4 - 12*tan(c)^5 - 300*d*x*tan(d*x)*tan(c) - 75*pi*sgn(2*tan(
d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)*tan(c
) + 20*tan(d*x)^3 - 75*pi*tan(d*x)*tan(c) + 150*arctan((tan(d*x)*tan(c) - 1
)/(tan(d*x) + tan(c)))*tan(d*x)*tan(c) + 150*arctan((tan(d*x) + tan(c))/(ta
n(d*x)*tan(c) - 1))*tan(d*x)*tan(c) + 300*tan(d*x)^2*tan(c) + 300*tan(d*x)*
tan(c)^2 + 20*tan(c)^3 + 60*d*x + 15*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x
)*tan(c)^2 - 2*tan(d*x) - 2*tan(c)) - 30*arctan((tan(d*x)*tan(c) - 1)/(tan(
d*x) + tan(c))) - 30*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1)) - 60
*tan(d*x) - 60*tan(c))*b^(3/2)/(d*tan(d*x)^5*tan(c)^5 - 5*d*tan(d*x)^4*tan(
c)^4 + 10*d*tan(d*x)^3*tan(c)^3 - 10*d*tan(d*x)^2*tan(c)^2 + 5*d*tan(d*x)*t
```

```
an(c) - d)
```

3.38 $\int \sqrt{b \tan^4(c+dx)}\,dx$

Optimal. Leaf size=50

$$\frac{\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d}-x\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}$$

```
[Out] (Cot[c + d*x]*Sqrt[b*Tan[c + d*x]^4])/d - x*Cot[c + d*x]^2*Sqrt[b*Tan[c + d
*x]^4]
```

Rubi [A] time = 0.020658, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$\frac{\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d}-x\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[c + d*x]^4],x]
```

```
[Out] (Cot[c + d*x]*Sqrt[b*Tan[c + d*x]^4])/d - x*Cot[c + d*x]^2*Sqrt[b*Tan[c + d
*x]^4]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\tan^4(c+dx)}\,dx &= \left(\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\right)\int \tan^2(c+dx)\,dx\\ &= \frac{\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d}-\left(\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\right)\int 1\,dx\\ &= \frac{\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d}-x\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\end{aligned}$$

Mathematica [A] time = 0.0884631, size = 41, normalized size = 0.82

$$-\frac{\cot(c+dx)\sqrt{b\tan^4(c+dx)}\left(\tan^{-1}(\tan(c+dx))\cot(c+dx)-1\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[c + d*x]^4],x]
```

```
[Out] -((Cot[c + d*x]*(-1 + ArcTan[Tan[c + d*x]]*Cot[c + d*x])*Sqrt[b*Tan[c + d*x
]^4])/d)
```

Maple [A] time = 0.022, size = 42, normalized size = 0.8

$$-\frac{-\tan(dx+c)+\arctan(\tan(dx+c))}{d(\tan(dx+c))^2}\sqrt{b(\tan(dx+c))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^4)^(1/2),x)
```

```
[Out] -1/d*(b*tan(d*x+c)^4)^(1/2)*(-tan(d*x+c)+arctan(tan(d*x+c)))/tan(d*x+c)^2
```

Maxima [A] time = 1.49983, size = 35, normalized size = 0.7

$$-\frac{(dx+c)\sqrt{b}-\sqrt{b}\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)^4*b)^(1/2),x, algorithm="maxima")
```

```
[Out] -((d*x + c)*sqrt(b) - sqrt(b)*tan(d*x + c))/d
```

Fricas [A] time = 1.34459, size = 88, normalized size = 1.76

$$-\frac{\sqrt{b\tan(dx+c)^4}(dx-\tan(dx+c))}{d\tan(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)^4*b)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(b*tan(d*x + c)^4)*(d*x - tan(d*x + c))/(d*tan(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\tan^4(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)**4*b)**(1/2),x)
```

```
[Out] Integral(sqrt(b*tan(c + d*x)**4), x)
```

Giac [B] time = 1.42438, size = 309, normalized size = 6.18

$$\frac{\left(\pi-4\,dx\tan(dx)\tan(c)-\pi\mathrm{sgn}\left(2\tan(dx)^2\tan(c)+2\tan(dx)\tan(c)^2-2\tan(dx)-2\tan(c)\right)\tan(dx)\tan(c)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)^4*b)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(pi - 4*d*x*tan(d*x)*tan(c) - pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*t
an(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)*tan(c) - pi*tan(d*x)*tan(c) + 2*a
rctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d*x)*tan(c) + 2*arctan
((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)*tan(c) + 4*d*x + pi*sg
n(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c)) - 2*ar
ctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c))) - 2*arctan((tan(d*x) + tan(
c))/(tan(d*x)*tan(c) - 1)) - 4*tan(d*x) - 4*tan(c))*sqrt(b)/(d*tan(d*x)*tan
(c) - d)
```

3.39 $\int \frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx$

Optimal. Leaf size=51

$$-\frac{x\tan^2(c+dx)}{\sqrt{b\tan^4(c+dx)}}-\frac{\tan(c+dx)}{d\sqrt{b\tan^4(c+dx)}}$$

```
[Out] -(Tan[c + d*x]/(d*Sqrt[b*Tan[c + d*x]^4])) - (x*Tan[c + d*x]^2)/Sqrt[b*Tan[
c + d*x]^4]
```

Rubi [A] time = 0.0217256, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$-\frac{x\tan^2(c+dx)}{\sqrt{b\tan^4(c+dx)}}-\frac{\tan(c+dx)}{d\sqrt{b\tan^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Tan[c + d*x]^4],x]
```


```
[Out] -(Tan[c + d*x]/(d*Sqrt[b*Tan[c + d*x]^4])) - (x*Tan[c + d*x]^2)/Sqrt[b*Tan[
c + d*x]^4]
```


Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx = \frac{\tan^2(c+dx)\int \cot^2(c+dx)\,dx}{\sqrt{b\tan^4(c+dx)}}$$
$$= -\frac{\tan(c+dx)}{d\sqrt{b\tan^4(c+dx)}} - \frac{\tan^2(c+dx)\int 1\,dx}{\sqrt{b\tan^4(c+dx)}}$$
$$= -\frac{\tan(c+dx)}{d\sqrt{b\tan^4(c+dx)}} - \frac{x\tan^2(c+dx)}{\sqrt{b\tan^4(c+dx)}}$$

Mathematica [C] time = 0.0549894, size = 43, normalized size = 0.84

$$-\frac{\tan(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)}{d\sqrt{b\tan^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Tan[c + d*x]^4],x]
```

```
[Out] -((Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2]*Tan[c + d*x])/(d*Sqrt[b
*Tan[c + d*x]^4]))
```

Maple [A] time = 0.026, size = 40, normalized size = 0.8

$$-\frac{\tan(dx+c)\left(\arctan(\tan(dx+c))\tan(dx+c)+1\right)}{d}\frac{1}{\sqrt{b\left(\tan(dx+c)\right)^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c)^4)^(1/2),x)
```

```
[Out] -1/d*tan(d*x+c)*(arctan(tan(d*x+c))*tan(d*x+c)+1)/(b*tan(d*x+c)^4)^(1/2)
```

Maxima [A] time = 1.40506, size = 36, normalized size = 0.71

$$-\frac{\frac{dx+c}{\sqrt{b}}+\frac{1}{\sqrt{b}\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(tan(d*x+c)^4*b)^(1/2),x, algorithm="maxima")
```

```
[Out] -((d*x + c)/sqrt(b) + 1/(sqrt(b)*tan(d*x + c)))/d
```

Fricas [A] time = 1.34884, size = 93, normalized size = 1.82

$$-\frac{\sqrt{b\tan\left(dx+c\right)^{4}}\left(dx\tan\left(dx+c\right)+1\right)}{bd\tan\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(tan(d*x+c)^4*b)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(b*tan(d*x + c)^4)*(d*x*tan(d*x + c) + 1)/(b*d*tan(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{b\tan^{4}\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(tan(d*x+c)**4*b)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*tan(c + d*x)**4), x)
```

Giac [A] time = 1.53974, size = 61, normalized size = 1.2

$$-\frac{\frac{2(dx+c)}{\sqrt{b}}-\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{b}}+\frac{1}{\sqrt{b}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(tan(d*x+c)^4*b)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*(d*x + c)/sqrt(b) - tan(1/2*d*x + 1/2*c)/sqrt(b) + 1/(sqrt(b)*tan(1
/2*d*x + 1/2*c)))/d
```

3.40 $\int \frac{1}{\left(b\tan^4(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=119

$$-\frac{x\tan^2(c+dx)}{b\sqrt{b\tan^4(c+dx)}}-\frac{\tan(c+dx)}{bd\sqrt{b\tan^4(c+dx)}}-\frac{\cot^3(c+dx)}{5bd\sqrt{b\tan^4(c+dx)}}+\frac{\cot(c+dx)}{3bd\sqrt{b\tan^4(c+dx)}}$$

```
[Out] Cot[c + d*x]/(3*b*d*Sqrt[b*Tan[c + d*x]^4]) - Cot[c + d*x]^3/(5*b*d*Sqrt[b*
Tan[c + d*x]^4]) - Tan[c + d*x]/(b*d*Sqrt[b*Tan[c + d*x]^4]) - (x*Tan[c + d
*x]^2)/(b*Sqrt[b*Tan[c + d*x]^4])
```

Rubi [A] time = 0.0430769, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$-\frac{x\tan^2(c+dx)}{b\sqrt{b\tan^4(c+dx)}}-\frac{\tan(c+dx)}{bd\sqrt{b\tan^4(c+dx)}}-\frac{\cot^3(c+dx)}{5bd\sqrt{b\tan^4(c+dx)}}+\frac{\cot(c+dx)}{3bd\sqrt{b\tan^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^4)^(-3/2),x]
```

```
[Out] Cot[c + d*x]/(3*b*d*Sqrt[b*Tan[c + d*x]^4]) - Cot[c + d*x]^3/(5*b*d*Sqrt[b*
Tan[c + d*x]^4]) - Tan[c + d*x]/(b*d*Sqrt[b*Tan[c + d*x]^4]) - (x*Tan[c + d
*x]^2)/(b*Sqrt[b*Tan[c + d*x]^4])
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(b\tan^4(c+dx)\right)^{3/2}}\,dx &= \frac{\tan^2(c+dx)\int \cot^6(c+dx)\,dx}{b\sqrt{b\tan^4(c+dx)}}\\ &= -\frac{\cot^3(c+dx)}{5bd\sqrt{b\tan^4(c+dx)}} - \frac{\tan^2(c+dx)\int \cot^4(c+dx)\,dx}{b\sqrt{b\tan^4(c+dx)}}\\ &= \frac{\cot(c+dx)}{3bd\sqrt{b\tan^4(c+dx)}} - \frac{\cot^3(c+dx)}{5bd\sqrt{b\tan^4(c+dx)}} + \frac{\tan^2(c+dx)\int \cot^2(c+dx)\,dx}{b\sqrt{b\tan^4(c+dx)}}\\ &= \frac{\cot(c+dx)}{3bd\sqrt{b\tan^4(c+dx)}} - \frac{\cot^3(c+dx)}{5bd\sqrt{b\tan^4(c+dx)}} - \frac{\tan(c+dx)}{bd\sqrt{b\tan^4(c+dx)}} - \frac{\tan^2(c+dx)\int 1\,dx}{b\sqrt{b\tan^4(c+dx)}}\\ &= \frac{\cot(c+dx)}{3bd\sqrt{b\tan^4(c+dx)}} - \frac{\cot^3(c+dx)}{5bd\sqrt{b\tan^4(c+dx)}} - \frac{\tan(c+dx)}{bd\sqrt{b\tan^4(c+dx)}} - \frac{x\tan^2(c+dx)}{b\sqrt{b\tan^4(c+dx)}}\end{aligned}$$

Mathematica [C] time = 0.04952, size = 45, normalized size = 0.38

$$-\frac{\tan(c+dx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(c+dx)\right)}{5d\left(b\tan^4(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^4)^(-3/2),x]
```

```
[Out] -(Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x]^2]*Tan[c + d*x])/(5*d*(b*T
an[c + d*x]^4)^(3/2))
```

Maple [A] time = 0.02, size = 63, normalized size = 0.5

$$-\frac{\tan(dx+c)\left(15\arctan(\tan(dx+c))(\tan(dx+c))^5+15(\tan(dx+c))^4-5(\tan(dx+c))^2+3\right)}{15d}\left(b(\tan(dx+c))^4\right)^-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c)^4)^(3/2),x)
```

```
[Out] -1/15/d*tan(d*x+c)*(15*arctan(tan(d*x+c))*tan(d*x+c)^5+15*tan(d*x+c)^4-5*ta
n(d*x+c)^2+3)/(b*tan(d*x+c)^4)^(3/2)
```

Maxima [A] time = 1.40729, size = 68, normalized size = 0.57

$$-\frac{\frac{15\,(dx+c)}{b^{\frac{3}{2}}}+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{b^{\frac{3}{2}}\,\tan(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(tan(d*x+c)^4*b)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*(d*x + c)/b^(3/2) + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/(b
^(3/2)*tan(d*x + c)^5))/d
```

Fricas [A] time = 1.37621, size = 162, normalized size = 1.36

$$-\frac{\left(15\,dx\tan\left(dx+c\right)^5+15\,\tan\left(dx+c\right)^4-5\,\tan\left(dx+c\right)^2+3\right)\sqrt{b\tan\left(dx+c\right)^4}}{15\,b^2d\tan\left(dx+c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(tan(d*x+c)^4*b)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(15*d*x*tan(d*x + c)^5 + 15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)*sq
rt(b*tan(d*x + c)^4)/(b^2*d*tan(d*x + c)^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(b\tan^4\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(tan(d*x+c)**4*b)**(3/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**4)**(-3/2), x)
```

Giac [A] time = 1.75923, size = 167, normalized size = 1.4

$$-\frac{\frac{480\,(dx+c)}{\sqrt{b}}-\frac{3\,b^{\frac{9}{2}}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-35\,b^{\frac{9}{2}}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+330\,b^{\frac{9}{2}}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{b^{5}}+\frac{330\,\sqrt{b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-35\,\sqrt{b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+3\,\sqrt{b}}{b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}}}{480\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(tan(d*x+c)^4*b)^(3/2),x, algorithm="giac")
```

```
[Out] -1/480*(480*(d*x + c)/sqrt(b) - (3*b^(9/2)*tan(1/2*d*x + 1/2*c)^5 - 35*b^(9
/2)*tan(1/2*d*x + 1/2*c)^3 + 330*b^(9/2)*tan(1/2*d*x + 1/2*c))/b^5 + (330*s
qrt(b)*tan(1/2*d*x + 1/2*c)^4 - 35*sqrt(b)*tan(1/2*d*x + 1/2*c)^2 + 3*sqrt(
b))/(b*tan(1/2*d*x + 1/2*c)^5))/(b*d)
```

3.41 $\int \frac{1}{\left(b\tan^4(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=183

$$-\frac{x\tan^2(c+dx)}{b^2\sqrt{b\tan^4(c+dx)}}-\frac{\tan(c+dx)}{b^2d\sqrt{b\tan^4(c+dx)}}-\frac{\cot^7(c+dx)}{9b^2d\sqrt{b\tan^4(c+dx)}}+\frac{\cot^5(c+dx)}{7b^2d\sqrt{b\tan^4(c+dx)}}-\frac{\cot^3(c+dx)}{5b^2d\sqrt{b\tan^4(c+dx)}}+$$

```
[Out] Cot[c + d*x]/(3*b^2*d*Sqrt[b*Tan[c + d*x]^4]) - Cot[c + d*x]^3/(5*b^2*d*Sqr
t[b*Tan[c + d*x]^4]) + Cot[c + d*x]^5/(7*b^2*d*Sqrt[b*Tan[c + d*x]^4]) - Co
t[c + d*x]^7/(9*b^2*d*Sqrt[b*Tan[c + d*x]^4]) - Tan[c + d*x]/(b^2*d*Sqrt[b*
Tan[c + d*x]^4]) - (x*Tan[c + d*x]^2)/(b^2*Sqrt[b*Tan[c + d*x]^4])
```

Rubi [A] time = 0.0640348, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$-\frac{x\tan^2(c+dx)}{b^2\sqrt{b\tan^4(c+dx)}}-\frac{\tan(c+dx)}{b^2d\sqrt{b\tan^4(c+dx)}}-\frac{\cot^7(c+dx)}{9b^2d\sqrt{b\tan^4(c+dx)}}+\frac{\cot^5(c+dx)}{7b^2d\sqrt{b\tan^4(c+dx)}}-\frac{\cot^3(c+dx)}{5b^2d\sqrt{b\tan^4(c+dx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^4)^(-5/2),x]
```


```
[Out] Cot[c + d*x]/(3*b^2*d*Sqrt[b*Tan[c + d*x]^4]) - Cot[c + d*x]^3/(5*b^2*d*Sqr
t[b*Tan[c + d*x]^4]) + Cot[c + d*x]^5/(7*b^2*d*Sqrt[b*Tan[c + d*x]^4]) - Co
t[c + d*x]^7/(9*b^2*d*Sqrt[b*Tan[c + d*x]^4]) - Tan[c + d*x]/(b^2*d*Sqrt[b*
Tan[c + d*x]^4]) - (x*Tan[c + d*x]^2)/(b^2*Sqrt[b*Tan[c + d*x]^4])
```


Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(b\tan^4(c+dx)\right)^{5/2}}\,dx &= \frac{\tan^2(c+dx)\int \cot^{10}(c+dx)\,dx}{b^2\sqrt{b\tan^4(c+dx)}} \\
&= -\frac{\cot^7(c+dx)}{9b^2d\sqrt{b\tan^4(c+dx)}} - \frac{\tan^2(c+dx)\int \cot^{8}(c+dx)\,dx}{b^2\sqrt{b\tan^4(c+dx)}} \\
&= \frac{\cot^5(c+dx)}{7b^2d\sqrt{b\tan^4(c+dx)}} - \frac{\cot^7(c+dx)}{9b^2d\sqrt{b\tan^4(c+dx)}} + \frac{\tan^2(c+dx)\int \cot^{6}(c+dx)\,dx}{b^2\sqrt{b\tan^4(c+dx)}} \\
&= -\frac{\cot^3(c+dx)}{5b^2d\sqrt{b\tan^4(c+dx)}} + \frac{\cot^5(c+dx)}{7b^2d\sqrt{b\tan^4(c+dx)}} - \frac{\cot^7(c+dx)}{9b^2d\sqrt{b\tan^4(c+dx)}} - \frac{\tan^2(c+dx)\int \mathrm{co}}{b^2\sqrt{b\tan^4(}} \\
&= \frac{\cot(c+dx)}{3b^2d\sqrt{b\tan^4(c+dx)}} - \frac{\cot^3(c+dx)}{5b^2d\sqrt{b\tan^4(c+dx)}} + \frac{\cot^5(c+dx)}{7b^2d\sqrt{b\tan^4(c+dx)}} - \frac{\cot^7(c+dx)}{9b^2d\sqrt{b\tan^4(c+d}} \\
&= \frac{\cot(c+dx)}{3b^2d\sqrt{b\tan^4(c+dx)}} - \frac{\cot^3(c+dx)}{5b^2d\sqrt{b\tan^4(c+dx)}} + \frac{\cot^5(c+dx)}{7b^2d\sqrt{b\tan^4(c+dx)}} - \frac{\cot^7(c+dx)}{9b^2d\sqrt{b\tan^4(c+d}} \\
&= \frac{\cot(c+dx)}{3b^2d\sqrt{b\tan^4(c+dx)}} - \frac{\cot^3(c+dx)}{5b^2d\sqrt{b\tan^4(c+dx)}} + \frac{\cot^5(c+dx)}{7b^2d\sqrt{b\tan^4(c+dx)}} - \frac{\cot^7(c+dx)}{9b^2d\sqrt{b\tan^4(c+d}}
\end{aligned}
$$

Mathematica [C] time = 0.0326356, size = 45, normalized size = 0.25

$$
-\frac{\tan(c+dx)\,{}_2F_1\left(-\frac{9}{2},1;-\frac{7}{2};-\tan^2(c+dx)\right)}{9d\left(b\tan^4(c+dx)\right)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^4)^(-5/2),x]
```

```
[Out] -(Hypergeometric2F1[-9/2, 1, -7/2, -Tan[c + d*x]^2]*Tan[c + d*x])/(9*d*(b*T
an[c + d*x]^4)^(5/2))
```

Maple [A] time = 0.021, size = 83, normalized size = 0.5

$$-\frac{\tan(dx+c)\left(315\arctan(\tan(dx+c))(\tan(dx+c))^9+315(\tan(dx+c))^8-105(\tan(dx+c))^6+63(\tan(dx+c)}{315d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c)^4)^(5/2),x)
```

```
[Out] -1/315/d*tan(d*x+c)*(315*arctan(tan(d*x+c))*tan(d*x+c)^9+315*tan(d*x+c)^8-1
05*tan(d*x+c)^6+63*tan(d*x+c)^4-45*tan(d*x+c)^2+35)/(b*tan(d*x+c)^4)^(5/2)
```

Maxima [A] time = 1.40725, size = 95, normalized size = 0.52

$$-\frac{\frac{315(dx+c)}{b^{\frac{5}{2}}}+\frac{315\tan(dx+c)^8-105\tan(dx+c)^6+63\tan(dx+c)^4-45\tan(dx+c)^2+35}{b^{\frac{5}{2}}\tan(dx+c)^9}}{315d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(tan(d*x+c)^4*b)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/315*(315*(d*x + c)/b^(5/2) + (315*tan(d*x + c)^8 - 105*tan(d*x + c)^6 +
63*tan(d*x + c)^4 - 45*tan(d*x + c)^2 + 35)/(b^(5/2)*tan(d*x + c)^9))/d
```

Fricas [A] time = 1.38451, size = 225, normalized size = 1.23

$$-\frac{\left(315\,dx\tan(dx+c)^9+315\tan(dx+c)^8-105\tan(dx+c)^6+63\tan(dx+c)^4-45\tan(dx+c)^2+35\right)\sqrt{b\tan(dx}}{315\,b^3d\tan(dx+c)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(tan(d*x+c)^4*b)^(5/2),x, algorithm="fricas")
```

[Out] -1/315*(315*d*x*tan(d*x + c)^9 + 315*tan(d*x + c)^8 - 105*tan(d*x + c)^6 + 63*tan(d*x + c)^4 - 45*tan(d*x + c)^2 + 35)*sqrt(b*tan(d*x + c)^4)/(b^3*d*tan(d*x + c)^11)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tan^4\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(tan(d*x+c)**4*b)**(5/2),x)

[Out] Integral((b*tan(c + d*x)**4)**(-5/2), x)

Giac [A] time = 3.0874, size = 250, normalized size = 1.37

$$-\frac{\frac{161280\,(dx+c)}{b^{\frac{5}{2}}}+\frac{121590\,\sqrt{b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-18480\,\sqrt{b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+3528\,\sqrt{b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-495\,\sqrt{b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+35\,\sqrt{b}}{b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}}-\frac{35\,b^{\frac{49}{2}}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{}}{161280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(tan(d*x+c)^4*b)^(5/2),x, algorithm="giac")

[Out] -1/161280*(161280*(d*x + c)/b^(5/2) + (121590*sqrt(b)*tan(1/2*d*x + 1/2*c)^8 - 18480*sqrt(b)*tan(1/2*d*x + 1/2*c)^6 + 3528*sqrt(b)*tan(1/2*d*x + 1/2*c)^4 - 495*sqrt(b)*tan(1/2*d*x + 1/2*c)^2 + 35*sqrt(b))/(b^3*tan(1/2*d*x + 1/2*c)^9) - (35*b^(49/2)*tan(1/2*d*x + 1/2*c)^9 - 495*b^(49/2)*tan(1/2*d*x + 1/2*c)^7 + 3528*b^(49/2)*tan(1/2*d*x + 1/2*c)^5 - 18480*b^(49/2)*tan(1/2*d*x + 1/2*c)^3 + 121590*b^(49/2)*tan(1/2*d*x + 1/2*c))/b^27)/d

3.42 $\int (b\tan^p(c+dx))^n\,dx$

Optimal. Leaf size=59

$$\frac{\tan(c+dx)\,(b\tan^p(c+dx))^n\ {}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(c+dx)\right)}{d(np+1)}$$

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]*(b*Tan[c + d*x]^p)^n)/(d*(1 + n*p))
```

Rubi [A] time = 0.0392326, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3659, 3476, 364}

$$\frac{\tan(c+dx)\,(b\tan^p(c+dx))^n\ {}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(c+dx)\right)}{d(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^p)^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]*(b*Tan[c + d*x]^p)^n)/(d*(1 + n*p))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (b\tan^p(c+dx))^n\,dx &= \left(\tan^{-np}(c+dx)\,(b\tan^p(c+dx))^n\right)\int \tan^{np}(c+dx)\,dx\\ &= \frac{\left(\tan^{-np}(c+dx)\,(b\tan^p(c+dx))^n\right)\text{Subst}\left(\int \frac{x^{np}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}\\ &= \frac{{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(c+dx)\right)\tan(c+dx)\,(b\tan^p(c+dx))^n}{d(1+np)}\end{aligned}$$

Mathematica [A] time = 0.0477217, size = 57, normalized size = 0.97

$$\frac{\tan(c+dx)\,(b\tan^p(c+dx))^n\ {}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(c+dx)\right)}{dnp+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^p)^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]*(b*Tan[c + d*x]^p)^n)/(d + d*n*p)
```

Maple [F] time = 12.661, size = 0, normalized size = 0.

$$\int \left(b\,(\tan(dx+c))^p\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^p)^n,x)
```

```
[Out] int((b*tan(d*x+c)^p)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \tan \left(dx + c\right)^{p}\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^p)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c)^p)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \tan \left(dx + c\right)^{p}\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^p)^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c)^p)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \tan^{p}{\left(c + dx \right)}\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)**p)**n,x)
```

```
[Out] Integral((b*tan(c + d*x)**p)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \tan \left(dx + c\right)^{p}\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^p)^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c)^p)^n, x)
```

3.43 $\int \left(b\tan^2(c+dx)\right)^n\,dx$

Optimal. Leaf size=59

$$\frac{\tan(c+dx)\left(b\tan^2(c+dx)\right)^n\,{}_2F_1\left(1,\frac{1}{2}(2n+1);\frac{1}{2}(2n+3);-\tan^2(c+dx)\right)}{d(2n+1)}$$

```
[Out] (Hypergeometric2F1[1, (1 + 2*n)/2, (3 + 2*n)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]*(b*Tan[c + d*x]^2)^n)/(d*(1 + 2*n))
```

Rubi [A] time = 0.0379199, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3658, 3476, 364}

$$\frac{\tan(c+dx)\left(b\tan^2(c+dx)\right)^n\,{}_2F_1\left(1,\frac{1}{2}(2n+1);\frac{1}{2}(2n+3);-\tan^2(c+dx)\right)}{d(2n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^2)^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + 2*n)/2, (3 + 2*n)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]*(b*Tan[c + d*x]^2)^n)/(d*(1 + 2*n))
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left(b\tan^2(c+dx)\right)^n\,dx &= \left(\tan^{-2n}(c+dx)\left(b\tan^2(c+dx)\right)^n\right)\int \tan^{2n}(c+dx)\,dx\\ &= \frac{\left(\tan^{-2n}(c+dx)\left(b\tan^2(c+dx)\right)^n\right)\text{Subst}\left(\int \frac{x^{2n}}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{{}_2F_1\left(1,\frac{1}{2}(1+2n);\frac{1}{2}(3+2n);-\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^2(c+dx)\right)^n}{d(1+2n)}\end{aligned}$$

Mathematica [A] time = 0.0455909, size = 49, normalized size = 0.83

$$\frac{\tan(c+dx)\left(b\tan^2(c+dx)\right)^n\,{}_2F_1\left(1,n+\frac{1}{2};n+\frac{3}{2};-\tan^2(c+dx)\right)}{2dn+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^2)^n,x]
```

```
[Out] (Hypergeometric2F1[1, 1/2 + n, 3/2 + n, -Tan[c + d*x]^2]*Tan[c + d*x]*(b*Ta
n[c + d*x]^2)^n)/(d + 2*d*n)
```

Maple [F] time = 0.265, size = 0, normalized size = 0.

$$\int \left(b\left(\tan(dx+c)\right)^2\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^2)^n,x)
```

```
[Out] int((b*tan(d*x+c)^2)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(dx+c\right)^{2}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^2)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c)^2)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan\left(dx+c\right)^{2}\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^2)^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c)^2)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan^{2}\left(c+dx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)**2)**n,x)
```

```
[Out] Integral((b*tan(c + d*x)**2)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(dx+c\right)^{2}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^2)^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c)^2)^n, x)
```

3.44 $\int \left(b\tan^3(c+dx)\right)^n\,dx$

Optimal. Leaf size=57

$$\frac{\tan(c+dx)\left(b\tan^3(c+dx)\right)^n\,{}_2F_1\left(1,\frac{1}{2}(3n+1);\frac{3(n+1)}{2};-\tan^2(c+dx)\right)}{d(3n+1)}$$

```
[Out] (Hypergeometric2F1[1, (1 + 3*n)/2, (3*(1 + n))/2, -Tan[c + d*x]^2]*Tan[c +
d*x]*(b*Tan[c + d*x]^3)^n)/(d*(1 + 3*n))
```

Rubi [A] time = 0.0380648, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3658, 3476, 364}

$$\frac{\tan(c+dx)\left(b\tan^3(c+dx)\right)^n\,{}_2F_1\left(1,\frac{1}{2}(3n+1);\frac{3(n+1)}{2};-\tan^2(c+dx)\right)}{d(3n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^3)^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + 3*n)/2, (3*(1 + n))/2, -Tan[c + d*x]^2]*Tan[c +
d*x]*(b*Tan[c + d*x]^3)^n)/(d*(1 + 3*n))
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left(b\tan^3(c+dx)\right)^n\,dx &= \left(\tan^{-3n}(c+dx)\left(b\tan^3(c+dx)\right)^n\right)\int \tan^{3n}(c+dx)\,dx\\ &= \frac{\left(\tan^{-3n}(c+dx)\left(b\tan^3(c+dx)\right)^n\right)\operatorname{Subst}\left(\int \frac{x^{3n}}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{{}_2F_1\left(1,\frac{1}{2}(1+3n);\frac{3(1+n)}{2};-\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^3(c+dx)\right)^n}{d(1+3n)}\end{aligned}$$

Mathematica [A] time = 0.0386418, size = 55, normalized size = 0.96

$$\frac{\tan(c+dx)\left(b\tan^3(c+dx)\right)^n\,{}_2F_1\left(1,\frac{1}{2}(3n+1);\frac{3(n+1)}{2};-\tan^2(c+dx)\right)}{3dn+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^3)^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + 3*n)/2, (3*(1 + n))/2, -Tan[c + d*x]^2]*Tan[c +
d*x]*(b*Tan[c + d*x]^3)^n)/(d + 3*d*n)
```

Maple [F] time = 0.484, size = 0, normalized size = 0.

$$\int \left(b\left(\tan\left(dx+c\right)\right)^3\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^3)^n,x)
```

```
[Out] int((b*tan(d*x+c)^3)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \tan (dx + c)^3\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^3)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c)^3)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \tan (dx + c)^3\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^3)^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c)^3)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \tan^3 (c + dx)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)**3)**n,x)
```

```
[Out] Integral((b*tan(c + d*x)**3)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \tan (dx + c)^3\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^3)^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c)^3)^n, x)
```

3.45 $\int \left(b\tan^4(c+dx)\right)^n\,dx$

Optimal. Leaf size=59

$$\frac{\tan(c+dx)\left(b\tan^4(c+dx)\right)^n\,{}_2F_1\left(1,\frac{1}{2}(4n+1);\frac{1}{2}(4n+3);-\tan^2(c+dx)\right)}{d(4n+1)}$$

```
[Out] (Hypergeometric2F1[1, (1 + 4*n)/2, (3 + 4*n)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]*(b*Tan[c + d*x]^4)^n)/(d*(1 + 4*n))
```

Rubi [A] time = 0.0396004, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3658, 3476, 364}

$$\frac{\tan(c+dx)\left(b\tan^4(c+dx)\right)^n\,{}_2F_1\left(1,\frac{1}{2}(4n+1);\frac{1}{2}(4n+3);-\tan^2(c+dx)\right)}{d(4n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^4)^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + 4*n)/2, (3 + 4*n)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]*(b*Tan[c + d*x]^4)^n)/(d*(1 + 4*n))
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(b\tan^4(c+dx)\right)^n\,dx=\left(\tan^{-4n}(c+dx)\left(b\tan^4(c+dx)\right)^n\right)\int \tan^{4n}(c+dx)\,dx$$
$$=\frac{\left(\tan^{-4n}(c+dx)\left(b\tan^4(c+dx)\right)^n\right)\text{Subst}\left(\int \frac{x^{4n}}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}$$
$$=\frac{{}_2F_1\left(1,\frac{1}{2}(1+4n);\frac{1}{2}(3+4n);-\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^4(c+dx)\right)^n}{d(1+4n)}$$

Mathematica [A] time = 0.0365274, size = 53, normalized size = 0.9

$$\frac{\tan(c+dx)\left(b\tan^4(c+dx)\right)^n\,{}_2F_1\left(1,2n+\frac{1}{2};2n+\frac{3}{2};-\tan^2(c+dx)\right)}{4dn+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^4)^n,x]
```

```
[Out] (Hypergeometric2F1[1, 1/2 + 2*n, 3/2 + 2*n, -Tan[c + d*x]^2]*Tan[c + d*x]*(
b*Tan[c + d*x]^4)^n)/(d + 4*d*n)
```

Maple [F] time = 0.262, size = 0, normalized size = 0.

$$\int \left(b\left(\tan(dx+c)\right)^4\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^4)^n,x)
```

```
[Out] int((b*tan(d*x+c)^4)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \tan \left(dx + c\right)^{4}\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)^4*b)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c)^4)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \tan \left(dx + c\right)^{4}\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)^4*b)^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c)^4)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \tan^{4}\left(c + dx\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)**4*b)**n,x)
```

```
[Out] Integral((b*tan(c + d*x)**4)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \tan \left(dx + c\right)^{4}\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)^4*b)^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c)^4)^n, x)
```

3.46 $\int (b\tan^p(c+dx))^{5/2}\,dx$

Optimal. Leaf size=71

$$\frac{2b^2\tan^{2p+1}(c+dx)\sqrt{b\tan^p(c+dx)}\,{}_2F_1\left(1,\frac{1}{4}(5p+2);\frac{1}{4}(5p+6);-\tan^2(c+dx)\right)}{d(5p+2)}$$

```
[Out] (2*b^2*Hypergeometric2F1[1, (2 + 5*p)/4, (6 + 5*p)/4, -Tan[c + d*x]^2]*Tan[
c + d*x]^(1 + 2*p)*Sqrt[b*Tan[c + d*x]^p])/(d*(2 + 5*p))
```

Rubi [A] time = 0.0484212, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{2b^2\tan^{2p+1}(c+dx)\sqrt{b\tan^p(c+dx)}\,{}_2F_1\left(1,\frac{1}{4}(5p+2);\frac{1}{4}(5p+6);-\tan^2(c+dx)\right)}{d(5p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^p)^(5/2),x]
```

```
[Out] (2*b^2*Hypergeometric2F1[1, (2 + 5*p)/4, (6 + 5*p)/4, -Tan[c + d*x]^2]*Tan[
c + d*x]^(1 + 2*p)*Sqrt[b*Tan[c + d*x]^p])/(d*(2 + 5*p))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (b\tan^p(c+dx))^{5/2}\,dx = \left(b^2\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\right)\int \tan^{\frac{5p}{2}}(c+dx)\,dx$$
$$= \frac{\left(b^2\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\right)\text{Subst}\left(\int \frac{x^{5p/2}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{2b^2\,{}_2F_1\left(1,\frac{1}{4}(2+5p);\frac{1}{4}(6+5p);-\tan^2(c+dx)\right)\tan^{1+2p}(c+dx)\sqrt{b\tan^p(c+dx)}}{d(2+5p)}$$

Mathematica [A] time = 0.0978874, size = 62, normalized size = 0.87

$$\frac{2\tan(c+dx)\,(b\tan^p(c+dx))^{5/2}\,{}_2F_1\left(1,\frac{1}{4}(5p+2);\frac{1}{4}(5p+6);-\tan^2(c+dx)\right)}{d(5p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^p)^(5/2),x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 + 5*p)/4, (6 + 5*p)/4, -Tan[c + d*x]^2]*Tan[c +
d*x]*(b*Tan[c + d*x]^p)^(5/2))/(d*(2 + 5*p))
```

Maple [F] time = 180., size = 0, normalized size = 0.

hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^p)^(5/2),x)
```

```
[Out] int((b*tan(d*x+c)^p)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \tan (dx + c)^p\right)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^p)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c)^p)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^p)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)**p)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \tan (dx + c)^p\right)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^p)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c)^p)^(5/2), x)
```

3.47 $\int (b\tan^p(c+dx))^{3/2}\,dx$

Optimal. Leaf size=65

$$\frac{2b\tan^{p+1}(c+dx)\sqrt{b\tan^p(c+dx)}\,{}_2F_1\left(1,\frac{1}{4}(3p+2);\frac{3(p+2)}{4};-\tan^2(c+dx)\right)}{d(3p+2)}$$

```
[Out] (2*b*Hypergeometric2F1[1, (2 + 3*p)/4, (3*(2 + p))/4, -Tan[c + d*x]^2]*Tan[
c + d*x]^(1 + p)*Sqrt[b*Tan[c + d*x]^p])/(d*(2 + 3*p))
```

Rubi [A] time = 0.0449443, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{2b\tan^{p+1}(c+dx)\sqrt{b\tan^p(c+dx)}\,{}_2F_1\left(1,\frac{1}{4}(3p+2);\frac{3(p+2)}{4};-\tan^2(c+dx)\right)}{d(3p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^p)^(3/2),x]
```

```
[Out] (2*b*Hypergeometric2F1[1, (2 + 3*p)/4, (3*(2 + p))/4, -Tan[c + d*x]^2]*Tan[
c + d*x]^(1 + p)*Sqrt[b*Tan[c + d*x]^p])/(d*(2 + 3*p))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (b\tan^p(c+dx))^{3/2}\,dx &= \left(b\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\right)\int \tan^{\frac{3p}{2}}(c+dx)\,dx\\ &= \frac{\left(b\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\right)\text{Subst}\left(\int \frac{x^{3p/2}}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{2b\,{}_2F_1\left(1,\frac{1}{4}(2+3p);\frac{3(2+p)}{4};-\tan^2(c+dx)\right)\tan^{1+p}(c+dx)\sqrt{b\tan^p(c+dx)}}{d(2+3p)}\end{aligned}$$

Mathematica [A] time = 0.0647902, size = 60, normalized size = 0.92

$$\frac{2\tan(c+dx)\,(b\tan^p(c+dx))^{3/2}\,{}_2F_1\left(1,\frac{1}{4}(3p+2);\frac{3(p+2)}{4};-\tan^2(c+dx)\right)}{d(3p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^p)^(3/2),x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 + 3*p)/4, (3*(2 + p))/4, -Tan[c + d*x]^2]*Tan[c
+ d*x]*(b*Tan[c + d*x]^p)^(3/2))/(d*(2 + 3*p))
```

Maple [F] time = 0.421, size = 0, normalized size = 0.

$$\int \left(b\left(\tan(dx+c)\right)^p\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^p)^(3/2),x)
```

```
[Out] int((b*tan(d*x+c)^p)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(dx+c\right)^{p}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^p)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c)^p)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^p)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan^{p}\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)**p)**(3/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**p)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(dx+c\right)^{p}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^p)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c)^p)^(3/2), x)
```

3.48 $\int \sqrt{b\tan^p(c+dx)}\,dx$

Optimal. Leaf size=56

$$\frac{2\tan(c+dx)\sqrt{b\tan^p(c+dx)}\,{}_2F_1\left(1,\frac{p+2}{4};\frac{p+6}{4};-\tan^2(c+dx)\right)}{d(p+2)}$$

```
[Out] (2*Hypergeometric2F1[1, (2 + p)/4, (6 + p)/4, -Tan[c + d*x]^2]*Tan[c + d*x]
*Sqrt[b*Tan[c + d*x]^p])/(d*(2 + p))
```

Rubi [A] time = 0.0415078, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{2\tan(c+dx)\sqrt{b\tan^p(c+dx)}\,{}_2F_1\left(1,\frac{p+2}{4};\frac{p+6}{4};-\tan^2(c+dx)\right)}{d(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[c + d*x]^p],x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 + p)/4, (6 + p)/4, -Tan[c + d*x]^2]*Tan[c + d*x]
*Sqrt[b*Tan[c + d*x]^p])/(d*(2 + p))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\tan^p(c+dx)}\,dx &= \left(\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\right)\int \tan^{\frac{p}{2}}(c+dx)\,dx\\ &= \frac{\left(\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\right)\operatorname{Subst}\left(\int \frac{x^{p/2}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}\\ &= \frac{2\,{}_2F_1\left(1,\frac{2+p}{4};\frac{6+p}{4};-\tan^2(c+dx)\right)\tan(c+dx)\sqrt{b\tan^p(c+dx)}}{d(2+p)}\end{aligned}$$

Mathematica [A] time = 0.0378778, size = 56, normalized size = 1.

$$\frac{2\tan(c+dx)\sqrt{b\tan^p(c+dx)}\,{}_2F_1\left(1,\frac{p+2}{4};\frac{p+6}{4};-\tan^2(c+dx)\right)}{d(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[c + d*x]^p],x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 + p)/4, (6 + p)/4, -Tan[c + d*x]^2]*Tan[c + d*x]
*Sqrt[b*Tan[c + d*x]^p])/(d*(2 + p))
```

Maple [F] time = 0.121, size = 0, normalized size = 0.

$$\int \sqrt{b\left(\tan\left(dx+c\right)\right)^{p}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^p)^(1/2),x)
```

```
[Out] int((b*tan(d*x+c)^p)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \tan{(dx+c)}^p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^p)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(d*x + c)^p), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^p)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \tan^{p}{\left(c + dx \right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)**p)**(1/2),x)
```

```
[Out] Integral(sqrt(b*tan(c + d*x)**p), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \tan{(dx+c)}^p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^p)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(d*x + c)^p), x)
```

3.49 $\int \frac{1}{\sqrt{b\tan^p(c+dx)}}\,dx$

Optimal. Leaf size=62

$$\frac{2\tan(c+dx)\,{}_2F_1\left(1,\frac{2-p}{4};\frac{6-p}{4};-\tan^2(c+dx)\right)}{d(2-p)\sqrt{b\tan^p(c+dx)}}$$

```
[Out] (2*Hypergeometric2F1[1, (2 - p)/4, (6 - p)/4, -Tan[c + d*x]^2]*Tan[c + d*x]
)/(d*(2 - p)*Sqrt[b*Tan[c + d*x]^p])
```

Rubi [A] time = 0.0485733, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{2\tan(c+dx)\,{}_2F_1\left(1,\frac{2-p}{4};\frac{6-p}{4};-\tan^2(c+dx)\right)}{d(2-p)\sqrt{b\tan^p(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Tan[c + d*x]^p],x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 - p)/4, (6 - p)/4, -Tan[c + d*x]^2]*Tan[c + d*x]
)/(d*(2 - p)*Sqrt[b*Tan[c + d*x]^p])
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{b\tan^p(c+dx)}}\,dx &= \frac{\tan^{\frac{p}{2}}(c+dx)\int \tan^{-\frac{p}{2}}(c+dx)\,dx}{\sqrt{b\tan^p(c+dx)}}\\ &= \frac{\tan^{\frac{p}{2}}(c+dx)\,\text{Subst}\left(\int \frac{x^{-p/2}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d\sqrt{b\tan^p(c+dx)}}\\ &= \frac{2\,{}_2F_1\left(1,\frac{2-p}{4};\frac{6-p}{4};-\tan^2(c+dx)\right)\tan(c+dx)}{d(2-p)\sqrt{b\tan^p(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0485459, size = 60, normalized size = 0.97

$$-\frac{2\tan(c+dx)\,{}_2F_1\left(1,\frac{2-p}{4};\frac{6-p}{4};-\tan^2(c+dx)\right)}{d(p-2)\sqrt{b\tan^p(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Tan[c + d*x]^p],x]
```

```
[Out] (-2*Hypergeometric2F1[1, (2 - p)/4, (6 - p)/4, -Tan[c + d*x]^2]*Tan[c + d*x
])/(d*(-2 + p)*Sqrt[b*Tan[c + d*x]^p])
```

Maple [F] time = 0.126, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\left(\tan\left(dx+c\right)\right)^p}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c)^p)^(1/2),x)
```

```
[Out] int(1/(b*tan(d*x+c)^p)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\tan{(dx+c)}^p}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^p)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*tan(d*x + c)^p), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^p)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\tan^{p}{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)**p)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*tan(c + d*x)**p), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \tan(dx+c)^p}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^p)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*tan(d*x + c)^p), x)
```

3.50 $\int \frac{1}{\left(b\tan^p(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=71

$$\frac{2\tan^{1-p}(c+dx)\,{}_2F_1\left(1,\frac{1}{4}(2-3p);\frac{3(2-p)}{4};-\tan^2(c+dx)\right)}{bd(2-3p)\sqrt{b\tan^p(c+dx)}}$$

```
[Out] (2*Hypergeometric2F1[1, (2 - 3*p)/4, (3*(2 - p))/4, -Tan[c + d*x]^2]*Tan[c
+ d*x]^(1 - p))/(b*d*(2 - 3*p)*Sqrt[b*Tan[c + d*x]^p])
```

Rubi [A] time = 0.0481324, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{2\tan^{1-p}(c+dx)\,{}_2F_1\left(1,\frac{1}{4}(2-3p);\frac{3(2-p)}{4};-\tan^2(c+dx)\right)}{bd(2-3p)\sqrt{b\tan^p(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^p)^(-3/2),x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 - 3*p)/4, (3*(2 - p))/4, -Tan[c + d*x]^2]*Tan[c
+ d*x]^(1 - p))/(b*d*(2 - 3*p)*Sqrt[b*Tan[c + d*x]^p])
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(b\tan^p(c+dx))^{3/2}}\,dx = \frac{\tan^{\frac{p}{2}}(c+dx)\int \tan^{-\frac{3p}{2}}(c+dx)\,dx}{b\sqrt{b\tan^p(c+dx)}}$$
$$= \frac{\tan^{\frac{p}{2}}(c+dx)\,\mathrm{Subst}\left(\int \frac{x^{-3p/2}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{bd\sqrt{b\tan^p(c+dx)}}$$
$$= \frac{2\,{}_2F_1\left(1, \frac{1}{4}(2-3p); \frac{3(2-p)}{4}; -\tan^2(c+dx)\right)\tan^{1-p}(c+dx)}{bd(2-3p)\sqrt{b\tan^p(c+dx)}}$$

Mathematica [A] time = 0.0678254, size = 60, normalized size = 0.85

$$-\frac{2\tan(c+dx)\,{}_2F_1\left(1, \frac{1}{4}(2-3p); -\frac{3}{4}(p-2); -\tan^2(c+dx)\right)}{d(3p-2)\,(b\tan^p(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^p)^(-3/2),x]
```

```
[Out] (-2*Hypergeometric2F1[1, (2 - 3*p)/4, (-3*(-2 + p))/4, -Tan[c + d*x]^2]*Tan
[c + d*x])/(d*(-2 + 3*p)*(b*Tan[c + d*x]^p)^(3/2))
```

Maple [F] time = 0.129, size = 0, normalized size = 0.

$$\int \left(b\left(\tan\left(dx+c\right)\right)^p\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c)^p)^(3/2),x)
```

```
[Out] int(1/(b*tan(d*x+c)^p)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tan\left(dx+c\right)^{p}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^p)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c)^p)^(-3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^p)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tan^{p}\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)**p)**(3/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**p)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tan\left(dx+c\right)^{p}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^p)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c)^p)^(-3/2), x)
```

3.51 $\int \frac{1}{\left(b\tan^p(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=71

$$\frac{2\tan^{1-2p}(c+dx)\,{}_2F_1\left(1,\frac{1}{4}(2-5p);\frac{1}{4}(6-5p);-\tan^2(c+dx)\right)}{b^2d(2-5p)\sqrt{b\tan^p(c+dx)}}$$

```
[Out] (2*Hypergeometric2F1[1, (2 - 5*p)/4, (6 - 5*p)/4, -Tan[c + d*x]^2]*Tan[c +
d*x]^(1 - 2*p))/(b^2*d*(2 - 5*p)*Sqrt[b*Tan[c + d*x]^p])
```

Rubi [A] time = 0.0466852, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{2\tan^{1-2p}(c+dx)\,{}_2F_1\left(1,\frac{1}{4}(2-5p);\frac{1}{4}(6-5p);-\tan^2(c+dx)\right)}{b^2d(2-5p)\sqrt{b\tan^p(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^p)^(-5/2),x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 - 5*p)/4, (6 - 5*p)/4, -Tan[c + d*x]^2]*Tan[c +
d*x]^(1 - 2*p))/(b^2*d*(2 - 5*p)*Sqrt[b*Tan[c + d*x]^p])
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(b\tan^p(c+dx))^{5/2}}\,dx &= \frac{\tan^{\frac{p}{2}}(c+dx)\int \tan^{-\frac{5p}{2}}(c+dx)\,dx}{b^2\sqrt{b\tan^p(c+dx)}}\\ &= \frac{\tan^{\frac{p}{2}}(c+dx)\,\mathrm{Subst}\left(\int \frac{x^{-5p/2}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{b^2 d\sqrt{b\tan^p(c+dx)}}\\ &= \frac{2\,{}_2F_1\left(1,\frac{1}{4}(2-5p);\frac{1}{4}(6-5p);-\tan^2(c+dx)\right)\tan^{1-2p}(c+dx)}{b^2 d(2-5p)\sqrt{b\tan^p(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0675633, size = 62, normalized size = 0.87

$$-\frac{2\tan(c+dx)\,{}_2F_1\left(1,\frac{1}{4}(2-5p);\frac{1}{4}(6-5p);-\tan^2(c+dx)\right)}{d(5p-2)\,(b\tan^p(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^p)^(-5/2),x]
```

```
[Out] (-2*Hypergeometric2F1[1, (2 - 5*p)/4, (6 - 5*p)/4, -Tan[c + d*x]^2]*Tan[c +
 d*x])/(d*(-2 + 5*p)*(b*Tan[c + d*x]^p)^(5/2))
```

Maple [F] time = 0.117, size = 0, normalized size = 0.

$$\int \left(b\left(\tan\left(dx+c\right)\right)^{p}\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c)^p)^(5/2),x)
```

```
[Out] int(1/(b*tan(d*x+c)^p)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tan\left(dx+c\right)^{p}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^p)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c)^p)^(-5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^p)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tan^{p}\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)**p)**(5/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**p)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\tan\left(dx+c\right)^{p}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^p)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c)^p)^(-5/2), x)
```

3.52 $\int (b \tan^p(c+dx))^{\frac{1}{p}}\, dx$

Optimal. Leaf size=32

$$-\frac{\cot(c+dx)\log(\cos(c+dx))\,(b\tan^p(c+dx))^{\frac{1}{p}}}{d}$$

```
[Out] -((Cot[c + d*x]*Log[Cos[c + d*x]]*(b*Tan[c + d*x]^p)^p^(-1))/d)
```

Rubi [A] time = 0.0191736, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3659, 3475}

$$-\frac{\cot(c+dx)\log(\cos(c+dx))\,(b\tan^p(c+dx))^{\frac{1}{p}}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^p)^p^(-1),x]
```

```
[Out] -((Cot[c + d*x]*Log[Cos[c + d*x]]*(b*Tan[c + d*x]^p)^p^(-1))/d)
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (b\tan^p(c+dx))^{\frac{1}{p}}\,dx = \left(\cot(c+dx)\,(b\tan^p(c+dx))^{\frac{1}{p}}\right)\int \tan(c+dx)\,dx$$
$$= -\frac{\cot(c+dx)\log(\cos(c+dx))\,(b\tan^p(c+dx))^{\frac{1}{p}}}{d}$$

Mathematica [A] time = 0.0220582, size = 32, normalized size = 1.

$$-\frac{\cot(c+dx)\log(\cos(c+dx))\,(b\tan^p(c+dx))^{\frac{1}{p}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^p)^p^(-1),x]
```

```
[Out] -((Cot[c + d*x]*Log[Cos[c + d*x]]*(b*Tan[c + d*x]^p)^p^(-1))/d)
```

Maple [C] time = 3.563, size = 18076, normalized size = 564.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^p)^(1/p),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(dx+c\right)^{p}\right)^{\left(\frac{1}{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^p)^(1/p),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c)^p)^(1/p), x)
```

Fricas [A] time = 1.09594, size = 59, normalized size = 1.84

$$-\frac{b^{\left(\frac{1}{p}\right)}\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^p)^(1/p),x, algorithm="fricas")
```

```
[Out] -1/2*b^(1/p)*log(1/(tan(d*x + c)^2 + 1))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tan^p(c+dx))^{\frac{1}{p}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)**p)**(1/p),x)
```

```
[Out] Integral((b*tan(c + d*x)**p)**(1/p), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan(dx+c)^p\right)^{\left(\frac{1}{p}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^p)^(1/p),x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c)^p)^(1/p), x)
```

3.53 $\int (a(b\tan(c+dx))^p)^n \, dx$

Optimal. Leaf size=61

$$\frac{\tan(c+dx)\,{}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(c+dx)\right)(a(b\tan(c+dx))^p)^n}{d(np+1)}$$

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]*(a*(b*Tan[c + d*x])^p)^n)/(d*(1 + n*p))
```

Rubi [A] time = 0.0453724, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{\tan(c+dx)\,{}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(c+dx)\right)(a(b\tan(c+dx))^p)^n}{d(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*(b*Tan[c + d*x])^p)^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]*(a*(b*Tan[c + d*x])^p)^n)/(d*(1 + n*p))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (a(b\tan(c+dx))^p)^n\,dx &= \left((b\tan(c+dx))^{-np}\left(a(b\tan(c+dx))^p\right)^n\right)\int (b\tan(c+dx))^{np}\,dx\\ &= \frac{\left(b(b\tan(c+dx))^{-np}\left(a(b\tan(c+dx))^p\right)^n\right)\mathrm{Subst}\left(\int \frac{x^{np}}{b^2+x^2}\,dx,x,b\tan(c+dx)\right)}{d}\\ &= \frac{{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(c+dx)\right)\tan(c+dx)\left(a(b\tan(c+dx))^p\right)^n}{d(1+np)}\end{aligned}$$

Mathematica [A] time = 0.0494637, size = 59, normalized size = 0.97

$$\frac{\tan(c+dx)\,{}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(c+dx)\right)\left(a(b\tan(c+dx))^p\right)^n}{dnp+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*(b*Tan[c + d*x])^p)^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]*(a*(b*Tan[c + d*x])^p)^n)/(d + d*n*p)
```

Maple [F] time = 5.166, size = 0, normalized size = 0.

$$\int \left(a\left(b\tan(dx+c)\right)^p\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(b*tan(d*x+c))^p)^n,x)
```

```
[Out] int((a*(b*tan(d*x+c))^p)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left((b\tan(dx+c))^p a\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*tan(d*x+c))^p)^n,x, algorithm="maxima")
```

```
[Out] integrate(((b*tan(d*x + c))^p*a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left((b\tan(dx+c))^p a\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*tan(d*x+c))^p)^n,x, algorithm="fricas")
```

```
[Out] integral(((b*tan(d*x + c))^p*a)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a(b\tan(c+dx))^p\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*tan(d*x+c))**p)**n,x)
```

```
[Out] Integral((a*(b*tan(c + d*x))**p)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left((b\tan(dx+c))^p a\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*tan(d*x+c))^p)^n,x, algorithm="giac")
```

```
[Out] integrate(((b*tan(d*x + c))^p*a)^n, x)
```

3.54 $\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal. Leaf size=257

$$-\frac{\cos^4(a+bx)(d\tan(a+bx))^{7/2}}{4bd^3}-\frac{21\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b}+\frac{21\sqrt{d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{32\sqrt{2}b}+\frac{21\sqrt{d}\log\left(\sqrt{d}\,\text{ta}\right.}{}$$

```
[Out] (-21*Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2
]*b) + (21*Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*
Sqrt[2]*b) + (21*Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[
d*Tan[a + b*x]]])/(64*Sqrt[2]*b) - (21*Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[a
+ b*x] + Sqrt[2]*Sqrt[d*Tan[a + b*x]]])/(64*Sqrt[2]*b) - (7*Cos[a + b*x]^2*
(d*Tan[a + b*x])^(3/2))/(16*b*d) - (Cos[a + b*x]^4*(d*Tan[a + b*x])^(7/2))/
(4*b*d^3)
```

Rubi [A] time = 0.195679, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2591, 288, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\cos^4(a+bx)(d\tan(a+bx))^{7/2}}{4bd^3}-\frac{21\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b}+\frac{21\sqrt{d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{32\sqrt{2}b}+\frac{21\sqrt{d}\log\left(\sqrt{d}\,\text{ta}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^4*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-21*Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2
]*b) + (21*Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*
Sqrt[2]*b) + (21*Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[
d*Tan[a + b*x]]])/(64*Sqrt[2]*b) - (21*Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[a
+ b*x] + Sqrt[2]*Sqrt[d*Tan[a + b*x]]])/(64*Sqrt[2]*b) - (7*Cos[a + b*x]^2*
(d*Tan[a + b*x])^(3/2))/(16*b*d) - (Cos[a + b*x]^4*(d*Tan[a + b*x])^(7/2))/
(4*b*d^3)
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx = \frac{d\,\text{Subst}\left(\int \frac{x^{9/2}}{\left(d^2+x^2\right)^3}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= -\frac{\cos^4(a+bx)(d\tan(a+bx))^{7/2}}{4bd^3} + \frac{(7d)\,\text{Subst}\left(\int \frac{x^{5/2}}{\left(d^2+x^2\right)^2}\,dx, x, d\tan(a+bx)\right)}{8b}$$

$$= -\frac{7\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{7/2}}{4bd^3} + \frac{(21d)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{7\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{7/2}}{4bd^3} + \frac{(21d)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{7\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{7/2}}{4bd^3} - \frac{(21d)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{7\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{7/2}}{4bd^3} + \frac{\left(21\sqrt{d}\right)\,\text{Subst}}{}$$

$$= \frac{21\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b} - \frac{21\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(a\right.}{64}$$

$$= -\frac{21\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} + \frac{21\sqrt{d}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} + \frac{21\sqrt{d}\log\left(\sqrt{d}\right.}{}$$

Mathematica [A] time = 0.216264, size = 122, normalized size = 0.47

$$-\frac{\sqrt{d\tan(a+bx)}\left(18\sin(2(a+bx))-2\sin(4(a+bx))+21\sqrt{\sin(2(a+bx))}\csc(a+bx)\sin^{-1}(\cos(a+bx)-\sin(a+bx))\right.}{64b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^4*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] -((21*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Csc[a + b*x]*Sqrt[Sin[2*(a + b*x)
]] + 21*Csc[a + b*x]*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)
]]]*Sqrt[Sin[2*(a + b*x)]] + 18*Sin[2*(a + b*x)] - 2*Sin[4*(a + b*x)])*Sqrt
[d*Tan[a + b*x]])/(64*b)
```

Maple [C] time = 0.279, size = 542, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^4*(d*tan(b*x+a))^(1/2),x)
```

```
[Out] -1/64/b*2^(1/2)*(cos(b*x+a)-1)*(21*I*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-
cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x
+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*
I,1/2*2^(1/2))-21*I*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*
x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*Ellipt
icPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-8*
cos(b*x+a)^4*2^(1/2)+8*cos(b*x+a)^3*2^(1/2)-21*((cos(b*x+a)-1)/sin(b*x+a))^
(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a
))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2
),1/2-1/2*I,1/2*2^(1/2))-21*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a
)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2
)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(
1/2))+22*cos(b*x+a)^2*2^(1/2)-22*cos(b*x+a)*2^(1/2))*(cos(b*x+a)+1)^2*(d*si
n(b*x+a)/cos(b*x+a))^(1/2)/sin(b*x+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**4*(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \tan (bx + a)} \sin (bx + a)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*tan(b*x + a))*sin(b*x + a)^4, x)
```

3.55 $\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal. Leaf size=227

$$-\frac{3\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b}+\frac{3\sqrt{d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}b}+\frac{3\sqrt{d}\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{\ldots}\right.}{8\sqrt{2}b}$$

```
[Out] (-3*Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*
b) + (3*Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt
[2]*b) + (3*Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan
[a + b*x]]])/(8*Sqrt[2]*b) - (3*Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x]
+ Sqrt[2]*Sqrt[d*Tan[a + b*x]]])/(8*Sqrt[2]*b) - (Cos[a + b*x]^2*(d*Tan[a +
 b*x])^(3/2))/(2*b*d)
```

Rubi [A] time = 0.160671, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2591, 288, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{3\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b}+\frac{3\sqrt{d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}b}+\frac{3\sqrt{d}\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{\ldots}\right.}{8\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-3*Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*
b) + (3*Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt
[2]*b) + (3*Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan
[a + b*x]]])/(8*Sqrt[2]*b) - (3*Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x]
+ Sqrt[2]*Sqrt[d*Tan[a + b*x]]])/(8*Sqrt[2]*b) - (Cos[a + b*x]^2*(d*Tan[a +
 b*x])^(3/2))/(2*b*d)
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = \frac{d\,\text{Subst}\left(\int \frac{x^{5/2}}{\left(d^2+x^2\right)^2}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= -\frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd} + \frac{(3d)\,\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\tan(a+bx)\right)}{4b}$$

$$= -\frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd} + \frac{(3d)\,\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{2b}$$

$$= -\frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd} - \frac{(3d)\,\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{4b} + \frac{(3}{}$$

$$= -\frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd} + \frac{\left(3\sqrt{d}\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+}\right.}{8\sqrt{2}b}$$

$$= \frac{3\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} - \frac{3\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(a+\right.}{8\sqrt{}}$$

$$= -\frac{3\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} + \frac{3\sqrt{d}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} + \frac{3\sqrt{d}\log\left(\sqrt{d}\right.}{}$$

Mathematica [A] time = 0.166263, size = 104, normalized size = 0.46

$$-\frac{\sqrt{\sin(2(a+bx))}\sqrt{d\tan(a+bx)}\left(2\sqrt{\sin(2(a+bx))}+3\csc(a+bx)\sin^{-1}(\cos(a+bx)-\sin(a+bx))+3\csc(a+bx)\log\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] -((3*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Csc[a + b*x] + 3*Csc[a + b*x]*Log[
Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]]] + 2*Sqrt[Sin[2*(a + b
*x)]])*Sqrt[Sin[2*(a + b*x)]]*Sqrt[d*Tan[a + b*x]])/(8*b)
```

Maple [C] time = 0.174, size = 524, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*(d*tan(b*x+a))^(1/2),x)
```

```
[Out] 1/8/b*2^(1/2)*(cos(b*x+a)-1)*(3*I*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b
*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a
))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I
,1/2*2^(1/2))-3*I*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+
a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*Ellipti
cPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))+3*
((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1
/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-
1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*((cos(b*x+a)-1)/si
n(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-
1-sin(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(
b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))-2*cos(b*x+a)^2*2^(1/2)+2*cos(b*x+a)*2^
(1/2))*(cos(b*x+a)+1)^2*(d*sin(b*x+a)/cos(b*x+a))^(1/2)/sin(b*x+a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\tan{(a + bx)}}\sin^{2}{(a + bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(d*tan(a + b*x))*sin(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\tan{(bx + a)}}\sin{(bx + a)}^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*tan(b*x + a))*sin(b*x + a)^2, x)
```

3.56 $\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal. Leaf size=18

$$-\frac{2d}{b\sqrt{d\tan(a+bx)}}$$

```
[Out] (-2*d)/(b*Sqrt[d*Tan[a + b*x]])
```

Rubi [A] time = 0.0414481, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2591, 30}

$$-\frac{2d}{b\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*d)/(b*Sqrt[d*Tan[a + b*x]])
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx &= \frac{d\,\text{Subst}\left(\int \frac{1}{x^{3/2}}\,dx, x, d\tan(a+bx)\right)}{b}\\ &= -\frac{2d}{b\sqrt{d\tan(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0727914, size = 18, normalized size = 1.

$$-\frac{2d}{b\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*d)/(b*Sqrt[d*Tan[a + b*x]])
```

Maple [B] time = 0.136, size = 38, normalized size = 2.1

$$-2\,\frac{\cos(bx+a)}{b\sin(bx+a)}\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*(d*tan(b*x+a))^(1/2),x)
```

```
[Out] -2/b*(d*sin(b*x+a)/cos(b*x+a))^(1/2)*cos(b*x+a)/sin(b*x+a)
```

Maxima [A] time = 2.26284, size = 31, normalized size = 1.72

$$-\frac{2\sqrt{d\tan(bx+a)}}{b\tan(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(d*tan(b*x + a))/(b*tan(b*x + a))
```

Fricas [B] time = 1.63919, size = 92, normalized size = 5.11

$$-\frac{2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)}{b\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)/(b*sin(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\tan\left(a+bx\right)}\csc^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(d*tan(a + b*x))*csc(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\tan\left(bx+a\right)}\csc\left(bx+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*tan(b*x + a))*csc(b*x + a)^2, x)
```

3.57 $\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal. Leaf size=41

$$-\frac{2d^3}{5b(d\tan(a+bx))^{5/2}}-\frac{2d}{b\sqrt{d\tan(a+bx)}}$$

```
[Out] (-2*d^3)/(5*b*(d*Tan[a + b*x])^(5/2)) - (2*d)/(b*Sqrt[d*Tan[a + b*x]])
```

Rubi [A] time = 0.0447967, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2591, 14}

$$-\frac{2d^3}{5b(d\tan(a+bx))^{5/2}}-\frac{2d}{b\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^4*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*d^3)/(5*b*(d*Tan[a + b*x])^(5/2)) - (2*d)/(b*Sqrt[d*Tan[a + b*x]])
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx = \frac{d\,\text{Subst}\left(\int \frac{d^2+x^2}{x^{7/2}}\,dx, x, d\tan(a+bx)\right)}{b}$$
$$= \frac{d\,\text{Subst}\left(\int \left(\frac{d^2}{x^{7/2}}+\frac{1}{x^{3/2}}\right)dx, x, d\tan(a+bx)\right)}{b}$$
$$= -\frac{2d^3}{5b(d\tan(a+bx))^{5/2}} - \frac{2d}{b\sqrt{d\tan(a+bx)}}$$

Mathematica [A] time = 0.116051, size = 30, normalized size = 0.73

$$-\frac{2d\left(\csc^2(a+bx)+4\right)}{5b\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^4*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*d*(4 + Csc[a + b*x]^2))/(5*b*Sqrt[d*Tan[a + b*x]])
```

Maple [A] time = 0.181, size = 50, normalized size = 1.2

$$\frac{\left(8\,(\cos(bx+a))^2-10\right)\cos(bx+a)}{5\,b\,(\sin(bx+a))^3}\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^4*(d*tan(b*x+a))^(1/2),x)
```

```
[Out] 2/5/b*(4*cos(b*x+a)^2-5)*cos(b*x+a)*(d*sin(b*x+a)/cos(b*x+a))^(1/2)/sin(b*x
+a)^3
```

Maxima [A] time = 2.99058, size = 45, normalized size = 1.1

$$-\frac{2\left(5\,d^2\tan(bx+a)^2+d^2\right)d}{5\,(d\tan(bx+a))^{\frac{5}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/5*(5*d^2*tan(b*x + a)^2 + d^2)*d/((d*tan(b*x + a))^(5/2)*b)
```

Fricas [A] time = 1.68853, size = 154, normalized size = 3.76

$$-\frac{2\left(4\cos\left(bx+a\right)^{3}-5\cos\left(bx+a\right)\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{5\left(b\cos\left(bx+a\right)^{2}-b\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*(4*cos(b*x + a)^3 - 5*cos(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))/
((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**4*(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\tan\left(bx+a\right)}\csc\left(bx+a\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*tan(b*x + a))*csc(b*x + a)^4, x)
```

3.58 $\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal. Leaf size=63

$$-\frac{2d^5}{9b(d\tan(a+bx))^{9/2}}-\frac{4d^3}{5b(d\tan(a+bx))^{5/2}}-\frac{2d}{b\sqrt{d\tan(a+bx)}}$$

```
[Out] (-2*d^5)/(9*b*(d*Tan[a + b*x])^(9/2)) - (4*d^3)/(5*b*(d*Tan[a + b*x])^(5/2)
) - (2*d)/(b*Sqrt[d*Tan[a + b*x]])
```

Rubi [A] time = 0.0515714, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2591, 270}

$$-\frac{2d^5}{9b(d\tan(a+bx))^{9/2}}-\frac{4d^3}{5b(d\tan(a+bx))^{5/2}}-\frac{2d}{b\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^6*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*d^5)/(9*b*(d*Tan[a + b*x])^(9/2)) - (4*d^3)/(5*b*(d*Tan[a + b*x])^(5/2)
) - (2*d)/(b*Sqrt[d*Tan[a + b*x]])
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx = \frac{d\,\text{Subst}\left(\int \frac{\left(d^2+x^2\right)^2}{x^{11/2}}\,dx, x, d\tan(a+bx)\right)}{b}$$
$$= \frac{d\,\text{Subst}\left(\int \left(\frac{d^4}{x^{11/2}}+\frac{2d^2}{x^{7/2}}+\frac{1}{x^{3/2}}\right)dx, x, d\tan(a+bx)\right)}{b}$$
$$= -\frac{2d^5}{9b(d\tan(a+bx))^{9/2}} - \frac{4d^3}{5b(d\tan(a+bx))^{5/2}} - \frac{2d}{b\sqrt{d\tan(a+bx)}}$$

Mathematica [A] time = 0.158643, size = 50, normalized size = 0.79

$$\frac{2d(20\cos(2(a+bx)) - 4\cos(4(a+bx)) - 21)\csc^4(a+bx)}{45b\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^6*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (2*d*(-21 + 20*Cos[2*(a + b*x)] - 4*Cos[4*(a + b*x)])*Csc[a + b*x]^4)/(45*b
*Sqrt[d*Tan[a + b*x]])
```

Maple [A] time = 0.187, size = 60, normalized size = 1.

$$-\frac{\left(64\,(\cos(bx+a))^4 - 144\,(\cos(bx+a))^2 + 90\right)\cos(bx+a)}{45\,b\,(\sin(bx+a))^5}\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^6*(d*tan(b*x+a))^(1/2),x)
```

```
[Out] -2/45/b*(32*cos(b*x+a)^4-72*cos(b*x+a)^2+45)*cos(b*x+a)*(d*sin(b*x+a)/cos(b
*x+a))^(1/2)/sin(b*x+a)^5
```

Maxima [A] time = 1.21788, size = 65, normalized size = 1.03

$$-\frac{2\left(45\,d^4\tan(bx+a)^4 + 18\,d^4\tan(bx+a)^2 + 5\,d^4\right)d}{45\,(d\tan(bx+a))^{\frac{9}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/45*(45*d^4*tan(b*x + a)^4 + 18*d^4*tan(b*x + a)^2 + 5*d^4)*d/((d*tan(b*x
 + a))^(9/2)*b)
```

Fricas [A] time = 1.79873, size = 213, normalized size = 3.38

$$-\frac{2\left(32\,\cos\left(bx+a\right)^{5}-72\,\cos\left(bx+a\right)^{3}+45\,\cos\left(bx+a\right)\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{45\left(b\cos\left(bx+a\right)^{4}-2\,b\cos\left(bx+a\right)^{2}+b\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/45*(32*cos(b*x + a)^5 - 72*cos(b*x + a)^3 + 45*cos(b*x + a))*sqrt(d*sin(
b*x + a)/cos(b*x + a))/((b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)*sin(b*x
 + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**6*(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{d\tan\left(bx+a\right)}\csc\left(bx+a\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*tan(b*x + a))*csc(b*x + a)^6, x)
```

3.59 $\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal. Leaf size=105

$$-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}-\frac{5d\sin(a+bx)}{6b\sqrt{d\tan(a+bx)}}+\frac{5\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{12b}$$

```
[Out] (-5*d*Sin[a + b*x])/(6*b*Sqrt[d*Tan[a + b*x]]) - (d*Sin[a + b*x]^3)/(3*b*Sq
rt[d*Tan[a + b*x]]) + (5*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin
[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(12*b)
```

Rubi [A] time = 0.133208, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2598, 2601, 2573, 2641}

$$-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}-\frac{5d\sin(a+bx)}{6b\sqrt{d\tan(a+bx)}}+\frac{5\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sqrt[d*Tan[a + b*x]],x]
```


```
[Out] (-5*d*Sin[a + b*x])/(6*b*Sqrt[d*Tan[a + b*x]]) - (d*Sin[a + b*x]^3)/(3*b*Sq
rt[d*Tan[a + b*x]]) + (5*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin
[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(12*b)
```


Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
```

```
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx &= -\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{5}{6}\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx\\ &= -\frac{5d\sin(a+bx)}{6b\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{5}{12}\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx\\ &= -\frac{5d\sin(a+bx)}{6b\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{\left(5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{}{\sqrt{\cos(a}}}{12\sqrt{\sin(a+bx)}}\\ &= -\frac{5d\sin(a+bx)}{6b\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{1}{12}\left(5\csc(a+bx)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(}\right.\\ &= -\frac{5d\sin(a+bx)}{6b\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{5\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+}}{12b}\end{aligned}$$

Mathematica [C] time = 1.82994, size = 139, normalized size = 1.32

$$-\frac{\cos(2(a+bx))\sec(a+bx)\sqrt{d\tan(a+bx)}\left((\cos(2(a+bx))-6)\sqrt{\tan(a+bx)}\sqrt{\sec^2(a+bx)}-5\sqrt[4]{-1}\sec^2(a+bx)F\left(i\sin\right.\right.}{6b\sqrt{\tan(a+bx)}\left(\tan^2(a+bx)-1\right)\sqrt{\sec^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] -(Cos[2*(a + b*x)]*Sec[a + b*x]*(-5*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/
4)*Sqrt[Tan[a + b*x]]], -1]*Sec[a + b*x]^2 + (-6 + Cos[2*(a + b*x)])*Sqrt[S
```

```
ec[a + b*x]^2]*Sqrt[Tan[a + b*x]])*Sqrt[d*Tan[a + b*x]])/(6*b*Sqrt[Sec[a +
b*x]^2]*Sqrt[Tan[a + b*x]]*(-1 + Tan[a + b*x]^2))
```

Maple [A] time = 0.207, size = 216, normalized size = 2.1

$$-\frac{\sqrt{2}\left(\cos\left(bx+a\right)-1\right)\left(\cos\left(bx+a\right)+1\right)^{2}}{12\,b\left(\sin\left(bx+a\right)\right)^{4}}\left(5\,\sin\left(bx+a\right)\sqrt{\frac{\cos\left(bx+a\right)-1}{\sin\left(bx+a\right)}}\sqrt{\frac{1-\cos\left(bx+a\right)+\sin\left(bx+a\right)}{\sin\left(bx+a\right)}}\sqrt{\frac{\cos\left(bx\right.}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*(d*tan(b*x+a))^(1/2),x)
```

```
[Out] -1/12/b*2^(1/2)*(cos(b*x+a)-1)*(5*sin(b*x+a)*((cos(b*x+a)-1)/sin(b*x+a))^(1
/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))
/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1
/2*2^(1/2))-2*cos(b*x+a)^4*2^(1/2)+2*cos(b*x+a)^3*2^(1/2)+7*cos(b*x+a)^2*2^
(1/2)-7*cos(b*x+a)*2^(1/2))*(cos(b*x+a)+1)^2*(d*sin(b*x+a)/cos(b*x+a))^(1/2
)/sin(b*x+a)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\tan\left(bx+a\right)}\sin\left(bx+a\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*tan(b*x + a))*sin(b*x + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos\left(bx+a\right)^{2}-1\right)\sqrt{d\tan\left(bx+a\right)}\sin\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sqrt(d*tan(b*x + a))*sin(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.60 $\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal. Leaf size=75

$$\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{2b}-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}$$

```
[Out] -((d*Sin[a + b*x])/(b*Sqrt[d*Tan[a + b*x]])) + (Csc[a + b*x]*EllipticF[a -
Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(2*b)
```

Rubi [A] time = 0.0901895, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2598, 2601, 2573, 2641}

$$\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{2b}-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] -((d*Sin[a + b*x])/(b*Sqrt[d*Tan[a + b*x]])) + (Csc[a + b*x]*EllipticF[a -
Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(2*b)
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx &= -\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}+\frac{1}{2}\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx\\
&= -\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}+\frac{\left(\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}\,dx}{2\sqrt{\sin(a+bx)}}\\
&= -\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}+\frac{1}{2}\left(\csc(a+bx)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2b}}\\
&= -\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}+\frac{\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{2b}
\end{aligned}$$

Mathematica [C] time = 0.96188, size = 57, normalized size = 0.76

$$\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}\left(\sqrt{\sec^2(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2(a+bx)\right)-1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (Cos[a + b*x]*(-1 + Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[a + b*x]^2]*Sqrt[
Sec[a + b*x]^2])*Sqrt[d*Tan[a + b*x]])/b
```

Maple [B] time = 0.14, size = 190, normalized size = 2.5

$$-\frac{\sqrt{2}\left(\cos(bx+a)-1\right)\left(\cos(bx+a)+1\right)^2}{2\,b\left(\sin(bx+a)\right)^4}\left(\sin(bx+a)\,\mathrm{EllipticF}\left(\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{\cos(bx+a}{\sin(bx+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*(d*tan(b*x+a))^(1/2),x)
```

```
[Out] -1/2/b*2^(1/2)*(cos(b*x+a)-1)*(sin(b*x+a)*EllipticF((-(cos(b*x+a)-1-sin(b*x
+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos
(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x
+a))^(1/2)+cos(b*x+a)^2*2^(1/2)-cos(b*x+a)*2^(1/2))*(cos(b*x+a)+1)^2*(d*sin
(b*x+a)/cos(b*x+a))^(1/2)/sin(b*x+a)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\tan(bx+a)}\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*tan(b*x + a))*sin(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\tan(bx+a)}\sin(bx+a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*sin(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\tan(a+bx)}\sin(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(d*tan(a + b*x))*sin(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\tan(bx+a)}\sin(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*tan(b*x + a))*sin(b*x + a), x)
```

3.61 $\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{b}$$

```
[Out] (Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Ta
n[a + b*x]])/b
```

Rubi [A] time = 0.0681022, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2601, 2573, 2641}

$$\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Ta
n[a + b*x]])/b
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx = \frac{\left(\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}\,dx}{\sqrt{\sin(a+bx)}}$$
$$= \left(\csc(a+bx)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx$$
$$= \frac{\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{b}$$

Mathematica [C] time = 0.14602, size = 73, normalized size = 1.55

$$-\frac{2\sqrt[4]{-1}\cos(a+bx)\sqrt{\sec^2(a+bx)}\sqrt{d\tan(a+bx)}F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right)\middle|-1\right)}{b\sqrt{\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*(-1)^(1/4)*Cos[a + b*x]*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x
]]], -1]*Sqrt[Sec[a + b*x]^2]*Sqrt[d*Tan[a + b*x]])/(b*Sqrt[Tan[a + b*x]])
```

Maple [B] time = 0.151, size = 159, normalized size = 3.4

$$-\frac{\sqrt{2}\left(\cos(bx+a)-1\right)\left(\cos(bx+a)+1\right)^2}{b\left(\sin(bx+a)\right)^3}\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a)-1+\sin(bx}{\sin(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*(d*tan(b*x+a))^(1/2),x)
```

```
[Out] -1/b*2^(1/2)*(d*sin(b*x+a)/cos(b*x+a))^(1/2)*(cos(b*x+a)-1)*(-(cos(b*x+a)-1
-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)
*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/si
```

```
n(b*x+a))^(1/2),1/2*2^(1/2))*(cos(b*x+a)+1)^2/sin(b*x+a)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\tan(bx+a)}\csc(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*tan(b*x + a))*csc(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{d\tan(bx+a)}\csc(bx+a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*csc(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\tan(a+bx)}\csc(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(d*tan(a + b*x))*csc(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\tan(bx+a)}\csc(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*tan(b*x + a))*csc(b*x + a), x)
```

3.62 $\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal. Leaf size=77

$$\frac{2\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{3b}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

```
[Out] (-2*d*Csc[a + b*x])/(3*b*Sqrt[d*Tan[a + b*x]]) + (2*Csc[a + b*x]*EllipticF[
a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(3*b)
```

Rubi [A] time = 0.10574, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {2599, 2601, 2573, 2641}

$$\frac{2\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{3b}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sqrt[d*Tan[a + b*x]],x]
```


```
[Out] (-2*d*Csc[a + b*x])/(3*b*Sqrt[d*Tan[a + b*x]]) + (2*Csc[a + b*x]*EllipticF[
a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(3*b)
```


Rule 2599

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^(n - 1)
)/(a^2*f*(m + n + 1)), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{2}{3}\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$= -\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{\left(2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}\,dx}{3\sqrt{\sin(a+bx)}}$$

$$= -\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{1}{3}\left(2\csc(a+bx)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\sin(2a+}}$$

$$= -\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{2\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{3b}$$

Mathematica [C] time = 0.566555, size = 115, normalized size = 1.49

$$\frac{2\cos(2(a+bx))\csc^3(a+bx)(d\tan(a+bx))^{3/2}\left(\sqrt{\sec^2(a+bx)}+2\sqrt[4]{-1}\tan^{\frac{3}{2}}(a+bx)F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right)\middle|-1\right.\right.}{3bd\left(\tan^2(a+bx)-1\right)\sqrt{\sec^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (2*Cos[2*(a + b*x)]*Csc[a + b*x]^3*(d*Tan[a + b*x])^(3/2)*(Sqrt[Sec[a + b*x
]^2] + 2*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1]
*Tan[a + b*x]^(3/2)))/(3*b*d*Sqrt[Sec[a + b*x]^2]*(-1 + Tan[a + b*x]^2))
```

Maple [B] time = 0.167, size = 301, normalized size = 3.9

$$\frac{\sqrt{2}\left(\cos(bx+a)-1\right)^2\left(\cos(bx+a)+1\right)^2}{3\,b\left(\sin(bx+a)\right)^6}\left(2\cos(bx+a)\sqrt{\frac{\cos(bx+a)-1}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\sqrt{-\frac{\cos(bx}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*(d*tan(b*x+a))^(1/2),x)
```

```
[Out] 1/3/b*2^(1/2)*(cos(b*x+a)-1)^2*(2*cos(b*x+a)*((cos(b*x+a)-1)/sin(b*x+a))^(1
/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a)
)/sin(b*x+a))^(1/2)*sin(b*x+a)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*
x+a))^(1/2),1/2*2^(1/2))+2*sin(b*x+a)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))
/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x
+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))
^(1/2)-cos(b*x+a)*2^(1/2))*(cos(b*x+a)+1)^2*(d*sin(b*x+a)/cos(b*x+a))^(1/2)
/sin(b*x+a)^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\tan(bx+a)}\csc(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*tan(b*x + a))*csc(b*x + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\tan(bx+a)}\csc(bx+a)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*csc(b*x + a)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\tan(bx+a)}\csc(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*tan(b*x + a))*csc(b*x + a)^3, x)
```

3.63 $\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal. Leaf size=105

$$-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}-\frac{4d\csc(a+bx)}{7b\sqrt{d\tan(a+bx)}}+\frac{4\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{7b}$$

```
[Out] (-4*d*Csc[a + b*x])/(7*b*Sqrt[d*Tan[a + b*x]]) - (2*d*Csc[a + b*x]^3)/(7*b*
Sqrt[d*Tan[a + b*x]]) + (4*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[S
in[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(7*b)
```

Rubi [A] time = 0.142963, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2599, 2601, 2573, 2641}

$$-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}-\frac{4d\csc(a+bx)}{7b\sqrt{d\tan(a+bx)}}+\frac{4\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^5*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-4*d*Csc[a + b*x])/(7*b*Sqrt[d*Tan[a + b*x]]) - (2*d*Csc[a + b*x]^3)/(7*b*
Sqrt[d*Tan[a + b*x]]) + (4*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[S
in[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(7*b)
```

Rule 2599

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^(n - 1)
)/(a^2*f*(m + n + 1)), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
```

```
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx &= -\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}+\frac{6}{7}\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx\\ &= -\frac{4d\csc(a+bx)}{7b\sqrt{d\tan(a+bx)}}-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}+\frac{4}{7}\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx\\ &= -\frac{4d\csc(a+bx)}{7b\sqrt{d\tan(a+bx)}}-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}+\frac{\left(4\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{}{\sqrt{\cos(a}}}{7\sqrt{\sin(a+bx)}}\\ &= -\frac{4d\csc(a+bx)}{7b\sqrt{d\tan(a+bx)}}-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}+\frac{1}{7}\left(4\csc(a+bx)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(}\right.\\ &= -\frac{4d\csc(a+bx)}{7b\sqrt{d\tan(a+bx)}}-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}+\frac{4\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+}}{7b}\end{aligned}$$

Mathematica [C] time = 1.42776, size = 124, normalized size = 1.18

$$-\frac{2d\cos(2(a+bx))\csc^3(a+bx)\left((\cos(2(a+bx))-2)\sec^2(a+bx)^{3/2}-4\sqrt[4]{-1}\tan^{\frac{7}{2}}(a+bx)F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx}\right.\right.\right.}{7b\left(\tan^2(a+bx)-1\right)\sqrt{\sec^2(a+bx)}\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^5*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*d*Cos[2*(a + b*x)]*Csc[a + b*x]^3*((-2 + Cos[2*(a + b*x)])*(Sec[a + b*x
]^2)^(3/2) - 4*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]
```

```
], -1]*Tan[a + b*x]^(7/2)))/(7*b*Sqrt[Sec[a + b*x]^2]*Sqrt[d*Tan[a + b*x]]*
(-1 + Tan[a + b*x]^2))
```

Maple [B] time = 0.189, size = 558, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^5*(d*tan(b*x+a))^(1/2),x)
```

```
[Out] -1/7/b*2^(1/2)*(cos(b*x+a)-1)^2*(4*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/si
n(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)
-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1
/2)*cos(b*x+a)^3*sin(b*x+a)+4*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x
+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+si
n(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*c
os(b*x+a)^2*sin(b*x+a)-4*cos(b*x+a)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos
(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x
+a))^(1/2)*sin(b*x+a)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/
2),1/2*2^(1/2))-4*sin(b*x+a)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+
a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin
(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-2*
cos(b*x+a)^3*2^(1/2)+3*cos(b*x+a)*2^(1/2))*(cos(b*x+a)+1)^2*(d*sin(b*x+a)/c
os(b*x+a))^(1/2)/sin(b*x+a)^8
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \tan(bx + a)} \csc(bx + a)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*tan(b*x + a))*csc(b*x + a)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\tan\left(bx+a\right)}\csc\left(bx+a\right)^{5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*csc(b*x + a)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**5*(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{d\tan\left(bx+a\right)}\csc\left(bx+a\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*tan(b*x + a))*csc(b*x + a)^5, x)
```

3.64 $\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=277

$$\frac{45d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b}-\frac{45d^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{32\sqrt{2}b}+\frac{45d^{3/2}\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right.}{64\sqrt{2}b}$$

```
[Out] (45*d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]
*b) - (45*d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*S
qrt[2]*b) + (45*d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d
*Tan[a + b*x]]])/(64*Sqrt[2]*b) - (45*d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a +
 b*x] + Sqrt[2]*Sqrt[d*Tan[a + b*x]]])/(64*Sqrt[2]*b) + (45*d*Sqrt[d*Tan[a
+ b*x]])/(16*b) - (9*Cos[a + b*x]^2*(d*Tan[a + b*x])^(5/2))/(16*b*d) - (Cos
[a + b*x]^4*(d*Tan[a + b*x])^(9/2))/(4*b*d^3)
```

Rubi [A] time = 0.19601, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2591, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{45d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b}-\frac{45d^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{32\sqrt{2}b}+\frac{45d^{3/2}\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right.}{64\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^4*(d*Tan[a + b*x])^(3/2),x]
```


```
[Out] (45*d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]
*b) - (45*d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*S
qrt[2]*b) + (45*d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d
*Tan[a + b*x]]])/(64*Sqrt[2]*b) - (45*d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a +
 b*x] + Sqrt[2]*Sqrt[d*Tan[a + b*x]]])/(64*Sqrt[2]*b) + (45*d*Sqrt[d*Tan[a
+ b*x]])/(16*b) - (9*Cos[a + b*x]^2*(d*Tan[a + b*x])^(5/2))/(16*b*d) - (Cos
[a + b*x]^4*(d*Tan[a + b*x])^(9/2))/(4*b*d^3)
```


Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{d\,\text{Subst}\left(\int \frac{x^{11/2}}{(d^2+x^2)^3}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= -\frac{\cos^4(a+bx)(d\tan(a+bx))^{9/2}}{4bd^3} + \frac{(9d)\,\text{Subst}\left(\int \frac{x^{7/2}}{(d^2+x^2)^2}\,dx, x, d\tan(a+bx)\right)}{8b}$$

$$= -\frac{9\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{9/2}}{4bd^3} + \frac{(45d)\,\text{Subst}}{}$$

$$= \frac{45d\sqrt{d\tan(a+bx)}}{16b} - \frac{9\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+b}{4bd^3}$$

$$= \frac{45d\sqrt{d\tan(a+bx)}}{16b} - \frac{9\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+b}{4bd^3}$$

$$= \frac{45d\sqrt{d\tan(a+bx)}}{16b} - \frac{9\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+b}{4bd^3}$$

$$= \frac{45d\sqrt{d\tan(a+bx)}}{16b} - \frac{9\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+b}{4bd^3}$$

$$= \frac{45d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b} - \frac{45d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan\right.}{}$$

$$= \frac{45d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} - \frac{45d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} + \frac{45d^{3/2}\log\left(\right.}{}$$

Mathematica [A] time = 0.805176, size = 123, normalized size = 0.44

$$-\frac{d\csc(a+bx)\sqrt{d\tan(a+bx)}\left(-143\sin(a+bx)-14\sin(3(a+bx))+\sin(5(a+bx))-45\sqrt{\sin(2(a+bx))}\sin^{-1}(\cos(a+\right.}{64b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^4*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -(d*Csc[a + b*x]*(-143*Sin[a + b*x] - 45*ArcSin[Cos[a + b*x] - Sin[a + b*x]
]*Sqrt[Sin[2*(a + b*x)]] + 45*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*
(a + b*x)]]]*Sqrt[Sin[2*(a + b*x)]] - 14*Sin[3*(a + b*x)] + Sin[5*(a + b*x)
])*Sqrt[d*Tan[a + b*x]])/(64*b)
```

Maple [C] time = 0.227, size = 712, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^4*(d*tan(b*x+a))^(3/2),x)
```

```
[Out] -1/64/b*2^(1/2)*(cos(b*x+a)-1)*(8*2^(1/2)*cos(b*x+a)^5+45*I*sin(b*x+a)*Elli
pticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))
*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(
1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-45*I*sin(b*x+a)*Elliptic
Pi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((c
os(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)
*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-8*cos(b*x+a)^4*2^(1/2)-45*si
n(b*x+a)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I
,1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/
sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-45*sin(b*x+
a)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2
^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*
x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)+90*sin(b*x+a)*Ell
ipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x
+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(co
s(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-34*cos(b*x+a)^3*2^(1/2)+34*cos(b*x
+a)^2*2^(1/2)-64*cos(b*x+a)*2^(1/2)+64*2^(1/2))*cos(b*x+a)*(cos(b*x+a)+1)^2
*(d*sin(b*x+a)/cos(b*x+a))^(3/2)/sin(b*x+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**4*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^{\frac{3}{2}}\sin(bx+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*sin(b*x + a)^4, x)
```

3.65 $\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=247

$$\frac{5d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b}-\frac{5d^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}b}+\frac{5d^{3/2}\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}b}$$

```
[Out] (5*d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*b
) - (5*d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[
2]*b) + (5*d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[
a + b*x]]])/(8*Sqrt[2]*b) - (5*d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] +
 Sqrt[2]*Sqrt[d*Tan[a + b*x]]])/(8*Sqrt[2]*b) + (5*d*Sqrt[d*Tan[a + b*x]])/
(2*b) - (Cos[a + b*x]^2*(d*Tan[a + b*x])^(5/2))/(2*b*d)
```

Rubi [A] time = 0.175975, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2591, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{5d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b}-\frac{5d^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}b}+\frac{5d^{3/2}\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```


```
[Out] (5*d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*b
) - (5*d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[
2]*b) + (5*d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[
a + b*x]]])/(8*Sqrt[2]*b) - (5*d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] +
 Sqrt[2]*Sqrt[d*Tan[a + b*x]]])/(8*Sqrt[2]*b) + (5*d*Sqrt[d*Tan[a + b*x]])/
(2*b) - (Cos[a + b*x]^2*(d*Tan[a + b*x])^(5/2))/(2*b*d)
```


Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{d\,\text{Subst}\left(\int \frac{x^{7/2}}{\left(d^2+x^2\right)^2}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= -\frac{\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{2bd} + \frac{(5d)\,\text{Subst}\left(\int \frac{x^{3/2}}{d^2+x^2}\,dx, x, d\tan(a+bx)\right)}{4b}$$

$$= \frac{5d\sqrt{d\tan(a+bx)}}{2b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{2bd} - \frac{\left(5d^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(d^2+x^2\right)}\,dx\right.}{4b}$$

$$= \frac{5d\sqrt{d\tan(a+bx)}}{2b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{2bd} - \frac{\left(5d^3\right)\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x,\right.}{2b}$$

$$= \frac{5d\sqrt{d\tan(a+bx)}}{2b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{2bd} - \frac{\left(5d^2\right)\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x,\right.}{4b}$$

$$= \frac{5d\sqrt{d\tan(a+bx)}}{2b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{2bd} + \frac{\left(5d^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-}\right.}{8\sqrt{2}}$$

$$= \frac{5d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} - \frac{5d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a\right.}{8}$$

$$= \frac{5d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} - \frac{5d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} + \frac{5d^{3/2}\log\left(\sqrt{d}\right.}{}$$

Mathematica [A] time = 0.549549, size = 113, normalized size = 0.46

$$\frac{d\csc(a+bx)\sqrt{d\tan(a+bx)}\left(17\sin(a+bx)+\sin(3(a+bx))+5\sqrt{\sin(2(a+bx))}\sin^{-1}(\cos(a+bx)-\sin(a+bx))-5\sqrt{\text{s}}\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (d*Csc[a + b*x]*(17*Sin[a + b*x] + 5*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Sq
rt[Sin[2*(a + b*x)]] - 5*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a +
b*x)]]]*Sqrt[Sin[2*(a + b*x)]] + Sin[3*(a + b*x)])*Sqrt[d*Tan[a + b*x]])/(8
*b)
```

Maple [C] time = 0.159, size = 676, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*(d*tan(b*x+a))^(3/2),x)
```

```
[Out] -1/8/b*2^(1/2)*(cos(b*x+a)-1)*(-5*I*sin(b*x+a)*EllipticPi(((1-cos(b*x+a)+si
n(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+
a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b
*x+a))/sin(b*x+a))^(1/2)+5*I*sin(b*x+a)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a
))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/
2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/
sin(b*x+a))^(1/2)+10*sin(b*x+a)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b
*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^
(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-5
*sin(b*x+a)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2
*I,1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a)
)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-5*sin(b*x+
a)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^
(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x
+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-2*cos(b*x+a)^3*2^(1
/2)+2*cos(b*x+a)^2*2^(1/2)-8*cos(b*x+a)*2^(1/2)+8*2^(1/2))*cos(b*x+a)*(cos(
b*x+a)+1)^2*(d*sin(b*x+a)/cos(b*x+a))^(3/2)/sin(b*x+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^{\frac{3}{2}}\sin(bx+a)^2\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*sin(b*x + a)^2, x)
```

3.66 $\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=18

$$\frac{2d\sqrt{d\tan(a+bx)}}{b}$$

```
[Out] (2*d*Sqrt[d*Tan[a + b*x]])/b
```

Rubi [A] time = 0.0429873, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2591, 30}

$$\frac{2d\sqrt{d\tan(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*d*Sqrt[d*Tan[a + b*x]])/b
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx &= \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, d\tan(a+bx)\right)}{b}\\ &= \frac{2d\sqrt{d\tan(a+bx)}}{b}\end{aligned}$$

Mathematica [A] time = 0.0496233, size = 18, normalized size = 1.

$$\frac{2d\sqrt{d\tan(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*d*Sqrt[d*Tan[a + b*x]])/b
```

Maple [B] time = 0.135, size = 58, normalized size = 3.2

$$2\,{\frac {\cos \left(bx+a \right) \left(\cos \left(bx+a \right) -1 \right) ^{2} \left(\cos \left(bx+a \right) +1 \right) ^{2}}{b \left(\sin \left(bx+a \right) \right) ^{5}}\left({\frac {d\sin \left(bx+a \right) }{\cos \left(bx+a \right) }} \right) ^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*(d*tan(b*x+a))^(3/2),x)
```

```
[Out] 2/b*(d*sin(b*x+a)/cos(b*x+a))^(3/2)*cos(b*x+a)*(cos(b*x+a)-1)^2*(cos(b*x+a)
+1)^2/sin(b*x+a)^5
```

Maxima [A] time = 1.50048, size = 31, normalized size = 1.72

$$\frac{2 \, \left(d \tan \left(b x + a\right)\right)^{\frac{3}{2}}}{b \tan \left(b x + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2*(d*tan(b*x + a))^(3/2)/(b*tan(b*x + a))
```

Fricas [A] time = 1.59553, size = 55, normalized size = 3.06

$$\frac{2 \, d \sqrt{\frac{d \sin \left(b x + a\right)}{\cos \left(b x + a\right)}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2*d*sqrt(d*sin(b*x + a)/cos(b*x + a))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^{\frac{3}{2}}\csc(bx+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*csc(b*x + a)^2, x)
```

3.67 $\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=41

$$\frac{2d\sqrt{d\tan(a+bx)}}{b}-\frac{2d^3}{3b(d\tan(a+bx))^{3/2}}$$

```
[Out] (-2*d^3)/(3*b*(d*Tan[a + b*x])^(3/2)) + (2*d*Sqrt[d*Tan[a + b*x]])/b
```

Rubi [A] time = 0.0478271, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2591, 14}

$$\frac{2d\sqrt{d\tan(a+bx)}}{b}-\frac{2d^3}{3b(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^4*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*d^3)/(3*b*(d*Tan[a + b*x])^(3/2)) + (2*d*Sqrt[d*Tan[a + b*x]])/b
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{d\,\text{Subst}\left(\int \frac{d^2+x^2}{x^{5/2}}\,dx, x, d\tan(a+bx)\right)}{b}$$
$$= \frac{d\,\text{Subst}\left(\int \left(\frac{d^2}{x^{5/2}} + \frac{1}{\sqrt{x}}\right)dx, x, d\tan(a+bx)\right)}{b}$$
$$= -\frac{2d^3}{3b(d\tan(a+bx))^{3/2}} + \frac{2d\sqrt{d\tan(a+bx)}}{b}$$

Mathematica [A] time = 0.0794664, size = 30, normalized size = 0.73

$$-\frac{2d\left(\csc^2(a+bx)-4\right)\sqrt{d\tan(a+bx)}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^4*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*d*(-4 + Csc[a + b*x]^2)*Sqrt[d*Tan[a + b*x]])/(3*b)
```

Maple [A] time = 0.155, size = 50, normalized size = 1.2

$$-\frac{\left(8\,(\cos(bx+a))^2-6\right)\cos(bx+a)}{3\,b\,(\sin(bx+a))^3}\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^4*(d*tan(b*x+a))^(3/2),x)
```

```
[Out] -2/3/b*(4*cos(b*x+a)^2-3)*cos(b*x+a)*(d*sin(b*x+a)/cos(b*x+a))^(3/2)/sin(b*
x+a)^3
```

Maxima [A] time = 1.69516, size = 46, normalized size = 1.12

$$-\frac{2\,d^3\left(\frac{1}{(d\tan(bx+a))^{\frac{3}{2}}} - \frac{3\sqrt{d\tan(bx+a)}}{d^2}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/3*d^3*(1/(d*tan(b*x + a))^(3/2) - 3*sqrt(d*tan(b*x + a))/d^2)/b
```

Fricas [A] time = 1.45203, size = 120, normalized size = 2.93

$$\frac{2 \left(4 \, d \cos \left(bx+a\right)^{2} - 3 \, d\right) \sqrt{\frac{d \sin(bx+a)}{\cos(bx+a)}}}{3 \left(b \cos \left(bx+a\right)^{2} - b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(4*d*cos(b*x + a)^2 - 3*d)*sqrt(d*sin(b*x + a)/cos(b*x + a))/(b*cos(b*x
 + a)^2 - b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**4*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d \tan \left(bx+a\right)\right)^{\frac{3}{2}} \csc \left(bx+a\right)^{4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*csc(b*x + a)^4, x)
```

3.68 $\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=63

$$-\frac{2d^5}{7b(d\tan(a+bx))^{7/2}}-\frac{4d^3}{3b(d\tan(a+bx))^{3/2}}+\frac{2d\sqrt{d\tan(a+bx)}}{b}$$

```
[Out] (-2*d^5)/(7*b*(d*Tan[a + b*x])^(7/2)) - (4*d^3)/(3*b*(d*Tan[a + b*x])^(3/2)
) + (2*d*Sqrt[d*Tan[a + b*x]])/b
```

Rubi [A] time = 0.0547959, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2591, 270}

$$-\frac{2d^5}{7b(d\tan(a+bx))^{7/2}}-\frac{4d^3}{3b(d\tan(a+bx))^{3/2}}+\frac{2d\sqrt{d\tan(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^6*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*d^5)/(7*b*(d*Tan[a + b*x])^(7/2)) - (4*d^3)/(3*b*(d*Tan[a + b*x])^(3/2)
) + (2*d*Sqrt[d*Tan[a + b*x]])/b
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{d\,\text{Subst}\left(\int \frac{\left(d^2+x^2\right)^2}{x^{9/2}}\,dx, x, d\tan(a+bx)\right)}{b}$$
$$= \frac{d\,\text{Subst}\left(\int \left(\frac{d^4}{x^{9/2}}+\frac{2d^2}{x^{5/2}}+\frac{1}{\sqrt{x}}\right)dx, x, d\tan(a+bx)\right)}{b}$$
$$= -\frac{2d^5}{7b(d\tan(a+bx))^{7/2}} - \frac{4d^3}{3b(d\tan(a+bx))^{3/2}} + \frac{2d\sqrt{d\tan(a+bx)}}{b}$$

Mathematica [A] time = 0.141006, size = 42, normalized size = 0.67

$$-\frac{2d\left(3\csc^4(a+bx)+8\csc^2(a+bx)-32\right)\sqrt{d\tan(a+bx)}}{21b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^6*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*d*(-32 + 8*Csc[a + b*x]^2 + 3*Csc[a + b*x]^4)*Sqrt[d*Tan[a + b*x]])/(21
*b)
```

Maple [A] time = 0.192, size = 60, normalized size = 1.

$$\frac{\left(64\,(\cos(bx+a))^4-112\,(\cos(bx+a))^2+42\right)\cos(bx+a)}{21\,b\,(\sin(bx+a))^5}\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^6*(d*tan(b*x+a))^(3/2),x)
```

```
[Out] 2/21/b*(32*cos(b*x+a)^4-56*cos(b*x+a)^2+21)*cos(b*x+a)*(d*sin(b*x+a)/cos(b*
x+a))^(3/2)/sin(b*x+a)^5
```

Maxima [A] time = 1.59114, size = 78, normalized size = 1.24

$$\frac{2\,d^5\left(\frac{21\,\sqrt{d\tan(bx+a)}}{d^4}-\frac{14\,d^2\tan(bx+a)^2+3\,d^2}{(d\tan(bx+a))^{\frac{7}{2}}\,d^2}\right)}{21\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/21*d^5*(21*sqrt(d*tan(b*x + a))/d^4 - (14*d^2*tan(b*x + a)^2 + 3*d^2)/((d
*tan(b*x + a))^(7/2)*d^2))/b
```

Fricas [A] time = 1.49198, size = 182, normalized size = 2.89

$$\frac{2\left(32\,d\cos\left(bx+a\right)^4-56\,d\cos\left(bx+a\right)^2+21\,d\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{21\left(b\cos\left(bx+a\right)^4-2\,b\cos\left(bx+a\right)^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*(32*d*cos(b*x + a)^4 - 56*d*cos(b*x + a)^2 + 21*d)*sqrt(d*sin(b*x + a)
/cos(b*x + a))/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**6*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}\csc\left(bx+a\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*csc(b*x + a)^6, x)
```

3.69 $\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=110

$$\frac{7d^3\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}-\frac{7d^2\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

```
[Out] (7*d^3*Sin[a + b*x]^3)/(3*b*(d*Tan[a + b*x])^(3/2)) - (7*d^2*EllipticE[a -
Pi/4 + b*x, 2]*Sin[a + b*x])/(2*b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x
]]) + (2*d*Sin[a + b*x]^3*Sqrt[d*Tan[a + b*x]])/b
```

Rubi [A] time = 0.141057, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2594, 2598, 2601, 2572, 2639}

$$\frac{7d^3\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}-\frac{7d^2\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (7*d^3*Sin[a + b*x]^3)/(3*b*(d*Tan[a + b*x])^(3/2)) - (7*d^2*EllipticE[a -
Pi/4 + b*x, 2]*Sin[a + b*x])/(2*b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x
]]) + (2*d*Sin[a + b*x]^3*Sqrt[d*Tan[a + b*x]])/b
```

Rule 2594

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
n - 1)), x] - Dist[(b^2*(m + n - 1))/(n - 1), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
```

```
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b} - \left(7d^2\right)\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

$$= \frac{7d^3\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}} + \frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b} - \frac{1}{2}\left(7d^2\right)\int \frac{\sin(a+bx}{\sqrt{d\tan(a+}}$$

$$= \frac{7d^3\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}} + \frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b} - \frac{\left(7d^2\sqrt{\sin(a+bx)}\right)\int\sqrt{}}{2\sqrt{\cos(a+bx}}$$

$$= \frac{7d^3\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}} + \frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b} - \frac{\left(7d^2\sin(a+bx)\right)\int\sqrt{\sin}}{2\sqrt{\sin(2a+2bx)}\sqrt{d}}$$

$$= \frac{7d^3\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}} - \frac{7d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} + \frac{2d\sin^3(a+bx)\sqrt{d\tan}}{b}$$

Mathematica [C] time = 0.580102, size = 90, normalized size = 0.82

$$\frac{(d\tan(a+bx))^{3/2}\left(2\cos(a+bx)(\cos(2(a+bx))+13)\sqrt{\sec^2(a+bx)}-28\sec(a+bx)\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)\right)}{12b\sqrt{\sec^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] ((-28*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x] + 2*Co
s[a + b*x]*(13 + Cos[2*(a + b*x)])*Sqrt[Sec[a + b*x]^2])*(d*Tan[a + b*x])^(
3/2))/(12*b*Sqrt[Sec[a + b*x]^2])
```

Maple [B] time = 0.155, size = 548, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*(d*tan(b*x+a))^(3/2),x)
```

```
[Out] -1/12/b*2^(1/2)*(cos(b*x+a)-1)^2*(2*cos(b*x+a)^4*2^(1/2)+21*cos(b*x+a)*Elli
pticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+
a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos
(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-42*cos(b*x+a)*EllipticE((-(cos(b*x+
a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))
^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x
+a))/sin(b*x+a))^(1/2)+21*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))
^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*
x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-42*El
lipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*
x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(c
os(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-11*cos(b*x+a)^2*2^(1/2)+21*cos(b*
x+a)*2^(1/2)-12*2^(1/2))*cos(b*x+a)*(cos(b*x+a)+1)^2*(d*sin(b*x+a)/cos(b*x+
a))^(3/2)/sin(b*x+a)^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d \tan (bx + a))^{\frac{3}{2}} \sin (bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*sin(b*x + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(d\cos\left(bx+a\right)^{2}-d\right)\sqrt{d\tan\left(bx+a\right)}\sin\left(bx+a\right)\tan\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(d*cos(b*x + a)^2 - d)*sqrt(d*tan(b*x + a))*sin(b*x + a)*tan(b*x
+ a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.70 $\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=76

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-\frac{3d^2\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[Out] (-3*d^2*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(b*Sqrt[Sin[2*a + 2*b*x]
]*Sqrt[d*Tan[a + b*x]]) + (2*d*Sin[a + b*x]*Sqrt[d*Tan[a + b*x]])/b
```

Rubi [A] time = 0.0868062, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2594, 2601, 2572, 2639}

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-\frac{3d^2\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-3*d^2*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(b*Sqrt[Sin[2*a + 2*b*x]
]*Sqrt[d*Tan[a + b*x]]) + (2*d*Sin[a + b*x]*Sqrt[d*Tan[a + b*x]])/b
```

Rule 2594

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
n - 1)), x] - Dist[(b^2*(m + n - 1))/(n - 1), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] && !(GtQ[m, 1] && !IntegerQ[(m - 1)/2])
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx &= \frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b} - \left(3d^2\right)\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx \\
&= \frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b} - \frac{\left(3d^2\sqrt{\sin(a+bx)}\right)\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,d}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}} \\
&= \frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b} - \frac{\left(3d^2\sin(a+bx)\right)\int \sqrt{\sin(2a+2bx)}\,dx}{\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} \\
&= -\frac{3d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} + \frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}
\end{aligned}$$

Mathematica [C] time = 0.283434, size = 58, normalized size = 0.76

$$-\frac{2\cos(a+bx)(d\tan(a+bx))^{3/2}\left(\sqrt{\sec^2(a+bx)}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)-1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x]*(-1 + Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sq
rt[Sec[a + b*x]^2])*(d*Tan[a + b*x])^(3/2))/b
```

Maple [B] time = 0.148, size = 526, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*(d*tan(b*x+a))^(3/2),x)
```

```
[Out] 1/2/b*2^(1/2)*(cos(b*x+a)-1)^2*(6*cos(b*x+a)*EllipticE(((1-cos(b*x+a)+sin(b
*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1
-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*
x+a))^(1/2)-3*cos(b*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(
1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+
a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+6*Ellipt
icE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-
1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x
+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-3*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/
sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*
x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(
1/2)+cos(b*x+a)^2*2^(1/2)-3*cos(b*x+a)*2^(1/2)+2*2^(1/2))*cos(b*x+a)*(cos(b
*x+a)+1)^2*(d*sin(b*x+a)/cos(b*x+a))^(3/2)/sin(b*x+a)^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^{\frac{3}{2}}\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*sin(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{d\tan(bx+a)}d\sin(bx+a)\tan(bx+a)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*d*sin(b*x + a)*tan(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.71 $\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=76

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-\frac{2d^2\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[Out] (-2*d^2*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(b*Sqrt[Sin[2*a + 2*b*x]
]*Sqrt[d*Tan[a + b*x]]) + (2*d*Sin[a + b*x]*Sqrt[d*Tan[a + b*x]])/b
```

Rubi [A] time = 0.0940729, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2593, 2601, 2572, 2639}

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-\frac{2d^2\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*d^2*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(b*Sqrt[Sin[2*a + 2*b*x]
]*Sqrt[d*Tan[a + b*x]]) + (2*d*Sin[a + b*x]*Sqrt[d*Tan[a + b*x]])/b
```

Rule 2593

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^(n - 1))
/(a^2*f*(n - 1)), x] - Dist[(b^2*(m + 2))/(a^2*(n - 1)), Int[(a*Sin[e + f*x
])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && Gt
Q[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*
n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx &= \frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b} - \left(2d^2\right)\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx \\
&= \frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b} - \frac{\left(2d^2\sqrt{\sin(a+bx)}\right)\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\, d}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}} \\
&= \frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b} - \frac{\left(2d^2\sin(a+bx)\right)\int \sqrt{\sin(2a+2bx)}\,dx}{\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} \\
&= -\frac{2d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} + \frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}
\end{aligned}
$$

Mathematica [C] time = 0.273229, size = 61, normalized size = 0.8

$$
-\frac{2\cos(a+bx)(d\tan(a+bx))^{3/2}\left(2\sqrt{\sec^2(a+bx)}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)-3\right)}{3b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x]*(-3 + 2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*
Sqrt[Sec[a + b*x]^2])*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Maple [B] time = 0.148, size = 519, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*(d*tan(b*x+a))^(3/2),x)
```

```
[Out] 1/b*2^(1/2)*(cos(b*x+a)-1)^2*(2*cos(b*x+a)*EllipticE((-(cos(b*x+a)-1-sin(b*
x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((co
s(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*
x+a))^(1/2)-cos(b*x+a)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1
/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a
))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)+2*Ellipt
icE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)
-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b
*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-EllipticF((-(cos(b*x+a)-1-sin(b*x+a))
/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x
+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))
^(1/2)-cos(b*x+a)*2^(1/2)+2^(1/2))*cos(b*x+a)*(cos(b*x+a)+1)^2*(d*sin(b*x+a
)/cos(b*x+a))^(3/2)/sin(b*x+a)^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^{\frac{3}{2}} \csc(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*csc(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\tan(bx+a)}d\csc(bx+a)\tan(bx+a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*d*csc(b*x + a)*tan(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^{\frac{3}{2}}\csc(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*csc(b*x + a), x)
```

3.72 $\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=102

$$-\frac{4d^2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}}-\frac{4d^2\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

```
[Out] (-4*d^2*Cos[a + b*x])/(b*Sqrt[d*Tan[a + b*x]]) - (4*d^2*EllipticE[a - Pi/4
+ b*x, 2]*Sin[a + b*x])/(b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]]) + (
2*d*Csc[a + b*x]*Sqrt[d*Tan[a + b*x]])/b
```

Rubi [A] time = 0.142878, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2593, 2601, 2570, 2572, 2639}

$$-\frac{4d^2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}}-\frac{4d^2\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-4*d^2*Cos[a + b*x])/(b*Sqrt[d*Tan[a + b*x]]) - (4*d^2*EllipticE[a - Pi/4
+ b*x, 2]*Sin[a + b*x])/(b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]]) + (
2*d*Csc[a + b*x]*Sqrt[d*Tan[a + b*x]])/b
```

Rule 2593

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^(n - 1))
/(a^2*f*(n - 1)), x] - Dist[(b^2*(m + 2))/(a^2*(n - 1)), Int[(a*Sin[e + f*x
])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && Gt
Q[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*
n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
```

```
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b} + \left(2d^2\right)\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

$$= \frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b} + \frac{\left(2d^2\sqrt{\sin(a+bx)}\right)\int \frac{\sqrt{\cos(a+bx)}}{\sin^{\frac{3}{2}}(a+bx)}\,dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$= -\frac{4d^2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}} + \frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b} - \frac{\left(4d^2\sqrt{\sin(a+bx)}\right)\int \sqrt{\cos \cdots}}{\sqrt{\cos(a+bx)}\sqrt{\cdots}}$$

$$= -\frac{4d^2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}} + \frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b} - \frac{\left(4d^2\sin(a+bx)\right)\int \sqrt{\sin(\cdots}}{\sqrt{\sin(2a+2bx)}\sqrt{d\tan\cdots}}$$

$$= -\frac{4d^2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}} - \frac{4d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} + \frac{2d\csc(a+bx)\sqrt{d\tan(\cdots}}{b}$$

Mathematica [C] time = 0.578046, size = 71, normalized size = 0.7

$$-\frac{2\cos(a+bx)(d\tan(a+bx))^{3/2}\left(4\sqrt{\sec^2(a+bx)}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)+3\csc^2(a+bx)-6\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x]*(-6 + 3*Csc[a + b*x]^2 + 4*Hypergeometric2F1[3/4, 3/2, 7/4
, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2])*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Maple [B] time = 0.174, size = 499, normalized size = 4.9

$$\frac{\cos(bx+a)\sqrt{2}}{b(\sin(bx+a))^2}\left(4\cos(bx+a)\,\mathrm{EllipticE}\left(\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}},1/2\sqrt{2}\right)\sqrt{\frac{\cos(bx+a)-1}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a)}{\sin}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*(d*tan(b*x+a))^(3/2),x)
```

```
[Out] 1/b*2^(1/2)*(4*cos(b*x+a)*EllipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))
^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*
x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-2*cos
(b*x+a)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2)
)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^
(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)+4*EllipticE((-(cos(b*x+
a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))
^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x
+a))/sin(b*x+a))^(1/2)-2*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^
(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x
+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-2*cos(
b*x+a)*2^(1/2)+2^(1/2))*cos(b*x+a)*(d*sin(b*x+a)/cos(b*x+a))^(3/2)/sin(b*x+
a)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^{\frac{3}{2}}\csc(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*csc(b*x + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\tan\left(bx+a\right)}d\csc\left(bx+a\right)^{3}\tan\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*d*csc(b*x + a)^3*tan(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}\csc\left(bx+a\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*csc(b*x + a)^3, x)
```

3.73 $\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal. Leaf size=277

$$\frac{77d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b}-\frac{77d^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{32\sqrt{2}b}-\frac{77d^{5/2}\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\right.}{64\sqrt{2}b}$$

```
[Out] (77*d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]
*b) - (77*d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*S
qrt[2]*b) - (77*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d
*Tan[a + b*x]]])/(64*Sqrt[2]*b) + (77*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a +
 b*x] + Sqrt[2]*Sqrt[d*Tan[a + b*x]]])/(64*Sqrt[2]*b) + (77*d*(d*Tan[a + b*
x])^(3/2))/(48*b) - (11*Cos[a + b*x]^2*(d*Tan[a + b*x])^(7/2))/(16*b*d) - (
Cos[a + b*x]^4*(d*Tan[a + b*x])^(11/2))/(4*b*d^3)
```

Rubi [A] time = 0.194189, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2591, 288, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{77d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b}-\frac{77d^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{32\sqrt{2}b}-\frac{77d^{5/2}\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\right.}{64\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^4*(d*Tan[a + b*x])^(5/2),x]
```


```
[Out] (77*d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]
*b) - (77*d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*S
qrt[2]*b) - (77*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d
*Tan[a + b*x]]])/(64*Sqrt[2]*b) + (77*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a +
 b*x] + Sqrt[2]*Sqrt[d*Tan[a + b*x]]])/(64*Sqrt[2]*b) + (77*d*(d*Tan[a + b*
x])^(3/2))/(48*b) - (11*Cos[a + b*x]^2*(d*Tan[a + b*x])^(7/2))/(16*b*d) - (
Cos[a + b*x]^4*(d*Tan[a + b*x])^(11/2))/(4*b*d^3)
```


Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{d\,\text{Subst}\left(\int \frac{x^{13/2}}{(d^2+x^2)^3}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= -\frac{\cos^4(a+bx)(d\tan(a+bx))^{11/2}}{4bd^3} + \frac{(11d)\,\text{Subst}\left(\int \frac{x^{9/2}}{(d^2+x^2)^2}\,dx, x, d\tan(a+bx)\right)}{8b}$$

$$= -\frac{11\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{11/2}}{4bd^3} + \frac{(77d)\,\text{Su}}{}$$

$$= \frac{77d(d\tan(a+bx))^{3/2}}{48b} - \frac{11\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(}{4bd^3}$$

$$= \frac{77d(d\tan(a+bx))^{3/2}}{48b} - \frac{11\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(}{4bd^3}$$

$$= \frac{77d(d\tan(a+bx))^{3/2}}{48b} - \frac{11\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(}{4bd^3}$$

$$= \frac{77d(d\tan(a+bx))^{3/2}}{48b} - \frac{11\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(}{4bd^3}$$

$$= -\frac{77d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b} + \frac{77d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\right.}{}$$

$$= \frac{77d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} - \frac{77d^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} - \frac{77d^{5/2}\log}{}$$

Mathematica [A] time = 0.568022, size = 142, normalized size = 0.51

$$\frac{d(d\tan(a+bx))^{3/2}\left(204\cos^2(a+bx) - 6\sin(4(a+bx))\cot(a+bx) + 231\sqrt{\sin(2(a+bx))}\cot(a+bx)\csc(a+bx)\sin^{-1}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^4*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (d*(128 + 204*Cos[a + b*x]^2 + 231*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Cot[
a + b*x]*Csc[a + b*x]*Sqrt[Sin[2*(a + b*x)]] + 231*Cot[a + b*x]*Csc[a + b*x
]*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]]]*Sqrt[Sin[2*(a +
 b*x)]] - 6*Cot[a + b*x]*Sin[4*(a + b*x)])*(d*Tan[a + b*x])^(3/2))/(192*b)
```

Maple [C] time = 0.196, size = 590, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^4*(d*tan(b*x+a))^(5/2),x)
```

```
[Out] -1/192/b*2^(1/2)*(cos(b*x+a)-1)*(24*2^(1/2)*cos(b*x+a)^5+231*I*cos(b*x+a)*(
(cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/
2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+s
in(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-231*I*cos(b*x+a)*((cos(
b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((
cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*
x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))-24*cos(b*x+a)^4*2^(1/2)+231*
cos(b*x+a)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin
(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-
cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))+231*cos(b*x
+a)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a)
)^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x
+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))-114*cos(b*x+a)^3*2
^(1/2)+114*cos(b*x+a)^2*2^(1/2)-64*cos(b*x+a)*2^(1/2)+64*2^(1/2))*cos(b*x+a
)*(cos(b*x+a)+1)^2*(d*sin(b*x+a)/cos(b*x+a))^(5/2)/sin(b*x+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**4*(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^{\frac{5}{2}}\sin(bx+a)^4\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^(5/2)*sin(b*x + a)^4, x)
```

3.74 $\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal. Leaf size=247

$$\frac{7d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b}-\frac{7d^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}b}-\frac{7d^{5/2}\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right.}{8\sqrt{2}b}$$

```
[Out] (7*d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*b
) - (7*d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[
2]*b) - (7*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[
a + b*x]]])/(8*Sqrt[2]*b) + (7*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] +
 Sqrt[2]*Sqrt[d*Tan[a + b*x]]])/(8*Sqrt[2]*b) + (7*d*(d*Tan[a + b*x])^(3/2)
)/(6*b) - (Cos[a + b*x]^2*(d*Tan[a + b*x])^(7/2))/(2*b*d)
```

Rubi [A] time = 0.175131, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2591, 288, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{7d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b}-\frac{7d^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}b}-\frac{7d^{5/2}\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right.}{8\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*(d*Tan[a + b*x])^(5/2),x]
```


```
[Out] (7*d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*b
) - (7*d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[
2]*b) - (7*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[
a + b*x]]])/(8*Sqrt[2]*b) + (7*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] +
 Sqrt[2]*Sqrt[d*Tan[a + b*x]]])/(8*Sqrt[2]*b) + (7*d*(d*Tan[a + b*x])^(3/2)
)/(6*b) - (Cos[a + b*x]^2*(d*Tan[a + b*x])^(7/2))/(2*b*d)
```


Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{d\,\text{Subst}\left(\int \frac{x^{9/2}}{\left(d^2+x^2\right)^2}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= -\frac{\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{2bd} + \frac{(7d)\,\text{Subst}\left(\int \frac{x^{5/2}}{d^2+x^2}\,dx, x, d\tan(a+bx)\right)}{4b}$$

$$= \frac{7d(d\tan(a+bx))^{3/2}}{6b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{2bd} - \frac{\left(7d^3\right)\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, \ldots\right)}{4b}$$

$$= \frac{7d(d\tan(a+bx))^{3/2}}{6b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{2bd} - \frac{\left(7d^3\right)\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, \ldots\right)}{2b}$$

$$= \frac{7d(d\tan(a+bx))^{3/2}}{6b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{2bd} + \frac{\left(7d^3\right)\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, \ldots\right)}{4b}$$

$$= \frac{7d(d\tan(a+bx))^{3/2}}{6b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{2bd} - \frac{\left(7d^{5/2}\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+\ldots}{-d-\sqrt{2}\sqrt{d}\ldots}\ldots\right)}{8\ldots}$$

$$= -\frac{7d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} + \frac{7d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan\ldots\right.}{\ldots}$$

$$= \frac{7d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} - \frac{7d^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} - \frac{7d^{5/2}\log\left(\sqrt{d}\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.384428, size = 126, normalized size = 0.51

$$\frac{d(d\tan(a+bx))^{3/2}\left(12\cos^2(a+bx)+21\sqrt{\sin(2(a+bx))}\cot(a+bx)\csc(a+bx)\sin^{-1}(\cos(a+bx)-\sin(a+bx))+21\cdots\right.}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (d*(16 + 12*Cos[a + b*x]^2 + 21*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Cot[a +
 b*x]*Csc[a + b*x]*Sqrt[Sin[2*(a + b*x)]] + 21*Cot[a + b*x]*Csc[a + b*x]*Lo
g[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]]]*Sqrt[Sin[2*(a + b*x
)]])*(d*Tan[a + b*x])^(3/2))/(24*b)
```

Maple [C] time = 0.131, size = 564, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*(d*tan(b*x+a))^(5/2),x)
```

```
[Out] 1/24/b*2^(1/2)*(cos(b*x+a)-1)*(21*I*cos(b*x+a)*((cos(b*x+a)-1)/sin(b*x+a))^
(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a
))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2
),1/2-1/2*I,1/2*2^(1/2))-21*I*cos(b*x+a)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*
((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin
(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+
1/2*I,1/2*2^(1/2))-21*cos(b*x+a)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(
b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))
^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/
2*2^(1/2))-21*cos(b*x+a)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+s
in(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*E
llipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2
))+6*cos(b*x+a)^3*2^(1/2)-6*cos(b*x+a)^2*2^(1/2)+8*cos(b*x+a)*2^(1/2)-8*2^(
1/2))*cos(b*x+a)*(cos(b*x+a)+1)^2*(d*sin(b*x+a)/cos(b*x+a))^(5/2)/sin(b*x+a
)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^{\frac{5}{2}}\sin(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^(5/2)*sin(b*x + a)^2, x)
```

3.75 $\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal. Leaf size=20

$$\frac{2d(d\tan(a+bx))^{3/2}}{3b}$$

```
[Out] (2*d*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Rubi [A] time = 0.0420846, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2591, 30}

$$\frac{2d(d\tan(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (2*d*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx &= \frac{d\,\text{Subst}\left(\int \sqrt{x}\,dx, x, d\tan(a+bx)\right)}{b}\\ &= \frac{2d(d\tan(a+bx))^{3/2}}{3b}\end{aligned}$$

Mathematica [A] time = 0.0751839, size = 20, normalized size = 1.

$$\frac{2d(d\tan(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (2*d*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Maple [B] time = 0.123, size = 38, normalized size = 1.9

$$\frac{2\cos(bx+a)}{3b\sin(bx+a)}\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*(d*tan(b*x+a))^(5/2),x)
```

```
[Out] 2/3/b*(d*sin(b*x+a)/cos(b*x+a))^(5/2)*cos(b*x+a)/sin(b*x+a)
```

Maxima [A] time = 1.39602, size = 31, normalized size = 1.55

$$\frac{2(d\tan(bx+a))^{\frac{5}{2}}}{3b\tan(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(d*tan(b*x + a))^(5/2)/(b*tan(b*x + a))
```

Fricas [B] time = 2.14551, size = 99, normalized size = 4.95

$$\frac{2d^2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\sin(bx+a)}{3b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*d^2*sqrt(d*sin(b*x + a)/cos(b*x + a))*sin(b*x + a)/(b*cos(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11302, size = 32, normalized size = 1.6

$$\frac{2\sqrt{d\tan\left(bx+a\right)}d^{2}\tan\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(d*tan(b*x + a))*d^2*tan(b*x + a)/b
```

3.76 $\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal. Leaf size=41

$$\frac{2d(d\tan(a+bx))^{3/2}}{3b}-\frac{2d^3}{b\sqrt{d\tan(a+bx)}}$$

```
[Out] (-2*d^3)/(b*Sqrt[d*Tan[a + b*x]]) + (2*d*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Rubi [A] time = 0.0482106, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2591, 14}

$$\frac{2d(d\tan(a+bx))^{3/2}}{3b}-\frac{2d^3}{b\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^4*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*d^3)/(b*Sqrt[d*Tan[a + b*x]]) + (2*d*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{d\,\text{Subst}\left(\int \frac{d^2+x^2}{x^{3/2}}\,dx, x, d\tan(a+bx)\right)}{b}$$
$$= \frac{d\,\text{Subst}\left(\int \left(\frac{d^2}{x^{3/2}}+\sqrt{x}\right)dx, x, d\tan(a+bx)\right)}{b}$$
$$= -\frac{2d^3}{b\sqrt{d\tan(a+bx)}} + \frac{2d(d\tan(a+bx))^{3/2}}{3b}$$

Mathematica [A] time = 0.112681, size = 32, normalized size = 0.78

$$-\frac{2d\left(3\cot^2(a+bx)-1\right)(d\tan(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^4*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*d*(-1 + 3*Cot[a + b*x]^2)*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Maple [A] time = 0.124, size = 50, normalized size = 1.2

$$-\frac{\left(8\,(\cos(bx+a))^2-2\right)\cos(bx+a)}{3\,b\,(\sin(bx+a))^3}\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^4*(d*tan(b*x+a))^(5/2),x)
```

```
[Out] -2/3/b*(4*cos(b*x+a)^2-1)*cos(b*x+a)*(d*sin(b*x+a)/cos(b*x+a))^(5/2)/sin(b*
x+a)^3
```

Maxima [A] time = 1.32078, size = 49, normalized size = 1.2

$$-\frac{2\,d^3\left(\frac{3}{\sqrt{d\tan(bx+a)}}-\frac{(d\tan(bx+a))^{\frac{3}{2}}}{d^2}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*d^3*(3/sqrt(d*tan(b*x + a)) - (d*tan(b*x + a))^(3/2)/d^2)/b
```

Fricas [A] time = 2.20697, size = 134, normalized size = 3.27

$$-\frac{2\left(4\,d^2\cos\left(bx+a\right)^2-d^2\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{3\,b\cos\left(bx+a\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(4*d^2*cos(b*x + a)^2 - d^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))/(b*cos(
b*x + a)*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**4*(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}\csc\left(bx+a\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^4, x)
```

3.77 $\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal. Leaf size=63

$$-\frac{2d^5}{5b(d\tan(a+bx))^{5/2}}-\frac{4d^3}{b\sqrt{d\tan(a+bx)}}+\frac{2d(d\tan(a+bx))^{3/2}}{3b}$$

```
[Out] (-2*d^5)/(5*b*(d*Tan[a + b*x])^(5/2)) - (4*d^3)/(b*Sqrt[d*Tan[a + b*x]]) +
(2*d*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Rubi [A] time = 0.0539081, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2591, 270}

$$-\frac{2d^5}{5b(d\tan(a+bx))^{5/2}}-\frac{4d^3}{b\sqrt{d\tan(a+bx)}}+\frac{2d(d\tan(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^6*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*d^5)/(5*b*(d*Tan[a + b*x])^(5/2)) - (4*d^3)/(b*Sqrt[d*Tan[a + b*x]]) +
(2*d*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{d\,\text{Subst}\left(\int \frac{(d^2+x^2)^2}{x^{7/2}}\,dx, x, d\tan(a+bx)\right)}{b}$$
$$= \frac{d\,\text{Subst}\left(\int \left(\frac{d^4}{x^{7/2}}+\frac{2d^2}{x^{3/2}}+\sqrt{x}\right)dx, x, d\tan(a+bx)\right)}{b}$$
$$= -\frac{2d^5}{5b(d\tan(a+bx))^{5/2}} - \frac{4d^3}{b\sqrt{d\tan(a+bx)}} + \frac{2d(d\tan(a+bx))^{3/2}}{3b}$$

Mathematica [A] time = 0.223922, size = 42, normalized size = 0.67

$$-\frac{2d(d\tan(a+bx))^{3/2}\left(3\cot^2(a+bx)\left(\csc^2(a+bx)+9\right)-5\right)}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^6*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*d*(-5 + 3*Cot[a + b*x]^2*(9 + Csc[a + b*x]^2))*(d*Tan[a + b*x])^(3/2))/
(15*b)
```

Maple [A] time = 0.154, size = 60, normalized size = 1.

$$\frac{\left(64\,(\cos(bx+a))^4-80\,(\cos(bx+a))^2+10\right)\cos(bx+a)}{15\,b\,(\sin(bx+a))^5}\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^6*(d*tan(b*x+a))^(5/2),x)
```

```
[Out] 2/15/b*(32*cos(b*x+a)^4-40*cos(b*x+a)^2+5)*cos(b*x+a)*(d*sin(b*x+a)/cos(b*x
+a))^(5/2)/sin(b*x+a)^5
```

Maxima [A] time = 1.51859, size = 76, normalized size = 1.21

$$\frac{2\,d^5\left(\frac{5\,(d\tan(bx+a))^{\frac{3}{2}}}{d^4}-\frac{3\left(10\,d^2\tan(bx+a)^2+d^2\right)}{(d\tan(bx+a))^{\frac{5}{2}}\,d^2}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*d^5*(5*(d*tan(b*x + a))^(3/2)/d^4 - 3*(10*d^2*tan(b*x + a)^2 + d^2)/((
d*tan(b*x + a))^(5/2)*d^2))/b
```

Fricas [A] time = 2.24406, size = 200, normalized size = 3.17

$$-\frac{2\left(32\,d^2\cos\left(bx+a\right)^4-40\,d^2\cos\left(bx+a\right)^2+5\,d^2\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{15\left(b\cos\left(bx+a\right)^3-b\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(32*d^2*cos(b*x + a)^4 - 40*d^2*cos(b*x + a)^2 + 5*d^2)*sqrt(d*sin(b*
x + a)/cos(b*x + a))/((b*cos(b*x + a)^3 - b*cos(b*x + a))*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**6*(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}\csc\left(bx+a\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^6, x)
```

3.78 $\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal. Leaf size=137

$$\frac{d^3\sin^3(a+bx)}{b\sqrt{d\tan(a+bx)}}+\frac{5d^3\sin(a+bx)}{2b\sqrt{d\tan(a+bx)}}-\frac{5d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{4b}+\frac{2d\sin^3(a+b}{}$$

```
[Out] (5*d^3*Sin[a + b*x])/(2*b*Sqrt[d*Tan[a + b*x]]) + (d^3*Sin[a + b*x]^3)/(b*S
qrt[d*Tan[a + b*x]]) - (5*d^2*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqr
t[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(4*b) + (2*d*Sin[a + b*x]^3*(d*Ta
n[a + b*x])^(3/2))/(3*b)
```

Rubi [A] time = 0.174526, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2594, 2598, 2601, 2573, 2641}

$$\frac{d^3\sin^3(a+bx)}{b\sqrt{d\tan(a+bx)}}+\frac{5d^3\sin(a+bx)}{2b\sqrt{d\tan(a+bx)}}-\frac{5d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{4b}+\frac{2d\sin^3(a+b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (5*d^3*Sin[a + b*x])/(2*b*Sqrt[d*Tan[a + b*x]]) + (d^3*Sin[a + b*x]^3)/(b*S
qrt[d*Tan[a + b*x]]) - (5*d^2*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqr
t[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(4*b) + (2*d*Sin[a + b*x]^3*(d*Ta
n[a + b*x])^(3/2))/(3*b)
```

Rule 2594

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
n - 1)), x] - Dist[(b^2*(m + n - 1))/(n - 1), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
```

```
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b} - \left(3d^2\right)\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$= \frac{d^3\sin^3(a+bx)}{b\sqrt{d\tan(a+bx)}} + \frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b} - \frac{1}{2}\left(5d^2\right)\int \sin(a+bx)\sqrt{d}$$

$$= \frac{5d^3\sin(a+bx)}{2b\sqrt{d\tan(a+bx)}} + \frac{d^3\sin^3(a+bx)}{b\sqrt{d\tan(a+bx)}} + \frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b} - \frac{1}{4}\big(5$$

$$= \frac{5d^3\sin(a+bx)}{2b\sqrt{d\tan(a+bx)}} + \frac{d^3\sin^3(a+bx)}{b\sqrt{d\tan(a+bx)}} + \frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b} - \big(5d$$

$$= \frac{5d^3\sin(a+bx)}{2b\sqrt{d\tan(a+bx)}} + \frac{d^3\sin^3(a+bx)}{b\sqrt{d\tan(a+bx)}} + \frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b} - \frac{1}{4}\big(5$$

$$= \frac{5d^3\sin(a+bx)}{2b\sqrt{d\tan(a+bx)}} + \frac{d^3\sin^3(a+bx)}{b\sqrt{d\tan(a+bx)}} - \frac{5d^2\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2}}{4b}$$

Mathematica [C] time = 3.24586, size = 153, normalized size = 1.12

$$-\frac{\csc(a+bx)\sqrt{\sec^2(a+bx)}(d\tan(a+bx))^{5/2}\left((77\cos(2(a+bx))+22\cos(4(a+bx))-\cos(6(a+bx))+22)\sqrt{\tan(a+bx}\right.}{48b\tan^{\frac{3}{2}}(a+bx)\left(\tan^2(a+bx)-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] -(Csc[a + b*x]*Sqrt[Sec[a + b*x]^2]*(120*(-1)^(1/4)*Cos[2*(a + b*x)]*Ellipt
icF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1] + (22 + 77*Cos[2*(a + b*x
)] + 22*Cos[4*(a + b*x)] - Cos[6*(a + b*x)])*Sqrt[Sec[a + b*x]^2]*Sqrt[Tan[
a + b*x]])*(d*Tan[a + b*x])^(5/2))/(48*b*Tan[a + b*x]^(3/2)*(-1 + Tan[a + b
*x]^2))
```

Maple [A] time = 0.128, size = 248, normalized size = 1.8

$$-\frac{\sqrt{2}(\cos(bx+a)-1)\cos(bx+a)(\cos(bx+a)+1)^2}{12\,b(\sin(bx+a))^6}\left(2\sqrt{2}(\cos(bx+a))^5-15\cos(bx+a)\sqrt{\frac{\cos(bx+a)-1}{\sin(bx+a)}}\sqrt{\frac{\cos(b}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*(d*tan(b*x+a))^(5/2),x)
```

```
[Out] -1/12/b*2^(1/2)*(cos(b*x+a)-1)*(2*2^(1/2)*cos(b*x+a)^5-15*cos(b*x+a)*((cos(
b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-
(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*sin(b*x+a)*EllipticF((-(cos(b*x
+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-2*cos(b*x+a)^4*2^(1/2)-13*
cos(b*x+a)^3*2^(1/2)+13*cos(b*x+a)^2*2^(1/2)-4*cos(b*x+a)*2^(1/2)+4*2^(1/2)
)*cos(b*x+a)*(cos(b*x+a)+1)^2*(d*sin(b*x+a)/cos(b*x+a))^(5/2)/sin(b*x+a)^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^{\frac{5}{2}}\sin(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(5/2)*sin(b*x + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(d^2\cos\left(bx+a\right)^2-d^2\right)\sqrt{d\tan\left(bx+a\right)}\sin\left(bx+a\right)\tan\left(bx+a\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(d^2*cos(b*x + a)^2 - d^2)*sqrt(d*tan(b*x + a))*sin(b*x + a)*tan(
b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.79 $\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal. Leaf size=108

$$\frac{5d^3\sin(a+bx)}{3b\sqrt{d\tan(a+bx)}}-\frac{5d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{6b}+\frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

```
[Out] (5*d^3*Sin[a + b*x])/(3*b*Sqrt[d*Tan[a + b*x]]) - (5*d^2*Csc[a + b*x]*Ellip
ticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(6*b)
+ (2*d*Sin[a + b*x]*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Rubi [A] time = 0.116395, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2594, 2598, 2601, 2573, 2641}

$$\frac{5d^3\sin(a+bx)}{3b\sqrt{d\tan(a+bx)}}-\frac{5d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{6b}+\frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*(d*Tan[a + b*x])^(5/2),x]
```


```
[Out] (5*d^3*Sin[a + b*x])/(3*b*Sqrt[d*Tan[a + b*x]]) - (5*d^2*Csc[a + b*x]*Ellip
ticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(6*b)
+ (2*d*Sin[a + b*x]*(d*Tan[a + b*x])^(3/2))/(3*b)
```


Rule 2594

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
n - 1)), x] - Dist[(b^2*(m + n - 1))/(n - 1), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
```

```
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx &= \frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{1}{3}\left(5d^2\right)\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx\\ &= \frac{5d^3\sin(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{1}{6}\left(5d^2\right)\int \csc(a+bx)\sqrt{d\\ &= \frac{5d^3\sin(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{\left(5d^2\sqrt{\cos(a+bx)}\sqrt{d\tan}\right.}{6\sqrt{}}\\ &= \frac{5d^3\sin(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{1}{6}\left(5d^2\csc(a+bx)\sqrt{\sin(2}\right.\\ &= \frac{5d^3\sin(a+bx)}{3b\sqrt{d\tan(a+bx)}}-\frac{5d^2\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+}}{6b}\end{aligned}$$

Mathematica [C] time = 2.19087, size = 133, normalized size = 1.23

$$-\frac{\cos(2(a+bx))\csc(a+bx)\sqrt{\sec^2(a+bx)}(d\tan(a+bx))^{5/2}\left((3\cos(2(a+bx))+7)\sqrt{\tan(a+bx)}\sqrt{\sec^2(a+bx)}+10\sqrt[4]{-}\right.}{6b\tan^{\frac{3}{2}}(a+bx)\left(\tan^2(a+bx)-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] -(Cos[2*(a + b*x)]*Csc[a + b*x]*Sqrt[Sec[a + b*x]^2]*(10*(-1)^(1/4)*Ellipti
cF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1] + (7 + 3*Cos[2*(a + b*x)])
*Sqrt[Sec[a + b*x]^2]*Sqrt[Tan[a + b*x]])*(d*Tan[a + b*x])^(5/2))/(6*b*Tan[
a + b*x]^(3/2)*(-1 + Tan[a + b*x]^2))
```

Maple [A] time = 0.141, size = 220, normalized size = 2.

$$\frac{\sqrt{2}\,(\cos(bx+a)-1)\cos(bx+a)\,(\cos(bx+a)+1)^2}{6\,b\,(\sin(bx+a))^6}\left(5\,\sin(bx+a)\cos(bx+a)\,\mathrm{EllipticF}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx}{\sin(bx+a)}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*(d*tan(b*x+a))^(5/2),x)
```

```
[Out] 1/6/b*2^(1/2)*(cos(b*x+a)-1)*(5*sin(b*x+a)*cos(b*x+a)*EllipticF(((1-cos(b*x
+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^
(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a
))/sin(b*x+a))^(1/2)+3*cos(b*x+a)^3*2^(1/2)-3*cos(b*x+a)^2*2^(1/2)+2*cos(b*
x+a)*2^(1/2)-2*2^(1/2))*cos(b*x+a)*(cos(b*x+a)+1)^2*(d*sin(b*x+a)/cos(b*x+a
))^(5/2)/sin(b*x+a)^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^{\frac{5}{2}}\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(5/2)*sin(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\tan\left(bx+a\right)}d^2\sin\left(bx+a\right)\tan\left(bx+a\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*d^2*sin(b*x + a)*tan(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.80 $\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal. Leaf size=80

$$\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{3b}$$

```
[Out] -(d^2*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt
[d*Tan[a + b*x]])/(3*b) + (2*d*Csc[a + b*x]*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Rubi [A] time = 0.101285, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2594, 2601, 2573, 2641}

$$\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] -(d^2*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt
[d*Tan[a + b*x]])/(3*b) + (2*d*Csc[a + b*x]*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Rule 2594

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
n - 1)), x] - Dist[(b^2*(m + n - 1))/(n - 1), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b} - \frac{1}{3}d^2\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$= \frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b} - \frac{\left(d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{}}}{3\sqrt{\sin(a+bx)}}$$

$$= \frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b} - \frac{1}{3}\left(d^2\csc(a+bx)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+b}\right.$$

$$= -\frac{d^2\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{3b} + \frac{2d\csc(a+bx}{}$$

Mathematica [C] time = 0.375554, size = 71, normalized size = 0.89

$$\frac{2d^2\cos(a+bx)\sqrt{d\tan(a+bx)}\left(\sec^2(a+bx)-\sqrt{\sec^2(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2(a+bx)\right)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (2*d^2*Cos[a + b*x]*(Sec[a + b*x]^2 - Hypergeometric2F1[1/4, 1/2, 5/4, -Tan
[a + b*x]^2]*Sqrt[Sec[a + b*x]^2])*Sqrt[d*Tan[a + b*x]])/(3*b)
```

Maple [B] time = 0.137, size = 192, normalized size = 2.4

$$\frac{\sqrt{2}(\cos(bx+a)-1)\cos(bx+a)(\cos(bx+a)+1)^2}{3b(\sin(bx+a))^6}\left(\sin(bx+a)\cos(bx+a)\,\mathrm{EllipticF}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx}{\sin(bx+a)}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*(d*tan(b*x+a))^(5/2),x)
```

```
[Out] 1/3/b*2^(1/2)*(cos(b*x+a)-1)*(sin(b*x+a)*cos(b*x+a)*EllipticF(((1-cos(b*x+a
)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1
/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))
/sin(b*x+a))^(1/2)+cos(b*x+a)*2^(1/2)-2^(1/2))*cos(b*x+a)*(cos(b*x+a)+1)^2*
(d*sin(b*x+a)/cos(b*x+a))^(5/2)/sin(b*x+a)^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^{\frac{5}{2}}\csc(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\tan(bx+a)}d^2\csc(bx+a)\tan(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*d^2*csc(b*x + a)*tan(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^{\frac{5}{2}}\csc(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a), x)
```

3.81 $\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal. Leaf size=80

$$\frac{2d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{3b}+\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

```
[Out] (2*d^2*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqr
t[d*Tan[a + b*x]])/(3*b) + (2*d*Csc[a + b*x]*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Rubi [A] time = 0.107196, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2593, 2601, 2573, 2641}

$$\frac{2d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{3b}+\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (2*d^2*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqr
t[d*Tan[a + b*x]])/(3*b) + (2*d*Csc[a + b*x]*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Rule 2593

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^(n - 1))
/(a^2*f*(n - 1)), x] - Dist[(b^2*(m + 2))/(a^2*(n - 1)), Int[(a*Sin[e + f*x
])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && Gt
Q[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*
n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b} + \frac{1}{3}\left(2d^2\right)\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$= \frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b} + \frac{\left(2d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{}{\sqrt{\cos(a+b}}}{3\sqrt{\sin(a+bx)}}$$

$$= \frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b} + \frac{1}{3}\left(2d^2\csc(a+bx)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+\right.$$

$$= \frac{2d^2\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{3b} + \frac{2d\csc(a+bx}{}$$

Mathematica [C] time = 0.340962, size = 71, normalized size = 0.89

$$\frac{2d^2\cos(a+bx)\sqrt{d\tan(a+bx)}\left(2\sqrt{\sec^2(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2(a+bx)\right)+\sec^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (2*d^2*Cos[a + b*x]*(Sec[a + b*x]^2 + 2*Hypergeometric2F1[1/4, 1/2, 5/4, -T
an[a + b*x]^2]*Sqrt[Sec[a + b*x]^2])*Sqrt[d*Tan[a + b*x]])/(3*b)
```

Maple [B] time = 0.146, size = 194, normalized size = 2.4

$$-\frac{\sqrt{2}\left(\cos(bx+a)-1\right)\cos(bx+a)\left(\cos(bx+a)+1\right)^2}{3\,b\left(\sin(bx+a)\right)^6}\left(2\cos(bx+a)\sqrt{\frac{\cos(bx+a)-1}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a)-1+\sin(bx+}{\sin(bx+a)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*(d*tan(b*x+a))^(5/2),x)
```

```
[Out] -1/3/b*2^(1/2)*(cos(b*x+a)-1)*(2*cos(b*x+a)*((cos(b*x+a)-1)/sin(b*x+a))^(1/
2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))
/sin(b*x+a))^(1/2)*sin(b*x+a)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x
+a))^(1/2),1/2*2^(1/2))-cos(b*x+a)*2^(1/2)+2^(1/2))*cos(b*x+a)*(cos(b*x+a)+
1)^2*(d*sin(b*x+a)/cos(b*x+a))^(5/2)/sin(b*x+a)^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^{\frac{5}{2}}\csc(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{d\tan(bx+a)}d^2\csc(bx+a)^3\tan(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*d^2*csc(b*x + a)^3*tan(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan (bx+a))^{\frac{5}{2}} \csc (bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^3, x)
```

3.82 $\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal. Leaf size=110

$$-\frac{4d^3\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{4d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{3b}+\frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

```
[Out] (-4*d^3*Csc[a + b*x])/(3*b*Sqrt[d*Tan[a + b*x]]) + (4*d^2*Csc[a + b*x]*Elli
pticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(3*b)
 + (2*d*Csc[a + b*x]^3*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Rubi [A] time = 0.146866, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2593, 2599, 2601, 2573, 2641}

$$-\frac{4d^3\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{4d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{3b}+\frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^5*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-4*d^3*Csc[a + b*x])/(3*b*Sqrt[d*Tan[a + b*x]]) + (4*d^2*Csc[a + b*x]*Elli
pticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(3*b)
 + (2*d*Csc[a + b*x]^3*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Rule 2593

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^(n - 1))
/(a^2*f*(n - 1)), x] - Dist[(b^2*(m + 2))/(a^2*(n - 1)), Int[(a*Sin[e + f*x
])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && Gt
Q[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*
n]
```

Rule 2599

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^(n - 1)
)/(a^2*f*(m + n + 1)), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
```

```
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx &= \frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}+\left(2d^2\right)\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx\\ &= -\frac{4d^3\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}+\frac{1}{3}\left(4d^2\right)\int \csc(a+bx\\ &= -\frac{4d^3\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}+\frac{\left(4d^2\sqrt{\cos(a+bx)}\sqrt{d}\right.}{}\\ &= -\frac{4d^3\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}+\frac{1}{3}\left(4d^2\csc(a+bx)\sqrt{s}\right.\\ &= -\frac{4d^3\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{4d^2\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a}}{3b}\end{aligned}$$

Mathematica [C] time = 0.469204, size = 110, normalized size = 1.

$$-\frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}\left(\cos(2(a+bx))\sqrt{\sec^2(a+bx)}+2\sqrt[4]{-1}\sin(2(a+bx))\sqrt{\tan(a+bx)}F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\right.\right.\right.}{3b\sqrt{\sec^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^5*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*d*Csc[a + b*x]^3*(Cos[2*(a + b*x)]*Sqrt[Sec[a + b*x]^2] + 2*(-1)^(1/4)*
EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1]*Sin[2*(a + b*x)]*Sq
rt[Tan[a + b*x]])*(d*Tan[a + b*x])^(3/2))/(3*b*Sqrt[Sec[a + b*x]^2])
```

Maple [B] time = 0.158, size = 320, normalized size = 2.9

$$\frac{\sqrt{2}\left(\cos\left(bx+a\right)-1\right)^{2}\cos\left(bx+a\right)\left(\cos\left(bx+a\right)+1\right)^{2}}{3\,b\left(\sin\left(bx+a\right)\right)^{8}}\left(4\,\mathrm{EllipticF}\left(\sqrt{-\frac{\cos\left(bx+a\right)-1-\sin\left(bx+a\right)}{\sin\left(bx+a\right)}},1/2\,\sqrt{2}\right)\sqrt{\frac{\cos\left(b\right.}{\sin\left(\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^5*(d*tan(b*x+a))^(5/2),x)
```

```
[Out] 1/3/b*2^(1/2)*(cos(b*x+a)-1)^2*(4*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin
(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-
1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/
2)*cos(b*x+a)^2*sin(b*x+a)+4*cos(b*x+a)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*(
(cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin
(b*x+a))^(1/2)*sin(b*x+a)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))
^(1/2),1/2*2^(1/2))-2*cos(b*x+a)^2*2^(1/2)+2^(1/2))*cos(b*x+a)*(cos(b*x+a)+
1)^2*(d*sin(b*x+a)/cos(b*x+a))^(5/2)/sin(b*x+a)^8
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}\csc\left(bx+a\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\tan(bx+a)}d^2\csc(bx+a)^5\tan(bx+a)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*d^2*csc(b*x + a)^5*tan(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**5*(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^{\frac{5}{2}}\csc(bx+a)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^5, x)
```

3.83 $\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal. Leaf size=140

$$-\frac{20d^3\csc^3(a+bx)}{21b\sqrt{d\tan(a+bx)}}-\frac{40d^3\csc(a+bx)}{21b\sqrt{d\tan(a+bx)}}+\frac{40d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{21b}+\frac{2d\csc$$

```
[Out] (-40*d^3*Csc[a + b*x])/(21*b*Sqrt[d*Tan[a + b*x]]) - (20*d^3*Csc[a + b*x]^3
)/(21*b*Sqrt[d*Tan[a + b*x]]) + (40*d^2*Csc[a + b*x]*EllipticF[a - Pi/4 + b
*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(21*b) + (2*d*Csc[a + b
*x]^5*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Rubi [A] time = 0.184841, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2593, 2599, 2601, 2573, 2641}

$$-\frac{20d^3\csc^3(a+bx)}{21b\sqrt{d\tan(a+bx)}}-\frac{40d^3\csc(a+bx)}{21b\sqrt{d\tan(a+bx)}}+\frac{40d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{21b}+\frac{2d\csc$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^7*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-40*d^3*Csc[a + b*x])/(21*b*Sqrt[d*Tan[a + b*x]]) - (20*d^3*Csc[a + b*x]^3
)/(21*b*Sqrt[d*Tan[a + b*x]]) + (40*d^2*Csc[a + b*x]*EllipticF[a - Pi/4 + b
*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(21*b) + (2*d*Csc[a + b
*x]^5*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Rule 2593

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^(n - 1))
/(a^2*f*(n - 1)), x] - Dist[(b^2*(m + 2))/(a^2*(n - 1)), Int[(a*Sin[e + f*x
])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && Gt
Q[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*
n]
```

Rule 2599

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^(n - 1)
```

```
)/(a^2*f*(m + n + 1)), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx &= \frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b}+\frac{1}{3}\left(10d^2\right)\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx \\
&= -\frac{20d^3\csc^3(a+bx)}{21b\sqrt{d\tan(a+bx)}}+\frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b}+\frac{1}{7}\left(20d^2\right)\int \csc^3(a+ \\
&= -\frac{40d^3\csc(a+bx)}{21b\sqrt{d\tan(a+bx)}}-\frac{20d^3\csc^3(a+bx)}{21b\sqrt{d\tan(a+bx)}}+\frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b} \\
&= -\frac{40d^3\csc(a+bx)}{21b\sqrt{d\tan(a+bx)}}-\frac{20d^3\csc^3(a+bx)}{21b\sqrt{d\tan(a+bx)}}+\frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b} \\
&= -\frac{40d^3\csc(a+bx)}{21b\sqrt{d\tan(a+bx)}}-\frac{20d^3\csc^3(a+bx)}{21b\sqrt{d\tan(a+bx)}}+\frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b} \\
&= -\frac{40d^3\csc(a+bx)}{21b\sqrt{d\tan(a+bx)}}-\frac{20d^3\csc^3(a+bx)}{21b\sqrt{d\tan(a+bx)}}+\frac{40d^2\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{21b}
\end{aligned}$$

Mathematica [C] time = 1.60996, size = 130, normalized size = 0.93

$$-\frac{d^2\csc(a+bx)\sqrt{d\tan(a+bx)}\left((10\cos(2(a+bx))-5\cos(4(a+bx))+1)\csc^3(a+bx)\sec(a+bx)\sqrt{\sec^2(a+bx)}+80\sqrt[4]{-}\right.}{21b\sqrt{\sec^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^7*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] -(d^2*Csc[a + b*x]*((1 + 10*Cos[2*(a + b*x)] - 5*Cos[4*(a + b*x)])*Csc[a +
b*x]^3*Sec[a + b*x]*Sqrt[Sec[a + b*x]^2] + 80*(-1)^(1/4)*EllipticF[I*ArcSin
h[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1]*Sqrt[Tan[a + b*x]])*Sqrt[d*Tan[a + b*
x]])/(21*b*Sqrt[Sec[a + b*x]^2])
```

Maple [B] time = 0.187, size = 571, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^7*(d*tan(b*x+a))^(5/2),x)
```

```
[Out] -1/21/b*2^(1/2)*(cos(b*x+a)-1)^2*(40*cos(b*x+a)^4*((cos(b*x+a)-1)/sin(b*x+a
))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*
x+a))/sin(b*x+a))^(1/2)*sin(b*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin
(b*x+a))^(1/2),1/2*2^(1/2))+40*cos(b*x+a)^3*((cos(b*x+a)-1)/sin(b*x+a))^(1/
2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/
sin(b*x+a))^(1/2)*sin(b*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a
))^(1/2),1/2*2^(1/2))-40*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+s
in(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*c
os(b*x+a)^2*sin(b*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/
2),1/2*2^(1/2))-40*sin(b*x+a)*cos(b*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a
))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos
(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a)
)^(1/2)-20*cos(b*x+a)^4*2^(1/2)+30*cos(b*x+a)^2*2^(1/2)-7*2^(1/2))*cos(b*x+
a)*(cos(b*x+a)+1)^2*(d*sin(b*x+a)/cos(b*x+a))^(5/2)/sin(b*x+a)^10
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^{\frac{5}{2}}\csc(bx+a)^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^7*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^7, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\tan\left(bx+a\right)}d^2\csc\left(bx+a\right)^7\tan\left(bx+a\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^7*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*d^2*csc(b*x + a)^7*tan(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**7*(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}\csc\left(bx+a\right)^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^7*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^7, x)
```

3.84 $\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal. Leaf size=257

$$-\frac{\cos^4(a+bx)(d\tan(a+bx))^{5/2}}{4bd^3}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b\sqrt{d}}+\frac{5\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{32\sqrt{2}b\sqrt{d}}-\frac{5\log\left(\sqrt{d}\tan(a+bx)-\right.}{64\sqrt{}}$$

```
[Out] (-5*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]*b*Sqrt[
d]) + (5*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]*b*
Sqrt[d]) - (5*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b
*x]]])/(64*Sqrt[2]*b*Sqrt[d]) + (5*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqr
t[2]*Sqrt[d*Tan[a + b*x]]])/(64*Sqrt[2]*b*Sqrt[d]) - (5*Cos[a + b*x]^2*Sqrt
[d*Tan[a + b*x]])/(16*b*d) - (Cos[a + b*x]^4*(d*Tan[a + b*x])^(5/2))/(4*b*d
^3)
```

Rubi [A] time = 0.173985, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2591, 288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\cos^4(a+bx)(d\tan(a+bx))^{5/2}}{4bd^3}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b\sqrt{d}}+\frac{5\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{32\sqrt{2}b\sqrt{d}}-\frac{5\log\left(\sqrt{d}\tan(a+bx)-\right.}{64\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^4/Sqrt[d*Tan[a + b*x]],x]
```


```
[Out] (-5*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]*b*Sqrt[
d]) + (5*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]*b*
Sqrt[d]) - (5*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b
*x]]])/(64*Sqrt[2]*b*Sqrt[d]) + (5*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqr
t[2]*Sqrt[d*Tan[a + b*x]]])/(64*Sqrt[2]*b*Sqrt[d]) - (5*Cos[a + b*x]^2*Sqrt
[d*Tan[a + b*x]])/(16*b*d) - (Cos[a + b*x]^4*(d*Tan[a + b*x])^(5/2))/(4*b*d
^3)
```


Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
```

```
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{d\,\text{Subst}\left(\int \frac{x^{7/2}}{\left(d^2+x^2\right)^3}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= -\frac{\cos^4(a+bx)(d\tan(a+bx))^{5/2}}{4bd^3} + \frac{(5d)\,\text{Subst}\left(\int \frac{x^{3/2}}{\left(d^2+x^2\right)^2}\,dx, x, d\tan(a+bx)\right)}{8b}$$

$$= -\frac{5\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{5/2}}{4bd^3} + \frac{(5d)\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(d^2+x^2\right)}\,dx, x\right.}{32b}$$

$$= -\frac{5\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{5/2}}{4bd^3} + \frac{(5d)\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{\right.}{16b}$$

$$= -\frac{5\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{5/2}}{4bd^3} + \frac{5\,\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\,\text{ta}\right.}{32b}$$

$$= -\frac{5\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{5/2}}{4bd^3} + \frac{5\,\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x\right.}{64b}$$

$$= -\frac{5\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b\sqrt{d}} + \frac{5\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan\right.}{64\sqrt{2}b\sqrt{d}}$$

$$= -\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b\sqrt{d}} + \frac{5\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b\sqrt{d}} - \frac{5\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\right.}{64\sqrt{2}b\sqrt{d}}$$

Mathematica [A] time = 0.705937, size = 122, normalized size = 0.47

$$\frac{\sec(a+bx)\left(-7\sin(a+bx)-6\sin(3(a+bx))+\sin(5(a+bx))-5\sqrt{\sin(2(a+bx))}\sin^{-1}(\cos(a+bx)-\sin(a+bx))+5\right.}{64b\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^4/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (Sec[a + b*x]*(-7*Sin[a + b*x] - 5*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Sqrt
[Sin[2*(a + b*x)]] + 5*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*
x)]]]*Sqrt[Sin[2*(a + b*x)]] - 6*Sin[3*(a + b*x)] + Sin[5*(a + b*x)]))/(64*
b*Sqrt[d*Tan[a + b*x]])
```

Maple [C] time = 0.163, size = 698, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^4/(d*tan(b*x+a))^(1/2),x)
```

```
[Out] -1/64/b*2^(1/2)*(cos(b*x+a)-1)*(5*I*sin(b*x+a)*((cos(b*x+a)-1)/sin(b*x+a))^
(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+
a))/sin(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1
/2),1/2-1/2*I,1/2*2^(1/2))-5*I*sin(b*x+a)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)
*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/s
in(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1
/2+1/2*I,1/2*2^(1/2))-8*2^(1/2)*cos(b*x+a)^5+8*cos(b*x+a)^4*2^(1/2)+5*sin(b
*x+a)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/
2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin
(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)+5*sin(b*x+a)*E
llipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/
2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a)
)^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-10*sin(b*x+a)*Ellipti
cF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-
1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*
x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)+18*cos(b*x+a)^3*2^(1/2)-18*cos(b*x+a)^
2*2^(1/2))*(cos(b*x+a)+1)^2/cos(b*x+a)/sin(b*x+a)^3/(d*sin(b*x+a)/cos(b*x+a
))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**4/(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26412, size = 336, normalized size = 1.31

$$\frac{1}{128}d\left(\frac{10\sqrt{2}\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^2}+\frac{10\sqrt{2}\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^2}+\frac{5\sqrt{2}\sqrt{|d|}\log\left(d\tan\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/128*d*(10*sqrt(2)*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) +
 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d^2) + 10*sqrt(2)*sqrt(abs(d))*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(
d)))/(b*d^2) + 5*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*t
an(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^2) - 5*sqrt(2)*sqrt(abs(d))*log(d*
tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^2)
- 8*(9*sqrt(d*tan(b*x + a))*d^2*tan(b*x + a)^2 + 5*sqrt(d*tan(b*x + a))*d^2
)/((d^2*tan(b*x + a)^2 + d^2)^2*b))
```

3.85 $\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal. Leaf size=227

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b\sqrt{d}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}b\sqrt{d}}-\frac{\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}b\sqrt{d}}+\frac{\log\left(\sqrt{d}\text{ta}\right.}{}$$

```
[Out] -ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(4*Sqrt[2]*b*Sqrt[d]) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(4*Sqrt[2]*b*Sqrt[d]) -
 Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b*x]]]/(8*Sqrt
[2]*b*Sqrt[d]) + Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqrt[2]*Sqrt[d*Tan[a
+ b*x]]]/(8*Sqrt[2]*b*Sqrt[d]) - (Cos[a + b*x]^2*Sqrt[d*Tan[a + b*x]])/(2*b
*d)
```

Rubi [A] time = 0.150287, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2591, 288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b\sqrt{d}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}b\sqrt{d}}-\frac{\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}b\sqrt{d}}+\frac{\log\left(\sqrt{d}\text{ta}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] -ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(4*Sqrt[2]*b*Sqrt[d]) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(4*Sqrt[2]*b*Sqrt[d]) -
 Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b*x]]]/(8*Sqrt
[2]*b*Sqrt[d]) + Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqrt[2]*Sqrt[d*Tan[a
+ b*x]]]/(8*Sqrt[2]*b*Sqrt[d]) - (Cos[a + b*x]^2*Sqrt[d*Tan[a + b*x]])/(2*b
*d)
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{d\,\text{Subst}\left(\int \frac{x^{3/2}}{\left(d^2+x^2\right)^2}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= -\frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd} + \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(d^2+x^2\right)}\,dx, x, d\tan(a+bx)\right)}{4b}$$

$$= -\frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd} + \frac{d\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{2b}$$

$$= -\frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd} + \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{4b} + \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x,\right.}{4b}$$

$$= -\frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd} + \frac{\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8b} + \frac{\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{}}\right.}{}$$

$$= -\frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b\sqrt{d}} + \frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+\right.}{8\sqrt{2}b\sqrt{d}}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b\sqrt{d}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b\sqrt{d}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\,\text{t}\right.}{8\sqrt{2}b\sqrt{d}}$$

Mathematica [A] time = 0.613381, size = 109, normalized size = 0.48

$$-\frac{\sec(a+bx)\left(\sin(a+bx)+\sin(3(a+bx))+\sqrt{\sin(2(a+bx))}\sin^{-1}(\cos(a+bx)-\sin(a+bx))-\sqrt{\sin(2(a+bx))}\log\left(\sin\right.\right.}{8b\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] -(Sec[a + b*x]*(Sin[a + b*x] + ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Sqrt[Sin
[2*(a + b*x)]] - Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]]]*
Sqrt[Sin[2*(a + b*x)]] + Sin[3*(a + b*x)]))/(8*b*Sqrt[d*Tan[a + b*x]])
```

Maple [C] time = 0.142, size = 672, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*tan(b*x+a))^(1/2),x)
```

```
[Out] 1/8/b*2^(1/2)*(cos(b*x+a)-1)*(I*sin(b*x+a)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2
)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/
sin(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),
1/2+1/2*I,1/2*2^(1/2))-I*sin(b*x+a)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos
(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x
+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2
*I,1/2*2^(1/2))-sin(b*x+a)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a
))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x
+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))
^(1/2)+2*sin(b*x+a)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)
,1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/
sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-sin(b*x+a)*
EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1
/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a
))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-2*cos(b*x+a)^3*2^(1/
2)+2*cos(b*x+a)^2*2^(1/2))*(cos(b*x+a)+1)^2/cos(b*x+a)/sin(b*x+a)^3/(d*sin(
b*x+a)/cos(b*x+a))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Integral(sin(a + b*x)**2/sqrt(d*tan(a + b*x)), x)
```

Giac [A] time = 1.18328, size = 296, normalized size = 1.3

$$\frac{1}{16}d\left(\frac{2\sqrt{2}\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^2}+\frac{2\sqrt{2}\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^2}+\frac{\sqrt{2}\sqrt{|d|}\log\left(d\tan(bx+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*d*(2*sqrt(2)*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2
*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d^2) + 2*sqrt(2)*sqrt(abs(d))*arcta
n(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d))
```

```
)/(b*d^2) + sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*
x + a))*sqrt(abs(d)) + abs(d))/(b*d^2) - sqrt(2)*sqrt(abs(d))*log(d*tan(b*x
 + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^2) - 8*sqr
t(d*tan(b*x + a))/((d^2*tan(b*x + a)^2 + d^2)*b))
```

3.86 $\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal. Leaf size=20

$$-\frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

```
[Out] (-2*d)/(3*b*(d*Tan[a + b*x])^(3/2))
```

Rubi [A] time = 0.0363285, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2591, 30}

$$-\frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2/Sqrt[d*Tan[a + b*x]],x]

[Out] (-2*d)/(3*b*(d*Tan[a + b*x])^(3/2))
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{d\,\mathrm{Subst}\left(\int \frac{1}{x^{5/2}}\,dx, x, d\tan(a+bx)\right)}{b}$$
$$= -\frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

Mathematica [A] time = 0.0997169, size = 20, normalized size = 1.

$$-\frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*d)/(3*b*(d*Tan[a + b*x])^(3/2))
```

Maple [B] time = 0.145, size = 38, normalized size = 1.9

$$-\frac{2\cos(bx+a)}{3b\sin(bx+a)}\frac{1}{\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/(d*tan(b*x+a))^(1/2),x)
```

```
[Out] -2/3/b*cos(b*x+a)/sin(b*x+a)/(d*sin(b*x+a)/cos(b*x+a))^(1/2)
```

Maxima [A] time = 1.09345, size = 31, normalized size = 1.55

$$-\frac{2}{3\sqrt{d\tan(bx+a)}b\tan(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3/(sqrt(d*tan(b*x + a))*b*tan(b*x + a))
```

Fricas [B] time = 2.14189, size = 109, normalized size = 5.45

$$\frac{2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)^2}{3\left(bd\cos(bx+a)^2-bd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)^2/(b*d*cos(b*x + a)^2 -
b*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Integral(csc(a + b*x)**2/sqrt(d*tan(a + b*x)), x)
```

Giac [A] time = 1.11735, size = 31, normalized size = 1.55

$$-\frac{2}{3\sqrt{d\tan(bx+a)}b\tan(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -2/3/(sqrt(d*tan(b*x + a))*b*tan(b*x + a))
```

3.87 $\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal. Leaf size=43

$$-\frac{2d^3}{7b(d\tan(a+bx))^{7/2}}-\frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

```
[Out] (-2*d^3)/(7*b*(d*Tan[a + b*x])^(7/2)) - (2*d)/(3*b*(d*Tan[a + b*x])^(3/2))
```

Rubi [A] time = 0.0427622, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2591, 14}

$$-\frac{2d^3}{7b(d\tan(a+bx))^{7/2}}-\frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^4/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*d^3)/(7*b*(d*Tan[a + b*x])^(7/2)) - (2*d)/(3*b*(d*Tan[a + b*x])^(3/2))
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{d\,\text{Subst}\left(\int \frac{d^2+x^2}{x^{9/2}}\,dx, x, d\tan(a+bx)\right)}{b}$$
$$= \frac{d\,\text{Subst}\left(\int \left(\frac{d^2}{x^{9/2}}+\frac{1}{x^{5/2}}\right)dx, x, d\tan(a+bx)\right)}{b}$$
$$= -\frac{2d^3}{7b(d\tan(a+bx))^{7/2}} - \frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

Mathematica [A] time = 0.127127, size = 40, normalized size = 0.93

$$\frac{2d(2\cos(2(a+bx))-5)\csc^2(a+bx)}{21b(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^4/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (2*d*(-5 + 2*Cos[2*(a + b*x)])*Csc[a + b*x]^2)/(21*b*(d*Tan[a + b*x])^(3/2)
)
```

Maple [A] time = 0.173, size = 50, normalized size = 1.2

$$\frac{\left(8\,(\cos(bx+a))^2-14\right)\cos(bx+a)}{21\,b\,(\sin(bx+a))^3}\frac{1}{\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^4/(d*tan(b*x+a))^(1/2),x)
```

```
[Out] 2/21/b*(4*cos(b*x+a)^2-7)*cos(b*x+a)/sin(b*x+a)^3/(d*sin(b*x+a)/cos(b*x+a))
^(1/2)
```

Maxima [A] time = 1.0649, size = 47, normalized size = 1.09

$$-\frac{2\left(7\,d^2\tan(bx+a)^2+3\,d^2\right)d}{21\,(d\tan(bx+a))^{\frac{7}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/21*(7*d^2*tan(b*x + a)^2 + 3*d^2)*d/((d*tan(b*x + a))^(7/2)*b)
```

Fricas [A] time = 2.31248, size = 173, normalized size = 4.02

$$\frac{2\left(4\cos\left(bx+a\right)^4-7\cos\left(bx+a\right)^2\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{21\left(bd\cos\left(bx+a\right)^4-2\,bd\cos\left(bx+a\right)^2+bd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*(4*cos(b*x + a)^4 - 7*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)
)/(b*d*cos(b*x + a)^4 - 2*b*d*cos(b*x + a)^2 + b*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**4/(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13789, size = 61, normalized size = 1.42

$$-\frac{2\left(7\,d^3\tan\left(bx+a\right)^2+3\,d^3\right)}{21\,\sqrt{d\tan\left(bx+a\right)}bd^3\tan\left(bx+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -2/21*(7*d^3*tan(b*x + a)^2 + 3*d^3)/(sqrt(d*tan(b*x + a))*b*d^3*tan(b*x +
a)^3)
```

3.88 $\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal. Leaf size=65

$$-\frac{2d^5}{11b(d\tan(a+bx))^{11/2}}-\frac{4d^3}{7b(d\tan(a+bx))^{7/2}}-\frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

```
[Out] (-2*d^5)/(11*b*(d*Tan[a + b*x])^(11/2)) - (4*d^3)/(7*b*(d*Tan[a + b*x])^(7/
2)) - (2*d)/(3*b*(d*Tan[a + b*x])^(3/2))
```

Rubi [A] time = 0.0489785, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2591, 270}

$$-\frac{2d^5}{11b(d\tan(a+bx))^{11/2}}-\frac{4d^3}{7b(d\tan(a+bx))^{7/2}}-\frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^6/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*d^5)/(11*b*(d*Tan[a + b*x])^(11/2)) - (4*d^3)/(7*b*(d*Tan[a + b*x])^(7/
2)) - (2*d)/(3*b*(d*Tan[a + b*x])^(3/2))
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{d\,\text{Subst}\left(\int \frac{\left(d^2+x^2\right)^2}{x^{13/2}}\,dx, x, d\tan(a+bx)\right)}{b}$$
$$= \frac{d\,\text{Subst}\left(\int \left(\frac{d^4}{x^{13/2}}+\frac{2d^2}{x^{9/2}}+\frac{1}{x^{5/2}}\right)dx, x, d\tan(a+bx)\right)}{b}$$
$$= -\frac{2d^5}{11b(d\tan(a+bx))^{11/2}} - \frac{4d^3}{7b(d\tan(a+bx))^{7/2}} - \frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

Mathematica [A] time = 0.157082, size = 50, normalized size = 0.77

$$\frac{2d(28\cos(2(a+bx))-4\cos(4(a+bx))-45)\csc^4(a+bx)}{231b(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^6/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (2*d*(-45 + 28*Cos[2*(a + b*x)] - 4*Cos[4*(a + b*x)])*Csc[a + b*x]^4)/(231*
b*(d*Tan[a + b*x])^(3/2))
```

Maple [A] time = 0.194, size = 60, normalized size = 0.9

$$-\frac{\left(64\ (\cos(bx+a))^4-176\ (\cos(bx+a))^2+154\right)\cos(bx+a)}{231\,b\,(\sin(bx+a))^5}\frac{1}{\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^6/(d*tan(b*x+a))^(1/2),x)
```

```
[Out] -2/231/b*(32*cos(b*x+a)^4-88*cos(b*x+a)^2+77)*cos(b*x+a)/sin(b*x+a)^5/(d*si
n(b*x+a)/cos(b*x+a))^(1/2)
```

Maxima [A] time = 1.14466, size = 65, normalized size = 1.

$$-\frac{2\left(77\,d^{4}\tan\left(bx+a\right)^{4}+66\,d^{4}\tan\left(bx+a\right)^{2}+21\,d^{4}\right)d}{231\,\left(d\tan\left(bx+a\right)\right)^{\frac{11}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/231*(77*d^4*tan(b*x + a)^4 + 66*d^4*tan(b*x + a)^2 + 21*d^4)*d/((d*tan(b
*x + a))^(11/2)*b)
```

Fricas [A] time = 2.60822, size = 235, normalized size = 3.62

$$\frac{2\left(32\,\cos\left(bx+a\right)^{6}-88\,\cos\left(bx+a\right)^{4}+77\,\cos\left(bx+a\right)^{2}\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{231\left(bd\cos\left(bx+a\right)^{6}-3\,bd\cos\left(bx+a\right)^{4}+3\,bd\cos\left(bx+a\right)^{2}-bd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/231*(32*cos(b*x + a)^6 - 88*cos(b*x + a)^4 + 77*cos(b*x + a)^2)*sqrt(d*si
n(b*x + a)/cos(b*x + a))/(b*d*cos(b*x + a)^6 - 3*b*d*cos(b*x + a)^4 + 3*b*d
*cos(b*x + a)^2 - b*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**6/(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10829, size = 78, normalized size = 1.2

$$-\frac{2\left(77\,d^{5}\tan\left(bx+a\right)^{4}+66\,d^{5}\tan\left(bx+a\right)^{2}+21\,d^{5}\right)}{231\,\sqrt{d\tan\left(bx+a\right)}bd^{5}\tan\left(bx+a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -2/231*(77*d^5*tan(b*x + a)^4 + 66*d^5*tan(b*x + a)^2 + 21*d^5)/(sqrt(d*tan
(b*x + a))*b*d^5*tan(b*x + a)^5)
```

3.89 $\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal. Leaf size=107

$$-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}-\frac{7d\sin^3(a+bx)}{30b(d\tan(a+bx))^{3/2}}+\frac{7\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{20b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[Out] (-7*d*Sin[a + b*x]^3)/(30*b*(d*Tan[a + b*x])^(3/2)) - (d*Sin[a + b*x]^5)/(5
*b*(d*Tan[a + b*x])^(3/2)) + (7*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/
(20*b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rubi [A] time = 0.132531, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2598, 2601, 2572, 2639}

$$-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}-\frac{7d\sin^3(a+bx)}{30b(d\tan(a+bx))^{3/2}}+\frac{7\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{20b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^5/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-7*d*Sin[a + b*x]^3)/(30*b*(d*Tan[a + b*x])^(3/2)) - (d*Sin[a + b*x]^5)/(5
*b*(d*Tan[a + b*x])^(3/2)) + (7*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/
(20*b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
```

```
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx &= -\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}+\frac{7}{10}\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx\\
&= -\frac{7d\sin^3(a+bx)}{30b(d\tan(a+bx))^{3/2}}-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}+\frac{7}{20}\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx\\
&= -\frac{7d\sin^3(a+bx)}{30b(d\tan(a+bx))^{3/2}}-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}+\frac{\left(7\sqrt{\sin(a+bx)}\right)\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,d}{20\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}\\
&= -\frac{7d\sin^3(a+bx)}{30b(d\tan(a+bx))^{3/2}}-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}+\frac{(7\sin(a+bx))\int \sqrt{\sin(2a+2bx)}\,dx}{20\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}\\
&= -\frac{7d\sin^3(a+bx)}{30b(d\tan(a+bx))^{3/2}}-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}+\frac{7E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{20b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}
\end{aligned}
$$

Mathematica [C] time = 0.840406, size = 86, normalized size = 0.8

$$
\frac{\sin(a+bx)\left(28\tan(a+bx)\sqrt{\sec^2(a+bx)}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)-20\sin(2(a+bx))+3\sin(4(a+bx))\right)}{120b\sqrt{d\tan(a+bx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^5/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (Sin[a + b*x]*(-20*Sin[2*(a + b*x)] + 3*Sin[4*(a + b*x)] + 28*Hypergeometri
c2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2]*Tan[a + b*x]))/(1
20*b*Sqrt[d*Tan[a + b*x]])
```

Maple [B] time = 0.177, size = 558, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^5/(d*tan(b*x+a))^(1/2),x)
```

```
[Out] -1/120/b*2^(1/2)*(cos(b*x+a)-1)^2*(12*cos(b*x+a)^6*2^(1/2)-38*cos(b*x+a)^4*
2^(1/2)-21*cos(b*x+a)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/
2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a)
)/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)+42*cos(b*
x+a)*EllipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*(
(cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/
2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-21*EllipticF((-(cos(b*x+a)
-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(
1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a
))/sin(b*x+a))^(1/2)+42*EllipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(
1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+
a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)+47*cos(
b*x+a)^2*2^(1/2)-21*cos(b*x+a)*2^(1/2))*(cos(b*x+a)+1)^2/cos(b*x+a)/sin(b*x
+a)^4/(d*sin(b*x+a)/cos(b*x+a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^5}{\sqrt{d\tan(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^5/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^5/sqrt(d*tan(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\cos(bx+a)^4 - 2\cos(bx+a)^2 + 1\right)\sqrt{d\tan(bx+a)}\sin(bx+a)}{d\tan(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^5/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqrt(d*tan(b*x + a))*sin(b
*x + a)/(d*tan(b*x + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**5/(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^5}{\sqrt{d\tan(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^5/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^5/sqrt(d*tan(b*x + a)), x)
```

3.90 $\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal. Leaf size=79

$$\frac{\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}$$

```
[Out] -(d*Sin[a + b*x]^3)/(3*b*(d*Tan[a + b*x])^(3/2)) + (EllipticE[a - Pi/4 + b*
x, 2]*Sin[a + b*x])/(2*b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rubi [A] time = 0.0943914, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2598, 2601, 2572, 2639}

$$\frac{\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] -(d*Sin[a + b*x]^3)/(3*b*(d*Tan[a + b*x])^(3/2)) + (EllipticE[a - Pi/4 + b*
x, 2]*Sin[a + b*x])/(2*b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx &= -\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}+\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx\\ &= -\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}+\frac{\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}\\ &= -\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}+\frac{\sin(a+bx)\int \sqrt{\sin(2a+2bx)}\,dx}{2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}\\ &= -\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}+\frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}\end{aligned}$$

Mathematica [C] time = 0.73202, size = 98, normalized size = 1.24

$$\frac{\sqrt{d\tan(a+bx)}\left(4\tan(a+bx)\sec(a+bx)\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)-(\sin(a+bx)+\sin(3(a+bx)))\sqrt{\sec^2(a+bx)}\right)}{12bd\sqrt{\sec^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (Sqrt[d*Tan[a + b*x]]*(-(Sqrt[Sec[a + b*x]^2]*(Sin[a + b*x] + Sin[3*(a + b*
x)])) + 4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]*Ta
n[a + b*x]))/(12*b*d*Sqrt[Sec[a + b*x]^2])
```

Maple [B] time = 0.173, size = 545, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/(d*tan(b*x+a))^(1/2),x)
```

```
[Out] 1/12/b*2^(1/2)*(cos(b*x+a)-1)^2*(2*cos(b*x+a)^4*2^(1/2)+3*cos(b*x+a)*Ellipt
icF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)
-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b
*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-6*cos(b*x+a)*EllipticE((-(cos(b*x+a)-
1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1
/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a)
)/sin(b*x+a))^(1/2)+3*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/
2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a)
)/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-6*Ellipti
cE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-
1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*
x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-5*cos(b*x+a)^2*2^(1/2)+3*cos(b*x+a)*2^
(1/2))*(cos(b*x+a)+1)^2/cos(b*x+a)/sin(b*x+a)^4/(d*sin(b*x+a)/cos(b*x+a))^(
1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^3}{\sqrt{d\tan(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^3/sqrt(d*tan(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(\cos(bx+a)^2-1\right)\sqrt{d\tan(bx+a)}\sin(bx+a)}{d\tan(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sqrt(d*tan(b*x + a))*sin(b*x + a)/(d*tan(b*x
 + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^3}{\sqrt{d\tan(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^3/sqrt(d*tan(b*x + a)), x)
```

3.91 $\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal. Leaf size=47

$$\frac{\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[Out] (EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[
d*Tan[a + b*x]])
```

Rubi [A] time = 0.0572691, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2601, 2572, 2639}

$$\frac{\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[
d*Tan[a + b*x]])
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx &= \frac{\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}} \\
&= \frac{\sin(a+bx)\int \sqrt{\sin(2a+2bx)}\,dx}{\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} \\
&= \frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}
\end{aligned}$$

Mathematica [C] time = 0.125529, size = 60, normalized size = 1.28

$$\frac{2\sin(a+bx)\sqrt{\sec^2(a+bx)}\sqrt{d\tan(a+bx)}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2]*S
in[a + b*x]*Sqrt[d*Tan[a + b*x]])/(3*b*d)
```

Maple [B] time = 0.147, size = 531, normalized size = 11.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*tan(b*x+a))^(1/2),x)
```

```
[Out] -1/2/b*2^(1/2)*(cos(b*x+a)-1)^2*(2*cos(b*x+a)*EllipticE((-(cos(b*x+a)-1-sin
(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*(
(cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin
(b*x+a))^(1/2)-cos(b*x+a)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))
^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*
```

```
x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)+2*Ell
ipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x
+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(co
s(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-EllipticF((-(cos(b*x+a)-1-sin(b*x+
a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(
b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+
a))^(1/2)+cos(b*x+a)^2*2^(1/2)-cos(b*x+a)*2^(1/2))*(cos(b*x+a)+1)^2/cos(b*x
+a)/sin(b*x+a)^4/(d*sin(b*x+a)/cos(b*x+a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)}{\sqrt{d\tan(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)/sqrt(d*tan(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{d\tan(bx+a)}\sin(bx+a)}{d\tan(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*sin(b*x + a)/(d*tan(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Integral(sin(a + b*x)/sqrt(d*tan(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)}{\sqrt{d\tan(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)/sqrt(d*tan(b*x + a)), x)
```

3.92 $\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal. Leaf size=72

$$-\frac{2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}}-\frac{2\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[Out] (-2*Cos[a + b*x])/(b*Sqrt[d*Tan[a + b*x]]) - (2*EllipticE[a - Pi/4 + b*x, 2
]*Sin[a + b*x])/(b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rubi [A] time = 0.0971656, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2601, 2570, 2572, 2639}

$$-\frac{2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}}-\frac{2\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*Cos[a + b*x])/(b*Sqrt[d*Tan[a + b*x]]) - (2*EllipticE[a - Pi/4 + b*x, 2
]*Sin[a + b*x])/(b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx &= \frac{\sqrt{\sin(a+bx)}\int \frac{\sqrt{\cos(a+bx)}}{\sin^{\frac{3}{2}}(a+bx)}\,dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}\\
&= -\frac{2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}} - \frac{\left(2\sqrt{\sin(a+bx)}\right)\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}\\
&= -\frac{2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}} - \frac{(2\sin(a+bx))\int \sqrt{\sin(2a+2bx)}\,dx}{\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}\\
&= -\frac{2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}} - \frac{2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}
\end{aligned}$$

Mathematica [C] time = 0.306319, size = 69, normalized size = 0.96

$$-\frac{2\cos(a+bx)\left(2\tan^2(a+bx)\sqrt{\sec^2(a+bx)}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)+3\right)}{3b\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*Cos[a + b*x]*(3 + 2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*S
qrt[Sec[a + b*x]^2]*Tan[a + b*x]^2))/(3*b*Sqrt[d*Tan[a + b*x]])
```

Maple [B] time = 0.164, size = 490, normalized size = 6.8

$$\frac{\sqrt{2}}{b\cos(bx+a)}\left(2\cos(bx+a)\,\mathrm{EllipticE}\left(\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}},1/2\sqrt{2}\right)\sqrt{\frac{\cos(bx+a)-1}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a)-}{\sin}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/(d*tan(b*x+a))^(1/2),x)
```

```
[Out] 1/b*2^(1/2)*(2*cos(b*x+a)*EllipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))
^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*
x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-cos(b
*x+a)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*
((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1
/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)+2*EllipticE((-(cos(b*x+a)
-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(
1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a
))/sin(b*x+a))^(1/2)-EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2
),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))
/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-cos(b*x+a)
*2^(1/2))/(d*sin(b*x+a)/cos(b*x+a))^(1/2)/cos(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)}{\sqrt{d\tan(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)/sqrt(d*tan(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{d\tan(bx+a)}\csc(bx+a)}{d\tan(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*csc(b*x + a)/(d*tan(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Integral(csc(a + b*x)/sqrt(d*tan(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)}{\sqrt{d\tan(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)/sqrt(d*tan(b*x + a)), x)
```

3.93 $\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal. Leaf size=102

$$-\frac{4\cos(a+bx)}{5b\sqrt{d\tan(a+bx)}}-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}-\frac{4\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[Out] (-2*d*Csc[a + b*x])/(5*b*(d*Tan[a + b*x])^(3/2)) - (4*Cos[a + b*x])/(5*b*Sq
rt[d*Tan[a + b*x]]) - (4*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(5*b*Sq
rt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rubi [A] time = 0.133896, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2599, 2601, 2570, 2572, 2639}

$$-\frac{4\cos(a+bx)}{5b\sqrt{d\tan(a+bx)}}-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}-\frac{4\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*d*Csc[a + b*x])/(5*b*(d*Tan[a + b*x])^(3/2)) - (4*Cos[a + b*x])/(5*b*Sq
rt[d*Tan[a + b*x]]) - (4*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(5*b*Sq
rt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rule 2599

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^(n - 1)
)/(a^2*f*(m + n + 1)), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
```

```
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx &= -\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}+\frac{2}{5}\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx\\
&= -\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}+\frac{\left(2\sqrt{\sin(a+bx)}\right)\int \frac{\sqrt{\cos(a+bx)}}{\sin^{\frac{3}{2}}(a+bx)}\,dx}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}\\
&= -\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}-\frac{4\cos(a+bx)}{5b\sqrt{d\tan(a+bx)}}-\frac{\left(4\sqrt{\sin(a+bx)}\right)\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}\\
&= -\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}-\frac{4\cos(a+bx)}{5b\sqrt{d\tan(a+bx)}}-\frac{(4\sin(a+bx))\int \sqrt{\sin(2a+2bx)}\,dx}{5\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}\\
&= -\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}-\frac{4\cos(a+bx)}{5b\sqrt{d\tan(a+bx)}}-\frac{4E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{5b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}
\end{aligned}
$$

Mathematica [C] time = 0.673756, size = 104, normalized size = 1.02

$$
\frac{6(\cos(2(a+bx))-2)\cot(a+bx)\csc(a+bx)\sqrt{\sec^2(a+bx)}-8\tan^2(a+bx)\sec(a+bx)\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)}{15b\sqrt{\sec^2(a+bx)}\sqrt{d\tan(a+bx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (6*(-2 + Cos[2*(a + b*x)])*Cot[a + b*x]*Csc[a + b*x]*Sqrt[Sec[a + b*x]^2] -
 8*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]*Tan[a + b
*x]^2)/(15*b*Sqrt[Sec[a + b*x]^2]*Sqrt[d*Tan[a + b*x]])
```

Maple [B] time = 0.186, size = 972, normalized size = 9.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3/(d*tan(b*x+a))^(1/2),x)
```

```
[Out] -1/5/b*2^(1/2)*(4*cos(b*x+a)^3*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)
*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1)/sin(b*x+a))^(
1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-2*
cos(b*x+a)^3*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+si
n(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*EllipticF(((1
-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+4*cos(b*x+a)^2*((1-c
os(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+
a))^(1/2)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*
x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-2*cos(b*x+a)^2*((1-cos(b*x+a)+sin(b*x+
a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*
x+a)-1)/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^
(1/2),1/2*2^(1/2))-4*cos(b*x+a)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*
x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+s
in(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+2
*cos(b*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1
/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a
))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-2*cos(b*x+a)^3*2^(1/2
)-4*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((c
os(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)
*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+2*EllipticF(((1-cos(b*x+a)+si
n(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*
((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin
(b*x+a))^(1/2)+cos(b*x+a)^2*2^(1/2)+2*cos(b*x+a)*2^(1/2))/sin(b*x+a)^2/(d*s
in(b*x+a)/cos(b*x+a))^(1/2)/cos(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^3}{\sqrt{d\tan(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^3/sqrt(d*tan(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\tan(bx+a)}\csc(bx+a)^3}{d\tan(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*csc(b*x + a)^3/(d*tan(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3/(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Integral(csc(a + b*x)**3/sqrt(d*tan(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^3}{\sqrt{d\tan(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3/sqrt(d*tan(b*x + a)), x)
```

3.94 $\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=257

$$-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}bd^{3/2}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{32\sqrt{2}bd^{3/2}}+\frac{3\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{64\sqrt{2}bd^{3/2}}-\frac{3\log}{}$$

```
[Out] (-3*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]*b*d^(3/
2)) + (3*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]*b*
d^(3/2)) + (3*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b
*x]]])/(64*Sqrt[2]*b*d^(3/2)) - (3*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqr
t[2]*Sqrt[d*Tan[a + b*x]]])/(64*Sqrt[2]*b*d^(3/2)) + (3*Cos[a + b*x]^2*(d*T
an[a + b*x])^(3/2))/(16*b*d^3) - (Cos[a + b*x]^4*(d*Tan[a + b*x])^(3/2))/(4
*b*d^3)
```

Rubi [A] time = 0.181913, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2591, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}bd^{3/2}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{32\sqrt{2}bd^{3/2}}+\frac{3\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{64\sqrt{2}bd^{3/2}}-\frac{3\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^4/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-3*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]*b*d^(3/
2)) + (3*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]*b*
d^(3/2)) + (3*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b
*x]]])/(64*Sqrt[2]*b*d^(3/2)) - (3*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqr
t[2]*Sqrt[d*Tan[a + b*x]]])/(64*Sqrt[2]*b*d^(3/2)) + (3*Cos[a + b*x]^2*(d*T
an[a + b*x])^(3/2))/(16*b*d^3) - (Cos[a + b*x]^4*(d*Tan[a + b*x])^(3/2))/(4
*b*d^3)
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
```

```
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{d\,\text{Subst}\left(\int \frac{x^{5/2}}{(d^2+x^2)^3}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= -\frac{\cos^4(a+bx)(d\tan(a+bx))^{3/2}}{4bd^3} + \frac{(3d)\,\text{Subst}\left(\int \frac{\sqrt{x}}{(d^2+x^2)^2}\,dx, x, d\tan(a+bx)\right)}{8b}$$

$$= \frac{3\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd^3} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{3/2}}{4bd^3} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\right.}{32bd}$$

$$= \frac{3\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd^3} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{3/2}}{4bd^3} + \frac{3\,\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x,\right.}{16bd}$$

$$= \frac{3\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd^3} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{3/2}}{4bd^3} - \frac{3\,\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x,\right.}{32bd}$$

$$= \frac{3\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd^3} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{3/2}}{4bd^3} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\right.}{64\sqrt{2}}$$

$$= \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}bd^{3/2}} - \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\,\text{ta}}\right.}{64\sqrt{2}bd^{3/2}}$$

$$= -\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}bd^{3/2}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}bd^{3/2}} + \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\right.}{64\sqrt{2}bd^{3/}}$$

Mathematica [A] time = 0.326889, size = 123, normalized size = 0.48

$$\frac{\csc(a+bx)\sqrt{d\tan(a+bx)}\left(\cos(a+bx)-2\cos(3(a+bx))+\cos(5(a+bx))-3\sqrt{\sin(2(a+bx))}\sin^{-1}(\cos(a+bx)-\sin\right.}{64bd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^4/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (Csc[a + b*x]*(Cos[a + b*x] - 2*Cos[3*(a + b*x)] + Cos[5*(a + b*x)] - 3*Arc
Sin[Cos[a + b*x] - Sin[a + b*x]]*Sqrt[Sin[2*(a + b*x)]] - 3*Log[Cos[a + b*x
] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]]]*Sqrt[Sin[2*(a + b*x)]])*Sqrt[d*T
an[a + b*x]])/(64*b*d^2)
```

Maple [C] time = 0.14, size = 558, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^4/(d*tan(b*x+a))^(3/2),x)
```

```
[Out] -1/64/b*2^(1/2)*(cos(b*x+a)-1)*(8*cos(b*x+a)^4*2^(1/2)+3*I*((cos(b*x+a)-1)/
sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a
)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/si
n(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*I*((cos(b*x+a)-1)/sin(b*x+a))^(1/2
)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/
sin(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),
1/2+1/2*I,1/2*2^(1/2))-8*cos(b*x+a)^3*2^(1/2)-3*((cos(b*x+a)-1)/sin(b*x+a))
^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x
+a))/sin(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(
1/2),1/2-1/2*I,1/2*2^(1/2))-3*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a
)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(
1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2
*2^(1/2))-6*cos(b*x+a)^2*2^(1/2)+6*cos(b*x+a)*2^(1/2))*(cos(b*x+a)+1)^2/cos
(b*x+a)^2/sin(b*x+a)/(d*sin(b*x+a)/cos(b*x+a))^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**4/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40565, size = 348, normalized size = 1.35

$$\frac{1}{128}d\left(\frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^4}+\frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^4}-\frac{3\sqrt{2}|d|^{\frac{3}{2}}\log\left(d\tan(b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/128*d*(6*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) +
2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d^4) + 6*sqrt(2)*abs(d)^(3/2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)
))/(b*d^4) - 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan
(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^4) + 3*sqrt(2)*abs(d)^(3/2)*log(d*ta
n(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^4) +
8*(3*sqrt(d*tan(b*x + a))*d^3*tan(b*x + a)^3 - sqrt(d*tan(b*x + a))*d^3*tan
(b*x + a))/((d^2*tan(b*x + a)^2 + d^2)^2*b*d^2))
```

3.95 $\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=227

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}bd^{3/2}}+\frac{\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}bd^{3/2}}-\frac{\log\left(\sqrt{d}\tan\ldots\right.}{}$$

```
[Out] -ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(4*Sqrt[2]*b*d^(3/2)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(4*Sqrt[2]*b*d^(3/2)) +
 Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b*x]]]/(8*Sqrt
[2]*b*d^(3/2)) - Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqrt[2]*Sqrt[d*Tan[a
+ b*x]]]/(8*Sqrt[2]*b*d^(3/2)) + (Cos[a + b*x]^2*(d*Tan[a + b*x])^(3/2))/(2
*b*d^3)
```

Rubi [A] time = 0.157729, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2591, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}bd^{3/2}}+\frac{\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}bd^{3/2}}-\frac{\log\left(\sqrt{d}\tan\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(4*Sqrt[2]*b*d^(3/2)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(4*Sqrt[2]*b*d^(3/2)) +
 Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b*x]]]/(8*Sqrt
[2]*b*d^(3/2)) - Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqrt[2]*Sqrt[d*Tan[a
+ b*x]]]/(8*Sqrt[2]*b*d^(3/2)) + (Cos[a + b*x]^2*(d*Tan[a + b*x])^(3/2))/(2
*b*d^3)
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{d\,\text{Subst}\left(\int \frac{\sqrt{x}}{\left(d^2+x^2\right)^2}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= \frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd^3} + \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\tan(a+bx)\right)}{4bd}$$

$$= \frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd^3} + \frac{\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{2bd}$$

$$= \frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd^3} - \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{4bd} + \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx,\right.}{4b}$$

$$= \frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd^3} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{3/2}} + \frac{\text{Subst}\left(\int \frac{}{-d}\right.}{}$$

$$= \frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{3/2}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a\right.}{8\sqrt{2}bd^{3/2}}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}} + \frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{\right.}{8\sqrt{2}bd^{3/2}}$$

Mathematica [A] time = 0.241129, size = 105, normalized size = 0.46

$$-\frac{\sqrt{\sin(2(a+bx))}\sqrt{d\tan(a+bx)}\left(-2\sqrt{\sin(2(a+bx))}+\csc(a+bx)\sin^{-1}(\cos(a+bx)-\sin(a+bx))+\csc(a+bx)\log\left(\text{si}\right.\right.}{8bd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -((ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Csc[a + b*x] + Csc[a + b*x]*Log[Cos[
a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]]] - 2*Sqrt[Sin[2*(a + b*x)]
])*Sqrt[Sin[2*(a + b*x)]]*Sqrt[d*Tan[a + b*x]])/(8*b*d^2)
```

Maple [C] time = 0.154, size = 530, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*tan(b*x+a))^(3/2),x)
```

```
[Out] 1/8/b*2^(1/2)*(cos(b*x+a)-1)*(-I*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin
(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((c
os(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b
*x+a))^(1/2)+I*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2
+1/2*I,1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*
x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)+((cos
(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(
-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin
(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))+((cos(b*x+a)-1)/sin(b*x+a
))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b
*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))
^(1/2),1/2+1/2*I,1/2*2^(1/2))+2*cos(b*x+a)^2*2^(1/2)-2*cos(b*x+a)*2^(1/2))*
(cos(b*x+a)+1)^2/cos(b*x+a)^2/sin(b*x+a)/(d*sin(b*x+a)/cos(b*x+a))^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(sin(a + b*x)**2/(d*tan(a + b*x))**(3/2), x)
```

Giac [A] time = 1.27878, size = 308, normalized size = 1.36

$$\frac{1}{16}d\left(\frac{8\sqrt{d\tan(bx+a)}\tan(bx+a)}{\left(d^2\tan(bx+a)^2+d^2\right)bd}+\frac{2\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^4}+\frac{2\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(b}{2\sqrt{|d|}}\right.}{bd^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*d*(8*sqrt(d*tan(b*x + a))*tan(b*x + a)/((d^2*tan(b*x + a)^2 + d^2)*b*d
) + 2*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqr
t(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d^4) + 2*sqrt(2)*abs(d)^(3/2)*arctan(-1
/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b
*d^4) - sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*x +
a))*sqrt(abs(d)) + abs(d))/(b*d^4) + sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a
) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^4))
```

3.96 $\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=20

$$-\frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

```
[Out] (-2*d)/(5*b*(d*Tan[a + b*x])^(5/2))
```

Rubi [A] time = 0.0429692, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2591, 30}

$$-\frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*d)/(5*b*(d*Tan[a + b*x])^(5/2))
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{d\,\mathrm{Subst}\left(\int \frac{1}{x^{7/2}}\,dx, x, d\tan(a+bx)\right)}{b}$$
$$= -\frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

Mathematica [A] time = 0.120176, size = 20, normalized size = 1.

$$-\frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*d)/(5*b*(d*Tan[a + b*x])^(5/2))
```

Maple [B] time = 0.129, size = 38, normalized size = 1.9

$$-\frac{2\cos(bx+a)}{5\,b\sin(bx+a)}\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/(d*tan(b*x+a))^(3/2),x)
```

```
[Out] -2/5/b*cos(b*x+a)/(d*sin(b*x+a)/cos(b*x+a))^(3/2)/sin(b*x+a)
```

Maxima [A] time = 1.10128, size = 31, normalized size = 1.55

$$-\frac{2}{5\,(d\tan(bx+a))^{\frac{3}{2}}\,b\tan(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/5/((d*tan(b*x + a))^(3/2)*b*tan(b*x + a))
```

Fricas [B] time = 2.1955, size = 135, normalized size = 6.75

$$\frac{2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos{(bx+a)}^3}{5\left(bd^2\cos{(bx+a)}^2-bd^2\right)\sin{(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)^3/((b*d^2*cos(b*x + a)^2
 - b*d^2)*sin(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc^2{(a+bx)}}{(d\tan{(a+bx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(csc(a + b*x)**2/(d*tan(a + b*x))**(3/2), x)
```

Giac [A] time = 1.15788, size = 35, normalized size = 1.75

$$-\frac{2}{5\sqrt{d\tan{(bx+a)}}bd\tan{(bx+a)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -2/5/(sqrt(d*tan(b*x + a))*b*d*tan(b*x + a)^2)
```

3.97 $\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=43

$$-\frac{2d^3}{9b(d\tan(a+bx))^{9/2}}-\frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

```
[Out] (-2*d^3)/(9*b*(d*Tan[a + b*x])^(9/2)) - (2*d)/(5*b*(d*Tan[a + b*x])^(5/2))
```

Rubi [A] time = 0.0496197, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2591, 14}

$$-\frac{2d^3}{9b(d\tan(a+bx))^{9/2}}-\frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^4/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*d^3)/(9*b*(d*Tan[a + b*x])^(9/2)) - (2*d)/(5*b*(d*Tan[a + b*x])^(5/2))
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{d\,\text{Subst}\left(\int \frac{d^2+x^2}{x^{11/2}}\,dx, x, d\tan(a+bx)\right)}{b}$$
$$= \frac{d\,\text{Subst}\left(\int \left(\frac{d^2}{x^{11/2}} + \frac{1}{x^{7/2}}\right)dx, x, d\tan(a+bx)\right)}{b}$$
$$= -\frac{2d^3}{9b(d\tan(a+bx))^{9/2}} - \frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

Mathematica [A] time = 0.0945412, size = 42, normalized size = 0.98

$$\frac{2\left(-5\csc^4(a+bx)+\csc^2(a+bx)+4\right)}{45bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^4/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*(4 + Csc[a + b*x]^2 - 5*Csc[a + b*x]^4))/(45*b*d*Sqrt[d*Tan[a + b*x]])
```

Maple [A] time = 0.149, size = 50, normalized size = 1.2

$$\frac{\left(8\,(\cos(bx+a))^2-18\right)\cos(bx+a)}{45\,b\,(\sin(bx+a))^3}\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^4/(d*tan(b*x+a))^(3/2),x)
```

```
[Out] 2/45/b*(4*cos(b*x+a)^2-9)*cos(b*x+a)/(d*sin(b*x+a)/cos(b*x+a))^(3/2)/sin(b*
x+a)^3
```

Maxima [A] time = 1.12163, size = 47, normalized size = 1.09

$$-\frac{2\left(9\,d^2\tan(bx+a)^2+5\,d^2\right)d}{45\,(d\tan(bx+a))^{\frac{9}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/45*(9*d^2*tan(b*x + a)^2 + 5*d^2)*d/((d*tan(b*x + a))^(9/2)*b)
```

Fricas [B] time = 2.13236, size = 201, normalized size = 4.67

$$\frac{2\left(4\cos\left(bx+a\right)^{5}-9\cos\left(bx+a\right)^{3}\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{45\left(bd^{2}\cos\left(bx+a\right)^{4}-2\,bd^{2}\cos\left(bx+a\right)^{2}+bd^{2}\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/45*(4*cos(b*x + a)^5 - 9*cos(b*x + a)^3)*sqrt(d*sin(b*x + a)/cos(b*x + a)
)/((b*d^2*cos(b*x + a)^4 - 2*b*d^2*cos(b*x + a)^2 + b*d^2)*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**4/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16506, size = 61, normalized size = 1.42

$$-\frac{2\left(9\,d^{4}\tan\left(bx+a\right)^{2}+5\,d^{4}\right)}{45\sqrt{d\tan\left(bx+a\right)}bd^{5}\tan\left(bx+a\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -2/45*(9*d^4*tan(b*x + a)^2 + 5*d^4)/(sqrt(d*tan(b*x + a))*b*d^5*tan(b*x +
a)^4)
```

3.98 $\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{2d^5}{13b(d\tan(a+bx))^{13/2}}-\frac{4d^3}{9b(d\tan(a+bx))^{9/2}}-\frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

```
[Out] (-2*d^5)/(13*b*(d*Tan[a + b*x])^(13/2)) - (4*d^3)/(9*b*(d*Tan[a + b*x])^(9/
2)) - (2*d)/(5*b*(d*Tan[a + b*x])^(5/2))
```

Rubi [A] time = 0.0541373, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2591, 270}

$$-\frac{2d^5}{13b(d\tan(a+bx))^{13/2}}-\frac{4d^3}{9b(d\tan(a+bx))^{9/2}}-\frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^6/(d*Tan[a + b*x])^(3/2),x]
```


```
[Out] (-2*d^5)/(13*b*(d*Tan[a + b*x])^(13/2)) - (4*d^3)/(9*b*(d*Tan[a + b*x])^(9/
2)) - (2*d)/(5*b*(d*Tan[a + b*x])^(5/2))
```


Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{d\,\mathrm{Subst}\left(\int \frac{(d^2+x^2)^2}{x^{15/2}}\,dx, x, d\tan(a+bx)\right)}{b}$$
$$= \frac{d\,\mathrm{Subst}\left(\int \left(\frac{d^4}{x^{15/2}} + \frac{2d^2}{x^{11/2}} + \frac{1}{x^{7/2}}\right)dx, x, d\tan(a+bx)\right)}{b}$$
$$= -\frac{2d^5}{13b(d\tan(a+bx))^{13/2}} - \frac{4d^3}{9b(d\tan(a+bx))^{9/2}} - \frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

Mathematica [A] time = 0.121051, size = 54, normalized size = 0.83

$$\frac{-90\csc^6(a+bx)+10\csc^4(a+bx)+16\csc^2(a+bx)+64}{585bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^6/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (64 + 16*Csc[a + b*x]^2 + 10*Csc[a + b*x]^4 - 90*Csc[a + b*x]^6)/(585*b*d*S
qrt[d*Tan[a + b*x]])
```

Maple [A] time = 0.2, size = 60, normalized size = 0.9

$$-\frac{\left(64\,(\cos(bx+a))^4 - 208\,(\cos(bx+a))^2 + 234\right)\cos(bx+a)}{585\,b\,(\sin(bx+a))^5}\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^6/(d*tan(b*x+a))^(3/2),x)
```

```
[Out] -2/585/b*(32*cos(b*x+a)^4-104*cos(b*x+a)^2+117)*cos(b*x+a)/sin(b*x+a)^5/(d*
sin(b*x+a)/cos(b*x+a))^(3/2)
```

Maxima [A] time = 1.16132, size = 65, normalized size = 1.

$$-\frac{2\left(117\,d^4\tan(bx+a)^4 + 130\,d^4\tan(bx+a)^2 + 45\,d^4\right)d}{585\,(d\tan(bx+a))^{\frac{13}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/585*(117*d^4*tan(b*x + a)^4 + 130*d^4*tan(b*x + a)^2 + 45*d^4)*d/((d*tan
(b*x + a))^(13/2)*b)
```

Fricas [B] time = 2.14254, size = 269, normalized size = 4.14

$$\frac{2\left(32\cos\left(bx+a\right)^{7}-104\cos\left(bx+a\right)^{5}+117\cos\left(bx+a\right)^{3}\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{585\left(bd^{2}\cos\left(bx+a\right)^{6}-3\,bd^{2}\cos\left(bx+a\right)^{4}+3\,bd^{2}\cos\left(bx+a\right)^{2}-bd^{2}\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/585*(32*cos(b*x + a)^7 - 104*cos(b*x + a)^5 + 117*cos(b*x + a)^3)*sqrt(d*
sin(b*x + a)/cos(b*x + a))/((b*d^2*cos(b*x + a)^6 - 3*b*d^2*cos(b*x + a)^4
+ 3*b*d^2*cos(b*x + a)^2 - b*d^2)*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**6/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15975, size = 78, normalized size = 1.2

$$-\frac{2\left(117\,d^{6}\tan\left(bx+a\right)^{4}+130\,d^{6}\tan\left(bx+a\right)^{2}+45\,d^{6}\right)}{585\sqrt{d\tan\left(bx+a\right)}bd^{7}\tan\left(bx+a\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -2/585*(117*d^6*tan(b*x + a)^4 + 130*d^6*tan(b*x + a)^2 + 45*d^6)/(sqrt(d*t
an(b*x + a))*b*d^7*tan(b*x + a)^6)
```

3.99 $\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=112

$$\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{12bd^2}+\frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}}-\frac{\sin(a+bx)}{6bd\sqrt{d\tan(a+bx)}}$$

```
[Out] -Sin[a + b*x]/(6*b*d*Sqrt[d*Tan[a + b*x]]) + Sin[a + b*x]^3/(3*b*d*Sqrt[d*T
an[a + b*x]]) + (Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2
*b*x]]*Sqrt[d*Tan[a + b*x]])/(12*b*d^2)
```

Rubi [A] time = 0.132623, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2596, 2598, 2601, 2573, 2641}

$$\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{12bd^2}+\frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}}-\frac{\sin(a+bx)}{6bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -Sin[a + b*x]/(6*b*d*Sqrt[d*Tan[a + b*x]]) + Sin[a + b*x]^3/(3*b*d*Sqrt[d*T
an[a + b*x]]) + (Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2
*b*x]]*Sqrt[d*Tan[a + b*x]])/(12*b*d^2)
```

Rule 2596

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*m)
, x] - Dist[(a^2*(n + 1))/(b^2*m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
&& IntegersQ[2*m, 2*n]
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
```

```
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx &= \frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}} + \frac{\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx}{6d^2} \\
&= -\frac{\sin(a+bx)}{6bd\sqrt{d\tan(a+bx)}} + \frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}} + \frac{\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx}{12d^2} \\
&= -\frac{\sin(a+bx)}{6bd\sqrt{d\tan(a+bx)}} + \frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}} + \frac{\left(\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin}}}{12d^2\sqrt{\sin(a+bx)}} \\
&= -\frac{\sin(a+bx)}{6bd\sqrt{d\tan(a+bx)}} + \frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}} + \frac{\left(\csc(a+bx)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}\right)\int}{12d^2} \\
&= -\frac{\sin(a+bx)}{6bd\sqrt{d\tan(a+bx)}} + \frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}} + \frac{\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}\sqrt{}}{12bd^2}
\end{aligned}$$

Mathematica [C] time = 0.359041, size = 102, normalized size = 0.91

$$-\frac{\csc(a+bx)\sqrt{d\tan(a+bx)}\left(\sin(4(a+bx))\sqrt{\sec^2(a+bx)}+4\sqrt[4]{-1}\sqrt{\tan(a+bx)}F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right)\middle|-1\right)\right)}{24bd^2\sqrt{\sec^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -(Csc[a + b*x]*(Sqrt[Sec[a + b*x]^2]*Sin[4*(a + b*x)] + 4*(-1)^(1/4)*Ellipt
icF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1]*Sqrt[Tan[a + b*x]])*Sqrt[
d*Tan[a + b*x]])/(24*b*d^2*Sqrt[Sec[a + b*x]^2])
```

Maple [A] time = 0.15, size = 222, normalized size = 2.

$$-\frac{\sqrt{2}\,(\cos(bx+a)-1)\,(\cos(bx+a)+1)^{2}}{12\,b\,(\sin(bx+a))^{2}\,(\cos(bx+a))^{2}}\left(\sin(bx+a)\sqrt{\frac{\cos(bx+a)-1}{\sin(bx+a)}}\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a}{\sin}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/(d*tan(b*x+a))^(3/2),x)
```

```
[Out] -1/12/b*2^(1/2)*(cos(b*x+a)-1)*(sin(b*x+a)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2
)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/s
in(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2
*2^(1/2))+2*cos(b*x+a)^4*2^(1/2)-2*cos(b*x+a)^3*2^(1/2)-cos(b*x+a)^2*2^(1/2
)+cos(b*x+a)*2^(1/2))*(cos(b*x+a)+1)^2/sin(b*x+a)^2/(d*sin(b*x+a)/cos(b*x+a
))^(3/2)/cos(b*x+a)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^{3}}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(\cos\left(bx+a\right)^{2}-1\right)\sqrt{d\tan\left(bx+a\right)}\sin\left(bx+a\right)}{d^{2}\tan\left(bx+a\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sqrt(d*tan(b*x + a))*sin(b*x + a)/(d^2*tan(b
*x + a)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(bx+a\right)^{3}}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

3.100 $\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{\sin(a+bx)}{bd\sqrt{d\tan(a+bx)}}+\frac{\sqrt{\sin(2a+2bx)}\sec(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2bd\sqrt{d\tan(a+bx)}}$$

```
[Out] Sin[a + b*x]/(b*d*Sqrt[d*Tan[a + b*x]]) + (EllipticF[a - Pi/4 + b*x, 2]*Sec
[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(2*b*d*Sqrt[d*Tan[a + b*x]])
```

Rubi [A] time = 0.0937112, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2602, 2569, 2573, 2641}

$$\frac{\sin(a+bx)}{bd\sqrt{d\tan(a+bx)}}+\frac{\sqrt{\sin(2a+2bx)}\sec(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] Sin[a + b*x]/(b*d*Sqrt[d*Tan[a + b*x]]) + (EllipticF[a - Pi/4 + b*x, 2]*Sec
[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(2*b*d*Sqrt[d*Tan[a + b*x]])
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx &= \frac{\sqrt{\sin(a+bx)}\int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sqrt{\sin(a+bx)}}\,dx}{d\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}\\
&= \frac{\sin(a+bx)}{bd\sqrt{d\tan(a+bx)}} + \frac{\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}\,dx}{2d\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}\\
&= \frac{\sin(a+bx)}{bd\sqrt{d\tan(a+bx)}} + \frac{\left(\sec(a+bx)\sqrt{\sin(2a+2bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{2d\sqrt{d\tan(a+bx)}}\\
&= \frac{\sin(a+bx)}{bd\sqrt{d\tan(a+bx)}} + \frac{F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{2bd\sqrt{d\tan(a+bx)}}
\end{aligned}
$$

Mathematica [C] time = 0.740658, size = 126, normalized size = 1.59

$$
\frac{\cos(2(a+bx))\tan^{\frac{3}{2}}(a+bx)\sec(a+bx)\left(-\sqrt{\tan(a+bx)}\sqrt{\sec^2(a+bx)}+\sqrt[4]{-1}\sec^2(a+bx)F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(a+b\right.\right.\right.}{b\left(\tan^2(a+bx)-1\right)\sqrt{\sec^2(a+bx)}(d\tan(a+bx))^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (Cos[2*(a + b*x)]*Sec[a + b*x]*((-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*S
qrt[Tan[a + b*x]]], -1]*Sec[a + b*x]^2 - Sqrt[Sec[a + b*x]^2]*Sqrt[Tan[a +
b*x]])*Tan[a + b*x]^(3/2))/(b*Sqrt[Sec[a + b*x]^2]*(d*Tan[a + b*x])^(3/2)*(
-1 + Tan[a + b*x]^2))
```

Maple [B] time = 0.125, size = 199, normalized size = 2.5

$$\frac{\sqrt{2}\,(\cos(bx+a)-1)\,(\cos(bx+a)+1)^2}{2\,b\,(\cos(bx+a))^2\,(\sin(bx+a))^2}\left(-\sin(bx+a)\,\mathrm{EllipticF}\left(\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{\cos(bx+a}{\sin(bx+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*tan(b*x+a))^(3/2),x)
```

```
[Out] 1/2/b*2^(1/2)*(cos(b*x+a)-1)*(-sin(b*x+a)*EllipticF((-(cos(b*x+a)-1-sin(b*x
+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos
(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x
+a))^(1/2)+cos(b*x+a)^2*2^(1/2)-cos(b*x+a)*2^(1/2))*(cos(b*x+a)+1)^2/cos(b*
x+a)^2/sin(b*x+a)^2/(d*sin(b*x+a)/cos(b*x+a))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{d\tan(bx+a)}\sin(bx+a)}{d^2\tan(bx+a)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*sin(b*x + a)/(d^2*tan(b*x + a)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(sin(a + b*x)/(d*tan(a + b*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

3.101 $\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=82

$$-\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{3bd^2}-\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

```
[Out] (-2*Csc[a + b*x])/(3*b*d*Sqrt[d*Tan[a + b*x]]) - (Csc[a + b*x]*EllipticF[a
- Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(3*b*d^2)
```

Rubi [A] time = 0.104005, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2597, 2601, 2573, 2641}

$$-\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{3bd^2}-\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Csc[a + b*x])/(3*b*d*Sqrt[d*Tan[a + b*x]]) - (Csc[a + b*x]*EllipticF[a
- Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(3*b*d^2)
```

Rule 2597

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
m + n + 1)), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx &= -\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}}-\frac{\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx}{3d^2}\\
&= -\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}}-\frac{\left(\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}\,dx}{3d^2\sqrt{\sin(a+bx)}}\\
&= -\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}}-\frac{\left(\csc(a+bx)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{3d^2}\\
&= -\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}}-\frac{\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{3bd^2}
\end{aligned}
$$

Mathematica [C] time = 0.691711, size = 110, normalized size = 1.34

$$
\frac{2\cos(2(a+bx))\sec(a+bx)\sqrt{\sec^2(a+bx)}\left(\sqrt{\sec^2(a+bx)}-\sqrt[4]{-1}\tan^{\frac{3}{2}}(a+bx)F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right)\middle|-1\right)\right)}{3b\left(\tan^2(a+bx)-1\right)(d\tan(a+bx))^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*Cos[2*(a + b*x)]*Sec[a + b*x]*Sqrt[Sec[a + b*x]^2]*(Sqrt[Sec[a + b*x]^2]
 - (-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1]*Tan[a
 + b*x]^(3/2)))/(3*b*(d*Tan[a + b*x])^(3/2)*(-1 + Tan[a + b*x]^2))
```

Maple [B] time = 0.135, size = 306, normalized size = 3.7

$$-\frac{\sqrt{2}\,(\cos(bx+a)-1)^2\,(\cos(bx+a)+1)^2}{3\,b\,(\cos(bx+a))^2\,(\sin(bx+a))^4}\left(\cos(bx+a)\sqrt{\frac{\cos(bx+a)-1}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\sqrt{-\frac{\cos(bx-}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/(d*tan(b*x+a))^(3/2),x)
```

```
[Out] -1/3/b*2^(1/2)*(cos(b*x+a)-1)^2*(cos(b*x+a)*((cos(b*x+a)-1)/sin(b*x+a))^(1/
2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))
/sin(b*x+a))^(1/2)*sin(b*x+a)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x
+a))^(1/2),1/2*2^(1/2))+sin(b*x+a)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/si
n(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)
-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1
/2)+cos(b*x+a)*2^(1/2))*(cos(b*x+a)+1)^2/cos(b*x+a)^2/sin(b*x+a)^4/(d*sin(b
*x+a)/cos(b*x+a))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\tan(bx+a)}\csc(bx+a)}{d^2\tan(bx+a)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*csc(b*x + a)/(d^2*tan(b*x + a)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(csc(a + b*x)/(d*tan(a + b*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

3.102 $\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=112

$$-\frac{2\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{21bd^2}-\frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}}+\frac{2\csc(a+bx)}{21bd\sqrt{d\tan(a+bx)}}$$

```
[Out] (2*Csc[a + b*x])/(21*b*d*Sqrt[d*Tan[a + b*x]]) - (2*Csc[a + b*x]^3)/(7*b*d*
Sqrt[d*Tan[a + b*x]]) - (2*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[S
in[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(21*b*d^2)
```

Rubi [A] time = 0.145929, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2597, 2599, 2601, 2573, 2641}

$$-\frac{2\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{21bd^2}-\frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}}+\frac{2\csc(a+bx)}{21bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```


```
[Out] (2*Csc[a + b*x])/(21*b*d*Sqrt[d*Tan[a + b*x]]) - (2*Csc[a + b*x]^3)/(7*b*d*
Sqrt[d*Tan[a + b*x]]) - (2*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[S
in[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(21*b*d^2)
```


Rule 2597

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
m + n + 1)), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2599

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^(n - 1)
)/(a^2*f*(m + n + 1)), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
```

```
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx &= -\frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}} - \frac{\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx}{7d^2} \\
&= \frac{2\csc(a+bx)}{21bd\sqrt{d\tan(a+bx)}} - \frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}} - \frac{2\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx}{21d^2} \\
&= \frac{2\csc(a+bx)}{21bd\sqrt{d\tan(a+bx)}} - \frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}} - \frac{\left(2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\mathrm{si}}}}{21d^2\sqrt{\sin(a+bx)}} \\
&= \frac{2\csc(a+bx)}{21bd\sqrt{d\tan(a+bx)}} - \frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}} - \frac{\left(2\csc(a+bx)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}\right)}{21d^2} \\
&= \frac{2\csc(a+bx)}{21bd\sqrt{d\tan(a+bx)}} - \frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}} - \frac{2\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}}{21bd^2}
\end{aligned}$$

Mathematica [C] time = 1.67093, size = 136, normalized size = 1.21

$$\frac{\csc^3(a+bx)\left(\left(10\cos(2(a+bx))+\cos(4(a+bx))+1\right)\sec^2(a+bx)^{3/2}-8\sqrt[4]{-1}\cos(2(a+bx))\tan^{\frac{7}{2}}(a+bx)F\left(i\sinh^{-1}\left(\right.\right.\right.}{42bd\left(\tan^2(a+bx)-1\right)\sqrt{\sec^2(a+bx)}\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (Csc[a + b*x]^3*((1 + 10*Cos[2*(a + b*x)] + Cos[4*(a + b*x)])*(Sec[a + b*x]
^2)^(3/2) - 8*(-1)^(1/4)*Cos[2*(a + b*x)]*EllipticF[I*ArcSinh[(-1)^(1/4)*Sq
rt[Tan[a + b*x]]], -1]*Tan[a + b*x]^(7/2)))/(42*b*d*Sqrt[Sec[a + b*x]^2]*Sq
rt[d*Tan[a + b*x]]*(-1 + Tan[a + b*x]^2))
```

Maple [B] time = 0.169, size = 566, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3/(d*tan(b*x+a))^(3/2),x)
```

```
[Out] 1/21/b*2^(1/2)*(cos(b*x+a)-1)^2*(2*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/si
n(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)
-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1
/2)*cos(b*x+a)^3*sin(b*x+a)+2*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x
+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+si
n(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*c
os(b*x+a)^2*sin(b*x+a)-2*cos(b*x+a)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos
(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x
+a))^(1/2)*sin(b*x+a)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/
2),1/2*2^(1/2))-2*sin(b*x+a)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+
a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin
(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-co
s(b*x+a)^3*2^(1/2)-2*cos(b*x+a)*2^(1/2))*(cos(b*x+a)+1)^2/cos(b*x+a)^2/sin(
b*x+a)^6/(d*sin(b*x+a)/cos(b*x+a))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^3}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\tan\left(bx+a\right)}\csc\left(bx+a\right)^{3}}{d^{2}\tan\left(bx+a\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*csc(b*x + a)^3/(d^2*tan(b*x + a)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^{3}\left(a+bx\right)}{\left(d\tan\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(csc(a + b*x)**3/(d*tan(a + b*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(bx+a\right)^{3}}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

3.103 $\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=257

$$-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}bd^{5/2}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{32\sqrt{2}bd^{5/2}}-\frac{3\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{64\sqrt{2}bd^{5/2}}+\frac{3\log}{}$$

```
[Out] (-3*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]*b*d^(5/
2)) + (3*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]*b*
d^(5/2)) - (3*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b
*x]]])/(64*Sqrt[2]*b*d^(5/2)) + (3*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqr
t[2]*Sqrt[d*Tan[a + b*x]]])/(64*Sqrt[2]*b*d^(5/2)) + (Cos[a + b*x]^2*Sqrt[d
*Tan[a + b*x]])/(16*b*d^3) - (Cos[a + b*x]^4*Sqrt[d*Tan[a + b*x]])/(4*b*d^3
)
```

Rubi [A] time = 0.176455, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2591, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}bd^{5/2}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{32\sqrt{2}bd^{5/2}}-\frac{3\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{64\sqrt{2}bd^{5/2}}+\frac{3\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^4/(d*Tan[a + b*x])^(5/2),x]
```


```
[Out] (-3*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]*b*d^(5/
2)) + (3*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]*b*
d^(5/2)) - (3*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b
*x]]])/(64*Sqrt[2]*b*d^(5/2)) + (3*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqr
t[2]*Sqrt[d*Tan[a + b*x]]])/(64*Sqrt[2]*b*d^(5/2)) + (Cos[a + b*x]^2*Sqrt[d
*Tan[a + b*x]])/(16*b*d^3) - (Cos[a + b*x]^4*Sqrt[d*Tan[a + b*x]])/(4*b*d^3
)
```


Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
```

```
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{d\,\text{Subst}\left(\int \frac{x^{3/2}}{(d^2+x^2)^3}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= -\frac{\cos^4(a+bx)\sqrt{d\tan(a+bx)}}{4bd^3} + \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)^2}\,dx, x, d\tan(a+bx)\right)}{8b}$$

$$= \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd^3} - \frac{\cos^4(a+bx)\sqrt{d\tan(a+bx)}}{4bd^3} + \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)}\,dx, x, d\,\text{t}\right.}{32bd}$$

$$= \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd^3} - \frac{\cos^4(a+bx)\sqrt{d\tan(a+bx)}}{4bd^3} + \frac{3\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\tan}\right.}{16bd}$$

$$= \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd^3} - \frac{\cos^4(a+bx)\sqrt{d\tan(a+bx)}}{4bd^3} + \frac{3\,\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan}\right.}{32bd^2}$$

$$= \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd^3} - \frac{\cos^4(a+bx)\sqrt{d\tan(a+bx)}}{4bd^3} - \frac{3\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x,\right.}{64\sqrt{2}bd^{5/2}}$$

$$= -\frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}bd^{5/2}} + \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d}\right.}{64\sqrt{2}bd^{5/2}}$$

$$= -\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}bd^{5/2}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}bd^{5/2}} - \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\right.}{64\sqrt{2}bd^{5/2}}$$

Mathematica [A] time = 0.741489, size = 123, normalized size = 0.48

$$-\frac{\csc(a+bx)\sqrt{d\tan(a+bx)}\left(\sin(a+bx)+2\sin(3(a+bx))+\sin(5(a+bx))+3\sqrt{\sin(2(a+bx))}\sin^{-1}(\cos(a+bx)-\text{si}\right.}{64bd^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^4/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] -(Csc[a + b*x]*(Sin[a + b*x] + 3*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Sqrt[S
in[2*(a + b*x)]] - 3*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)
]]]*Sqrt[Sin[2*(a + b*x)]] + 2*Sin[3*(a + b*x)] + Sin[5*(a + b*x)])*Sqrt[d*
Tan[a + b*x]])/(64*b*d^3)
```

Maple [C] time = 0.148, size = 698, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^4/(d*tan(b*x+a))^(5/2),x)
```

```
[Out] -1/64/b*2^(1/2)*(cos(b*x+a)-1)*(8*2^(1/2)*cos(b*x+a)^5-3*I*sin(b*x+a)*((cos
(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(
-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin
(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*I*sin(b*x+a)*((cos(b*x+
a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos
(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+
a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))-8*cos(b*x+a)^4*2^(1/2)+3*sin(b
*x+a)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/
2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin
(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)+3*sin(b*x+a)*E
llipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/
2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a)
)^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-6*sin(b*x+a)*Elliptic
F((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1
)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x
+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-2*cos(b*x+a)^3*2^(1/2)+2*cos(b*x+a)^2*2
^(1/2))*(cos(b*x+a)+1)^2/cos(b*x+a)^3/sin(b*x+a)/(d*sin(b*x+a)/cos(b*x+a))^
(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**4/(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25871, size = 339, normalized size = 1.32

$$\frac{1}{128}d\left(\frac{6\sqrt{2}\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^4}+\frac{6\sqrt{2}\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^4}+\frac{3\sqrt{2}\sqrt{|d|}\log\left(d\tan\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/128*d*(6*sqrt(2)*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) +
2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d^4) + 6*sqrt(2)*sqrt(abs(d))*arct
an(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)
))/(b*d^4) + 3*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan
(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^4) - 3*sqrt(2)*sqrt(abs(d))*log(d*ta
n(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^4) +
8*(sqrt(d*tan(b*x + a))*d^2*tan(b*x + a)^2 - 3*sqrt(d*tan(b*x + a))*d^2)/((
d^2*tan(b*x + a)^2 + d^2)^2*b*d^2))
```

3.104 $\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=227

$$-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{5/2}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}bd^{5/2}}-\frac{3\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}bd^{5/2}}+\frac{3\log}{}$$

```
[Out] (-3*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*b*d^(5/2
)) + (3*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*b*d^
(5/2)) - (3*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b*x
]]])/(8*Sqrt[2]*b*d^(5/2)) + (3*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqrt[2
]*Sqrt[d*Tan[a + b*x]]])/(8*Sqrt[2]*b*d^(5/2)) + (Cos[a + b*x]^2*Sqrt[d*Tan
[a + b*x]])/(2*b*d^3)
```

Rubi [A] time = 0.162345, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2591, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{5/2}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}bd^{5/2}}-\frac{3\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}bd^{5/2}}+\frac{3\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/(d*Tan[a + b*x])^(5/2),x]
```



```
[Out] (-3*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*b*d^(5/2
)) + (3*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*b*d^
(5/2)) - (3*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b*x
]]])/(8*Sqrt[2]*b*d^(5/2)) + (3*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqrt[2
]*Sqrt[d*Tan[a + b*x]]])/(8*Sqrt[2]*b*d^(5/2)) + (Cos[a + b*x]^2*Sqrt[d*Tan
[a + b*x]])/(2*b*d^3)
```


Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(d^2+x^2\right)^2}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd^3} + \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(d^2+x^2\right)}\,dx, x, d\tan(a+bx)\right)}{4bd}$$

$$= \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd^3} + \frac{3\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{2bd}$$

$$= \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd^3} + \frac{3\,\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{4bd^2} + \frac{3\,\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx\right.}{4b}$$

$$= \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd^3} - \frac{3\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{5/2}} - \frac{3\,\text{Subst}\left(\int -\right.}{}$$

$$= -\frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{5/2}} + \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\,\text{ta}}\right.}{8\sqrt{2}bd^{5/2}}$$

$$= -\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{5/2}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{5/2}} - \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\right.}{8\sqrt{2}bd^{5/2}}$$

Mathematica [A] time = 0.509314, size = 113, normalized size = 0.5

$$\frac{\csc(a+bx)\sqrt{d\tan(a+bx)}\left(\sin(a+bx)+\sin(3(a+bx))-3\sqrt{\sin(2(a+bx))}\sin^{-1}(\cos(a+bx)-\sin(a+bx))+3\sqrt{\sin(2(\right.}{8bd^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (Csc[a + b*x]*(Sin[a + b*x] - 3*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Sqrt[Si
n[2*(a + b*x)]] + 3*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]
]]*Sqrt[Sin[2*(a + b*x)]] + Sin[3*(a + b*x)])*Sqrt[d*Tan[a + b*x]])/(8*b*d^
3)
```

Maple [C] time = 0.129, size = 672, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*tan(b*x+a))^(5/2),x)
```

```
[Out] 1/8/b*2^(1/2)*(cos(b*x+a)-1)*(-3*I*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/s
in(b*x+a))^(1/2)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a
))/sin(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/
2),1/2-1/2*I,1/2*2^(1/2))+3*I*sin(b*x+a)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*
((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/si
n(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/
2+1/2*I,1/2*2^(1/2))-3*sin(b*x+a)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/si
n(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((
cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(
b*x+a))^(1/2)+6*sin(b*x+a)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a)
)^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b
*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-3*si
n(b*x+a)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I
,1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/
sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)+2*cos(b*x+a
)^3*2^(1/2)-2*cos(b*x+a)^2*2^(1/2))*(cos(b*x+a)+1)^2/cos(b*x+a)^3/sin(b*x+a
)/(d*sin(b*x+a)/cos(b*x+a))^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21178, size = 301, normalized size = 1.33

$$\frac{1}{16}d\left(\frac{6\sqrt{2}\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^4}+\frac{6\sqrt{2}\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^4}+\frac{3\sqrt{2}\sqrt{|d|}\log\left(d\tan(bx\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*d*(6*sqrt(2)*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2
*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d^4) + 6*sqrt(2)*sqrt(abs(d))*arcta
```

```
n(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d))
)/(b*d^4) + 3*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(
b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^4) - 3*sqrt(2)*sqrt(abs(d))*log(d*tan
(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^4) + 8
*sqrt(d*tan(b*x + a))/((d^2*tan(b*x + a)^2 + d^2)*b*d^2))
```

3.105 $\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=20

$$-\frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

```
[Out] (-2*d)/(7*b*(d*Tan[a + b*x])^(7/2))
```

Rubi [A] time = 0.0428568, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2591, 30}

$$-\frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2/(d*Tan[a + b*x])^(5/2),x]

[Out] (-2*d)/(7*b*(d*Tan[a + b*x])^(7/2))
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{d\,\text{Subst}\left(\int \frac{1}{x^{9/2}}\,dx, x, d\tan(a+bx)\right)}{b}$$
$$= -\frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

Mathematica [A] time = 0.159704, size = 20, normalized size = 1.

$$-\frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*d)/(7*b*(d*Tan[a + b*x])^(7/2))
```

Maple [B] time = 0.13, size = 38, normalized size = 1.9

$$-\frac{2\cos(bx+a)}{7b\sin(bx+a)}\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/(d*tan(b*x+a))^(5/2),x)
```

```
[Out] -2/7/b*cos(b*x+a)/sin(b*x+a)/(d*sin(b*x+a)/cos(b*x+a))^(5/2)
```

Maxima [A] time = 1.08351, size = 31, normalized size = 1.55

$$-\frac{2}{7\,(d\tan(bx+a))^{\frac{5}{2}}\,b\tan(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/7/((d*tan(b*x + a))^(5/2)*b*tan(b*x + a))
```

Fricas [B] time = 2.37278, size = 150, normalized size = 7.5

$$-\frac{2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos{(bx+a)}^4}{7\left(bd^3\cos{(bx+a)}^4-2\,bd^3\cos{(bx+a)}^2+bd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/7*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)^4/(b*d^3*cos(b*x + a)^4
 - 2*b*d^3*cos(b*x + a)^2 + b*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14885, size = 35, normalized size = 1.75

$$-\frac{2}{7\sqrt{d\tan{(bx+a)}}bd^2\tan{(bx+a)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] -2/7/(sqrt(d*tan(b*x + a))*b*d^2*tan(b*x + a)^3)
```

3.106 $\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=43

$$-\frac{2d^3}{11b(d\tan(a+bx))^{11/2}}-\frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

```
[Out] (-2*d^3)/(11*b*(d*Tan[a + b*x])^(11/2)) - (2*d)/(7*b*(d*Tan[a + b*x])^(7/2)
)
```

Rubi [A] time = 0.0496892, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2591, 14}

$$-\frac{2d^3}{11b(d\tan(a+bx))^{11/2}}-\frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^4/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*d^3)/(11*b*(d*Tan[a + b*x])^(11/2)) - (2*d)/(7*b*(d*Tan[a + b*x])^(7/2)
)
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{d\,\text{Subst}\left(\int \frac{d^2+x^2}{x^{13/2}}\,dx, x, d\tan(a+bx)\right)}{b}$$
$$= \frac{d\,\text{Subst}\left(\int \left(\frac{d^2}{x^{13/2}}+\frac{1}{x^{9/2}}\right)dx, x, d\tan(a+bx)\right)}{b}$$
$$= -\frac{2d^3}{11b(d\tan(a+bx))^{11/2}} - \frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

Mathematica [A] time = 0.172509, size = 50, normalized size = 1.16

$$\frac{2(2\cos(2(a+bx))-9)\cot^4(a+bx)\csc^2(a+bx)\sqrt{d\tan(a+bx)}}{77bd^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^4/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (2*(-9 + 2*Cos[2*(a + b*x)])*Cot[a + b*x]^4*Csc[a + b*x]^2*Sqrt[d*Tan[a + b
*x]])/(77*b*d^3)
```

Maple [A] time = 0.148, size = 50, normalized size = 1.2

$$\frac{\left(8\,(\cos(bx+a))^2-22\right)\cos(bx+a)}{77\,b\,(\sin(bx+a))^3}\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^4/(d*tan(b*x+a))^(5/2),x)
```

```
[Out] 2/77/b*(4*cos(b*x+a)^2-11)*cos(b*x+a)/sin(b*x+a)^3/(d*sin(b*x+a)/cos(b*x+a)
)^(5/2)
```

Maxima [A] time = 1.12031, size = 47, normalized size = 1.09

$$-\frac{2\left(11\,d^2\tan(bx+a)^2+7\,d^2\right)d}{77\,(d\tan(bx+a))^{\frac{11}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/77*(11*d^2*tan(b*x + a)^2 + 7*d^2)*d/((d*tan(b*x + a))^(11/2)*b)
```

Fricas [B] time = 2.80336, size = 217, normalized size = 5.05

$$-\frac{2\left(4\cos(bx+a)^{6}-11\cos(bx+a)^{4}\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{77\left(bd^{3}\cos(bx+a)^{6}-3\,bd^{3}\cos(bx+a)^{4}+3\,bd^{3}\cos(bx+a)^{2}-bd^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/77*(4*cos(b*x + a)^6 - 11*cos(b*x + a)^4)*sqrt(d*sin(b*x + a)/cos(b*x +
a))/(b*d^3*cos(b*x + a)^6 - 3*b*d^3*cos(b*x + a)^4 + 3*b*d^3*cos(b*x + a)^2
 - b*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**4/(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14505, size = 61, normalized size = 1.42

$$-\frac{2\left(11\,d^{3}\tan(bx+a)^{2}+7\,d^{3}\right)}{77\sqrt{d\tan(bx+a)}bd^{5}\tan(bx+a)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] -2/77*(11*d^3*tan(b*x + a)^2 + 7*d^3)/(sqrt(d*tan(b*x + a))*b*d^5*tan(b*x +
 a)^5)
```

3.107 $\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{2d^5}{15b(d\tan(a+bx))^{15/2}}-\frac{4d^3}{11b(d\tan(a+bx))^{11/2}}-\frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

```
[Out] (-2*d^5)/(15*b*(d*Tan[a + b*x])^(15/2)) - (4*d^3)/(11*b*(d*Tan[a + b*x])^(1
1/2)) - (2*d)/(7*b*(d*Tan[a + b*x])^(7/2))
```

Rubi [A] time = 0.0562098, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2591, 270}

$$-\frac{2d^5}{15b(d\tan(a+bx))^{15/2}}-\frac{4d^3}{11b(d\tan(a+bx))^{11/2}}-\frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^6/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*d^5)/(15*b*(d*Tan[a + b*x])^(15/2)) - (4*d^3)/(11*b*(d*Tan[a + b*x])^(1
1/2)) - (2*d)/(7*b*(d*Tan[a + b*x])^(7/2))
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{d\,\text{Subst}\left(\int \frac{\left(d^2+x^2\right)^2}{x^{17/2}}\,dx, x, d\tan(a+bx)\right)}{b}$$
$$= \frac{d\,\text{Subst}\left(\int \left(\frac{d^4}{x^{17/2}}+\frac{2d^2}{x^{13/2}}+\frac{1}{x^{9/2}}\right)dx, x, d\tan(a+bx)\right)}{b}$$
$$= -\frac{2d^5}{15b(d\tan(a+bx))^{15/2}}-\frac{4d^3}{11b(d\tan(a+bx))^{11/2}}-\frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

Mathematica [A] time = 0.232077, size = 60, normalized size = 0.92

$$\frac{2(44\cos(2(a+bx))-4\cos(4(a+bx))-117)\cot^4(a+bx)\csc^4(a+bx)\sqrt{d\tan(a+bx)}}{1155bd^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^6/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (2*(-117 + 44*Cos[2*(a + b*x)] - 4*Cos[4*(a + b*x)])*Cot[a + b*x]^4*Csc[a +
 b*x]^4*Sqrt[d*Tan[a + b*x]])/(1155*b*d^3)
```

Maple [A] time = 0.172, size = 60, normalized size = 0.9

$$-\frac{\left(64\,(\cos(bx+a))^4-240\,(\cos(bx+a))^2+330\right)\cos(bx+a)}{1155\,b\,(\sin(bx+a))^5}\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^6/(d*tan(b*x+a))^(5/2),x)
```

```
[Out] -2/1155/b*(32*cos(b*x+a)^4-120*cos(b*x+a)^2+165)*cos(b*x+a)/sin(b*x+a)^5/(d
*sin(b*x+a)/cos(b*x+a))^(5/2)
```

Maxima [A] time = 1.0186, size = 65, normalized size = 1.

$$-\frac{2\left(165\,d^4\tan(bx+a)^4+210\,d^4\tan(bx+a)^2+77\,d^4\right)d}{1155\,(d\tan(bx+a))^{\frac{15}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/1155*(165*d^4*tan(b*x + a)^4 + 210*d^4*tan(b*x + a)^2 + 77*d^4)*d/((d*ta
n(b*x + a))^(15/2)*b)
```

Fricas [B] time = 3.20698, size = 285, normalized size = 4.38

$$-\frac{2\left(32\cos\left(bx+a\right)^{8}-120\cos\left(bx+a\right)^{6}+165\cos\left(bx+a\right)^{4}\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{1155\left(bd^{3}\cos\left(bx+a\right)^{8}-4\,bd^{3}\cos\left(bx+a\right)^{6}+6\,bd^{3}\cos\left(bx+a\right)^{4}-4\,bd^{3}\cos\left(bx+a\right)^{2}+bd^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/1155*(32*cos(b*x + a)^8 - 120*cos(b*x + a)^6 + 165*cos(b*x + a)^4)*sqrt(
d*sin(b*x + a)/cos(b*x + a))/(b*d^3*cos(b*x + a)^8 - 4*b*d^3*cos(b*x + a)^6
 + 6*b*d^3*cos(b*x + a)^4 - 4*b*d^3*cos(b*x + a)^2 + b*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**6/(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14282, size = 78, normalized size = 1.2

$$-\frac{2\left(165\,d^{5}\tan\left(bx+a\right)^{4}+210\,d^{5}\tan\left(bx+a\right)^{2}+77\,d^{5}\right)}{1155\,\sqrt{d\tan\left(bx+a\right)}bd^{7}\tan\left(bx+a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] -2/1155*(165*d^5*tan(b*x + a)^4 + 210*d^5*tan(b*x + a)^2 + 77*d^5)/(sqrt(d*
tan(b*x + a))*b*d^7*tan(b*x + a)^7)
```

3.108 $\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=144

$$\frac{3\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{40bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}}-\frac{3\sin^5(a+bx)}{70bd(d\tan(a+bx))^{3/2}}-\frac{\sin^3(a+bx)}{20bd(d\tan(a+bx))^{3/2}}$$

```
[Out] -Sin[a + b*x]^3/(20*b*d*(d*Tan[a + b*x])^(3/2)) - (3*Sin[a + b*x]^5)/(70*b*
d*(d*Tan[a + b*x])^(3/2)) + Sin[a + b*x]^7/(7*b*d*(d*Tan[a + b*x])^(3/2)) +
 (3*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(40*b*d^2*Sqrt[Sin[2*a + 2*b
*x]]*Sqrt[d*Tan[a + b*x]])
```

Rubi [A] time = 0.186241, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2596, 2598, 2601, 2572, 2639}

$$\frac{3\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{40bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}}-\frac{3\sin^5(a+bx)}{70bd(d\tan(a+bx))^{3/2}}-\frac{\sin^3(a+bx)}{20bd(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^7/(d*Tan[a + b*x])^(5/2),x]
```


```
[Out] -Sin[a + b*x]^3/(20*b*d*(d*Tan[a + b*x])^(3/2)) - (3*Sin[a + b*x]^5)/(70*b*
d*(d*Tan[a + b*x])^(3/2)) + Sin[a + b*x]^7/(7*b*d*(d*Tan[a + b*x])^(3/2)) +
 (3*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(40*b*d^2*Sqrt[Sin[2*a + 2*b
*x]]*Sqrt[d*Tan[a + b*x]])
```


Rule 2596

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*m)
, x] - Dist[(a^2*(n + 1))/(b^2*m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
&& IntegersQ[2*m, 2*n]
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
```

```
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}} + \frac{3\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx}{14d^2}$$

$$= -\frac{3\sin^5(a+bx)}{70bd(d\tan(a+bx))^{3/2}} + \frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}} + \frac{3\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx}{20d^2}$$

$$= -\frac{\sin^3(a+bx)}{20bd(d\tan(a+bx))^{3/2}} - \frac{3\sin^5(a+bx)}{70bd(d\tan(a+bx))^{3/2}} + \frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}} + \frac{3\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,d}{40d^2}$$

$$= -\frac{\sin^3(a+bx)}{20bd(d\tan(a+bx))^{3/2}} - \frac{3\sin^5(a+bx)}{70bd(d\tan(a+bx))^{3/2}} + \frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}} + \frac{\left(3\sqrt{\sin(a+bx)}\right)}{40d^2\sqrt{\mathrm{co}}}$$

$$= -\frac{\sin^3(a+bx)}{20bd(d\tan(a+bx))^{3/2}} - \frac{3\sin^5(a+bx)}{70bd(d\tan(a+bx))^{3/2}} + \frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}} + \frac{(3\sin(a+bx))\int}{40d^2\sqrt{\sin(2a+}}$$

$$= -\frac{\sin^3(a+bx)}{20bd(d\tan(a+bx))^{3/2}} - \frac{3\sin^5(a+bx)}{70bd(d\tan(a+bx))^{3/2}} + \frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}} + \frac{3E\left(a-\frac{\pi}{4}+\right.}{40bd^2\sqrt{\sin(2a+}}$$

Mathematica [C] time = 1.54922, size = 122, normalized size = 0.85

$$\frac{\sqrt{d\tan(a+bx)}\left(112\tan(a+bx)\sec(a+bx)\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)-(15\sin(a+bx)+29\sin(3(a+bx))+9\sin(5\ldots}{2240bd^3\sqrt{\sec^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^7/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (Sqrt[d*Tan[a + b*x]]*(-(Sqrt[Sec[a + b*x]^2]*(15*Sin[a + b*x] + 29*Sin[3*(
a + b*x)] + 9*Sin[5*(a + b*x)] - 5*Sin[7*(a + b*x)])) + 112*Hypergeometric2
F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]*Tan[a + b*x]))/(2240*b*d^3*
Sqrt[Sec[a + b*x]^2])
```

Maple [B] time = 0.187, size = 571, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^7/(d*tan(b*x+a))^(5/2),x)
```

```
[Out] -1/560/b*2^(1/2)*(cos(b*x+a)-1)^2*(40*2^(1/2)*cos(b*x+a)^8-108*cos(b*x+a)^6
*2^(1/2)+82*cos(b*x+a)^4*2^(1/2)-21*cos(b*x+a)*EllipticF((-(cos(b*x+a)-1-si
n(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*
((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/si
n(b*x+a))^(1/2)+42*cos(b*x+a)*EllipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x
+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+si
n(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-2
1*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((co
s(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*
(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)+42*EllipticE((-(cos(b*x+a)-1-
sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2
)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/
sin(b*x+a))^(1/2)+7*cos(b*x+a)^2*2^(1/2)-21*cos(b*x+a)*2^(1/2))*(cos(b*x+a)
+1)^2/cos(b*x+a)^3/sin(b*x+a)^2/(d*sin(b*x+a)/cos(b*x+a))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin (bx+a)^7}{(d\tan (bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^7/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^7/(d*tan(b*x + a))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\left(\cos (bx+a)^6-3\,\cos (bx+a)^4+3\,\cos (bx+a)^2-1\right)\sqrt{d\tan (bx+a)}\sin (bx+a)}{d^3\tan (bx+a)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^7/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^6 - 3*cos(b*x + a)^4 + 3*cos(b*x + a)^2 - 1)*sqrt(d
*tan(b*x + a))*sin(b*x + a)/(d^3*tan(b*x + a)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**7/(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin (bx+a)^7}{(d\tan (bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^7/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^7/(d*tan(b*x + a))^(5/2), x)
```

3.109 $\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=114

$$\frac{3\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{20bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}}-\frac{\sin^3(a+bx)}{10bd(d\tan(a+bx))^{3/2}}$$

```
[Out] -Sin[a + b*x]^3/(10*b*d*(d*Tan[a + b*x])^(3/2)) + Sin[a + b*x]^5/(5*b*d*(d*
Tan[a + b*x])^(3/2)) + (3*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(20*b*
d^2*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rubi [A] time = 0.142571, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2596, 2598, 2601, 2572, 2639}

$$\frac{3\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{20bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}}-\frac{\sin^3(a+bx)}{10bd(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^5/(d*Tan[a + b*x])^(5/2),x]
```


```
[Out] -Sin[a + b*x]^3/(10*b*d*(d*Tan[a + b*x])^(3/2)) + Sin[a + b*x]^5/(5*b*d*(d*
Tan[a + b*x])^(3/2)) + (3*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(20*b*
d^2*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```


Rule 2596

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*m)
, x] - Dist[(a^2*(n + 1))/(b^2*m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
&& IntegersQ[2*m, 2*n]
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
```

```
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx &= \frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}}+\frac{3\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx}{10d^2}\\
&= -\frac{\sin^3(a+bx)}{10bd(d\tan(a+bx))^{3/2}}+\frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}}+\frac{3\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx}{20d^2}\\
&= -\frac{\sin^3(a+bx)}{10bd(d\tan(a+bx))^{3/2}}+\frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}}+\frac{\left(3\sqrt{\sin(a+bx)}\right)\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{20d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}\\
&= -\frac{\sin^3(a+bx)}{10bd(d\tan(a+bx))^{3/2}}+\frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}}+\frac{(3\sin(a+bx))\int \sqrt{\sin(2a+2bx)}\,dx}{20d^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}\\
&= -\frac{\sin^3(a+bx)}{10bd(d\tan(a+bx))^{3/2}}+\frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}}+\frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{20bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}
\end{aligned}
$$

Mathematica [C] time = 1.06887, size = 100, normalized size = 0.88

$$
\frac{\sqrt{d\tan(a+bx)}\left(8\tan(a+bx)\sec(a+bx)\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)-(\sin(3(a+bx))+\sin(5(a+bx)))\sqrt{\sec^2(a+bx)}\right)}{80bd^3\sqrt{\sec^2(a+bx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^5/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (Sqrt[d*Tan[a + b*x]]*(-(Sqrt[Sec[a + b*x]^2]*(Sin[3*(a + b*x)] + Sin[5*(a
+ b*x)])) + 8*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x
]*Tan[a + b*x]))/(80*b*d^3*Sqrt[Sec[a + b*x]^2])
```

Maple [B] time = 0.138, size = 550, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^5/(d*tan(b*x+a))^(5/2),x)
```

```
[Out] 1/40/b*2^(1/2)*(cos(b*x+a)-1)^2*(4*cos(b*x+a)^6*2^(1/2)-6*cos(b*x+a)^4*2^(1
/2)-6*cos(b*x+a)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2
*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(
b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+3*cos(b*x+a)*Ell
ipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+
a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(
b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-6*EllipticE(((1-cos(b*x+a)+sin(b*x+a
))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos
(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a)
)^(1/2)+3*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2
))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))
^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-cos(b*x+a)^2*2^(1/2)+3*
cos(b*x+a)*2^(1/2))*(cos(b*x+a)+1)^2/sin(b*x+a)^2/(d*sin(b*x+a)/cos(b*x+a))
^(5/2)/cos(b*x+a)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^5}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^5/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^5/(d*tan(b*x + a))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\cos\left(bx+a\right)^{4}-2\,\cos\left(bx+a\right)^{2}+1\right)\sqrt{d\tan\left(bx+a\right)}\sin\left(bx+a\right)}{d^{3}\tan\left(bx+a\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^5/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqrt(d*tan(b*x + a))*sin(b
*x + a)/(d^3*tan(b*x + a)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**5/(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(bx+a\right)^{5}}{\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^5/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^5/(d*tan(b*x + a))^(5/2), x)
```

3.110 $\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=84

$$\frac{\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$

```
[Out] Sin[a + b*x]^3/(3*b*d*(d*Tan[a + b*x])^(3/2)) + (EllipticE[a - Pi/4 + b*x,
2]*Sin[a + b*x])/(2*b*d^2*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rubi [A] time = 0.102488, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2596, 2601, 2572, 2639}

$$\frac{\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] Sin[a + b*x]^3/(3*b*d*(d*Tan[a + b*x])^(3/2)) + (EllipticE[a - Pi/4 + b*x,
2]*Sin[a + b*x])/(2*b*d^2*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rule 2596

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*m)
, x] - Dist[(a^2*(n + 1))/(b^2*m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
&& IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx &= \frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}}+\frac{\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx}{2d^2}\\ &= \frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}}+\frac{\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{2d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}\\ &= \frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}}+\frac{\sin(a+bx)\int \sqrt{\sin(2a+2bx)}\,dx}{2d^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}\\ &= \frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}}+\frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{2bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}\end{aligned}$$

Mathematica [C] time = 0.617591, size = 97, normalized size = 1.15

$$\frac{\sqrt{d\tan(a+bx)}\left(4\tan(a+bx)\sec(a+bx)\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)+(\sin(a+bx)+\sin(3(a+bx)))\sqrt{\sec^2(a+bx)}\right)}{12bd^3\sqrt{\sec^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (Sqrt[d*Tan[a + b*x]]*(Sqrt[Sec[a + b*x]^2]*(Sin[a + b*x] + Sin[3*(a + b*x)
]) + 4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]*Tan[a
 + b*x]))/(12*b*d^3*Sqrt[Sec[a + b*x]^2])
```

Maple [B] time = 0.167, size = 544, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/(d*tan(b*x+a))^(5/2),x)
```

```
[Out] -1/12/b*2^(1/2)*(cos(b*x+a)-1)^2*(2*cos(b*x+a)^4*2^(1/2)+6*cos(b*x+a)*Ellip
ticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a
)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(
b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-3*cos(b*x+a)*EllipticF((-(cos(b*x+a)
-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(
1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a
))/sin(b*x+a))^(1/2)+6*EllipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1
/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a
))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-3*Ellipt
icF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)
-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b
*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)+cos(b*x+a)^2*2^(1/2)-3*cos(b*x+a)*2^(
1/2))*(cos(b*x+a)+1)^2/cos(b*x+a)^3/sin(b*x+a)^2/(d*sin(b*x+a)/cos(b*x+a))^
(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^3}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^3/(d*tan(b*x + a))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(\cos(bx+a)^2-1\right)\sqrt{d\tan(bx+a)}\sin(bx+a)}{d^3\tan(bx+a)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sqrt(d*tan(b*x + a))*sin(b*x + a)/(d^3*tan(b
*x + a)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)^3}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^3/(d*tan(b*x + a))^(5/2), x)
```

3.111 $\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}} dx$

Optimal. Leaf size=78

$$-\frac{3\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}}$$

```
[Out] (-2*Sin[a + b*x])/(b*d*(d*Tan[a + b*x])^(3/2)) - (3*EllipticE[a - Pi/4 + b*
x, 2]*Sin[a + b*x])/(b*d^2*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rubi [A] time = 0.0865126, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2597, 2601, 2572, 2639}

$$-\frac{3\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*Sin[a + b*x])/(b*d*(d*Tan[a + b*x])^(3/2)) - (3*EllipticE[a - Pi/4 + b*
x, 2]*Sin[a + b*x])/(b*d^2*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rule 2597

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
m + n + 1)), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx &= -\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}} - \frac{3\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx}{d^2} \\
&= -\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}} - \frac{\left(3\sqrt{\sin(a+bx)}\right)\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}} \\
&= -\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}} - \frac{(3\sin(a+bx))\int \sqrt{\sin(2a+2bx)}\,dx}{d^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} \\
&= -\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}} - \frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}
\end{aligned}$$

Mathematica [C] time = 0.376484, size = 69, normalized size = 0.88

$$-\frac{2\cos(a+bx)\left(\tan^2(a+bx)\sqrt{\sec^2(a+bx)}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)+1\right)}{bd^2\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*Cos[a + b*x]*(1 + Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sqr
t[Sec[a + b*x]^2]*Tan[a + b*x]^2))/(b*d^2*Sqrt[d*Tan[a + b*x]])
```

Maple [B] time = 0.138, size = 511, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*tan(b*x+a))^(5/2),x)
```

```
[Out] 1/2/b*2^(1/2)*(6*cos(b*x+a)*EllipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a
))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(
b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-3*c
os(b*x+a)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/
2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a)
)^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)+6*EllipticE((-(cos(b*
x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a
))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b
*x+a))/sin(b*x+a))^(1/2)-3*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a)
)^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b
*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)+cos(
b*x+a)^2*2^(1/2)-3*cos(b*x+a)*2^(1/2))*sin(b*x+a)^2/cos(b*x+a)^3/(d*sin(b*x
+a)/cos(b*x+a))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)/(d*tan(b*x + a))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\tan(bx+a)}\sin(bx+a)}{d^3\tan(bx+a)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*sin(b*x + a)/(d^3*tan(b*x + a)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)/(d*tan(b*x + a))^(5/2), x)
```

3.112 $\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=110

$$\frac{6\cos(a+bx)}{5bd^2\sqrt{d\tan(a+bx)}}+\frac{6\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

```
[Out] (-2*Csc[a + b*x])/(5*b*d*(d*Tan[a + b*x])^(3/2)) + (6*Cos[a + b*x])/(5*b*d^
2*Sqrt[d*Tan[a + b*x]]) + (6*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(5*
b*d^2*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rubi [A] time = 0.136172, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2597, 2601, 2570, 2572, 2639}

$$\frac{6\cos(a+bx)}{5bd^2\sqrt{d\tan(a+bx)}}+\frac{6\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*Csc[a + b*x])/(5*b*d*(d*Tan[a + b*x])^(3/2)) + (6*Cos[a + b*x])/(5*b*d^
2*Sqrt[d*Tan[a + b*x]]) + (6*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(5*
b*d^2*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rule 2597

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
m + n + 1)), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
```

```
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx &= -\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}} - \frac{3\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx}{5d^2} \\
&= -\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}} - \frac{\left(3\sqrt{\sin(a+bx)}\right)\int \frac{\sqrt{\cos(a+bx)}}{\sin^{\frac{3}{2}}(a+bx)}\,dx}{5d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}} \\
&= -\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}} + \frac{6\cos(a+bx)}{5bd^2\sqrt{d\tan(a+bx)}} + \frac{\left(6\sqrt{\sin(a+bx)}\right)\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+b}}{5d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}} \\
&= -\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}} + \frac{6\cos(a+bx)}{5bd^2\sqrt{d\tan(a+bx)}} + \frac{(6\sin(a+bx))\int \sqrt{\sin(2a+2bx)}\,dx}{5d^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} \\
&= -\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}} + \frac{6\cos(a+bx)}{5bd^2\sqrt{d\tan(a+bx)}} + \frac{6E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{5bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}\end{aligned}$$

Mathematica [C] time = 1.76854, size = 105, normalized size = 0.95

$$\frac{2\sin(a+bx)\sqrt{d\tan(a+bx)}\left(2\sec^2(a+bx)\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)-\left(\csc^4(a+bx)-4\csc^2(a+bx)+3\right)\sqrt{\sec^2(a}\right.}{5bd^3\sqrt{\sec^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (2*(2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]^2 - (3
 - 4*Csc[a + b*x]^2 + Csc[a + b*x]^4)*Sqrt[Sec[a + b*x]^2])*Sin[a + b*x]*Sq
rt[d*Tan[a + b*x]])/(5*b*d^3*Sqrt[Sec[a + b*x]^2])
```

Maple [B] time = 0.164, size = 980, normalized size = 8.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/(d*tan(b*x+a))^(5/2),x)
```

```
[Out] -1/5/b*2^(1/2)*(3*cos(b*x+a)^3*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*
x+a))^(1/2),1/2*2^(1/2))*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((co
s(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)-
6*cos(b*x+a)^3*EllipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*
2^(1/2))*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b
*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)+3*cos(b*x+a)^2*E
llipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*(-(cos(
b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a
))^(1/2)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)-6*cos(b*x+a)^2*EllipticE((-(cos(
b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*(-(cos(b*x+a)-1-sin(b*x
+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b
*x+a)-1)/sin(b*x+a))^(1/2)-3*cos(b*x+a)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a
))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b
*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a
))^(1/2)+6*cos(b*x+a)*EllipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/
2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a)
)/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)+3*cos(b*x
+a)^3*2^(1/2)-3*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2
*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(
b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)+6*EllipticE((-(
cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin
(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1
-sin(b*x+a))/sin(b*x+a))^(1/2)+cos(b*x+a)^2*2^(1/2)-3*cos(b*x+a)*2^(1/2))/c
os(b*x+a)^3/(d*sin(b*x+a)/cos(b*x+a))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)/(d*tan(b*x + a))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\tan(bx+a)}\csc(bx+a)}{d^3\tan(bx+a)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*csc(b*x + a)/(d^3*tan(b*x + a)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)/(d*tan(b*x + a))^(5/2), x)
```

3.113 $\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}} dx$

Optimal. Leaf size=140

$$\frac{4\cos(a+bx)}{15bd^2\sqrt{d\tan(a+bx)}}+\frac{4\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{15bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}}+\frac{2\csc(a+bx)}{15bd(d\tan(a+bx))^{3/2}}$$

```
[Out] (2*Csc[a + b*x])/(15*b*d*(d*Tan[a + b*x])^(3/2)) - (2*Csc[a + b*x]^3)/(9*b*
d*(d*Tan[a + b*x])^(3/2)) + (4*Cos[a + b*x])/(15*b*d^2*Sqrt[d*Tan[a + b*x]]
) + (4*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(15*b*d^2*Sqrt[Sin[2*a +
2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rubi [A] time = 0.183669, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2597, 2599, 2601, 2570, 2572, 2639}

$$\frac{4\cos(a+bx)}{15bd^2\sqrt{d\tan(a+bx)}}+\frac{4\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{15bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}}+\frac{2\csc(a+bx)}{15bd(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3/(d*Tan[a + b*x])^(5/2),x]
```


```
[Out] (2*Csc[a + b*x])/(15*b*d*(d*Tan[a + b*x])^(3/2)) - (2*Csc[a + b*x]^3)/(9*b*
d*(d*Tan[a + b*x])^(3/2)) + (4*Cos[a + b*x])/(15*b*d^2*Sqrt[d*Tan[a + b*x]]
) + (4*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(15*b*d^2*Sqrt[Sin[2*a +
2*b*x]]*Sqrt[d*Tan[a + b*x]])
```


Rule 2597

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
m + n + 1)), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2599

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^(n - 1)
```

```
)/(a^2*f*(m + n + 1)), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}} - \frac{\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx}{3d^2}$$

$$= \frac{2\csc(a+bx)}{15bd(d\tan(a+bx))^{3/2}} - \frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}} - \frac{2\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx}{15d^2}$$

$$= \frac{2\csc(a+bx)}{15bd(d\tan(a+bx))^{3/2}} - \frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}} - \frac{\left(2\sqrt{\sin(a+bx)}\right)\int \frac{\sqrt{\cos(a+bx)}}{\sin^{\frac{3}{2}}(a+bx)}\,dx}{15d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$= \frac{2\csc(a+bx)}{15bd(d\tan(a+bx))^{3/2}} - \frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}} + \frac{4\cos(a+bx)}{15bd^2\sqrt{d\tan(a+bx)}} + \frac{\left(4\sqrt{\sin(a+bx)}\right)\int}{15d^2\sqrt{\cos}}$$

$$= \frac{2\csc(a+bx)}{15bd(d\tan(a+bx))^{3/2}} - \frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}} + \frac{4\cos(a+bx)}{15bd^2\sqrt{d\tan(a+bx)}} + \frac{(4\sin(a+bx))\int}{15d^2\sqrt{\sin(2a+2b}}$$

$$= \frac{2\csc(a+bx)}{15bd(d\tan(a+bx))^{3/2}} - \frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}} + \frac{4\cos(a+bx)}{15bd^2\sqrt{d\tan(a+bx)}} + \frac{4E\left(a-\frac{\pi}{4}+b\right.}{15bd^2\sqrt{\sin(2a+2}}$$

<u>Mathematica</u> [C] time = 0.8006, size = 116, normalized size = 0.83

$$\frac{2\sin(a+bx)\sqrt{d\tan(a+bx)}\left(4\sec^2(a+bx)\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)+\left(-5\csc^6(a+bx)+8\csc^4(a+bx)+3\csc^2(a\right.\right.}{45bd^3\sqrt{\sec^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (2*(4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]^2 + (-
6 + 3*Csc[a + b*x]^2 + 8*Csc[a + b*x]^4 - 5*Csc[a + b*x]^6)*Sqrt[Sec[a + b*
x]^2])*Sin[a + b*x]*Sqrt[d*Tan[a + b*x]])/(45*b*d^3*Sqrt[Sec[a + b*x]^2])
```

<u>Maple</u> [B] time = 0.2, size = 1479, normalized size = 10.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3/(d*tan(b*x+a))^(5/2),x)
```

```
[Out] -1/45/b*2^(1/2)*(12*cos(b*x+a)^5*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1
/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1)/sin(b*x+a)
)^(1/2)*EllipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2)
)-6*cos(b*x+a)^5*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)
-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*Elliptic
F((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+12*cos(b*x+a)^
4*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/
sin(b*x+a))^(1/2)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*EllipticE((-(cos(b*x+a)
-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-6*cos(b*x+a)^4*(-(cos(b*x+a)-
1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2
)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/s
in(b*x+a))^(1/2),1/2*2^(1/2))-24*cos(b*x+a)^3*EllipticE((-(cos(b*x+a)-1-sin
(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x
+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1)/sin
(b*x+a))^(1/2)+12*cos(b*x+a)^3*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*
x+a))^(1/2),1/2*2^(1/2))*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((co
s(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)-
6*2^(1/2)*cos(b*x+a)^5-24*cos(b*x+a)^2*EllipticE((-(cos(b*x+a)-1-sin(b*x+a)
)/sin(b*x+a))^(1/2),1/2*2^(1/2))*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1
/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1)/sin(b*x+a)
)^(1/2)+12*cos(b*x+a)^2*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(
1/2),1/2*2^(1/2))*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a
)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)+3*cos(b
*x+a)^4*2^(1/2)+12*cos(b*x+a)*EllipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x
+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+si
n(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-6
*cos(b*x+a)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(
1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+
a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)+12*cos(b*x+a)^3*2^(
1/2)+12*EllipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2)
)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^
(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-6*EllipticF((-(cos(b*x+
a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))
^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x
+a))/sin(b*x+a))^(1/2)+2*cos(b*x+a)^2*2^(1/2)-6*cos(b*x+a)*2^(1/2))/cos(b*x
+a)^3/sin(b*x+a)^2/(d*sin(b*x+a)/cos(b*x+a))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^3}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^3/(d*tan(b*x + a))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\tan\left(bx+a\right)}\csc\left(bx+a\right)^{3}}{d^{3}\tan\left(bx+a\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*csc(b*x + a)^3/(d^3*tan(b*x + a)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3/(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(bx+a\right)^{3}}{\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3/(d*tan(b*x + a))^(5/2), x)
```

3.114 $\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx$

Optimal. Leaf size=68

$$-\frac{8a^2b\sqrt{a\sin(e+fx)}}{5f\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}}$$

```
[Out] (-8*a^2*b*Sqrt[a*Sin[e + f*x]])/(5*f*Sqrt[b*Tan[e + f*x]]) - (2*b*(a*Sin[e
+ f*x])^(5/2))/(5*f*Sqrt[b*Tan[e + f*x]])
```

Rubi [A] time = 0.0901038, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2598, 2589}

$$-\frac{8a^2b\sqrt{a\sin(e+fx)}}{5f\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (-8*a^2*b*Sqrt[a*Sin[e + f*x]])/(5*f*Sqrt[b*Tan[e + f*x]]) - (2*b*(a*Sin[e
+ f*x])^(5/2))/(5*f*Sqrt[b*Tan[e + f*x]])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = -\frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}} + \frac{1}{5}\left(4a^2\right)\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx$$
$$= -\frac{8a^2b\sqrt{a\sin(e+fx)}}{5f\sqrt{b\tan(e+fx)}} - \frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}}$$

Mathematica [A] time = 0.186618, size = 51, normalized size = 0.75

$$-\frac{a^2\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}(\sin(2(e+fx))+8\cot(e+fx))}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] -(a^2*Sqrt[a*Sin[e + f*x]]*(8*Cot[e + f*x] + Sin[2*(e + f*x)])*Sqrt[b*Tan[e
 + f*x]])/(5*f)
```

Maple [B] time = 0.303, size = 493, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x)
```

```
[Out] -1/10/f*(a*sin(f*x+e))^(5/2)*(5*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)*ln(-(2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^2-cos(f*x+e)^2-
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*cos(f*x+e)-1)/sin(f*x+e)^2)-5*cos(
f*x+e)*ln(-2*(2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^2-cos(f*x+e
)^2-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*cos(f*x+e)-1)/sin(f*x+e)^2)*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-4*cos(f*x+e)^3+5*ln(-(2*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^2-cos(f*x+e)^2-2*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)+2*cos(f*x+e)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)-5*ln(-2*(2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^2-cos(f*x+e)^
2-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*cos(f*x+e)-1)/sin(f*x+e)^2)*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+20*cos(f*x+e))*(b*sin(f*x+e)/cos(f*x+e))^(
1/2)/sin(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}\sqrt{b\tan\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(5/2)*sqrt(b*tan(f*x + e)), x)
```

Fricas [A] time = 1.63554, size = 161, normalized size = 2.37

$$\frac{2\left(a^2\cos\left(fx+e\right)^3-5\,a^2\cos\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}}{5\,f\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(a^2*cos(f*x + e)^3 - 5*a^2*cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*s
in(f*x + e)/cos(f*x + e))/(f*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(5/2)*(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.115 $\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx$

Optimal. Leaf size=88

$$\frac{4a^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}}-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}$$

```
[Out] (-2*b*(a*Sin[e + f*x])^(3/2))/(3*f*Sqrt[b*Tan[e + f*x]]) + (4*a^2*Sqrt[Cos[
e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(3*f*Sqrt[a*Sin[e
 + f*x]])
```

Rubi [A] time = 0.105774, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2598, 2601, 2641}

$$\frac{4a^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}}-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]],x]
```


```
[Out] (-2*b*(a*Sin[e + f*x])^(3/2))/(3*f*Sqrt[b*Tan[e + f*x]]) + (4*a^2*Sqrt[Cos[
e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(3*f*Sqrt[a*Sin[e
 + f*x]])
```


Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
```

```
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx &= -\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}+\frac{1}{3}\left(2a^2\right)\int\frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx\\
&= -\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}+\frac{\left(2a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int\frac{1}{\sqrt{\cos(e+fx)}}\,dx}{3\sqrt{a\sin(e+fx)}}\\
&= -\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}+\frac{4a^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.312267, size = 80, normalized size = 0.91

$$-\frac{2ab\sqrt{a\sin(e+fx)}\left(\sin(e+fx)\sqrt[4]{\cos^2(e+fx)}-2F\left(\frac{1}{2}\sin^{-1}(\sin(e+fx))\middle|2\right)\right)}{3f\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (-2*a*b*Sqrt[a*Sin[e + f*x]]*(-2*EllipticF[ArcSin[Sin[e + f*x]]/2, 2] + (Co
s[e + f*x]^2)^(1/4)*Sin[e + f*x]))/(3*f*(Cos[e + f*x]^2)^(1/4)*Sqrt[b*Tan[e
 + f*x]])
```

Maple [C] time = 0.199, size = 131, normalized size = 1.5

$$-\frac{2}{3f\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\left(2i\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\text{EllipticF}\left(\frac{i\left(\cos\left(fx+e\right)-1\right)}{\sin\left(fx+e\right)},i\right)\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x)
```

```
[Out] -2/3/f*(2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elli
pticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)+cos(f*x+e)^2-cos(f*x+e))*(a
*sin(f*x+e))^(3/2)*(b*sin(f*x+e)/cos(f*x+e))^(1/2)/(cos(f*x+e)-1)/sin(f*x+e
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(fx+e))^{\frac{3}{2}}\sqrt{b\tan(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(3/2)*sqrt(b*tan(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a\sin(fx+e)}\sqrt{b\tan(fx+e)}a\sin(fx+e), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sin(f*x + e))*sqrt(b*tan(f*x + e))*a*sin(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(3/2)*(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.116 $\int \sqrt{a \sin(e+fx)}\sqrt{b \tan(e+fx)}\,dx$

Optimal. Leaf size=30

$$-\frac{2b\sqrt{a\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

```
[Out] (-2*b*Sqrt[a*Sin[e + f*x]])/(f*Sqrt[b*Tan[e + f*x]])
```

Rubi [A] time = 0.041555, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2589}

$$-\frac{2b\sqrt{a\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (-2*b*Sqrt[a*Sin[e + f*x]])/(f*Sqrt[b*Tan[e + f*x]])
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx = -\frac{2b\sqrt{a\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

Mathematica [A] time = 0.129498, size = 30, normalized size = 1.

$$-\frac{2b\sqrt{a\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (-2*b*Sqrt[a*Sin[e + f*x]])/(f*Sqrt[b*Tan[e + f*x]])
```

Maple [B] time = 0.197, size = 295, normalized size = 9.8

$$\frac{\left(\cos\left(fx+e\right)-1\right)\cos\left(fx+e\right)}{2f\left(\sin\left(fx+e\right)\right)^{3}}\left(4\cos\left(fx+e\right)\sqrt{-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^{2}}}+\ln\left(-2\frac{1}{\left(\sin\left(fx+e\right)\right)^{2}}\left(2\sqrt{-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+\right.}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*(cos(f*x+e)-1)*(4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+ln(
-2*(2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^2-cos(f*x+e)^2-2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*cos(f*x+e)-1)/sin(f*x+e)^2)-ln(-(2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^2-cos(f*x+e)^2-2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)+2*cos(f*x+e)-1)/sin(f*x+e)^2)+4*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2))*cos(f*x+e)*(a*sin(f*x+e))^(1/2)*(b*sin(f*x+e)/cos(f*x+e))^
(1/2)/sin(f*x+e)^3/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\sin\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e))*sqrt(b*tan(f*x + e)), x)
```

Fricas [A] time = 1.5758, size = 120, normalized size = 4.

$$-\frac{2\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)}{f\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)/(f*s
in(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(1/2)*(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(f*x + e))*sqrt(b*tan(f*x + e)), x)
```

3.117 $\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx$

Optimal. Leaf size=50

$$\frac{2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

```
[Out] (2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(f*Sq
rt[a*Sin[e + f*x]])
```

Rubi [A] time = 0.0528332, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2601, 2641}

$$\frac{2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[e + f*x]]/Sqrt[a*Sin[e + f*x]],x]
```

```
[Out] (2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(f*Sq
rt[a*Sin[e + f*x]])
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}}\,dx}{\sqrt{a\sin(e+fx)}}$$
$$= \frac{2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

Mathematica [A] time = 0.115375, size = 60, normalized size = 1.2

$$\frac{2\cos(e+fx)\sqrt{b\tan(e+fx)}F\left(\frac{1}{2}\sin^{-1}(\sin(e+fx))\middle|2\right)}{f\sqrt[4]{\cos^2(e+fx)}\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[e + f*x]]/Sqrt[a*Sin[e + f*x]],x]
```

```
[Out] (2*Cos[e + f*x]*EllipticF[ArcSin[Sin[e + f*x]]/2, 2]*Sqrt[b*Tan[e + f*x]])/
(f*(Cos[e + f*x]^2)^(1/4)*Sqrt[a*Sin[e + f*x]])
```

Maple [C] time = 0.138, size = 88, normalized size = 1.8

$$\frac{2i}{f}\mathrm{EllipticF}\left(\frac{i\left(\cos\left(fx+e\right)-1\right)}{\sin\left(fx+e\right)},i\right)\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\frac{1}{\sqrt{a\sin\left(fx+e\right)}}\frac{1}{\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x)
```

```
[Out] 2*I/f/(a*sin(f*x+e))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)/(1/(cos(f*x+e)+1))^(1/2)*(b*sin(f*x+e)/cos(f*x+e
))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\tan\left(fx+e\right)}}{\sqrt{a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))/sqrt(a*sin(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\sin\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}}{a\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sin(f*x + e))*sqrt(b*tan(f*x + e))/(a*sin(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\tan\left(e+fx\right)}}{\sqrt{a\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(1/2)/(a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(b*tan(e + f*x))/sqrt(a*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\tan\left(fx+e\right)}}{\sqrt{a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))/sqrt(a*sin(f*x + e)), x)
```

3.118 $\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=107

$$-\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\tan^{-1}\left(\sqrt{\cos(e+fx)}\right)}{af\sqrt{a\sin(e+fx)}}-\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\tanh^{-1}\left(\sqrt{\cos(e+fx)}\right)}{af\sqrt{a\sin(e+fx)}}$$

```
[Out] -((ArcTan[Sqrt[Cos[e + f*x]]]*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/(a*f
*Sqrt[a*Sin[e + f*x]])) - (ArcTanh[Sqrt[Cos[e + f*x]]]*Sqrt[Cos[e + f*x]]*S
qrt[b*Tan[e + f*x]])/(a*f*Sqrt[a*Sin[e + f*x]])
```

Rubi [A] time = 0.0875813, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2601, 12, 2565, 329, 212, 206, 203}

$$-\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\tan^{-1}\left(\sqrt{\cos(e+fx)}\right)}{af\sqrt{a\sin(e+fx)}}-\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\tanh^{-1}\left(\sqrt{\cos(e+fx)}\right)}{af\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[e + f*x]]/(a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((ArcTan[Sqrt[Cos[e + f*x]]]*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/(a*f
*Sqrt[a*Sin[e + f*x]])) - (ArcTanh[Sqrt[Cos[e + f*x]]]*Sqrt[Cos[e + f*x]]*S
qrt[b*Tan[e + f*x]])/(a*f*Sqrt[a*Sin[e + f*x]])
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = \frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int \frac{\csc(e+fx)}{a\sqrt{\cos(e+fx)}}\,dx}{\sqrt{a\sin(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int \frac{\csc(e+fx)}{\sqrt{\cos(e+fx)}}\,dx}{a\sqrt{a\sin(e+fx)}}$$

$$= -\frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}(1-x^2)}\,dx, x, \cos(e+fx)\right)}{af\sqrt{a\sin(e+fx)}}$$

$$= -\frac{\left(2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \sqrt{\cos(e+fx)}\right)}{af\sqrt{a\sin(e+fx)}}$$

$$= -\frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\cos(e+fx)}\right)}{af\sqrt{a\sin(e+fx)}} - \frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan\right.}{}$$

$$= -\frac{\tan^{-1}\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{af\sqrt{a\sin(e+fx)}} - \frac{\tanh^{-1}\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx}}{af\sqrt{a\sin(e+fx)}}$$

Mathematica [A] time = 0.252972, size = 72, normalized size = 0.67

$$-\frac{b\sqrt{a\sin(e+fx)}\left(\tan^{-1}\left(\sqrt[4]{\cos^2(e+fx)}\right)+\tanh^{-1}\left(\sqrt[4]{\cos^2(e+fx)}\right)\right)}{a^2f\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[e + f*x]]/(a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((b*(ArcTan[(Cos[e + f*x]^2)^(1/4)] + ArcTanh[(Cos[e + f*x]^2)^(1/4)])*Sqr
t[a*Sin[e + f*x]])/(a^2*f*(Cos[e + f*x]^2)^(1/4)*Sqrt[b*Tan[e + f*x]]))
```

Maple [B] time = 0.167, size = 185, normalized size = 1.7

$$\frac{(\cos(fx+e)-1)\cos(fx+e)}{2f\sin(fx+e)}\left(\ln\left(-2\frac{1}{(\sin(fx+e))^2}\left(2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}(\cos(fx+e))^2-(\cos(fx+e))^2-2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/2/f*(ln(-2*(2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^2-cos(f*x+e
)^2-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*cos(f*x+e)-1)/sin(f*x+e)^2)-ar
ctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)))*(cos(f*x+e)-1)*cos(f*x+e)*(
b*sin(f*x+e)/cos(f*x+e))^(1/2)/sin(f*x+e)/(a*sin(f*x+e))^(3/2)/(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b \tan\left(fx+e\right)}}{\left(a \sin\left(fx+e\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))/(a*sin(f*x + e))^(3/2), x)
```

Fricas [B] time = 3.93726, size = 1035, normalized size = 9.67

$$\left[\frac{2\sqrt{-\frac{b}{a}}\arctan\left(\frac{2\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{-\frac{b}{a}}\cos\left(fx+e\right)}{\left(b\cos\left(fx+e\right)+b\right)\sin\left(fx+e\right)}\right)+\sqrt{-\frac{b}{a}}\log\left(-\frac{b\cos\left(fx+e\right)^{3}+4\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{-\frac{b}{a}}\cos\left(fx+e\right)\sin\left(fx+e\right)\cdots}{\cos\left(fx+e\right)^{3}+3\cos\left(fx+e\right)^{2}+3\cos\left(fx+e\right)\cdots}\right.}{4af}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-b/a)*arctan(2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*
x + e))*sqrt(-b/a)*cos(f*x + e)/((b*cos(f*x + e) + b)*sin(f*x + e))) + sqrt
```

```
(-b/a)*log(-(b*cos(f*x + e)^3 + 4*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/
cos(f*x + e))*sqrt(-b/a)*cos(f*x + e)*sin(f*x + e) - 5*b*cos(f*x + e)^2 - 5
*b*cos(f*x + e) + b)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + 3*cos(f*x + e) +
1)))/(a*f), 1/4*(2*sqrt(b/a)*arctan(2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x +
 e)/cos(f*x + e))*sqrt(b/a)*cos(f*x + e)/((b*cos(f*x + e) - b)*sin(f*x + e)
)) + sqrt(b/a)*log((4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(a*sin(f*x + e))*
sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(b/a) - (b*cos(f*x + e)^2 + 6*b*cos(f
*x + e) + b)*sin(f*x + e))/((cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sin(f*x +
 e))))/(a*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(1/2)/(a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b \tan \left(fx+e\right)}}{\left(a \sin \left(fx+e\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))/(a*sin(f*x + e))^(3/2), x)
```

3.119 $\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=86

$$\frac{\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[Out] -(b/(a^2*f*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])) + (Sqrt[Cos[e + f*x]
]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(a^2*f*Sqrt[a*Sin[e + f*x
]])
```

Rubi [A] time = 0.103392, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2599, 2601, 2641}

$$\frac{\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[e + f*x]]/(a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -(b/(a^2*f*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])) + (Sqrt[Cos[e + f*x]
]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(a^2*f*Sqrt[a*Sin[e + f*x
]])
```

Rule 2599

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^(n - 1)
)/(a^2*f*(m + n + 1)), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
```

```
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx &= -\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}+\frac{\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx}{2a^2}\\ &= -\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}+\frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}}\,dx}{2a^2\sqrt{a\sin(e+fx)}}\\ &= -\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}+\frac{\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.243335, size = 79, normalized size = 0.92

$$\frac{b\left(\sin(e+fx)F\left(\frac{1}{2}\sin^{-1}(\sin(e+fx))\middle|2\right)-\sqrt[4]{\cos^2(e+fx)}\right)}{a^2f\sqrt[4]{\cos^2(e+fx)}\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[e + f*x]]/(a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (b*(-(Cos[e + f*x]^2)^(1/4) + EllipticF[ArcSin[Sin[e + f*x]]/2, 2]*Sin[e +
f*x]))/(a^2*f*(Cos[e + f*x]^2)^(1/4)*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*
x]])
```

Maple [C] time = 0.167, size = 178, normalized size = 2.1

$$\frac{\sin\left(fx+e\right)}{f}\left(i\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\text{EllipticF}\left(\frac{i\left(\cos\left(fx+e\right)-1\right)}{\sin\left(fx+e\right)},i\right)\sin\left(fx+e\right)\cos\left(fx+e\right)+i\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/f*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF
(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)+I*(1/(cos(f*x+e)+1))^
(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+
e),I)*sin(f*x+e)-cos(f*x+e))*(b*sin(f*x+e)/cos(f*x+e))^(1/2)*sin(f*x+e)/(a*
sin(f*x+e))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))/(a*sin(f*x + e))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{a\sin\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}}{\left(a^{3}\cos\left(fx+e\right)^{2}-a^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e))*sqrt(b*tan(f*x + e))/((a^3*cos(f*x + e)^2 -
a^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(1/2)/(a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))/(a*sin(f*x + e))^(5/2), x)
```

3.120 $\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=126

$$-\frac{24a^2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{5f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}+\frac{12a^2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{5f}-\frac{2b(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{5f}$$

```
[Out] (-24*a^2*b^2*EllipticE[(e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(5*f*Sqrt[Cos[
e + f*x]]*Sqrt[b*Tan[e + f*x]]) + (12*a^2*b*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan
[e + f*x]])/(5*f) - (2*b*(a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]])/(5*f)
```

Rubi [A] time = 0.169058, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2598, 2594, 2601, 2639}

$$-\frac{24a^2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{5f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}+\frac{12a^2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{5f}-\frac{2b(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-24*a^2*b^2*EllipticE[(e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(5*f*Sqrt[Cos[
e + f*x]]*Sqrt[b*Tan[e + f*x]]) + (12*a^2*b*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan
[e + f*x]])/(5*f) - (2*b*(a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]])/(5*f)
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2594

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
n - 1)), x] - Dist[(b^2*(m + n - 1))/(n - 1), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
```

```
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = -\frac{2b(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{5f}+\frac{1}{5}\left(6a^2\right)\int\sqrt{a\sin(e+fx)}(b\tan(e+$$

$$=\frac{12a^2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{5f}-\frac{2b(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{5f}-$$

$$=\frac{12a^2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{5f}-\frac{2b(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{5f}-$$

$$=-\frac{24a^2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{5f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}+\frac{12a^2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+}}{5f}$$

Mathematica [C] time = 0.324921, size = 99, normalized size = 0.79

$$\frac{a^2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\left(\cos^2(e+fx)^{3/4}(\cos(2(e+fx))+11)-12\cos^2(e+fx)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)\right)}{5f\cos^2(e+fx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (a^2*b*((Cos[e + f*x]^2)^(3/4)*(11 + Cos[2*(e + f*x)]) - 12*Cos[e + f*x]^2*
Hypergeometric2F1[1/4, 1/2, 3/2, Sin[e + f*x]^2])*Sqrt[a*Sin[e + f*x]]*Sqrt
```

```
[b*Tan[e + f*x]])/(5*f*(Cos[e + f*x]^2)^(3/4))
```

Maple [C] time = 0.217, size = 336, normalized size = 2.7

$$-\frac{2\cos\left(fx+e\right)}{5f\left(\sin\left(fx+e\right)\right)^{5}}\left(12i\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\mathrm{EllipticF}\left(\frac{i\left(\cos\left(fx+e\right)-1\right)}{\sin\left(fx+e\right)},i\right)\sin\left(fx+e\right)\cos\left(f\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x)
```

```
[Out] -2/5/f*(12*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ell
ipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)-12*I*sin(f*x+e)
*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elli
pticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)+12*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+
e)-12*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elliptic
E(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)+cos(f*x+e)^4-8*cos(f*x+e)^2+12*
cos(f*x+e)-5)*cos(f*x+e)*(a*sin(f*x+e))^(5/2)*(b*sin(f*x+e)/cos(f*x+e))^(3/
2)/sin(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(a^{2}b\cos\left(fx+e\right)^{2}-a^{2}b\right)\sqrt{a\sin\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}\tan\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*b*cos(f*x + e)^2 - a^2*b)*sqrt(a*sin(f*x + e))*sqrt(b*tan(f*
x + e))*tan(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(5/2)*(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.121 $\int (a \sin(e + fx))^{3/2} (b \tan(e + fx))^{3/2} \, dx$

Optimal. Leaf size=68

$$\frac{8a^2b\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}} - \frac{2b(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{3f}$$

```
[Out] (8*a^2*b*Sqrt[b*Tan[e + f*x]])/(3*f*Sqrt[a*Sin[e + f*x]]) - (2*b*(a*Sin[e +
 f*x])^(3/2)*Sqrt[b*Tan[e + f*x]])/(3*f)
```

Rubi [A] time = 0.104562, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2598, 2589}

$$\frac{8a^2b\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}} - \frac{2b(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (8*a^2*b*Sqrt[b*Tan[e + f*x]])/(3*f*Sqrt[a*Sin[e + f*x]]) - (2*b*(a*Sin[e +
 f*x])^(3/2)*Sqrt[b*Tan[e + f*x]])/(3*f)
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = -\frac{2b(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{3f} + \frac{1}{3}\left(4a^2\right)\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx$$
$$= \frac{8a^2b\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}} - \frac{2b(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{3f}$$

Mathematica [A] time = 0.15863, size = 45, normalized size = 0.66

$$\frac{a^2b(\cos(2(e+fx))+7)\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (a^2*b*(7 + Cos[2*(e + f*x)])*Sqrt[b*Tan[e + f*x]])/(3*f*Sqrt[a*Sin[e + f*x
]])
```

Maple [B] time = 0.155, size = 492, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x)
```

```
[Out] 1/6/f*(-3*cos(f*x+e)*ln(-2*(2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+
e)^2-cos(f*x+e)^2-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*cos(f*x+e)-1)/si
n(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+3*cos(f*x+e)*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)*ln(-(2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x
+e)^2-cos(f*x+e)^2-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*cos(f*x+e)-1)/s
in(f*x+e)^2)+4*cos(f*x+e)^2-3*ln(-2*(2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
*cos(f*x+e)^2-cos(f*x+e)^2-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*cos(f*x
+e)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+3*ln(-(2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^2-cos(f*x+e)^2-2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)+2*cos(f*x+e)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)+12)*cos(f*x+e)*(a*sin(f*x+e))^(3/2)*(b*sin(f*x+e)/cos(f*x+e))^(3
/2)/sin(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(fx+e))^{\frac{3}{2}} (b\tan(fx+e))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2), x)
```

Fricas [A] time = 1.65223, size = 143, normalized size = 2.1

$$\frac{2\left(ab\cos(fx+e)^{2}+3\,ab\right)\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}{3\,f\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a*b*cos(f*x + e)^2 + 3*a*b)*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/c
os(f*x + e))/(f*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(3/2)*(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.122 $\int \sqrt{a \sin(e+fx)}(b \tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=84

$$\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f}-\frac{4b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[Out] (-4*b^2*EllipticE[(e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(f*Sqrt[Cos[e + f*x
]]*Sqrt[b*Tan[e + f*x]]) + (2*b*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])/
f
```

Rubi [A] time = 0.101287, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2594, 2601, 2639}

$$\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f}-\frac{4b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(3/2),x]
```


```
[Out] (-4*b^2*EllipticE[(e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(f*Sqrt[Cos[e + f*x
]]*Sqrt[b*Tan[e + f*x]]) + (2*b*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])/
f
```


Rule 2594

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
n - 1)), x] - Dist[(b^2*(m + n - 1))/(n - 1), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
```

```
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}\,dx &= \frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f}-\left(2b^2\right)\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx\\ &= \frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f}-\frac{\left(2b^2\sqrt{a\sin(e+fx)}\right)\int \sqrt{\cos(e+fx)}\,dx}{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\\ &= -\frac{4b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}+\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f}\end{aligned}$$

Mathematica [C] time = 0.194348, size = 83, normalized size = 0.99

$$\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\left(\cos^2(e+fx)^{3/4}-\cos^2(e+fx)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)\right)}{f\cos^2(e+fx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*b*((Cos[e + f*x]^2)^(3/4) - Cos[e + f*x]^2*Hypergeometric2F1[1/4, 1/2, 3
/2, Sin[e + f*x]^2])*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])/(f*(Cos[e +
 f*x]^2)^(3/4))
```

Maple [C] time = 0.172, size = 328, normalized size = 3.9

$$-2\,\frac{\cos\left(fx+e\right)\sqrt{a\sin\left(fx+e\right)}}{f\left(\sin\left(fx+e\right)\right)^{3}}\left(2\,i\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\mathrm{EllipticF}\left(\frac{i\left(\cos\left(fx+e\right)-1\right)}{\sin\left(fx+e\right)},i\right)\sin\left(fx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x)
```

```
[Out] -2/f*(2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellipt
icF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)-2*I*sin(f*x+e)*cos
(f*x+e)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)-2*
I*sin(f*x+e)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-1)*cos(f*x+
e)*(a*sin(f*x+e))^(1/2)*(b*sin(f*x+e)/cos(f*x+e))^(3/2)/sin(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin\left(fx+e\right)}\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a\sin\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}b\tan\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sin(f*x + e))*sqrt(b*tan(f*x + e))*b*tan(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(1/2)*(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.123 $\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx$

Optimal. Leaf size=30

$$\frac{2b\sqrt{b\tan(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

```
[Out] (2*b*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[a*Sin[e + f*x]])
```

Rubi [A] time = 0.0487723, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2589}

$$\frac{2b\sqrt{b\tan(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x])^(3/2)/Sqrt[a*Sin[e + f*x]],x]

[Out] (2*b*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[a*Sin[e + f*x]])
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{2b\sqrt{b\tan(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

Mathematica [A] time = 0.065601, size = 30, normalized size = 1.

$$\frac{2b\sqrt{b\tan(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x])^(3/2)/Sqrt[a*Sin[e + f*x]],x]
```

```
[Out] (2*b*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[a*Sin[e + f*x]])
```

Maple [B] time = 0.155, size = 308, normalized size = 10.3

$$-\frac{\left(\cos\left(fx+e\right)-1\right)\cos\left(fx+e\right)}{2f\left(\sin\left(fx+e\right)\right)^{3}}\left(\cos\left(fx+e\right)\ln\left(-2\frac{1}{\left(\sin\left(fx+e\right)\right)^{2}}\left(2\sqrt{-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^{2}}}\left(\cos\left(fx+e\right)\right)^{2}-\left(\cos\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*(cos(f*x+e)-1)*(cos(f*x+e)*ln(-2*(2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)*cos(f*x+e)^2-cos(f*x+e)^2-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*cos
(f*x+e)-1)/sin(f*x+e)^2)-cos(f*x+e)*ln(-(2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)*cos(f*x+e)^2-cos(f*x+e)^2-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*cos
(f*x+e)-1)/sin(f*x+e)^2)+4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+
4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*cos(f*x+e)*(b*sin(f*x+e)/cos(f*x+e)
)^(3/2)/sin(f*x+e)^3/(a*sin(f*x+e))^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^(3/2)/sqrt(a*sin(f*x + e)), x)
```

Fricas [A] time = 1.79891, size = 107, normalized size = 3.57

$$\frac{2\sqrt{a\sin\left(fx+e\right)}b\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}}{af\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(a*sin(f*x + e))*b*sqrt(b*sin(f*x + e)/cos(f*x + e))/(a*f*sin(f*x + e
))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(3/2)/(a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^(3/2)/sqrt(a*sin(f*x + e)), x)
```

3.124 $\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=90

$$\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{a^2f}-\frac{2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{a^2f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[Out] (-2*b^2*EllipticE[(e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(a^2*f*Sqrt[Cos[e +
 f*x]]*Sqrt[b*Tan[e + f*x]]) + (2*b*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x
]])/(a^2*f)
```

Rubi [A] time = 0.111241, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2593, 2601, 2639}

$$\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{a^2f}-\frac{2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{a^2f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x])^(3/2)/(a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*b^2*EllipticE[(e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(a^2*f*Sqrt[Cos[e +
 f*x]]*Sqrt[b*Tan[e + f*x]]) + (2*b*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x
]])/(a^2*f)
```

Rule 2593

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^(n - 1))
/(a^2*f*(n - 1)), x] - Dist[(b^2*(m + 2))/(a^2*(n - 1)), Int[(a*Sin[e + f*x
])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && Gt
Q[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*
n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
```

```
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx &= \frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{a^2 f} - \frac{b^2\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx}{a^2} \\
&= \frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{a^2 f} - \frac{\left(b^2\sqrt{a\sin(e+fx)}\right)\int \sqrt{\cos(e+fx)}\,dx}{a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} \\
&= -\frac{2b^2 E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{a^2 f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} + \frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{a^2 f}
\end{aligned}$$

Mathematica [C] time = 0.264575, size = 92, normalized size = 1.02

$$\frac{(b\tan(e+fx))^{3/2}\left(2\cos(e+fx)\cos^2(e+fx)^{3/4}-\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)\right)}{af\cos^2(e+fx)^{3/4}\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x])^(3/2)/(a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((2*Cos[e + f*x]*(Cos[e + f*x]^2)^(3/4) - Cos[e + f*x]^3*Hypergeometric2F1[
1/4, 1/2, 3/2, Sin[e + f*x]^2])*(b*Tan[e + f*x])^(3/2))/(a*f*(Cos[e + f*x]^
2)^(3/4)*Sqrt[a*Sin[e + f*x]])
```

Maple [C] time = 0.163, size = 316, normalized size = 3.5

$$-2\,\frac{\cos(fx+e)}{f\left(a\sin(fx+e)\right)^{3/2}\sin(fx+e)}\left(i\sqrt{\left(\cos(fx+e)+1\right)^{-1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\mathrm{EllipticF}\left(\frac{i\left(\cos(fx+e)-1\right)}{\sin(fx+e)},i\right)\sin(f$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x)
```

```
[Out] -2/f*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elliptic
F(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)-I*sin(f*x+e)*cos(f*x
+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*
(cos(f*x+e)-1)/sin(f*x+e),I)+I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)-I*(1/(co
s(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e
)-1)/sin(f*x+e),I)*sin(f*x+e)+cos(f*x+e)-1)*cos(f*x+e)*(b*sin(f*x+e)/cos(f*
x+e))^(3/2)/(a*sin(f*x+e))^(3/2)/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^(3/2)/(a*sin(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{a\sin\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}b\tan\left(fx+e\right)}{a^2\cos\left(fx+e\right)^2-a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e))*sqrt(b*tan(f*x + e))*b*tan(f*x + e)/(a^2*cos
(f*x + e)^2 - a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(3/2)/(a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \tan \left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a \sin \left(fx+e\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^(3/2)/(a*sin(f*x + e))^(3/2), x)
```

3.125 $\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=145

$$\frac{b^2\sqrt{a\sin(e+fx)}\tan^{-1}\left(\sqrt{\cos(e+fx)}\right)}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{b^2\sqrt{a\sin(e+fx)}\tanh^{-1}\left(\sqrt{\cos(e+fx)}\right)}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}+\frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}$$

```
[Out] (b^2*ArcTan[Sqrt[Cos[e + f*x]]]*Sqrt[a*Sin[e + f*x]])/(a^3*f*Sqrt[Cos[e + f
*x]]*Sqrt[b*Tan[e + f*x]]) - (b^2*ArcTanh[Sqrt[Cos[e + f*x]]]*Sqrt[a*Sin[e
+ f*x]])/(a^3*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]) + (2*b*Sqrt[b*Tan[
e + f*x]])/(a^2*f*Sqrt[a*Sin[e + f*x]])
```

Rubi [A] time = 0.150169, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {2593, 2601, 12, 2565, 329, 298, 203, 206}

$$\frac{b^2\sqrt{a\sin(e+fx)}\tan^{-1}\left(\sqrt{\cos(e+fx)}\right)}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{b^2\sqrt{a\sin(e+fx)}\tanh^{-1}\left(\sqrt{\cos(e+fx)}\right)}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}+\frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x])^(3/2)/(a*Sin[e + f*x])^(5/2),x]
```


```
[Out] (b^2*ArcTan[Sqrt[Cos[e + f*x]]]*Sqrt[a*Sin[e + f*x]])/(a^3*f*Sqrt[Cos[e + f
*x]]*Sqrt[b*Tan[e + f*x]]) - (b^2*ArcTanh[Sqrt[Cos[e + f*x]]]*Sqrt[a*Sin[e
+ f*x]])/(a^3*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]) + (2*b*Sqrt[b*Tan[
e + f*x]])/(a^2*f*Sqrt[a*Sin[e + f*x]])
```


Rule 2593

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^(n - 1))
/(a^2*f*(n - 1)), x] - Dist[(b^2*(m + 2))/(a^2*(n - 1)), Int[(a*Sin[e + f*x
])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && Gt
Q[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*
n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
```

```
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx = \frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}} + \frac{b^2\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx}{a^2}$$

$$= \frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}} + \frac{\left(b^2\sqrt{a\sin(e+fx)}\right)\int \frac{\sqrt{\cos(e+fx)}\csc(e+fx)}{a}\,dx}{a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$= \frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}} + \frac{\left(b^2\sqrt{a\sin(e+fx)}\right)\int \sqrt{\cos(e+fx)}\csc(e+fx)\,dx}{a^3\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$= \frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}} - \frac{\left(b^2\sqrt{a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{1-x^2}\,dx, x, \cos(e+fx)\right)}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$= \frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}} - \frac{\left(2b^2\sqrt{a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt{\cos(e+fx)}\right)}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$= \frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}} - \frac{\left(b^2\sqrt{a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\cos(e+fx)}\right)}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} + \frac{\left(b^2\sqrt{a\sin(e\right.}{a^3}$$

$$= \frac{b^2\tan^{-1}\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{b^2\tanh^{-1}\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} + \frac{2b\sqrt{}}{a^2f\sqrt{}}$$

Mathematica [A] time = 0.34573, size = 104, normalized size = 0.72

$$\frac{b\sqrt{b\tan(e+fx)}\left(2\cos^2(e+fx)^{3/4} + \cos^2(e+fx)\tan^{-1}\left(\sqrt[4]{\cos^2(e+fx)}\right) - \cos^2(e+fx)\tanh^{-1}\left(\sqrt[4]{\cos^2(e+fx)}\right)\right)}{a^2f\cos^2(e+fx)^{3/4}\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x])^(3/2)/(a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (b*(ArcTan[(Cos[e + f*x]^2)^(1/4)]*Cos[e + f*x]^2 - ArcTanh[(Cos[e + f*x]^2
)^(1/4)]*Cos[e + f*x]^2 + 2*(Cos[e + f*x]^2)^(3/4))*Sqrt[b*Tan[e + f*x]])/(
a^2*f*(Cos[e + f*x]^2)^(3/4)*Sqrt[a*Sin[e + f*x]])
```

Maple [A] time = 0.148, size = 247, normalized size = 1.7

$$-\frac{\left(\cos\left(fx+e\right)-1\right)\cos\left(fx+e\right)}{2 f \sin\left(fx+e\right)}\left(\cos\left(fx+e\right)\ln\left(-2\frac{1}{\left(\sin\left(fx+e\right)\right)^{2}}\left(2\sqrt{-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^{2}}}\left(\cos\left(fx+e\right)\right)^{2}-\left(\cos\left(\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/2/f*(cos(f*x+e)-1)*(cos(f*x+e)*ln(-2*(2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)*cos(f*x+e)^2-cos(f*x+e)^2-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*cos
(f*x+e)-1)/sin(f*x+e)^2)+cos(f*x+e)*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2))+4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+4*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2))*cos(f*x+e)*(b*sin(f*x+e)/cos(f*x+e))^(3/2)/sin(f*x
+e)/(a*sin(f*x+e))^(5/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^(3/2)/(a*sin(f*x + e))^(5/2), x)
```

Fricas [B] time = 5.00055, size = 1334, normalized size = 9.2

$$\left[\frac{2ab\sqrt{-\frac{b}{a}}\arctan\left(\frac{2\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{-\frac{b}{a}}\cos\left(fx+e\right)}{\left(b\cos\left(fx+e\right)+b\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)+ab\sqrt{-\frac{b}{a}}\log\left(-\frac{b\cos\left(fx+e\right)^{3}-4\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{-\frac{b}{a}}\right.}{\cos\left(fx+e\right)^{3}+3\cos\right.}}{4a^{3}f\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b*sqrt(-b/a)*arctan(2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/co
s(f*x + e))*sqrt(-b/a)*cos(f*x + e)/((b*cos(f*x + e) + b)*sin(f*x + e)))*si
n(f*x + e) + a*b*sqrt(-b/a)*log(-(b*cos(f*x + e)^3 - 4*sqrt(a*sin(f*x + e))
*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-b/a)*cos(f*x + e)*sin(f*x + e) - 5
*b*cos(f*x + e)^2 - 5*b*cos(f*x + e) + b)/(cos(f*x + e)^3 + 3*cos(f*x + e)^
2 + 3*cos(f*x + e) + 1))*sin(f*x + e) + 8*sqrt(a*sin(f*x + e))*b*sqrt(b*sin
(f*x + e)/cos(f*x + e)))/(a^3*f*sin(f*x + e)), -1/4*(2*a*b*sqrt(b/a)*arctan
(2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(b/a)*cos(f*x
 + e)/((b*cos(f*x + e) - b)*sin(f*x + e)))*sin(f*x + e) - a*b*sqrt(b/a)*log
((4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e
)/cos(f*x + e))*sqrt(b/a) - (b*cos(f*x + e)^2 + 6*b*cos(f*x + e) + b)*sin(f
*x + e))/((cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e)))*sin(f*x + e)
 - 8*sqrt(a*sin(f*x + e))*b*sqrt(b*sin(f*x + e)/cos(f*x + e)))/(a^3*f*sin(f
*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(3/2)/(a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^(3/2)/(a*sin(f*x + e))^(5/2), x)
```

3.126 $\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal. Leaf size=123

$$-\frac{4a^2b(a\sin(e+fx))^{5/2}}{15f(b\tan(e+fx))^{3/2}}+\frac{8a^4E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{15f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}}$$

```
[Out] (-4*a^2*b*(a*Sin[e + f*x])^(5/2))/(15*f*(b*Tan[e + f*x])^(3/2)) - (2*b*(a*S
in[e + f*x])^(9/2))/(9*f*(b*Tan[e + f*x])^(3/2)) + (8*a^4*EllipticE[(e + f*
x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(15*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]
])
```

Rubi [A] time = 0.161566, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2598, 2601, 2639}

$$-\frac{4a^2b(a\sin(e+fx))^{5/2}}{15f(b\tan(e+fx))^{3/2}}+\frac{8a^4E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{15f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(9/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (-4*a^2*b*(a*Sin[e + f*x])^(5/2))/(15*f*(b*Tan[e + f*x])^(3/2)) - (2*b*(a*S
in[e + f*x])^(9/2))/(9*f*(b*Tan[e + f*x])^(3/2)) + (8*a^4*EllipticE[(e + f*
x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(15*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]
])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
```

```
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx &= -\frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}}+\frac{1}{3}\left(2a^2\right)\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx\\ &= -\frac{4a^2b(a\sin(e+fx))^{5/2}}{15f(b\tan(e+fx))^{3/2}}-\frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}}+\frac{1}{15}\left(4a^4\right)\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx\\ &= -\frac{4a^2b(a\sin(e+fx))^{5/2}}{15f(b\tan(e+fx))^{3/2}}-\frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}}+\frac{\left(4a^4\sqrt{a\sin(e+fx)}\right)\int \sqrt{\cos(e+fx)}\,dx}{15\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\\ &= -\frac{4a^2b(a\sin(e+fx))^{5/2}}{15f(b\tan(e+fx))^{3/2}}-\frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}}+\frac{8a^4E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{15f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\end{aligned}$$

Mathematica [C] time = 0.515641, size = 100, normalized size = 0.81

$$\frac{a^4\sin(2(e+fx))\sqrt{a\sin(e+fx)}\left(12\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)+\cos^2(e+fx)^{3/4}(5\cos(2(e+fx))-17)\right)}{90f\cos^2(e+fx)^{3/4}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(9/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (a^4*((Cos[e + f*x]^2)^(3/4)*(-17 + 5*Cos[2*(e + f*x)]) + 12*Hypergeometric
2F1[1/4, 1/2, 3/2, Sin[e + f*x]^2])*Sqrt[a*Sin[e + f*x]]*Sin[2*(e + f*x)])/
(90*f*(Cos[e + f*x]^2)^(3/4)*Sqrt[b*Tan[e + f*x]])
```

Maple [C] time = 0.203, size = 349, normalized size = 2.8

$$\frac{2\left(12i\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\mathrm{EllipticF}\left(\frac{i\left(\cos\left(fx+e\right)-1\right)}{\sin\left(fx+e\right)},i\right)\sin\left(\right.\right.}{45f\left(\sin\left(fx+e\right)\right)^{5}\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(1/2),x)
```

```
[Out] 2/45/f*(12*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ell
ipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)-12*I*sin(f*x+e)
*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elli
pticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)-5*cos(f*x+e)^6+12*I*(1/(cos(f*x+e)+1))
^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x
+e),I)*sin(f*x+e)-12*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)+16*cos(f*x+e)^4-
23*cos(f*x+e)^2+12*cos(f*x+e))*(a*sin(f*x+e))^(9/2)/sin(f*x+e)^5/(b*sin(f*x
+e)/cos(f*x+e))^(1/2)/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)\right)^{\frac{9}{2}}}{\sqrt{b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(9/2)/sqrt(b*tan(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^{4}\cos\left(fx+e\right)^{4}-2\,a^{4}\cos\left(fx+e\right)^{2}+a^{4}\right)\sqrt{a\sin\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}}{b\tan\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(f*x + e)^4 - 2*a^4*cos(f*x + e)^2 + a^4)*sqrt(a*sin(f*x +
 e))*sqrt(b*tan(f*x + e))/(b*tan(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(9/2)/(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a \sin \left(f x+e\right)\right)^{\frac{9}{2}}}{\sqrt{b \tan \left(f x+e\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^(9/2)/sqrt(b*tan(f*x + e)), x)
```

3.127 $\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal. Leaf size=68

$$-\frac{8a^2b(a\sin(e+fx))^{3/2}}{21f(b\tan(e+fx))^{3/2}}-\frac{2b(a\sin(e+fx))^{7/2}}{7f(b\tan(e+fx))^{3/2}}$$

```
[Out] (-8*a^2*b*(a*Sin[e + f*x])^(3/2))/(21*f*(b*Tan[e + f*x])^(3/2)) - (2*b*(a*S
in[e + f*x])^(7/2))/(7*f*(b*Tan[e + f*x])^(3/2))
```

Rubi [A] time = 0.101757, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2598, 2589}

$$-\frac{8a^2b(a\sin(e+fx))^{3/2}}{21f(b\tan(e+fx))^{3/2}}-\frac{2b(a\sin(e+fx))^{7/2}}{7f(b\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(7/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (-8*a^2*b*(a*Sin[e + f*x])^(3/2))/(21*f*(b*Tan[e + f*x])^(3/2)) - (2*b*(a*S
in[e + f*x])^(7/2))/(7*f*(b*Tan[e + f*x])^(3/2))
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{2b(a\sin(e+fx))^{7/2}}{7f(b\tan(e+fx))^{3/2}} + \frac{1}{7}\left(4a^2\right)\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx$$
$$= -\frac{8a^2b(a\sin(e+fx))^{3/2}}{21f(b\tan(e+fx))^{3/2}} - \frac{2b(a\sin(e+fx))^{7/2}}{7f(b\tan(e+fx))^{3/2}}$$

Mathematica [A] time = 0.163352, size = 52, normalized size = 0.76

$$\frac{a^3\cos(e+fx)(3\cos(2(e+fx))-11)\sqrt{a\sin(e+fx)}}{21f\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(7/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (a^3*Cos[e + f*x]*(-11 + 3*Cos[2*(e + f*x)])*Sqrt[a*Sin[e + f*x]])/(21*f*Sq
rt[b*Tan[e + f*x]])
```

Maple [A] time = 0.148, size = 60, normalized size = 0.9

$$\frac{\left(6\left(\cos\left(fx+e\right)\right)^2-14\right)\cos\left(fx+e\right)}{21f\left(\sin\left(fx+e\right)\right)^3}\left(a\sin\left(fx+e\right)\right)^{\frac{7}{2}}\frac{1}{\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x)
```

```
[Out] 2/21/f*(3*cos(f*x+e)^2-7)*(a*sin(f*x+e))^(7/2)*cos(f*x+e)/(b*sin(f*x+e)/cos
(f*x+e))^(1/2)/sin(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)\right)^{\frac{7}{2}}}{\sqrt{b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(7/2)/sqrt(b*tan(f*x + e)), x)
```

Fricas [A] time = 1.65726, size = 170, normalized size = 2.5

$$\frac{2\left(3\,a^3\cos\left(fx+e\right)^4-7\,a^3\cos\left(fx+e\right)^2\right)\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}}{21\,bf\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*a^3*cos(f*x + e)^4 - 7*a^3*cos(f*x + e)^2)*sqrt(a*sin(f*x + e))*sqr
t(b*sin(f*x + e)/cos(f*x + e))/(b*f*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(7/2)/(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)\right)^{\frac{7}{2}}}{\sqrt{b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^(7/2)/sqrt(b*tan(f*x + e)), x)
```

3.128 $\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal. Leaf size=88

$$\frac{4a^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{5f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}$$

```
[Out] (-2*b*(a*Sin[e + f*x])^(5/2))/(5*f*(b*Tan[e + f*x])^(3/2)) + (4*a^2*Ellipti
cE[(e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(5*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan
[e + f*x]])
```

Rubi [A] time = 0.104979, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2598, 2601, 2639}

$$\frac{4a^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{5f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(5/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (-2*b*(a*Sin[e + f*x])^(5/2))/(5*f*(b*Tan[e + f*x])^(3/2)) + (4*a^2*Ellipti
cE[(e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(5*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan
[e + f*x]])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
```

```
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx &= -\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}+\frac{1}{5}\left(2a^2\right)\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx\\ &= -\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}+\frac{\left(2a^2\sqrt{a\sin(e+fx)}\right)\int \sqrt{\cos(e+fx)}\,dx}{5\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\\ &= -\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}+\frac{4a^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{5f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\end{aligned}$$

Mathematica [C] time = 0.233148, size = 87, normalized size = 0.99

$$-\frac{a^2\sin(2(e+fx))\sqrt{a\sin(e+fx)}\left(\cos^2(e+fx)^{3/4}-{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)\right)}{5f\cos^2(e+fx)^{3/4}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(5/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] -(a^2*((Cos[e + f*x]^2)^(3/4) - Hypergeometric2F1[1/4, 1/2, 3/2, Sin[e + f*
x]^2])*Sqrt[a*Sin[e + f*x]]*Sin[2*(e + f*x)])/(5*f*(Cos[e + f*x]^2)^(3/4)*S
qrt[b*Tan[e + f*x]])
```

Maple [C] time = 0.196, size = 337, normalized size = 3.8

$$\frac{2}{5f\left(\sin\left(fx+e\right)\right)^3\cos\left(fx+e\right)}\left(2i\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\mathrm{EllipticF}\left(\frac{i\left(\cos\left(fx+e\right)-1\right)}{\sin\left(fx+e\right)},i\right)\sin\left(fx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x)
```

```
[Out] 2/5/f*(2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellip
ticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)-2*I*sin(f*x+e)*co
s(f*x+e)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)-2
*I*sin(f*x+e)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(
1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e)^4-3*cos(f*x+e)^2+2*cos(f*
x+e))*(a*sin(f*x+e))^(5/2)/sin(f*x+e)^3/cos(f*x+e)/(b*sin(f*x+e)/cos(f*x+e)
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}}{\sqrt{b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(5/2)/sqrt(b*tan(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(a^2\cos\left(fx+e\right)^2-a^2\right)\sqrt{a\sin\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}}{b\tan\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^2 - a^2)*sqrt(a*sin(f*x + e))*sqrt(b*tan(f*x +
e))/(b*tan(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(5/2)/(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}}{\sqrt{b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^(5/2)/sqrt(b*tan(f*x + e)), x)
```

3.129 $\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal. Leaf size=32

$$-\frac{2b(a\sin(e+fx))^{3/2}}{3f(b\tan(e+fx))^{3/2}}$$

```
[Out] (-2*b*(a*Sin[e + f*x])^(3/2))/(3*f*(b*Tan[e + f*x])^(3/2))
```

Rubi [A] time = 0.0493257, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2589}

$$-\frac{2b(a\sin(e+fx))^{3/2}}{3f(b\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(3/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (-2*b*(a*Sin[e + f*x])^(3/2))/(3*f*(b*Tan[e + f*x])^(3/2))
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{2b(a\sin(e+fx))^{3/2}}{3f(b\tan(e+fx))^{3/2}}$$

Mathematica [A] time = 0.131211, size = 32, normalized size = 1.

$$-\frac{2b(a\sin(e+fx))^{3/2}}{3f(b\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(3/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (-2*b*(a*Sin[e + f*x])^(3/2))/(3*f*(b*Tan[e + f*x])^(3/2))
```

Maple [A] time = 0.135, size = 48, normalized size = 1.5

$$-\frac{2\cos\left(fx+e\right)}{3f\sin\left(fx+e\right)}\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\frac{1}{\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x)
```

```
[Out] -2/3/f*(a*sin(f*x+e))^(3/2)*cos(f*x+e)/(b*sin(f*x+e)/cos(f*x+e))^(1/2)/sin(
f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(3/2)/sqrt(b*tan(f*x + e)), x)
```

Fricas [B] time = 1.63189, size = 131, normalized size = 4.09

$$-\frac{2\sqrt{a\sin\left(fx+e\right)}a\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{2}}{3\,bf\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(a*sin(f*x + e))*a*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)^
2/(b*f*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(3/2)/(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^(3/2)/sqrt(b*tan(f*x + e)), x)
```

3.130 $\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal. Leaf size=50

$$\frac{2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[Out] (2*EllipticE[(e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(f*Sqrt[Cos[e + f*x]]*Sq
rt[b*Tan[e + f*x]])
```

Rubi [A] time = 0.0511875, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2601, 2639}

$$\frac{2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sin[e + f*x]]/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (2*EllipticE[(e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(f*Sqrt[Cos[e + f*x]]*Sq
rt[b*Tan[e + f*x]])
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{\sqrt{a\sin(e+fx)}\int\sqrt{\cos(e+fx)}\,dx}{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$
$$= \frac{2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

Mathematica [C] time = 0.148261, size = 69, normalized size = 1.38

$$\frac{\sin(2(e+fx))\sqrt{a\sin(e+fx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)}{2f\cos^2(e+fx)^{3/4}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sin[e + f*x]]/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (Hypergeometric2F1[1/4, 1/2, 3/2, Sin[e + f*x]^2]*Sqrt[a*Sin[e + f*x]]*Sin[
2*(e + f*x)])/(2*f*(Cos[e + f*x]^2)^(3/4)*Sqrt[b*Tan[e + f*x]])
```

Maple [C] time = 0.166, size = 327, normalized size = 6.5

$$2\,\frac{\sqrt{a\sin(fx+e)}}{f\sin(fx+e)\cos(fx+e)}\left(i\sqrt{(\cos(fx+e)+1)^{-1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\mathrm{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\sin(fx+e)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x)
```

```
[Out] 2/f*(a*sin(f*x+e))^(1/2)*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)-
I*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)+I*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)
*sin(f*x+e)-I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*El
lipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)-cos(f*x+e)^2+cos(f*x+e))/
```

```
(b*sin(f*x+e)/cos(f*x+e))^(1/2)/sin(f*x+e)/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a \sin\left(fx+e\right)}}{\sqrt{b \tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e))/sqrt(b*tan(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a \sin\left(fx+e\right)}\sqrt{b \tan\left(fx+e\right)}}{b \tan\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sin(f*x + e))*sqrt(b*tan(f*x + e))/(b*tan(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a \sin\left(e+fx\right)}}{\sqrt{b \tan\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(1/2)/(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*sin(e + f*x))/sqrt(b*tan(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a \sin \left(fx+e\right)}}{\sqrt{b \tan \left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(f*x + e))/sqrt(b*tan(f*x + e)), x)
```

3.131 $\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx$

Optimal. Leaf size=106

$$\frac{\sqrt{a\sin(e+fx)}\tan^{-1}\left(\sqrt{\cos(e+fx)}\right)}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{\sqrt{a\sin(e+fx)}\tanh^{-1}\left(\sqrt{\cos(e+fx)}\right)}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[Out] (ArcTan[Sqrt[Cos[e + f*x]]]*Sqrt[a*Sin[e + f*x]])/(a*f*Sqrt[Cos[e + f*x]]*S
qrt[b*Tan[e + f*x]]) - (ArcTanh[Sqrt[Cos[e + f*x]]]*Sqrt[a*Sin[e + f*x]])/(
a*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

Rubi [A] time = 0.0829008, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2601, 12, 2565, 329, 298, 203, 206}

$$\frac{\sqrt{a\sin(e+fx)}\tan^{-1}\left(\sqrt{\cos(e+fx)}\right)}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{\sqrt{a\sin(e+fx)}\tanh^{-1}\left(\sqrt{\cos(e+fx)}\right)}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]]),x]
```

```
[Out] (ArcTan[Sqrt[Cos[e + f*x]]]*Sqrt[a*Sin[e + f*x]])/(a*f*Sqrt[Cos[e + f*x]]*S
qrt[b*Tan[e + f*x]]) - (ArcTanh[Sqrt[Cos[e + f*x]]]*Sqrt[a*Sin[e + f*x]])/(
a*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \frac{\sqrt{a\sin(e+fx)}\int \frac{\sqrt{\cos(e+fx)}\csc(e+fx)}{a}\,dx}{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$
$$= \frac{\sqrt{a\sin(e+fx)}\int \sqrt{\cos(e+fx)}\csc(e+fx)\,dx}{a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$
$$= -\frac{\sqrt{a\sin(e+fx)}\,\mathrm{Subst}\left(\int \frac{\sqrt{x}}{1-x^2}\,dx, x, \cos(e+fx)\right)}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$
$$= -\frac{\left(2\sqrt{a\sin(e+fx)}\right)\mathrm{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt{\cos(e+fx)}\right)}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$
$$= -\frac{\sqrt{a\sin(e+fx)}\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\cos(e+fx)}\right)}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} + \frac{\sqrt{a\sin(e+fx)}\,\mathrm{Subst}\left(\int \ldots\right.}{af\sqrt{\cos(e+fx)}\ldots}$$
$$= \frac{\tan^{-1}\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{\tanh^{-1}\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

Mathematica [A] time = 0.144943, size = 80, normalized size = 0.75

$$\frac{\sin(2(e+fx))\left(\tan^{-1}\left(\sqrt[4]{\cos^2(e+fx)}\right) - \tanh^{-1}\left(\sqrt[4]{\cos^2(e+fx)}\right)\right)}{2f\cos^2(e+fx)^{3/4}\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]]),x]
```

```
[Out] ((ArcTan[(Cos[e + f*x]^2)^(1/4)] - ArcTanh[(Cos[e + f*x]^2)^(1/4)])*Sin[2*(
e + f*x)])/(2*f*(Cos[e + f*x]^2)^(3/4)*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e +
f*x]])
```

Maple [A] time = 0.119, size = 177, normalized size = 1.7

$$-\frac{\cos(fx+e)-1}{2f\sin(fx+e)}\left(\ln\left(-\frac{1}{(\sin(fx+e))^2}\left(2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}(\cos(fx+e))^2-(\cos(fx+e))^2-2\sqrt{-\frac{\cos(fx\ldots}{(\cos(fx+\ldots}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*(cos(f*x+e)-1)*(ln(-(2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+
e)^2-cos(f*x+e)^2-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*cos(f*x+e)-1)/si
n(f*x+e)^2)+arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)))/(a*sin(f*x+e)
)^(1/2)/sin(f*x+e)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(b*sin(f*x+e)/cos(f
*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sin\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e))*sqrt(b*tan(f*x + e))), x)
```

Fricas [B] time = 3.72479, size = 1072, normalized size = 10.11

$$\left[\frac{2\sqrt{-ab}\arctan\left(\frac{2\sqrt{-ab}\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)}{\left(ab\cos\left(fx+e\right)+ab\right)\sin\left(fx+e\right)}\right)-\sqrt{-ab}\log\left(-\frac{ab\cos\left(fx+e\right)^{3}-5\,ab\cos\left(fx+e\right)^{2}+4\sqrt{-ab}\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b\sin\left(fx+e\right.}{\cos\left(fx+e\right)}}}{\cos\left(fx+e\right)^{3}+3\cos\left(fx+e\right)^{2}+3\cos}\right.}{4\,abf}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/4*(2*sqrt(-a*b)*arctan(2*sqrt(-a*b)*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x
+ e)/cos(f*x + e))*cos(f*x + e)/((a*b*cos(f*x + e) + a*b)*sin(f*x + e))) -
```

```
sqrt(-a*b)*log(-(a*b*cos(f*x + e)^3 - 5*a*b*cos(f*x + e)^2 + 4*sqrt(-a*b)*s
qrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)*sin(f*x
+ e) - 5*a*b*cos(f*x + e) + a*b)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + 3*cos
(f*x + e) + 1)))/(a*b*f), -1/4*(2*sqrt(a*b)*arctan(2*sqrt(a*b)*sqrt(a*sin(f
*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)/((a*b*cos(f*x + e)
- a*b)*sin(f*x + e))) - sqrt(a*b)*log((4*sqrt(a*b)*(cos(f*x + e)^2 + cos(f*
x + e))*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e)) - (a*b*cos(f
*x + e)^2 + 6*a*b*cos(f*x + e) + a*b)*sin(f*x + e))/((cos(f*x + e)^2 - 2*co
s(f*x + e) + 1)*sin(f*x + e))))/(a*b*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))**(1/2)/(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sin\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e))*sqrt(b*tan(f*x + e))), x)
```

3.132 $\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx$

Optimal. Leaf size=87

$$-\frac{b\sqrt{a\sin(e+fx)}}{a^2f(b\tan(e+fx))^{3/2}}-\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{a^2f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[Out] -((b*Sqrt[a*Sin[e + f*x]])/(a^2*f*(b*Tan[e + f*x])^(3/2))) - (EllipticE[(e
+ f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(a^2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e +
 f*x]])
```

Rubi [A] time = 0.105374, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2599, 2601, 2639}

$$-\frac{b\sqrt{a\sin(e+fx)}}{a^2f(b\tan(e+fx))^{3/2}}-\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{a^2f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a*Sin[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]),x]
```

```
[Out] -((b*Sqrt[a*Sin[e + f*x]])/(a^2*f*(b*Tan[e + f*x])^(3/2))) - (EllipticE[(e
+ f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(a^2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e +
 f*x]])
```

Rule 2599

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^(n - 1)
)/(a^2*f*(m + n + 1)), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
```

```
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx &= -\frac{b\sqrt{a\sin(e+fx)}}{a^2f(b\tan(e+fx))^{3/2}}-\frac{\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx}{2a^2}\\ &= -\frac{b\sqrt{a\sin(e+fx)}}{a^2f(b\tan(e+fx))^{3/2}}-\frac{\sqrt{a\sin(e+fx)}\int \sqrt{\cos(e+fx)}\,dx}{2a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\\ &= -\frac{b\sqrt{a\sin(e+fx)}}{a^2f(b\tan(e+fx))^{3/2}}-\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{a^2f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\end{aligned}$$

Mathematica [C] time = 0.337826, size = 89, normalized size = 1.02

$$-\frac{b\sqrt{a\sin(e+fx)}\left(\sin^2(e+fx)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)+2\cos^2(e+fx)^{3/4}\right)}{2a^2f\cos^2(e+fx)^{3/4}(b\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*Sin[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]),x]
```

```
[Out] -(b*Sqrt[a*Sin[e + f*x]]*(2*(Cos[e + f*x]^2)^(3/4) + Hypergeometric2F1[1/4,
 1/2, 3/2, Sin[e + f*x]^2]*Sin[e + f*x]^2))/(2*a^2*f*(Cos[e + f*x]^2)^(3/4)
*(b*Tan[e + f*x])^(3/2))
```

Maple [C] time = 0.171, size = 315, normalized size = 3.6

$$-\frac{\sin\left(fx+e\right)}{f\cos\left(fx+e\right)}\left(i\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\mathrm{EllipticF}\left(\frac{i\left(\cos\left(fx+e\right)-1\right)}{\sin\left(fx+e\right)},i\right)\sin\left(fx+e\right)\cos\left(fx+e\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elliptic
F(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)-I*sin(f*x+e)*cos(f*x
+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*
(cos(f*x+e)-1)/sin(f*x+e),I)+I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)-I*(1/(co
s(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e
)-1)/sin(f*x+e),I)*sin(f*x+e)+cos(f*x+e))*sin(f*x+e)/(a*sin(f*x+e))^(3/2)/(
b*sin(f*x+e)/cos(f*x+e))^(1/2)/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((a*sin(f*x + e))^(3/2)*sqrt(b*tan(f*x + e))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{a\sin\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}}{\left(a^2b\cos\left(fx+e\right)^2-a^2b\right)\tan\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(a*sin(f*x + e))*sqrt(b*tan(f*x + e))/((a^2*b*cos(f*x + e)^2
- a^2*b)*tan(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))**(3/2)/(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sin(f*x + e))^(3/2)*sqrt(b*tan(f*x + e))), x)
```

3.133 $\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx$

Optimal. Leaf size=146

$$-\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}+\frac{\sqrt{a\sin(e+fx)}\tan^{-1}\left(\sqrt{\cos(e+fx)}\right)}{4a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{\sqrt{a\sin(e+fx)}\tanh^{-1}\left(\sqrt{\cos(e+fx)}\right)}{4a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[Out] -b/(2*a^2*f*Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(3/2)) + (ArcTan[Sqrt[Cos
[e + f*x]]]*Sqrt[a*Sin[e + f*x]])/(4*a^3*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e
+ f*x]]) - (ArcTanh[Sqrt[Cos[e + f*x]]]*Sqrt[a*Sin[e + f*x]])/(4*a^3*f*Sqrt
[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

Rubi [A] time = 0.141696, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {2599, 2601, 12, 2565, 329, 298, 203, 206}

$$-\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}+\frac{\sqrt{a\sin(e+fx)}\tan^{-1}\left(\sqrt{\cos(e+fx)}\right)}{4a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{\sqrt{a\sin(e+fx)}\tanh^{-1}\left(\sqrt{\cos(e+fx)}\right)}{4a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]),x]
```


```
[Out] -b/(2*a^2*f*Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(3/2)) + (ArcTan[Sqrt[Cos
[e + f*x]]]*Sqrt[a*Sin[e + f*x]])/(4*a^3*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e
+ f*x]]) - (ArcTanh[Sqrt[Cos[e + f*x]]]*Sqrt[a*Sin[e + f*x]])/(4*a^3*f*Sqrt
[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```


Rule 2599

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^(n - 1)
)/(a^2*f*(m + n + 1)), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
```

```
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = -\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}} + \frac{\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx}{4a^2}$$

$$= -\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}} + \frac{\sqrt{a\sin(e+fx)}\int \frac{\sqrt{\cos(e+fx)}\csc(e+fx)}{a}}{4a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$= -\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}} + \frac{\sqrt{a\sin(e+fx)}\int \sqrt{\cos(e+fx)}\csc(}{4a^3\sqrt{\cos(e+fx)}\sqrt{b\tan(e+}}$$

$$= -\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}} - \frac{\sqrt{a\sin(e+fx)}\,\mathrm{Subst}\left(\int \frac{\sqrt{x}}{1-x^2}\,dx, x,\right.}{4a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e}}$$

$$= -\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}} - \frac{\sqrt{a\sin(e+fx)}\,\mathrm{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x,\right.}{2a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e}}$$

$$= -\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}} - \frac{\sqrt{a\sin(e+fx)}\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x,\right.}{4a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e}}$$

$$= -\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}} + \frac{\tan^{-1}\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+f}}{4a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

Mathematica [A] time = 0.598743, size = 112, normalized size = 0.77

$$\frac{-4\cos^2(e+fx)^{3/4}\cot(e+fx)+\sin(2(e+fx))\tan^{-1}\left(\sqrt[4]{\cos^2(e+fx)}\right)-\sin(2(e+fx))\tanh^{-1}\left(\sqrt[4]{\cos^2(e+fx)}\right)}{8a^2f\cos^2(e+fx)^{3/4}\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]),x]
```

```
[Out] (-4*(Cos[e + f*x]^2)^(3/4)*Cot[e + f*x] + ArcTan[(Cos[e + f*x]^2)^(1/4)]*Si
n[2*(e + f*x)] - ArcTanh[(Cos[e + f*x]^2)^(1/4)]*Sin[2*(e + f*x)])/(8*a^2*f
*(Cos[e + f*x]^2)^(3/4)*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [B] time = 0.165, size = 319, normalized size = 2.2

$$-\frac{\sin\left(fx+e\right)}{8f}\left(4\cos\left(fx+e\right)\sqrt{-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^{2}}}+\cos\left(fx+e\right)\ln\left(-\frac{1}{\left(\sin\left(fx+e\right)\right)^{2}}\left(2\sqrt{-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^{2}}}\left(\mathrm{co}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x)
```

```
[Out] -1/8/f*(4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+cos(f*x+e)*ln(-(2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^2-cos(f*x+e)^2-2*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)+2*cos(f*x+e)-1)/sin(f*x+e)^2)+cos(f*x+e)*arctan
(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-ln(-(2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)*cos(f*x+e)^2-cos(f*x+e)^2-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)+2*cos(f*x+e)-1)/sin(f*x+e)^2)-arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)))*sin(f*x+e)/(a*sin(f*x+e))^(5/2)/(b*sin(f*x+e)/cos(f*x+e))^(1/2)/(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}\sqrt{b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((a*sin(f*x + e))^(5/2)*sqrt(b*tan(f*x + e))), x)
```

Fricas [B] time = 4.46479, size = 1571, normalized size = 10.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/16*(2*sqrt(-a*b)*(cos(f*x + e)^2 - 1)*arctan(2*sqrt(-a*b)*sqrt(a*sin(f*x
 + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)/((a*b*cos(f*x + e) +
a*b)*sin(f*x + e)))*sin(f*x + e) - sqrt(-a*b)*(cos(f*x + e)^2 - 1)*log(-(a*
b*cos(f*x + e)^3 - 5*a*b*cos(f*x + e)^2 + 4*sqrt(-a*b)*sqrt(a*sin(f*x + e))
*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 5*a*b*cos(f*
x + e) + a*b)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + 3*cos(f*x + e) + 1))*sin
(f*x + e) + 8*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*
x + e)^2)/((a^3*b*f*cos(f*x + e)^2 - a^3*b*f)*sin(f*x + e)), -1/16*(2*sqrt(
a*b)*(cos(f*x + e)^2 - 1)*arctan(2*sqrt(a*b)*sqrt(a*sin(f*x + e))*sqrt(b*si
n(f*x + e)/cos(f*x + e))*cos(f*x + e)/((a*b*cos(f*x + e) - a*b)*sin(f*x + e
)))*sin(f*x + e) - sqrt(a*b)*(cos(f*x + e)^2 - 1)*log((4*sqrt(a*b)*(cos(f*x
 + e)^2 + cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x +
e)) - (a*b*cos(f*x + e)^2 + 6*a*b*cos(f*x + e) + a*b)*sin(f*x + e))/((cos(f
*x + e)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e)))*sin(f*x + e) - 8*sqrt(a*sin(
f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)^2)/((a^3*b*f*cos(f
*x + e)^2 - a^3*b*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))**(5/2)/(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}\sqrt{b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sin(f*x + e))^(5/2)*sqrt(b*tan(f*x + e))), x)
```

3.134 $\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{2a^2(a\sin(e+fx))^{9/2}}{117bf\sqrt{b\tan(e+fx)}}-\frac{16a^4(a\sin(e+fx))^{5/2}}{585bf\sqrt{b\tan(e+fx)}}-\frac{64a^6\sqrt{a\sin(e+fx)}}{585bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}}$$

```
[Out] (-64*a^6*Sqrt[a*Sin[e + f*x]])/(585*b*f*Sqrt[b*Tan[e + f*x]]) - (16*a^4*(a*
Sin[e + f*x])^(5/2))/(585*b*f*Sqrt[b*Tan[e + f*x]]) - (2*a^2*(a*Sin[e + f*x
])^(9/2))/(117*b*f*Sqrt[b*Tan[e + f*x]]) + (2*(a*Sin[e + f*x])^(13/2))/(13*
b*f*Sqrt[b*Tan[e + f*x]])
```

Rubi [A] time = 0.20672, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2596, 2598, 2589}

$$-\frac{2a^2(a\sin(e+fx))^{9/2}}{117bf\sqrt{b\tan(e+fx)}}-\frac{16a^4(a\sin(e+fx))^{5/2}}{585bf\sqrt{b\tan(e+fx)}}-\frac{64a^6\sqrt{a\sin(e+fx)}}{585bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(13/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-64*a^6*Sqrt[a*Sin[e + f*x]])/(585*b*f*Sqrt[b*Tan[e + f*x]]) - (16*a^4*(a*
Sin[e + f*x])^(5/2))/(585*b*f*Sqrt[b*Tan[e + f*x]]) - (2*a^2*(a*Sin[e + f*x
])^(9/2))/(117*b*f*Sqrt[b*Tan[e + f*x]]) + (2*(a*Sin[e + f*x])^(13/2))/(13*
b*f*Sqrt[b*Tan[e + f*x]])
```

Rule 2596

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*m)
, x] - Dist[(a^2*(n + 1))/(b^2*m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
&& IntegersQ[2*m, 2*n]
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
```

```
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx &= \frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}} + \frac{a^2\int (a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}\,dx}{13b^2}\\
&= -\frac{2a^2(a\sin(e+fx))^{9/2}}{117bf\sqrt{b\tan(e+fx)}} + \frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}} + \frac{\left(8a^4\right)\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx}{117b^2}\\
&= -\frac{16a^4(a\sin(e+fx))^{5/2}}{585bf\sqrt{b\tan(e+fx)}} - \frac{2a^2(a\sin(e+fx))^{9/2}}{117bf\sqrt{b\tan(e+fx)}} + \frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}} + \frac{\left(32a^6\right)\int \sqrt{a\sin(e
\end{aligned}
$$

$$
= -\frac{64a^6\sqrt{a\sin(e+fx)}}{585bf\sqrt{b\tan(e+fx)}} - \frac{16a^4(a\sin(e+fx))^{5/2}}{585bf\sqrt{b\tan(e+fx)}} - \frac{2a^2(a\sin(e+fx))^{9/2}}{117bf\sqrt{b\tan(e+fx)}} + \frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+f}
$$

Mathematica [A] time = 0.423783, size = 67, normalized size = 0.46

$$
\frac{a^6\cos^2(e+fx)(340\cos(2(e+fx))-45\cos(4(e+fx))-551)\sqrt{a\sin(e+fx)}}{2340bf\sqrt{b\tan(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(13/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (a^6*Cos[e + f*x]^2*(-551 + 340*Cos[2*(e + f*x)] - 45*Cos[4*(e + f*x)])*Sqr
t[a*Sin[e + f*x]])/(2340*b*f*Sqrt[b*Tan[e + f*x]])
```

Maple [A] time = 0.151, size = 70, normalized size = 0.5

$$
-{\frac{ \left(90\, \left(\cos \left(fx+e \right) \right) ^{4}-260\, \left(\cos \left(fx+e \right) \right) ^{2}+234 \right) \cos \left(fx+e \right) }{585\,f \left(\sin \left(fx+e \right) \right) ^{5}} \left(a\sin \left(fx+e \right) \right) ^{{\frac{13}{2}}} \left({\frac{b\sin \left(fx+e \right) }{\cos \left(fx+e \right) }} \right) ^{-{\frac{3}{2}}}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(13/2)/(b*tan(f*x+e))^(3/2),x)
```

```
[Out] -2/585/f*(45*cos(f*x+e)^4-130*cos(f*x+e)^2+117)*(a*sin(f*x+e))^(13/2)*cos(f
*x+e)/(b*sin(f*x+e)/cos(f*x+e))^(3/2)/sin(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)\right)^{\frac{13}{2}}}{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(13/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(13/2)/(b*tan(f*x + e))^(3/2), x)
```

Fricas [A] time = 1.81169, size = 213, normalized size = 1.46

$$-\frac{2\left(45\,a^6\cos\left(fx+e\right)^7-130\,a^6\cos\left(fx+e\right)^5+117\,a^6\cos\left(fx+e\right)^3\right)\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}{585\,b^2 f\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(13/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/585*(45*a^6*cos(f*x + e)^7 - 130*a^6*cos(f*x + e)^5 + 117*a^6*cos(f*x +
e)^3)*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))/(b^2*f*sin(f*x
 + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(13/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)\right)^{\frac{13}{2}}}{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(13/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^(13/2)/(b*tan(f*x + e))^(3/2), x)
```

3.135 $\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=109

$$-\frac{2a^2(a\sin(e+fx))^{5/2}}{45bf\sqrt{b\tan(e+fx)}}-\frac{8a^4\sqrt{a\sin(e+fx)}}{45bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{9/2}}{9bf\sqrt{b\tan(e+fx)}}$$

```
[Out] (-8*a^4*Sqrt[a*Sin[e + f*x]])/(45*b*f*Sqrt[b*Tan[e + f*x]]) - (2*a^2*(a*Sin
[e + f*x])^(5/2))/(45*b*f*Sqrt[b*Tan[e + f*x]]) + (2*(a*Sin[e + f*x])^(9/2)
)/(9*b*f*Sqrt[b*Tan[e + f*x]])
```

Rubi [A] time = 0.150827, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2596, 2598, 2589}

$$-\frac{2a^2(a\sin(e+fx))^{5/2}}{45bf\sqrt{b\tan(e+fx)}}-\frac{8a^4\sqrt{a\sin(e+fx)}}{45bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{9/2}}{9bf\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(9/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-8*a^4*Sqrt[a*Sin[e + f*x]])/(45*b*f*Sqrt[b*Tan[e + f*x]]) - (2*a^2*(a*Sin
[e + f*x])^(5/2))/(45*b*f*Sqrt[b*Tan[e + f*x]]) + (2*(a*Sin[e + f*x])^(9/2)
)/(9*b*f*Sqrt[b*Tan[e + f*x]])
```

Rule 2596

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*m)
, x] - Dist[(a^2*(n + 1))/(b^2*m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
&& IntegersQ[2*m, 2*n]
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
```

```
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx &= \frac{2(a\sin(e+fx))^{9/2}}{9bf\sqrt{b\tan(e+fx)}}+\frac{a^2\int(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx}{9b^2}\\ &= -\frac{2a^2(a\sin(e+fx))^{5/2}}{45bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{9/2}}{9bf\sqrt{b\tan(e+fx)}}+\frac{\left(4a^4\right)\int\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx}{45b^2}\\ &= -\frac{8a^4\sqrt{a\sin(e+fx)}}{45bf\sqrt{b\tan(e+fx)}}-\frac{2a^2(a\sin(e+fx))^{5/2}}{45bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{9/2}}{9bf\sqrt{b\tan(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.212795, size = 57, normalized size = 0.52

$$\frac{a^4\cos^2(e+fx)(5\cos(2(e+fx))-13)\sqrt{a\sin(e+fx)}}{45bf\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(9/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (a^4*Cos[e + f*x]^2*(-13 + 5*Cos[2*(e + f*x)])*Sqrt[a*Sin[e + f*x]])/(45*b*
f*Sqrt[b*Tan[e + f*x]])
```

Maple [A] time = 0.115, size = 60, normalized size = 0.6

$$\frac{\left(10\left(\cos\left(fx+e\right)\right)^2-18\right)\cos\left(fx+e\right)}{45f\left(\sin\left(fx+e\right)\right)^3}\left(a\sin\left(fx+e\right)\right)^{\frac{9}{2}}\left(\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x)
```

```
[Out] 2/45/f*(a*sin(f*x+e))^(9/2)*(5*cos(f*x+e)^2-9)*cos(f*x+e)/(b*sin(f*x+e)/cos
(f*x+e))^(3/2)/sin(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a \sin\left(fx+e\right)\right)^{\frac{9}{2}}}{\left(b \tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(9/2)/(b*tan(f*x + e))^(3/2), x)
```

Fricas [A] time = 1.70836, size = 173, normalized size = 1.59

$$\frac{2\left(5\,a^4 \cos\left(fx+e\right)^5 - 9\,a^4 \cos\left(fx+e\right)^3\right)\sqrt{a \sin\left(fx+e\right)}\sqrt{\frac{b \sin\left(fx+e\right)}{\cos\left(fx+e\right)}}}{45\,b^2 f \sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*a^4*cos(f*x + e)^5 - 9*a^4*cos(f*x + e)^3)*sqrt(a*sin(f*x + e))*sqr
t(b*sin(f*x + e)/cos(f*x + e))/(b^2*f*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(9/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)\right)^{\frac{9}{2}}}{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^(9/2)/(b*tan(f*x + e))^(3/2), x)
```

3.136 $\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=32

$$-\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{5/2}}$$

```
[Out] (-2*b*(a*Sin[e + f*x])^(5/2))/(5*f*(b*Tan[e + f*x])^(5/2))
```

Rubi [A] time = 0.0547281, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2589}

$$-\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(5/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*b*(a*Sin[e + f*x])^(5/2))/(5*f*(b*Tan[e + f*x])^(5/2))
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{5/2}}$$

Mathematica [A] time = 0.131872, size = 45, normalized size = 1.41

$$-\frac{2a^2\cos^2(e+fx)\sqrt{a\sin(e+fx)}}{5bf\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(5/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*a^2*Cos[e + f*x]^2*Sqrt[a*Sin[e + f*x]])/(5*b*f*Sqrt[b*Tan[e + f*x]])
```

Maple [A] time = 0.112, size = 48, normalized size = 1.5

$$-\frac{2\cos\left(fx+e\right)}{5f\sin\left(fx+e\right)}\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}\left(\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x)
```

```
[Out] -2/5/f*(a*sin(f*x+e))^(5/2)*cos(f*x+e)/(b*sin(f*x+e)/cos(f*x+e))^(3/2)/sin(
f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(5/2)/(b*tan(f*x + e))^(3/2), x)
```

Fricas [B] time = 1.60006, size = 136, normalized size = 4.25

$$-\frac{2\sqrt{a\sin\left(fx+e\right)}a^{2}\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{3}}{5\,b^{2}f\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*sqrt(a*sin(f*x + e))*a^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e
)^3/(b^2*f*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(5/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a \sin\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b \tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^(5/2)/(b*tan(f*x + e))^(3/2), x)
```

3.137 $\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=141

$$-\frac{a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\tan^{-1}\left(\sqrt{\cos(e+fx)}\right)}{b^2f\sqrt{a\sin(e+fx)}}-\frac{a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\tanh^{-1}\left(\sqrt{\cos(e+fx)}\right)}{b^2f\sqrt{a\sin(e+fx)}}+\frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

```
[Out] (2*Sqrt[a*Sin[e + f*x]])/(b*f*Sqrt[b*Tan[e + f*x]]) - (a*ArcTan[Sqrt[Cos[e
+ f*x]]]*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/(b^2*f*Sqrt[a*Sin[e + f*x
]]) - (a*ArcTanh[Sqrt[Cos[e + f*x]]]*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]
])/(b^2*f*Sqrt[a*Sin[e + f*x]])
```

Rubi [A] time = 0.143315, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {2595, 2601, 12, 2565, 329, 212, 206, 203}

$$-\frac{a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\tan^{-1}\left(\sqrt{\cos(e+fx)}\right)}{b^2f\sqrt{a\sin(e+fx)}}-\frac{a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\tanh^{-1}\left(\sqrt{\cos(e+fx)}\right)}{b^2f\sqrt{a\sin(e+fx)}}+\frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sin[e + f*x]]/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*Sqrt[a*Sin[e + f*x]])/(b*f*Sqrt[b*Tan[e + f*x]]) - (a*ArcTan[Sqrt[Cos[e
+ f*x]]]*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/(b^2*f*Sqrt[a*Sin[e + f*x
]]) - (a*ArcTanh[Sqrt[Cos[e + f*x]]]*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]
])/(b^2*f*Sqrt[a*Sin[e + f*x]])
```

Rule 2595

```
Int[Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]/((b_.)*tan[(e_.) + (f_.)*(x_)])^(3/
2), x_Symbol] :> Simp[(2*Sqrt[a*Sin[e + f*x]])/(b*f*Sqrt[b*Tan[e + f*x]]),
x] + Dist[a^2/b^2, Int[Sqrt[b*Tan[e + f*x]]/(a*Sin[e + f*x])^(3/2), x], x]
/; FreeQ[{a, b, e, f}, x]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
```

```
f, m, n}, x] && !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}} + \frac{a^2\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx}{b^2}$$

$$= \frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}} + \frac{\left(a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int \frac{\csc(e+fx)}{a\sqrt{\cos(e+fx)}}\,dx}{b^2\sqrt{a\sin(e+fx)}}$$

$$= \frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}} + \frac{\left(a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int \frac{\csc(e+fx)}{\sqrt{\cos(e+fx)}}\,dx}{b^2\sqrt{a\sin(e+fx)}}$$

$$= \frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}} - \frac{\left(a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}(1-x^2)}\,dx, x, \cos(e+fx)\right)}{b^2f\sqrt{a\sin(e+fx)}}$$

$$= \frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}} - \frac{\left(2a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\mathrm{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \sqrt{\cos(e+fx)}\right)}{b^2f\sqrt{a\sin(e+fx)}}$$

$$= \frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}} - \frac{\left(a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\cos(e+fx)}\right)}{b^2f\sqrt{a\sin(e+fx)}} - \frac{(a}{}$$

$$= \frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}} - \frac{a\tan^{-1}\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{b^2f\sqrt{a\sin(e+fx)}} - \frac{a\tanh^{-1}\left(\sqrt{\cos(}\right.}{}$$

Mathematica [A] time = 0.326026, size = 88, normalized size = 0.62

$$\frac{\sqrt{a\sin(e+fx)}\left(2\sqrt[4]{\cos^2(e+fx)} - \tan^{-1}\left(\sqrt[4]{\cos^2(e+fx)}\right) - \tanh^{-1}\left(\sqrt[4]{\cos^2(e+fx)}\right)\right)}{bf\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sin[e + f*x]]/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-ArcTan[(Cos[e + f*x]^2)^(1/4)] - ArcTanh[(Cos[e + f*x]^2)^(1/4)] + 2*(Co
s[e + f*x]^2)^(1/4))*Sqrt[a*Sin[e + f*x]])/(b*f*(Cos[e + f*x]^2)^(1/4)*Sqrt
[b*Tan[e + f*x]])
```

Maple [A] time = 0.151, size = 237, normalized size = 1.7

$$-\frac{\cos\left(fx+e\right)-1}{2 f \sin\left(fx+e\right)\cos\left(fx+e\right)}\left(4\cos\left(fx+e\right)\sqrt{-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^{2}}}-\ln\left(-\frac{1}{\left(\sin\left(fx+e\right)\right)^{2}}\left(2\sqrt{-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x)
```

```
[Out] -1/2/f*(cos(f*x+e)-1)*(4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-ln
(-(2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^2-cos(f*x+e)^2-2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*cos(f*x+e)-1)/sin(f*x+e)^2)+arctan(1/2/(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))+4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))
*(a*sin(f*x+e))^(1/2)/(b*sin(f*x+e)/cos(f*x+e))^(3/2)/sin(f*x+e)/cos(f*x+e)
/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sin\left(fx+e\right)}}{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e))/(b*tan(f*x + e))^(3/2), x)
```

Fricas [B] time = 3.89498, size = 1351, normalized size = 9.58

$$\left[\frac{2b\sqrt{-\frac{a}{b}}\arctan\left(\frac{2\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{-\frac{a}{b}}\cos\left(fx+e\right)}{\left(a\cos\left(fx+e\right)+a\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)+b\sqrt{-\frac{a}{b}}\log\left(-\frac{a\cos\left(fx+e\right)^{3}+4\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{-\frac{a}{b}}}{\cos\left(fx+e\right)^{3}+3\cos}\right.}{4b^{2}f\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*b*sqrt(-a/b)*arctan(2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(
f*x + e))*sqrt(-a/b)*cos(f*x + e)/((a*cos(f*x + e) + a)*sin(f*x + e)))*sin(
f*x + e) + b*sqrt(-a/b)*log(-(a*cos(f*x + e)^3 + 4*sqrt(a*sin(f*x + e))*sqr
t(b*sin(f*x + e)/cos(f*x + e))*sqrt(-a/b)*cos(f*x + e)*sin(f*x + e) - 5*a*c
os(f*x + e)^2 - 5*a*cos(f*x + e) + a)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 +
3*cos(f*x + e) + 1))*sin(f*x + e) + 8*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x +
 e)/cos(f*x + e))*cos(f*x + e))/(b^2*f*sin(f*x + e)), 1/4*(2*b*sqrt(a/b)*ar
ctan(2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(a/b)*cos
(f*x + e)/((a*cos(f*x + e) - a)*sin(f*x + e)))*sin(f*x + e) + b*sqrt(a/b)*l
og((4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x +
 e)/cos(f*x + e))*sqrt(a/b) - (a*cos(f*x + e)^2 + 6*a*cos(f*x + e) + a)*sin
(f*x + e))/((cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e)))*sin(f*x +
e) + 8*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e))
/(b^2*f*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(1/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a \sin\left(fx+e\right)}}{\left(b \tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(f*x + e))/(b*tan(f*x + e))^(3/2), x)
```

3.138 $\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=151

$$\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\tan^{-1}\left(\sqrt{\cos(e+fx)}\right)}{4ab^2f\sqrt{a\sin(e+fx)}}+\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\tanh^{-1}\left(\sqrt{\cos(e+fx)}\right)}{4ab^2f\sqrt{a\sin(e+fx)}}-\frac{\cdots}{2bf(a\sin(\cdots}$$

```
[Out] -1/(2*b*f*(a*Sin[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]) + (ArcTan[Sqrt[Cos[e
 + f*x]]]*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/(4*a*b^2*f*Sqrt[a*Sin[e
+ f*x]]) + (ArcTanh[Sqrt[Cos[e + f*x]]]*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f
*x]])/(4*a*b^2*f*Sqrt[a*Sin[e + f*x]])
```

Rubi [A] time = 0.154978, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {2597, 2601, 12, 2565, 329, 212, 206, 203}

$$\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\tan^{-1}\left(\sqrt{\cos(e+fx)}\right)}{4ab^2f\sqrt{a\sin(e+fx)}}+\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\tanh^{-1}\left(\sqrt{\cos(e+fx)}\right)}{4ab^2f\sqrt{a\sin(e+fx)}}-\frac{\cdots}{2bf(a\sin(\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -1/(2*b*f*(a*Sin[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]) + (ArcTan[Sqrt[Cos[e
 + f*x]]]*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/(4*a*b^2*f*Sqrt[a*Sin[e
+ f*x]]) + (ArcTanh[Sqrt[Cos[e + f*x]]]*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f
*x]])/(4*a*b^2*f*Sqrt[a*Sin[e + f*x]])
```

Rule 2597

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
m + n + 1)), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
```

```
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = -\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}} - \frac{\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx}{4b^2}$$

$$= -\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}} - \frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int \frac{\text{c}}{a\sqrt{}}}{4b^2\sqrt{a\sin(e+fx)}}$$

$$= -\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}} - \frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int \frac{\text{cs}}{\sqrt{\text{c}}}}{4ab^2\sqrt{a\sin(e+fx)}}$$

$$= -\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}} + \frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\text{Sub}}{4ab^2f\sqrt{a\sin}}$$

$$= -\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}} + \frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\text{Sub}}{2ab^2f\sqrt{a\sin}}$$

$$= -\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}} + \frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\text{Sub}}{4ab^2f\sqrt{a\sin}}$$

$$= -\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}} + \frac{\tan^{-1}\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}}{4ab^2f\sqrt{a\sin(e+fx)}}$$

Mathematica [A] time = 0.330586, size = 103, normalized size = 0.68

$$\frac{\sin^2(e+fx)\left(\tan^{-1}\left(\sqrt[4]{\cos^2(e+fx)}\right)-2\sqrt[4]{\cos^2(e+fx)}\csc^2(e+fx)+\tanh^{-1}\left(\sqrt[4]{\cos^2(e+fx)}\right)\right)}{4bf\sqrt[4]{\cos^2(e+fx)}(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] ((ArcTan[(Cos[e + f*x]^2)^(1/4)] + ArcTanh[(Cos[e + f*x]^2)^(1/4)] - 2*(Cos
[e + f*x]^2)^(1/4)*Csc[e + f*x]^2)*Sin[e + f*x]^2)/(4*b*f*(Cos[e + f*x]^2)^
(1/4)*(a*Sin[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]])
```

Maple [B] time = 0.138, size = 320, normalized size = 2.1

$$-\frac{\sin\left(fx+e\right)}{8f\cos\left(fx+e\right)}\left(\cos\left(fx+e\right)\ln\left(-\frac{1}{\left(\sin\left(fx+e\right)\right)^{2}}\left(2\sqrt{-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^{2}}}\left(\cos\left(fx+e\right)\right)^{2}-\left(\cos\left(fx+e\right)\right)^{2}-2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x)
```

```
[Out] -1/8/f*(cos(f*x+e)*ln(-(2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^2
-cos(f*x+e)^2-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*cos(f*x+e)-1)/sin(f*
x+e)^2)-cos(f*x+e)*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))+4*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-ln(-(2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
*cos(f*x+e)^2-cos(f*x+e)^2-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*cos(f*x
+e)-1)/sin(f*x+e)^2)+arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)))*sin(
f*x+e)/cos(f*x+e)/(a*sin(f*x+e))^(3/2)/(b*sin(f*x+e)/cos(f*x+e))^(3/2)/(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((a*sin(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2)), x)
```

Fricas [B] time = 4.49621, size = 1580, normalized size = 10.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] [-1/16*(2*sqrt(-a*b)*(cos(f*x + e)^2 - 1)*arctan(2*sqrt(-a*b)*sqrt(a*sin(f*
x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)/((a*b*cos(f*x + e) +
 a*b)*sin(f*x + e)))*sin(f*x + e) + sqrt(-a*b)*(cos(f*x + e)^2 - 1)*log(-(a
*b*cos(f*x + e)^3 - 5*a*b*cos(f*x + e)^2 + 4*sqrt(-a*b)*sqrt(a*sin(f*x + e)
)*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 5*a*b*cos(f
*x + e) + a*b)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + 3*cos(f*x + e) + 1))*si
n(f*x + e) - 8*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f
*x + e))/((a^2*b^2*f*cos(f*x + e)^2 - a^2*b^2*f)*sin(f*x + e)), -1/16*(2*sq
rt(a*b)*(cos(f*x + e)^2 - 1)*arctan(2*sqrt(a*b)*sqrt(a*sin(f*x + e))*sqrt(b
*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)/((a*b*cos(f*x + e) - a*b)*sin(f*x
+ e)))*sin(f*x + e) - sqrt(a*b)*(cos(f*x + e)^2 - 1)*log(-(4*sqrt(a*b)*(cos
(f*x + e)^2 + cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*
x + e)) + (a*b*cos(f*x + e)^2 + 6*a*b*cos(f*x + e) + a*b)*sin(f*x + e))/((c
os(f*x + e)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e)))*sin(f*x + e) - 8*sqrt(a*
sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e))/((a^2*b^2*f*c
os(f*x + e)^2 - a^2*b^2*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))**(3/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sin(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2)), x)
```

3.139 $\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=167

$$\frac{8a^6\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{77b^2f\sqrt{a\sin(e+fx)}}-\frac{4a^4(a\sin(e+fx))^{3/2}}{77bf\sqrt{b\tan(e+fx)}}-\frac{2a^2(a\sin(e+fx))^{7/2}}{77bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}}$$

```
[Out] (-4*a^4*(a*Sin[e + f*x])^(3/2))/(77*b*f*Sqrt[b*Tan[e + f*x]]) - (2*a^2*(a*S
in[e + f*x])^(7/2))/(77*b*f*Sqrt[b*Tan[e + f*x]]) + (2*(a*Sin[e + f*x])^(11
/2))/(11*b*f*Sqrt[b*Tan[e + f*x]]) + (8*a^6*Sqrt[Cos[e + f*x]]*EllipticF[(e
 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(77*b^2*f*Sqrt[a*Sin[e + f*x]])
```

Rubi [A] time = 0.226937, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2596, 2598, 2601, 2641}

$$\frac{8a^6\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{77b^2f\sqrt{a\sin(e+fx)}}-\frac{4a^4(a\sin(e+fx))^{3/2}}{77bf\sqrt{b\tan(e+fx)}}-\frac{2a^2(a\sin(e+fx))^{7/2}}{77bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(11/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-4*a^4*(a*Sin[e + f*x])^(3/2))/(77*b*f*Sqrt[b*Tan[e + f*x]]) - (2*a^2*(a*S
in[e + f*x])^(7/2))/(77*b*f*Sqrt[b*Tan[e + f*x]]) + (2*(a*Sin[e + f*x])^(11
/2))/(11*b*f*Sqrt[b*Tan[e + f*x]]) + (8*a^6*Sqrt[Cos[e + f*x]]*EllipticF[(e
 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(77*b^2*f*Sqrt[a*Sin[e + f*x]])
```

Rule 2596

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*m)
, x] - Dist[(a^2*(n + 1))/(b^2*m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
&& IntegersQ[2*m, 2*n]
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
```

```
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}} + \frac{a^2\int (a\sin(e+fx))^{7/2}\sqrt{b\tan(e+fx)}\,dx}{11b^2}$$

$$= -\frac{2a^2(a\sin(e+fx))^{7/2}}{77bf\sqrt{b\tan(e+fx)}} + \frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}} + \frac{\left(6a^4\right)\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx}{77b^2}$$

$$= -\frac{4a^4(a\sin(e+fx))^{3/2}}{77bf\sqrt{b\tan(e+fx)}} - \frac{2a^2(a\sin(e+fx))^{7/2}}{77bf\sqrt{b\tan(e+fx)}} + \frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}} + \frac{\left(4a^6\right)\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}}{77b^2}$$

$$= -\frac{4a^4(a\sin(e+fx))^{3/2}}{77bf\sqrt{b\tan(e+fx)}} - \frac{2a^2(a\sin(e+fx))^{7/2}}{77bf\sqrt{b\tan(e+fx)}} + \frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}} + \frac{\left(4a^6\sqrt{\cos(e+fx)}\sqrt{}\right.}{77b^2}$$

$$= -\frac{4a^4(a\sin(e+fx))^{3/2}}{77bf\sqrt{b\tan(e+fx)}} - \frac{2a^2(a\sin(e+fx))^{7/2}}{77bf\sqrt{b\tan(e+fx)}} + \frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}} + \frac{8a^6\sqrt{\cos(e+fx)}F\left(\right.}{77b^2f}$$

Mathematica [A] time = 0.749951, size = 118, normalized size = 0.71

$$\frac{a^5\tan^2(e+fx)\sqrt{a\sin(e+fx)}\left(\sqrt[4]{\cos^2(e+fx)}(-22\cos(e+fx)-17\cos(3(e+fx))+7\cos(5(e+fx)))+64\cot(e+fx\right.}{616f\sqrt[4]{\cos^2(e+fx)}(b\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(11/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (a^5*((Cos[e + f*x]^2)^(1/4)*(-22*Cos[e + f*x] - 17*Cos[3*(e + f*x)] + 7*Co
s[5*(e + f*x)]) + 64*Cot[e + f*x]*EllipticF[ArcSin[Sin[e + f*x]]/2, 2])*Sqr
t[a*Sin[e + f*x]]*Tan[e + f*x]^2)/(616*f*(Cos[e + f*x]^2)^(1/4)*(b*Tan[e +
f*x])^(3/2))
```

Maple [C] time = 0.221, size = 181, normalized size = 1.1

$$-\frac{2}{77 f \left(\cos \left(f x+e\right)-1\right) \left(\sin \left(f x+e\right)\right)^{3} \left(\cos \left(f x+e\right)\right)^{2}}\left(-7 \left(\cos \left(f x+e\right)\right)^{6}+4 i\sqrt{\left(\cos \left(f x+e\right)+1\right)^{-1}}\sqrt{\frac{\cos \left(f x+e\right)}{\cos \left(f x+e\right)+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(11/2)/(b*tan(f*x+e))^(3/2),x)
```

```
[Out] -2/77/f*(-7*cos(f*x+e)^6+4*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)+7*cos(f*x+
e)^5+13*cos(f*x+e)^4-13*cos(f*x+e)^3-4*cos(f*x+e)^2+4*cos(f*x+e))*(a*sin(f*
x+e))^(11/2)/(cos(f*x+e)-1)/sin(f*x+e)^3/(b*sin(f*x+e)/cos(f*x+e))^(3/2)/co
s(f*x+e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a \sin \left(f x+e\right)\right)^{\frac{11}{2}}}{\left(b \tan \left(f x+e\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(11/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(11/2)/(b*tan(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^{5} \cos\left(fx+e\right)^{4}-2\,a^{5} \cos\left(fx+e\right)^{2}+a^{5}\right)\sqrt{a \sin\left(fx+e\right)}\sqrt{b \tan\left(fx+e\right)}\sin\left(fx+e\right)}{b^{2} \tan\left(fx+e\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(11/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^5*cos(f*x + e)^4 - 2*a^5*cos(f*x + e)^2 + a^5)*sqrt(a*sin(f*x +
 e))*sqrt(b*tan(f*x + e))*sin(f*x + e)/(b^2*tan(f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(11/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a \sin\left(fx+e\right)\right)^{\frac{11}{2}}}{\left(b \tan\left(fx+e\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(11/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^(11/2)/(b*tan(f*x + e))^(3/2), x)
```

3.140 $\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=130

$$\frac{4a^4\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{21b^2f\sqrt{a\sin(e+fx)}}-\frac{2a^2(a\sin(e+fx))^{3/2}}{21bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}}$$

```
[Out] (-2*a^2*(a*Sin[e + f*x])^(3/2))/(21*b*f*Sqrt[b*Tan[e + f*x]]) + (2*(a*Sin[e
 + f*x])^(7/2))/(7*b*f*Sqrt[b*Tan[e + f*x]]) + (4*a^4*Sqrt[Cos[e + f*x]]*El
lipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(21*b^2*f*Sqrt[a*Sin[e + f*x]
])
```

Rubi [A] time = 0.167287, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2596, 2598, 2601, 2641}

$$\frac{4a^4\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{21b^2f\sqrt{a\sin(e+fx)}}-\frac{2a^2(a\sin(e+fx))^{3/2}}{21bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(7/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*a^2*(a*Sin[e + f*x])^(3/2))/(21*b*f*Sqrt[b*Tan[e + f*x]]) + (2*(a*Sin[e
 + f*x])^(7/2))/(7*b*f*Sqrt[b*Tan[e + f*x]]) + (4*a^4*Sqrt[Cos[e + f*x]]*El
lipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(21*b^2*f*Sqrt[a*Sin[e + f*x]
])
```

Rule 2596

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*m)
, x] - Dist[(a^2*(n + 1))/(b^2*m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
&& IntegersQ[2*m, 2*n]
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
```

```
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}} + \frac{a^2\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx}{7b^2}$$

$$= -\frac{2a^2(a\sin(e+fx))^{3/2}}{21bf\sqrt{b\tan(e+fx)}} + \frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}} + \frac{\left(2a^4\right)\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx}{21b^2}$$

$$= -\frac{2a^2(a\sin(e+fx))^{3/2}}{21bf\sqrt{b\tan(e+fx)}} + \frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}} + \frac{\left(2a^4\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+f}}}{21b^2\sqrt{a\sin(e+fx)}}$$

$$= -\frac{2a^2(a\sin(e+fx))^{3/2}}{21bf\sqrt{b\tan(e+fx)}} + \frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}} + \frac{4a^4\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+}}{21b^2f\sqrt{a\sin(e+fx)}}$$

Mathematica [A] time = 0.354352, size = 97, normalized size = 0.75

$$\frac{a^3\sqrt{a\sin(e+fx)}\left((5\sin(e+fx)-3\sin(3(e+fx)))\sqrt[4]{\cos^2(e+fx)}+8F\left(\frac{1}{2}\sin^{-1}(\sin(e+fx))\middle|2\right)\right)}{42bf\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(7/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (a^3*Sqrt[a*Sin[e + f*x]]*(8*EllipticF[ArcSin[Sin[e + f*x]]/2, 2] + (Cos[e
+ f*x]^2)^(1/4)*(5*Sin[e + f*x] - 3*Sin[3*(e + f*x)])))/(42*b*f*(Cos[e + f*
x]^2)^(1/4)*Sqrt[b*Tan[e + f*x]])
```

Maple [C] time = 0.179, size = 161, normalized size = 1.2

$$-\frac{2}{21\,f\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{2}}\left(2\,i\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\text{EllipticF}\left(\frac{i\left(\cos\left(fx\right.\right.}{\sin\left(fx\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(3/2),x)
```

```
[Out] -2/21/f*(2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ell
ipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)+3*cos(f*x+e)^4-3*cos(f*x+e
)^3-2*cos(f*x+e)^2+2*cos(f*x+e))*(a*sin(f*x+e))^(7/2)/(cos(f*x+e)-1)/sin(f*
x+e)/cos(f*x+e)^2/(b*sin(f*x+e)/cos(f*x+e))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)\right)^{\frac{7}{2}}}{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(7/2)/(b*tan(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a^{3}\cos\left(fx+e\right)^{2}-a^{3}\right)\sqrt{a\sin\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}\sin\left(fx+e\right)}{b^{2}\tan\left(fx+e\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^3*cos(f*x + e)^2 - a^3)*sqrt(a*sin(f*x + e))*sqrt(b*tan(f*x +
e))*sin(f*x + e)/(b^2*tan(f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(7/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)\right)^{\frac{7}{2}}}{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^(7/2)/(b*tan(f*x + e))^(3/2), x)
```

3.141 $\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=93

$$\frac{2a^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{3b^2f\sqrt{a\sin(e+fx)}}+\frac{2(a\sin(e+fx))^{3/2}}{3bf\sqrt{b\tan(e+fx)}}$$

```
[Out] (2*(a*Sin[e + f*x])^(3/2))/(3*b*f*Sqrt[b*Tan[e + f*x]]) + (2*a^2*Sqrt[Cos[e
 + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(3*b^2*f*Sqrt[a*Si
n[e + f*x]])
```

Rubi [A] time = 0.112447, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2596, 2601, 2641}

$$\frac{2a^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{3b^2f\sqrt{a\sin(e+fx)}}+\frac{2(a\sin(e+fx))^{3/2}}{3bf\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(3/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*(a*Sin[e + f*x])^(3/2))/(3*b*f*Sqrt[b*Tan[e + f*x]]) + (2*a^2*Sqrt[Cos[e
 + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(3*b^2*f*Sqrt[a*Si
n[e + f*x]])
```

Rule 2596

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*m)
, x] - Dist[(a^2*(n + 1))/(b^2*m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
&& IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
```

```
f, m, n}, x] && !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx &= \frac{2(a\sin(e+fx))^{3/2}}{3bf\sqrt{b\tan(e+fx)}}+\frac{a^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx}{3b^2}\\ &= \frac{2(a\sin(e+fx))^{3/2}}{3bf\sqrt{b\tan(e+fx)}}+\frac{\left(a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}}\,dx}{3b^2\sqrt{a\sin(e+fx)}}\\ &= \frac{2(a\sin(e+fx))^{3/2}}{3bf\sqrt{b\tan(e+fx)}}+\frac{2a^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{3b^2f\sqrt{a\sin(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.232295, size = 80, normalized size = 0.86

$$\frac{2a\sqrt{a\sin(e+fx)}\left(\sin(e+fx)\sqrt[4]{\cos^2(e+fx)}+F\left(\frac{1}{2}\sin^{-1}(\sin(e+fx))\middle|2\right)\right)}{3bf\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(3/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*a*Sqrt[a*Sin[e + f*x]]*(EllipticF[ArcSin[Sin[e + f*x]]/2, 2] + (Cos[e +
f*x]^2)^(1/4)*Sin[e + f*x]))/(3*b*f*(Cos[e + f*x]^2)^(1/4)*Sqrt[b*Tan[e + f
*x]])
```

Maple [C] time = 0.15, size = 137, normalized size = 1.5

$$-\frac{2\sin\left(fx+e\right)}{3f\left(\cos\left(fx+e\right)-1\right)\left(\cos\left(fx+e\right)\right)^2}\left(i\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\text{EllipticF}\left(\frac{i\left(\cos\left(fx+e\right)-1\right)}{\sin\left(fx+e\right)},i\right)\text{si}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x)
```

```
[Out] -2/3/f*sin(f*x+e)*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)-cos(f*x+e)^2+cos(f
*x+e))*(a*sin(f*x+e))^(3/2)/(cos(f*x+e)-1)/cos(f*x+e)^2/(b*sin(f*x+e)/cos(f
*x+e))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(fx+e))^{\frac{3}{2}}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(3/2)/(b*tan(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\sin(fx+e)}\sqrt{b\tan(fx+e)}a\sin(fx+e)}{b^2\tan(fx+e)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sin(f*x + e))*sqrt(b*tan(f*x + e))*a*sin(f*x + e)/(b^2*tan(
f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(3/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^(3/2)/(b*tan(f*x + e))^(3/2), x)
```

3.142 $\int \frac{1}{\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=86

$$-\frac{\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{a\sin(e+fx)}}-\frac{1}{bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[Out] -(1/(b*f*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])) - (Sqrt[Cos[e + f*x]]*
EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(b^2*f*Sqrt[a*Sin[e + f*x]]
)
```

Rubi [A] time = 0.104479, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2597, 2601, 2641}

$$-\frac{\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{a\sin(e+fx)}}-\frac{1}{bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -(1/(b*f*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])) - (Sqrt[Cos[e + f*x]]*
EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(b^2*f*Sqrt[a*Sin[e + f*x]]
)
```

Rule 2597

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
m + n + 1)), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
```

```
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = -\frac{1}{bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx}{2b^2}$$

$$= -\frac{1}{bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+}}}{2b^2\sqrt{a\sin(e+fx)}}$$

$$= -\frac{1}{bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e}}{b^2f\sqrt{a\sin(e+fx)}}$$

Mathematica [A] time = 0.184657, size = 79, normalized size = 0.92

$$\frac{\sin(e+fx)\left(-F\left(\frac{1}{2}\sin^{-1}(\sin(e+fx))\middle|2\right)\right)-\sqrt[4]{\cos^2(e+fx)}}{bf\sqrt[4]{\cos^2(e+fx)}\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] (-(Cos[e + f*x]^2)^(1/4) - EllipticF[ArcSin[Sin[e + f*x]]/2, 2]*Sin[e + f*x
])/(b*f*(Cos[e + f*x]^2)^(1/4)*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [C] time = 0.16, size = 185, normalized size = 2.2

$$-\frac{\sin(fx+e)}{f(\cos(fx+e))^2}\left(i\sqrt{(\cos(fx+e)+1)^{-1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\text{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\sin(fx+e)\cos(fx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x)
```

```
[Out] -1/f*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elliptic
F(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)+I*(1/(cos(f*x+e)+1))
^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x
+e),I)*sin(f*x+e)+cos(f*x+e))*sin(f*x+e)/cos(f*x+e)^2/(a*sin(f*x+e))^(1/2)/
(b*sin(f*x+e)/cos(f*x+e))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sin\left(fx+e\right)}\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\sin\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}}{ab^2\sin\left(fx+e\right)\tan\left(fx+e\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(a*sin(f*x + e))*sqrt(b*tan(f*x + e))/(a*b^2*sin(f*x + e)*tan(
f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))**(1/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sin\left(fx+e\right)}\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^(3/2)), x)
```

3.143 $\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=130

$$-\frac{\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{6a^2b^2f\sqrt{a\sin(e+fx)}}+\frac{1}{6a^2bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}$$

```
[Out] -1/(3*b*f*(a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]) + 1/(6*a^2*b*f*Sqrt[
a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]]) - (Sqrt[Cos[e + f*x]]*EllipticF[(e +
f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(6*a^2*b^2*f*Sqrt[a*Sin[e + f*x]])
```

Rubi [A] time = 0.168532, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2597, 2599, 2601, 2641}

$$-\frac{\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{6a^2b^2f\sqrt{a\sin(e+fx)}}+\frac{1}{6a^2bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a*Sin[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -1/(3*b*f*(a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]) + 1/(6*a^2*b*f*Sqrt[
a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]]) - (Sqrt[Cos[e + f*x]]*EllipticF[(e +
f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(6*a^2*b^2*f*Sqrt[a*Sin[e + f*x]])
```

Rule 2597

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
m + n + 1)), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2599

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^(n - 1)
)/(a^2*f*(m + n + 1)), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
```

```
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = -\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} - \frac{\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx}{6b^2}$$

$$= -\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} + \frac{1}{6a^2bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+f}}$$

$$= -\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} + \frac{1}{6a^2bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+f}}$$

$$= -\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} + \frac{1}{6a^2bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+f}}$$

Mathematica [A] time = 0.342652, size = 96, normalized size = 0.74

$$\frac{\sqrt[4]{\cos^2(e+fx)}\left(1-2\csc^2(e+fx)\right)-\sin(e+fx)F\left(\frac{1}{2}\sin^{-1}(\sin(e+fx))\middle|2\right)}{6a^2bf\sqrt[4]{\cos^2(e+fx)}\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*Sin[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] ((Cos[e + f*x]^2)^(1/4)*(1 - 2*Csc[e + f*x]^2) - EllipticF[ArcSin[Sin[e + f
*x]]/2, 2]*Sin[e + f*x])/(6*a^2*b*f*(Cos[e + f*x]^2)^(1/4)*Sqrt[a*Sin[e + f
*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [C] time = 0.172, size = 337, normalized size = 2.6

$$\frac{\sin\left(fx+e\right)}{6f\left(\cos\left(fx+e\right)\right)^{2}}\left(i\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{3}\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\text{EllipticF}\left(\frac{i\left(\cos\left(fx+e\right)-\right.}{\sin\left(fx+e\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x)
```

```
[Out] 1/6/f*(I*sin(f*x+e)*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)+I*sin(f*x+e)*cos(
f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellipti
cF(I*(cos(f*x+e)-1)/sin(f*x+e),I)-I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*cos
(f*x+e)-I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellipt
icF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)-cos(f*x+e)^3-cos(f*x+e))*sin(
f*x+e)/cos(f*x+e)^2/(a*sin(f*x+e))^(5/2)/(b*sin(f*x+e)/cos(f*x+e))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((a*sin(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{a\sin\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}}{\left(a^3b^2\cos\left(fx+e\right)^2-a^3b^2\right)\sin\left(fx+e\right)\tan\left(fx+e\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(a*sin(f*x + e))*sqrt(b*tan(f*x + e))/((a^3*b^2*cos(f*x + e)^
2 - a^3*b^2)*sin(f*x + e)*tan(f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))**(5/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sin(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2)), x)
```

3.144 $\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=167

$$-\frac{\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{12a^4b^2f\sqrt{a\sin(e+fx)}}+\frac{1}{12a^4bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}+\frac{1}{30a^2bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}$$

```
[Out] -1/(5*b*f*(a*Sin[e + f*x])^(9/2)*Sqrt[b*Tan[e + f*x]]) + 1/(30*a^2*b*f*(a*S
in[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]) + 1/(12*a^4*b*f*Sqrt[a*Sin[e + f*x
]]*Sqrt[b*Tan[e + f*x]]) - (Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sq
rt[b*Tan[e + f*x]])/(12*a^4*b^2*f*Sqrt[a*Sin[e + f*x]])
```

Rubi [A] time = 0.233063, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2597, 2599, 2601, 2641}

$$-\frac{\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{12a^4b^2f\sqrt{a\sin(e+fx)}}+\frac{1}{12a^4bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}+\frac{1}{30a^2bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a*Sin[e + f*x])^(9/2)*(b*Tan[e + f*x])^(3/2)),x]
```


```
[Out] -1/(5*b*f*(a*Sin[e + f*x])^(9/2)*Sqrt[b*Tan[e + f*x]]) + 1/(30*a^2*b*f*(a*S
in[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]) + 1/(12*a^4*b*f*Sqrt[a*Sin[e + f*x
]]*Sqrt[b*Tan[e + f*x]]) - (Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sq
rt[b*Tan[e + f*x]])/(12*a^4*b^2*f*Sqrt[a*Sin[e + f*x]])
```


Rule 2597

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
m + n + 1)), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2599

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^(n - 1)
```

```
)/(a^2*f*(m + n + 1)), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx = -\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}} - \frac{\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{9/2}}\,dx}{10b^2}$$

$$= -\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}} + \frac{1}{30a^2bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e}$$

$$= -\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}} + \frac{1}{30a^2bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e}$$

$$= -\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}} + \frac{1}{30a^2bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e}$$

$$= -\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}} + \frac{1}{30a^2bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e}$$

Mathematica [A] time = 0.399519, size = 106, normalized size = 0.63

$$\frac{\sqrt[4]{\cos^2(e+fx)}\left(-12\csc^4(e+fx)+2\csc^2(e+fx)+5\right)-5\sin(e+fx)F\left(\frac{1}{2}\sin^{-1}(\sin(e+fx))\middle|2\right)}{60a^4bf\sqrt[4]{\cos^2(e+fx)}\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*Sin[e + f*x])^(9/2)*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] ((Cos[e + f*x]^2)^(1/4)*(5 + 2*Csc[e + f*x]^2 - 12*Csc[e + f*x]^4) - 5*Elli
pticF[ArcSin[Sin[e + f*x]]/2, 2]*Sin[e + f*x])/(60*a^4*b*f*(Cos[e + f*x]^2)
^(1/4)*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [C] time = 0.218, size = 487, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x)
```

```
[Out] -1/60/f*(5*I*cos(f*x+e)^5*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)+5*I*cos(f*x+e
)^4*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*E
llipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)-10*I*sin(f*x+e)*cos(f*x+e)^3*(1/(co
s(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e
)-1)/sin(f*x+e),I)-10*I*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)+5*
I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(c
os(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)-5*cos(f*x+e)^5+5*I*(1/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)
-1)/sin(f*x+e),I)*sin(f*x+e)+12*cos(f*x+e)^3+5*cos(f*x+e))*sin(f*x+e)/(a*si
n(f*x+e))^(9/2)/(b*sin(f*x+e)/cos(f*x+e))^(3/2)/cos(f*x+e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)\right)^{\frac{9}{2}}\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((a*sin(f*x + e))^(9/2)*(b*tan(f*x + e))^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a\sin\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}}{\left(a^5b^2\cos\left(fx+e\right)^4-2\,a^5b^2\cos\left(fx+e\right)^2+a^5b^2\right)\sin\left(fx+e\right)\tan\left(fx+e\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(a*sin(f*x + e))*sqrt(b*tan(f*x + e))/((a^5*b^2*cos(f*x + e)^4
 - 2*a^5*b^2*cos(f*x + e)^2 + a^5*b^2)*sin(f*x + e)*tan(f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))**(9/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(fx+e\right)\right)^{\frac{9}{2}}\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sin(f*x + e))^(9/2)*(b*tan(f*x + e))^(3/2)), x)
```

3.145 $\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx$

Optimal. Leaf size=64

$$\frac{6\cos^2(e+fx)^{3/4}(b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{17}{12};\frac{29}{12};\sin^2(e+fx)\right)}{17df}$$

```
[Out] (6*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[3/4, 17/12, 29/12, Sin[e + f*x]
^2]*(b*Sin[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2))/(17*d*f)
```

Rubi [A] time = 0.0951208, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2602, 2577}

$$\frac{6\cos^2(e+fx)^{3/4}(b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{17}{12};\frac{29}{12};\sin^2(e+fx)\right)}{17df}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sin[e + f*x])^(4/3)*Sqrt[d*Tan[e + f*x]],x]
```


```
[Out] (6*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[3/4, 17/12, 29/12, Sin[e + f*x]
^2]*(b*Sin[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2))/(17*d*f)
```


Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \frac{\left(b\cos^{\frac{3}{2}}(e+fx)(d\tan(e+fx))^{3/2}\right)\int \frac{(b\sin(e+fx))^{11/6}}{\sqrt{\cos(e+fx)}}\,dx}{d(b\sin(e+fx))^{3/2}}$$
$$= \frac{6\cos^2(e+fx)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{17}{12};\frac{29}{12};\sin^2(e+fx)\right)(b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3}}{17df}$$

Mathematica [A] time = 0.376948, size = 69, normalized size = 1.08

$$\frac{3\sin(2(e+fx))(b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,{}_2F_1\left(\frac{3}{4},\frac{17}{12};\frac{29}{12};\sin^2(e+fx)\right)}{17f\sqrt[4]{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sin[e + f*x])^(4/3)*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (3*Hypergeometric2F1[3/4, 17/12, 29/12, Sin[e + f*x]^2]*(b*Sin[e + f*x])^(4
/3)*Sin[2*(e + f*x)]*Sqrt[d*Tan[e + f*x]])/(17*f*(Cos[e + f*x]^2)^(1/4))
```

Maple [F] time = 0.327, size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)\right)^{\frac{4}{3}}\sqrt{d\tan\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x)
```

```
[Out] int((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)\right)^{\frac{4}{3}}\sqrt{d\tan\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(4/3)*sqrt(d*tan(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\sqrt{d\tan\left(fx+e\right)}b\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(1/3)*sqrt(d*tan(f*x + e))*b*sin(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))**(4/3)*(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.146 $\int \sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}\,dx$

Optimal. Leaf size=64

$$\frac{6\cos^2(e+fx)^{3/4}\sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{11}{12};\frac{23}{12};\sin^2(e+fx)\right)}{11df}$$

```
[Out] (6*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[3/4, 11/12, 23/12, Sin[e + f*x]
^2]*(b*Sin[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2))/(11*d*f)
```

Rubi [A] time = 0.0886116, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2602, 2577}

$$\frac{6\cos^2(e+fx)^{3/4}\sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{11}{12};\frac{23}{12};\sin^2(e+fx)\right)}{11df}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sin[e + f*x])^(1/3)*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[3/4, 11/12, 23/12, Sin[e + f*x]
^2]*(b*Sin[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2))/(11*d*f)
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \frac{\left(b\cos^{\frac{3}{2}}(e+fx)(d\tan(e+fx))^{3/2}\right)\int \frac{(b\sin(e+fx))^{5/6}}{\sqrt{\cos(e+fx)}}\,dx}{d(b\sin(e+fx))^{3/2}}$$
$$= \frac{6\cos^2(e+fx)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{11}{12};\frac{23}{12};\sin^2(e+fx)\right)\sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}}{11df}$$

Mathematica [A] time = 0.341838, size = 69, normalized size = 1.08

$$\frac{3\sin(2(e+fx))\sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}\,{}_2F_1\left(\frac{3}{4},\frac{11}{12};\frac{23}{12};\sin^2(e+fx)\right)}{11f\sqrt[4]{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sin[e + f*x])^(1/3)*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (3*Hypergeometric2F1[3/4, 11/12, 23/12, Sin[e + f*x]^2]*(b*Sin[e + f*x])^(1
/3)*Sin[2*(e + f*x)]*Sqrt[d*Tan[e + f*x]])/(11*f*(Cos[e + f*x]^2)^(1/4))
```

Maple [F] time = 0.175, size = 0, normalized size = 0.

$$\int \sqrt[3]{b\sin\left(fx+e\right)}\sqrt{d\tan\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x)
```

```
[Out] int((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\sqrt{d\tan\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(1/3)*sqrt(d*tan(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\sqrt{d\tan\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(1/3)*sqrt(d*tan(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))**(1/3)*(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\sqrt{d\tan\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e))^(1/3)*sqrt(d*tan(f*x + e)), x)
```

3.147 $\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx$

Optimal. Leaf size=64

$$\frac{6\cos^2(e+fx)^{3/4}(d\tan(e+fx))^{3/2}\,{}_2F_1\left(\frac{7}{12},\frac{3}{4};\frac{19}{12};\sin^2(e+fx)\right)}{7df\sqrt[3]{b\sin(e+fx)}}$$

```
[Out] (6*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[7/12, 3/4, 19/12, Sin[e + f*x]^
2]*(d*Tan[e + f*x])^(3/2))/(7*d*f*(b*Sin[e + f*x])^(1/3))
```

Rubi [A] time = 0.082222, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2602, 2577}

$$\frac{6\cos^2(e+fx)^{3/4}(d\tan(e+fx))^{3/2}\,{}_2F_1\left(\frac{7}{12},\frac{3}{4};\frac{19}{12};\sin^2(e+fx)\right)}{7df\sqrt[3]{b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Tan[e + f*x]]/(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[7/12, 3/4, 19/12, Sin[e + f*x]^
2]*(d*Tan[e + f*x])^(3/2))/(7*d*f*(b*Sin[e + f*x])^(1/3))
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{\left(b\cos^{\frac{3}{2}}(e+fx)(d\tan(e+fx))^{3/2}\right)\int \frac{\sqrt[6]{b\sin(e+fx)}}{\sqrt{\cos(e+fx)}}\,dx}{d(b\sin(e+fx))^{3/2}}$$
$$= \frac{6\cos^2(e+fx)^{3/4}\,{}_2F_1\left(\frac{7}{12},\frac{3}{4};\frac{19}{12};\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{7df\sqrt[3]{b\sin(e+fx)}}$$

Mathematica [A] time = 0.313252, size = 64, normalized size = 1.

$$\frac{6\cos^2(e+fx)^{3/4}(d\tan(e+fx))^{3/2}\,{}_2F_1\left(\frac{7}{12},\frac{3}{4};\frac{19}{12};\sin^2(e+fx)\right)}{7df\sqrt[3]{b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Tan[e + f*x]]/(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[7/12, 3/4, 19/12, Sin[e + f*x]^
2]*(d*Tan[e + f*x])^(3/2))/(7*d*f*(b*Sin[e + f*x])^(1/3))
```

Maple [F] time = 0.221, size = 0, normalized size = 0.

$$\int \sqrt{d\tan\left(fx+e\right)}\frac{1}{\sqrt[3]{b\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/3),x)
```

```
[Out] int((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\tan\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*tan(f*x + e))/(b*sin(f*x + e))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\sin\left(fx+e\right)\right)^{\frac{2}{3}}\sqrt{d\tan\left(fx+e\right)}}{b\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))/(b*sin(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\tan\left(e+fx\right)}}{\sqrt[3]{b\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(1/2)/(b*sin(f*x+e))**(1/3),x)
```

```
[Out] Integral(sqrt(d*tan(e + f*x))/(b*sin(e + f*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\tan\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*tan(f*x + e))/(b*sin(f*x + e))^(1/3), x)
```

3.148 $\int \frac{\sqrt{d \tan(e+fx)}}{(b \sin(e+fx))^{4/3}} \, dx$

Optimal. Leaf size=62

$$\frac{6 \cos^2(e+fx)^{3/4} (d \tan(e+fx))^{3/2} \, _2F_1\left(\frac{1}{12},\frac{3}{4};\frac{13}{12};\sin^2(e+fx)\right)}{df (b \sin(e+fx))^{4/3}}$$

```
[Out] (6*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[1/12, 3/4, 13/12, Sin[e + f*x]^
2]*(d*Tan[e + f*x])^(3/2))/(d*f*(b*Sin[e + f*x])^(4/3))
```

Rubi [A] time = 0.0971231, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2602, 2577}

$$\frac{6 \cos^2(e+fx)^{3/4} (d \tan(e+fx))^{3/2} \, _2F_1\left(\frac{1}{12},\frac{3}{4};\frac{13}{12};\sin^2(e+fx)\right)}{df (b \sin(e+fx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Tan[e + f*x]]/(b*Sin[e + f*x])^(4/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[1/12, 3/4, 13/12, Sin[e + f*x]^
2]*(d*Tan[e + f*x])^(3/2))/(d*f*(b*Sin[e + f*x])^(4/3))
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}\,dx = \frac{\left(b\cos^{\frac{3}{2}}(e+fx)(d\tan(e+fx))^{3/2}\right)\int \frac{1}{\sqrt{\cos(e+fx)}(b\sin(e+fx))^{5/6}}\,dx}{d(b\sin(e+fx))^{3/2}}$$
$$= \frac{6\cos^2(e+fx)^{3/4}\,{}_2F_1\left(\frac{1}{12},\frac{3}{4};\frac{13}{12};\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{df(b\sin(e+fx))^{4/3}}$$

Mathematica [A] time = 0.314855, size = 67, normalized size = 1.08

$$\frac{3\sin(2(e+fx))\sqrt{d\tan(e+fx)}\,{}_2F_1\left(\frac{1}{12},\frac{3}{4};\frac{13}{12};\sin^2(e+fx)\right)}{f\sqrt[4]{\cos^2(e+fx)}(b\sin(e+fx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Tan[e + f*x]]/(b*Sin[e + f*x])^(4/3),x]
```

```
[Out] (3*Hypergeometric2F1[1/12, 3/4, 13/12, Sin[e + f*x]^2]*Sin[2*(e + f*x)]*Sqr
t[d*Tan[e + f*x]])/(f*(Cos[e + f*x]^2)^(1/4)*(b*Sin[e + f*x])^(4/3))
```

Maple [F] time = 0.173, size = 0, normalized size = 0.

$$\int \sqrt{d\tan\left(fx+e\right)}\left(b\sin\left(fx+e\right)\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(4/3),x)
```

```
[Out] int((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\tan\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*tan(f*x + e))/(b*sin(f*x + e))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(b\sin\left(fx+e\right)\right)^{\frac{2}{3}}\sqrt{d\tan\left(fx+e\right)}}{b^2\cos\left(fx+e\right)^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(-(b*sin(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))/(b^2*cos(f*x + e)^2 -
 b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(1/2)/(b*sin(f*x+e))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\tan\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*tan(f*x + e))/(b*sin(f*x + e))^(4/3), x)
```

3.149 $\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=64

$$\frac{6\cos^2(e+fx)^{5/4}(b\sin(e+fx))^{4/3}(d\tan(e+fx))^{5/2}\,{}_2F_1\left(\frac{5}{4},\frac{23}{12};\frac{35}{12};\sin^2(e+fx)\right)}{23df}$$

```
[Out] (6*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[5/4, 23/12, 35/12, Sin[e + f*x]
^2]*(b*Sin[e + f*x])^(4/3)*(d*Tan[e + f*x])^(5/2))/(23*d*f)
```

Rubi [A] time = 0.0992829, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2602, 2577}

$$\frac{6\cos^2(e+fx)^{5/4}(b\sin(e+fx))^{4/3}(d\tan(e+fx))^{5/2}\,{}_2F_1\left(\frac{5}{4},\frac{23}{12};\frac{35}{12};\sin^2(e+fx)\right)}{23df}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sin[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[5/4, 23/12, 35/12, Sin[e + f*x]
^2]*(b*Sin[e + f*x])^(4/3)*(d*Tan[e + f*x])^(5/2))/(23*d*f)
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \frac{\left(b\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\right)\int \frac{(b\sin(e+fx))^{17/6}}{\cos^{\frac{3}{2}}(e+fx)}\,dx}{d(b\sin(e+fx))^{5/2}}$$
$$= \frac{6\cos^2(e+fx)^{5/4}\,{}_2F_1\left(\frac{5}{4},\frac{23}{12};\frac{35}{12};\sin^2(e+fx)\right)(b\sin(e+fx))^{4/3}(d\tan(e+fx)}{23df}$$

Mathematica [A] time = 0.527215, size = 63, normalized size = 0.98

$$-\frac{2d(b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\left(\sqrt[4]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{4},\frac{11}{12};\frac{23}{12};\sin^2(e+fx)\right)-1\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sin[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*d*(-1 + (Cos[e + f*x]^2)^(1/4)*Hypergeometric2F1[1/4, 11/12, 23/12, Sin
[e + f*x]^2])*(b*Sin[e + f*x])^(4/3)*Sqrt[d*Tan[e + f*x]])/f
```

Maple [F] time = 0.13, size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)\right)^{\frac{4}{3}}\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x)
```

```
[Out] int((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)\right)^{\frac{4}{3}}\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(4/3)*(d*tan(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\sqrt{d\tan\left(fx+e\right)}bd\sin\left(fx+e\right)\tan\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(1/3)*sqrt(d*tan(f*x + e))*b*d*sin(f*x + e)*tan(f
*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))**(4/3)*(d*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.150 $\int \sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=64

$$\frac{6\cos^2(e+fx)^{5/4}\sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{5/2}\,{}_2F_1\left(\frac{5}{4},\frac{17}{12};\frac{29}{12};\sin^2(e+fx)\right)}{17df}$$

```
[Out] (6*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[5/4, 17/12, 29/12, Sin[e + f*x]
^2]*(b*Sin[e + f*x])^(1/3)*(d*Tan[e + f*x])^(5/2))/(17*d*f)
```

Rubi [A] time = 0.0944806, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2602, 2577}

$$\frac{6\cos^2(e+fx)^{5/4}\sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{5/2}\,{}_2F_1\left(\frac{5}{4},\frac{17}{12};\frac{29}{12};\sin^2(e+fx)\right)}{17df}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sin[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[5/4, 17/12, 29/12, Sin[e + f*x]
^2]*(b*Sin[e + f*x])^(1/3)*(d*Tan[e + f*x])^(5/2))/(17*d*f)
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}\,dx = \frac{\left(b\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\right)\int \frac{(b\sin(e+fx))^{11/6}}{\cos^{\frac{3}{2}}(e+fx)}\,dx}{d(b\sin(e+fx))^{5/2}}$$
$$= \frac{6\cos^2(e+fx)^{5/4}\,{}_2F_1\left(\frac{5}{4},\frac{17}{12};\frac{29}{12};\sin^2(e+fx)\right)\sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{5/2}}{17df}$$

Mathematica [A] time = 0.409215, size = 63, normalized size = 0.98

$$-\frac{2d\sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}\left(\sqrt[4]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{4},\frac{5}{12};\frac{17}{12};\sin^2(e+fx)\right)-1\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sin[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*d*(-1 + (Cos[e + f*x]^2)^(1/4)*Hypergeometric2F1[1/4, 5/12, 17/12, Sin[
e + f*x]^2])*(b*Sin[e + f*x])^(1/3)*Sqrt[d*Tan[e + f*x]])/f
```

Maple [F] time = 0.125, size = 0, normalized size = 0.

$$\int \sqrt[3]{b\sin\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x)
```

```
[Out] int((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(1/3)*(d*tan(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\sqrt{d\tan\left(fx+e\right)}d\tan\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(1/3)*sqrt(d*tan(f*x + e))*d*tan(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))**(1/3)*(d*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e))^(1/3)*(d*tan(f*x + e))^(3/2), x)
```

3.151 $\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx$

Optimal. Leaf size=64

$$\frac{6\cos^2(e+fx)^{5/4}(d\tan(e+fx))^{5/2}\,{}_2F_1\left(\frac{13}{12},\frac{5}{4};\frac{25}{12};\sin^2(e+fx)\right)}{13df\sqrt[3]{b\sin(e+fx)}}$$

```
[Out] (6*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[13/12, 5/4, 25/12, Sin[e + f*x]
^2]*(d*Tan[e + f*x])^(5/2))/(13*d*f*(b*Sin[e + f*x])^(1/3))
```

Rubi [A] time = 0.0960796, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2602, 2577}

$$\frac{6\cos^2(e+fx)^{5/4}(d\tan(e+fx))^{5/2}\,{}_2F_1\left(\frac{13}{12},\frac{5}{4};\frac{25}{12};\sin^2(e+fx)\right)}{13df\sqrt[3]{b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(3/2)/(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[13/12, 5/4, 25/12, Sin[e + f*x]
^2]*(d*Tan[e + f*x])^(5/2))/(13*d*f*(b*Sin[e + f*x])^(1/3))
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{\left(b\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\right)\int \frac{(b\sin(e+fx))^{7/6}}{\cos^{\frac{3}{2}}(e+fx)}\,dx}{d(b\sin(e+fx))^{5/2}}$$
$$= \frac{6\cos^2(e+fx)^{5/4}\,{}_2F_1\left(\frac{13}{12},\frac{5}{4};\frac{25}{12};\sin^2(e+fx)\right)(d\tan(e+fx))^{5/2}}{13df\sqrt[3]{b\sin(e+fx)}}$$

Mathematica [A] time = 0.436655, size = 63, normalized size = 0.98

$$-\frac{2d\sqrt{d\tan(e+fx)}\left(\sqrt[4]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{12},\frac{1}{4};\frac{13}{12};\sin^2(e+fx)\right)-1\right)}{f\sqrt[3]{b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(3/2)/(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (-2*d*(-1 + (Cos[e + f*x]^2)^(1/4)*Hypergeometric2F1[1/12, 1/4, 13/12, Sin[
e + f*x]^2])*Sqrt[d*Tan[e + f*x]])/(f*(b*Sin[e + f*x])^(1/3))
```

Maple [F] time = 0.114, size = 0, normalized size = 0.

$$\int \left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}\frac{1}{\sqrt[3]{b\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(1/3),x)
```

```
[Out] int((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(f*x + e))^(3/2)/(b*sin(f*x + e))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\sin\left(fx+e\right)\right)^{\frac{2}{3}}\sqrt{d\tan\left(fx+e\right)}d\tan\left(fx+e\right)}{b\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))*d*tan(f*x + e)/(b*sin(
f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(3/2)/(b*sin(f*x+e))**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(1/3),x, algorithm="giac")

[Out] integrate((d*tan(f*x + e))^(3/2)/(b*sin(f*x + e))^(1/3), x)
```

3.152 $\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx$

Optimal. Leaf size=64

$$\frac{6\cos^2(e+fx)^{5/4}(d\tan(e+fx))^{5/2}\,{}_2F_1\left(\frac{7}{12},\frac{5}{4};\frac{19}{12};\sin^2(e+fx)\right)}{7df(b\sin(e+fx))^{4/3}}$$

```
[Out] (6*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[7/12, 5/4, 19/12, Sin[e + f*x]^
2]*(d*Tan[e + f*x])^(5/2))/(7*d*f*(b*Sin[e + f*x])^(4/3))
```

Rubi [A] time = 0.0941235, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2602, 2577}

$$\frac{6\cos^2(e+fx)^{5/4}(d\tan(e+fx))^{5/2}\,{}_2F_1\left(\frac{7}{12},\frac{5}{4};\frac{19}{12};\sin^2(e+fx)\right)}{7df(b\sin(e+fx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(3/2)/(b*Sin[e + f*x])^(4/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[7/12, 5/4, 19/12, Sin[e + f*x]^
2]*(d*Tan[e + f*x])^(5/2))/(7*d*f*(b*Sin[e + f*x])^(4/3))
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx = \frac{\left(b\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\right)\int \frac{\sqrt[6]{b\sin(e+fx)}}{\cos^{\frac{3}{2}}(e+fx)}\,dx}{d(b\sin(e+fx))^{5/2}}$$
$$= \frac{6\cos^2(e+fx)^{5/4}\,{}_2F_1\left(\frac{7}{12},\frac{5}{4};\frac{19}{12};\sin^2(e+fx)\right)(d\tan(e+fx))^{5/2}}{7df(b\sin(e+fx))^{4/3}}$$

Mathematica [A] time = 0.478017, size = 69, normalized size = 1.08

$$-\frac{2d(b\sin(e+fx))^{2/3}\sqrt{d\tan(e+fx)}\left(4\sqrt[4]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{4},\frac{7}{12};\frac{19}{12};\sin^2(e+fx)\right)-7\right)}{7b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(3/2)/(b*Sin[e + f*x])^(4/3),x]

[Out] (-2*d*(-7 + 4*(Cos[e + f*x]^2)^(1/4)*Hypergeometric2F1[1/4, 7/12, 19/12, Si
n[e + f*x]^2])*(b*Sin[e + f*x])^(2/3)*Sqrt[d*Tan[e + f*x]])/(7*b^2*f)
```

Maple [F] time = 0.116, size = 0, normalized size = 0.

$$\int \left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(4/3),x)

[Out] int((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(f*x + e))^(3/2)/(b*sin(f*x + e))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(b\sin\left(fx+e\right)\right)^{\frac{2}{3}}\sqrt{d\tan\left(fx+e\right)}d\tan\left(fx+e\right)}{b^2\cos\left(fx+e\right)^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(-(b*sin(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))*d*tan(f*x + e)/(b^2*c
os(f*x + e)^2 - b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(3/2)/(b*sin(f*x+e))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(4/3),x, algorithm="giac")

[Out] integrate((d*tan(f*x + e))^(3/2)/(b*sin(f*x + e))^(4/3), x)
```

3.153 $\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx$

Optimal. Leaf size=64

$$\frac{6\cos^2(e+fx)^{7/6}\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{7/3}\,{}_2F_1\left(\frac{7}{6},\frac{17}{12};\frac{29}{12};\sin^2(e+fx)\right)}{17df}$$

```
[Out] (6*(Cos[e + f*x]^2)^(7/6)*Hypergeometric2F1[7/6, 17/12, 29/12, Sin[e + f*x]
^2]*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(7/3))/(17*d*f)
```

Rubi [A] time = 0.0926452, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2602, 2577}

$$\frac{6\cos^2(e+fx)^{7/6}\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{7/3}\,{}_2F_1\left(\frac{7}{6},\frac{17}{12};\frac{29}{12};\sin^2(e+fx)\right)}{17df}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(7/6)*Hypergeometric2F1[7/6, 17/12, 29/12, Sin[e + f*x]
^2]*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(7/3))/(17*d*f)
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \frac{\left(b\cos^{\frac{7}{3}}(e+fx)(d\tan(e+fx))^{7/3}\right)\int \frac{(b\sin(e+fx))^{11/6}}{\cos^{\frac{4}{3}}(e+fx)}\,dx}{d(b\sin(e+fx))^{7/3}}$$
$$= \frac{6\cos^2(e+fx)^{7/6}\,{}_2F_1\left(\frac{7}{6},\frac{17}{12};\frac{29}{12};\sin^2(e+fx)\right)\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{7/3}}{17df}$$

Mathematica [A] time = 0.423323, size = 65, normalized size = 1.02

$$-\frac{3d\sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+fx)}\left(\sqrt[4]{\sec^2(e+fx)}\,{}_2F_1\left(\frac{5}{12},\frac{5}{4};\frac{17}{12};-\tan^2(e+fx)\right)-1\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (-3*d*(-1 + Hypergeometric2F1[5/12, 5/4, 17/12, -Tan[e + f*x]^2]*(Sec[e + f
*x]^2)^(1/4))*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(1/3))/f
```

Maple [F] time = 0.136, size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x)
```

```
[Out] int((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sin\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}d\tan\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(1/3)*d*tan(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))**(1/2)*(d*tan(f*x+e))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.154 $\int \sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx$

Optimal. Leaf size=64

$$\frac{6\cos^2(e+fx)^{2/3}\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{11}{12};\frac{23}{12};\sin^2(e+fx)\right)}{11df}$$

```
[Out] (6*(Cos[e + f*x]^2)^(2/3)*Hypergeometric2F1[2/3, 11/12, 23/12, Sin[e + f*x]
^2]*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(4/3))/(11*d*f)
```

Rubi [A] time = 0.0865125, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2602, 2577}

$$\frac{6\cos^2(e+fx)^{2/3}\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{11}{12};\frac{23}{12};\sin^2(e+fx)\right)}{11df}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(2/3)*Hypergeometric2F1[2/3, 11/12, 23/12, Sin[e + f*x]
^2]*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(4/3))/(11*d*f)
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \frac{\left(b\cos^{\frac{4}{3}}(e+fx)(d\tan(e+fx))^{4/3}\right)\int \frac{(b\sin(e+fx))^{5/6}}{\sqrt[3]{\cos(e+fx)}}\,dx}{d(b\sin(e+fx))^{4/3}}$$
$$= \frac{6\cos^2(e+fx)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{11}{12};\frac{23}{12};\sin^2(e+fx)\right)\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}}{11df}$$

Mathematica [A] time = 0.335994, size = 66, normalized size = 1.03

$$\frac{6\sqrt[4]{\sec^2(e+fx)}\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,{}_2F_1\left(\frac{11}{12},\frac{5}{4};\frac{23}{12};-\tan^2(e+fx)\right)}{11df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (6*Hypergeometric2F1[11/12, 5/4, 23/12, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(
1/4)*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(4/3))/(11*d*f)
```

Maple [F] time = 0.211, size = 0, normalized size = 0.

$$\int \sqrt{b\sin(fx+e)}\sqrt[3]{d\tan(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x)
```

```
[Out] int((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin(fx+e)}\left(d\tan(fx+e)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sin\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))**(1/2)*(d*tan(f*x+e))**(1/3),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.155 $\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx$

Optimal. Leaf size=64

$$\frac{6\sqrt[3]{\cos^2(e+fx)}\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{7}{12};\frac{19}{12};\sin^2(e+fx)\right)}{7df}$$

```
[Out] (6*(Cos[e + f*x]^2)^(1/3)*Hypergeometric2F1[1/3, 7/12, 19/12, Sin[e + f*x]^
2]*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(2/3))/(7*d*f)
```

Rubi [A] time = 0.0830445, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2602, 2577}

$$\frac{6\sqrt[3]{\cos^2(e+fx)}\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{7}{12};\frac{19}{12};\sin^2(e+fx)\right)}{7df}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sin[e + f*x]]/(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(1/3)*Hypergeometric2F1[1/3, 7/12, 19/12, Sin[e + f*x]^
2]*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(2/3))/(7*d*f)
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{\left(b\cos^{\frac{2}{3}}(e+fx)(d\tan(e+fx))^{2/3}\right)\int \sqrt[3]{\cos(e+fx)}\sqrt[6]{b\sin(e+fx)}\,dx}{d(b\sin(e+fx))^{2/3}}$$
$$= \frac{6\sqrt[3]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{3},\frac{7}{12};\frac{19}{12};\sin^2(e+fx)\right)\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{2/3}}{7df}$$

Mathematica [A] time = 0.318824, size = 66, normalized size = 1.03

$$\frac{6\sqrt[4]{\sec^2(e+fx)}\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{2/3}\,{}_2F_1\left(\frac{7}{12},\frac{5}{4};\frac{19}{12};-\tan^2(e+fx)\right)}{7df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sin[e + f*x]]/(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (6*Hypergeometric2F1[7/12, 5/4, 19/12, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(1
/4)*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(2/3))/(7*d*f)
```

Maple [F] time = 0.275, size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)}\frac{1}{\sqrt[3]{d\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x)
```

```
[Out] int((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sin\left(fx+e\right)}}{\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e))/(d*tan(f*x + e))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sin\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{2}{3}}}{d\tan\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(2/3)/(d*tan(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sin\left(e+fx\right)}}{\sqrt[3]{d\tan\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))**(1/2)/(d*tan(f*x+e))**(1/3),x)
```

```
[Out] Integral(sqrt(b*sin(e + f*x))/(d*tan(e + f*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sin\left(fx+e\right)}}{\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e))/(d*tan(f*x + e))^(1/3), x)
```

3.156 $\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx$

Optimal. Leaf size=62

$$\frac{6\sqrt{b\sin(e+fx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{12};\frac{13}{12};\sin^2(e+fx)\right)}{df\sqrt[6]{\cos^2(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$

```
[Out] (6*Hypergeometric2F1[-1/6, 1/12, 13/12, Sin[e + f*x]^2]*Sqrt[b*Sin[e + f*x]
])/(d*f*(Cos[e + f*x]^2)^(1/6)*(d*Tan[e + f*x])^(1/3))
```

Rubi [A] time = 0.0959553, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2602, 2577}

$$\frac{6\sqrt{b\sin(e+fx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{12};\frac{13}{12};\sin^2(e+fx)\right)}{df\sqrt[6]{\cos^2(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sin[e + f*x]]/(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (6*Hypergeometric2F1[-1/6, 1/12, 13/12, Sin[e + f*x]^2]*Sqrt[b*Sin[e + f*x]
])/(d*f*(Cos[e + f*x]^2)^(1/6)*(d*Tan[e + f*x])^(1/3))
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \frac{\left(b\sqrt[3]{b\sin(e+fx)}\right)\int \frac{\cos^{\frac{4}{3}}(e+fx)}{(b\sin(e+fx))^{5/6}}\,dx}{d\sqrt[3]{\cos(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$
$$= \frac{6\,{}_2F_1\left(-\frac{1}{6},\frac{1}{12};\frac{13}{12};\sin^2(e+fx)\right)\sqrt{b\sin(e+fx)}}{df\sqrt[6]{\cos^2(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$

Mathematica [A] time = 0.389723, size = 64, normalized size = 1.03

$$\frac{6\sqrt[4]{\sec^2(e+fx)}\sqrt{b\sin(e+fx)}\,{}_2F_1\left(\frac{1}{12},\frac{5}{4};\frac{13}{12};-\tan^2(e+fx)\right)}{df\sqrt[3]{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sin[e + f*x]]/(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (6*Hypergeometric2F1[1/12, 5/4, 13/12, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(1
/4)*Sqrt[b*Sin[e + f*x]])/(d*f*(d*Tan[e + f*x])^(1/3))
```

Maple [F] time = 0.183, size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x)
```

```
[Out] int((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sin\left(fx+e\right)}}{\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e))/(d*tan(f*x + e))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sin\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{2}{3}}}{d^2\tan\left(fx+e\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(2/3)/(d^2*tan(f*x + e)^2),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))**(1/2)/(d*tan(f*x+e))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sin\left(fx+e\right)}}{\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x, algorithm="giac")

[Out] integrate(sqrt(b*sin(f*x + e))/(d*tan(f*x + e))^(4/3), x)
```

3.157 $\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx$

Optimal. Leaf size=64

$$\frac{6\cos^2(e+fx)^{7/6}(b\sin(e+fx))^{3/2}(d\tan(e+fx))^{7/3}\,{}_2F_1\left(\frac{7}{6},\frac{23}{12};\frac{35}{12};\sin^2(e+fx)\right)}{23df}$$

```
[Out] (6*(Cos[e + f*x]^2)^(7/6)*Hypergeometric2F1[7/6, 23/12, 35/12, Sin[e + f*x]
^2]*(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(7/3))/(23*d*f)
```

Rubi [A] time = 0.0986516, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2602, 2577}

$$\frac{6\cos^2(e+fx)^{7/6}(b\sin(e+fx))^{3/2}(d\tan(e+fx))^{7/3}\,{}_2F_1\left(\frac{7}{6},\frac{23}{12};\frac{35}{12};\sin^2(e+fx)\right)}{23df}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(7/6)*Hypergeometric2F1[7/6, 23/12, 35/12, Sin[e + f*x]
^2]*(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(7/3))/(23*d*f)
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \frac{\left(b\cos^{\frac{7}{3}}(e+fx)(d\tan(e+fx))^{7/3}\right)\int \frac{(b\sin(e+fx))^{17/6}}{\cos^{\frac{4}{3}}(e+fx)}\,dx}{d(b\sin(e+fx))^{7/3}}$$
$$= \frac{6\cos^2(e+fx)^{7/6}\,{}_2F_1\left(\frac{7}{6},\frac{23}{12};\frac{35}{12};\sin^2(e+fx)\right)(b\sin(e+fx))^{3/2}(d\tan(e+fx)}{23df}$$

Mathematica [A] time = 0.576644, size = 85, normalized size = 1.33

$$\frac{3d(b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\left(\sqrt[4]{\sec^2(e+fx)}-\sec^2(e+fx)\,{}_2F_1\left(\frac{11}{12},\frac{7}{4};\frac{23}{12};-\tan^2(e+fx)\right)\right)}{f\sqrt[4]{\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (3*d*(-(Hypergeometric2F1[11/12, 7/4, 23/12, -Tan[e + f*x]^2]*Sec[e + f*x]^
2) + (Sec[e + f*x]^2)^(1/4))*(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(1/3))
/(f*(Sec[e + f*x]^2)^(1/4))
```

Maple [F] time = 0.111, size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)\right)^{\frac{3}{2}}\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x)
```

```
[Out] int((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)\right)^{\frac{3}{2}}\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(3/2)*(d*tan(f*x + e))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sin\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}bd\sin\left(fx+e\right)\tan\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(1/3)*b*d*sin(f*x + e)*tan(f
*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))**(3/2)*(d*tan(f*x+e))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.158 $\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx$

Optimal. Leaf size=64

$$\frac{6\cos^2(e+fx)^{2/3}(b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{17}{12};\frac{29}{12};\sin^2(e+fx)\right)}{17df}$$

```
[Out] (6*(Cos[e + f*x]^2)^(2/3)*Hypergeometric2F1[2/3, 17/12, 29/12, Sin[e + f*x]
^2]*(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3))/(17*d*f)
```

Rubi [A] time = 0.0919098, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2602, 2577}

$$\frac{6\cos^2(e+fx)^{2/3}(b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{17}{12};\frac{29}{12};\sin^2(e+fx)\right)}{17df}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(2/3)*Hypergeometric2F1[2/3, 17/12, 29/12, Sin[e + f*x]
^2]*(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3))/(17*d*f)
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \frac{\left(b\cos^{\frac{4}{3}}(e+fx)(d\tan(e+fx))^{4/3}\right)\int \frac{(b\sin(e+fx))^{11/6}}{\sqrt[3]{\cos(e+fx)}}\,dx}{d(b\sin(e+fx))^{4/3}}$$
$$= \frac{6\cos^2(e+fx)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{17}{12};\frac{29}{12};\sin^2(e+fx)\right)(b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}}{17df}$$

Mathematica [A] time = 0.476477, size = 72, normalized size = 1.12

$$\frac{6\cos(e+fx)\sec^2(e+fx)^{7/4}(b\sin(e+fx))^{5/2}\sqrt[3]{d\tan(e+fx)}\,{}_2F_1\left(\frac{17}{12},\frac{7}{4};\frac{29}{12};-\tan^2(e+fx)\right)}{17bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (6*Cos[e + f*x]*Hypergeometric2F1[17/12, 7/4, 29/12, -Tan[e + f*x]^2]*(Sec[
e + f*x]^2)^(7/4)*(b*Sin[e + f*x])^(5/2)*(d*Tan[e + f*x])^(1/3))/(17*b*f)
```

Maple [F] time = 0.153, size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt[3]{d\tan\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x)
```

```
[Out] int((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)\right)^{\frac{3}{2}}\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(3/2)*(d*tan(f*x + e))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sin\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}b\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(1/3)*b*sin(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))**(3/2)*(d*tan(f*x+e))**(1/3),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.159 $\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx$

Optimal. Leaf size=64

$$\frac{6\sqrt[3]{\cos^2(e+fx)}(b\sin(e+fx))^{3/2}(d\tan(e+fx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{13}{12};\frac{25}{12};\sin^2(e+fx)\right)}{13df}$$

```
[Out] (6*(Cos[e + f*x]^2)^(1/3)*Hypergeometric2F1[1/3, 13/12, 25/12, Sin[e + f*x]
^2]*(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(2/3))/(13*d*f)
```

Rubi [A] time = 0.0943887, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2602, 2577}

$$\frac{6\sqrt[3]{\cos^2(e+fx)}(b\sin(e+fx))^{3/2}(d\tan(e+fx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{13}{12};\frac{25}{12};\sin^2(e+fx)\right)}{13df}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sin[e + f*x])^(3/2)/(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(1/3)*Hypergeometric2F1[1/3, 13/12, 25/12, Sin[e + f*x]
^2]*(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(2/3))/(13*d*f)
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{\left(b\cos^{\frac{2}{3}}(e+fx)(d\tan(e+fx))^{2/3}\right)\int \sqrt[3]{\cos(e+fx)}(b\sin(e+fx))^{7/6}\,dx}{d(b\sin(e+fx))^{2/3}}$$
$$= \frac{6\sqrt[3]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{3},\frac{13}{12};\frac{25}{12};\sin^2(e+fx)\right)(b\sin(e+fx))^{3/2}(d\tan(e+fx))^{2/3}}{13df}$$

Mathematica [A] time = 0.730304, size = 67, normalized size = 1.05

$$\frac{2d(b\sin(e+fx))^{3/2}\left(\sec^2(e+fx)^{3/4}\,{}_2F_1\left(\frac{1}{12},\frac{3}{4};\frac{13}{12};-\tan^2(e+fx)\right)-1\right)}{3f(d\tan(e+fx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sin[e + f*x])^(3/2)/(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (2*d*(-1 + Hypergeometric2F1[1/12, 3/4, 13/12, -Tan[e + f*x]^2]*(Sec[e + f*
x]^2)^(3/4))*(b*Sin[e + f*x])^(3/2))/(3*f*(d*Tan[e + f*x])^(4/3))
```

Maple [F] time = 0.101, size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)\right)^{\frac{3}{2}}\frac{1}{\sqrt[3]{d\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x)
```

```
[Out] int((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(3/2)/(d*tan(f*x + e))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sin\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{2}{3}}b\sin\left(fx+e\right)}{d\tan\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(2/3)*b*sin(f*x + e)/(d*tan(
f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))**(3/2)/(d*tan(f*x+e))**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x, algorithm="giac")

[Out] integrate((b*sin(f*x + e))^(3/2)/(d*tan(f*x + e))^(1/3), x)
```

3.160 $\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx$

Optimal. Leaf size=64

$$\frac{6(b\sin(e+fx))^{3/2}\,{}_2F_1\left(-\frac{1}{6},\frac{7}{12};\frac{19}{12};\sin^2(e+fx)\right)}{7df\sqrt[6]{\cos^2(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$

```
[Out] (6*Hypergeometric2F1[-1/6, 7/12, 19/12, Sin[e + f*x]^2]*(b*Sin[e + f*x])^(3
/2))/(7*d*f*(Cos[e + f*x]^2)^(1/6)*(d*Tan[e + f*x])^(1/3))
```

Rubi [A] time = 0.0934154, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2602, 2577}

$$\frac{6(b\sin(e+fx))^{3/2}\,{}_2F_1\left(-\frac{1}{6},\frac{7}{12};\frac{19}{12};\sin^2(e+fx)\right)}{7df\sqrt[6]{\cos^2(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sin[e + f*x])^(3/2)/(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (6*Hypergeometric2F1[-1/6, 7/12, 19/12, Sin[e + f*x]^2]*(b*Sin[e + f*x])^(3
/2))/(7*d*f*(Cos[e + f*x]^2)^(1/6)*(d*Tan[e + f*x])^(1/3))
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \frac{\left(b\sqrt[3]{b\sin(e+fx)}\right)\int \cos^{\frac{4}{3}}(e+fx)\sqrt[6]{b\sin(e+fx)}\,dx}{d\sqrt[3]{\cos(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$
$$= \frac{6\,{}_2F_1\left(-\frac{1}{6},\frac{7}{12};\frac{19}{12};\sin^2(e+fx)\right)(b\sin(e+fx))^{3/2}}{7df\sqrt[6]{\cos^2(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$

Mathematica [A] time = 0.380261, size = 70, normalized size = 1.09

$$\frac{2(b\sin(e+fx))^{3/2}\left(2\sec^2(e+fx)^{3/4}\,{}_2F_1\left(\frac{7}{12},\frac{3}{4};\frac{19}{12};-\tan^2(e+fx)\right)+7\right)}{21df\sqrt[3]{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sin[e + f*x])^(3/2)/(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (2*(7 + 2*Hypergeometric2F1[7/12, 3/4, 19/12, -Tan[e + f*x]^2]*(Sec[e + f*x
]^2)^(3/4))*(b*Sin[e + f*x])^(3/2))/(21*d*f*(d*Tan[e + f*x])^(1/3))
```

Maple [F] time = 0.104, size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)\right)^{\frac{3}{2}}\left(d\tan\left(fx+e\right)\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x)
```

```
[Out] int((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(3/2)/(d*tan(f*x + e))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sin\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{2}{3}}b\sin\left(fx+e\right)}{d^2\tan\left(fx+e\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(2/3)*b*sin(f*x + e)/(d^2*ta
n(f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))**(3/2)/(d*tan(f*x+e))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\sin\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e))^(3/2)/(d*tan(f*x + e))^(4/3), x)
```

3.161 $\int (a\sin(e+fx))^m \tan^3(e+fx)\,dx$

Optimal. Leaf size=48

$$\frac{(a\sin(e+fx))^{m+4}\,{}_2F_1\left(2,\frac{m+4}{2};\frac{m+6}{2};\sin^2(e+fx)\right)}{a^4f(m+4)}$$

```
[Out] (Hypergeometric2F1[2, (4 + m)/2, (6 + m)/2, Sin[e + f*x]^2]*(a*Sin[e + f*x]
)^(4 + m))/(a^4*f*(4 + m))
```

Rubi [A] time = 0.0487703, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2592, 364}

$$\frac{(a\sin(e+fx))^{m+4}\,{}_2F_1\left(2,\frac{m+4}{2};\frac{m+6}{2};\sin^2(e+fx)\right)}{a^4f(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^m*Tan[e + f*x]^3,x]
```

```
[Out] (Hypergeometric2F1[2, (4 + m)/2, (6 + m)/2, Sin[e + f*x]^2]*(a*Sin[e + f*x]
)^(4 + m))/(a^4*f*(4 + m))
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (a\sin(e+fx))^m \tan^3(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^{3+m}}{(a^2-x^2)^2}\,dx, x, a\sin(e+fx)\right)}{f}$$
$$= \frac{{}_2F_1\left(2, \frac{4+m}{2}; \frac{6+m}{2}; \sin^2(e+fx)\right)(a\sin(e+fx))^{4+m}}{a^4 f(4+m)}$$

Mathematica [A] time = 0.0614199, size = 53, normalized size = 1.1

$$\frac{\sin^4(e+fx)(a\sin(e+fx))^m\,{}_2F_1\left(2, \frac{m+4}{2}; \frac{m+4}{2}+1; \sin^2(e+fx)\right)}{f(m+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^m*Tan[e + f*x]^3,x]
```

```
[Out] (Hypergeometric2F1[2, (4 + m)/2, 1 + (4 + m)/2, Sin[e + f*x]^2]*Sin[e + f*x
]^4*(a*Sin[e + f*x])^m)/(f*(4 + m))
```

Maple [F] time = 0.308, size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)\right)^m \left(\tan\left(fx+e\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^m*tan(f*x+e)^3,x)
```

```
[Out] int((a*sin(f*x+e))^m*tan(f*x+e)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)\right)^m \tan\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^m*tan(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sin\left(fx+e\right)\right)^{m}\tan\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e))^m*tan(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**m*tan(f*x+e)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)\right)^{m}\tan\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^m*tan(f*x + e)^3, x)
```

3.162 $\int (a\sin(e+fx))^m \tan(e+fx)\,dx$

Optimal. Leaf size=48

$$\frac{(a\sin(e+fx))^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};\sin^2(e+fx)\right)}{a^2f(m+2)}$$

```
[Out] (Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[e + f*x]^2]*(a*Sin[e + f*x]
)^(2 + m))/(a^2*f*(2 + m))
```

Rubi [A] time = 0.0348985, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2592, 364}

$$\frac{(a\sin(e+fx))^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};\sin^2(e+fx)\right)}{a^2f(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^m*Tan[e + f*x],x]
```

```
[Out] (Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[e + f*x]^2]*(a*Sin[e + f*x]
)^(2 + m))/(a^2*f*(2 + m))
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (a\sin(e+fx))^m \tan(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^{1+m}}{a^2-x^2}\,dx, x, a\sin(e+fx)\right)}{f}$$
$$= \frac{{}_2F_1\left(1, \frac{2+m}{2}; \frac{4+m}{2}; \sin^2(e+fx)\right)(a\sin(e+fx))^{2+m}}{a^2 f(2+m)}$$

Mathematica [A] time = 0.0325041, size = 53, normalized size = 1.1

$$\frac{\sin^2(e+fx)(a\sin(e+fx))^m\,{}_2F_1\left(1, \frac{m+2}{2}; \frac{m+2}{2}+1; \sin^2(e+fx)\right)}{f(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^m*Tan[e + f*x],x]
```

```
[Out] (Hypergeometric2F1[1, (2 + m)/2, 1 + (2 + m)/2, Sin[e + f*x]^2]*Sin[e + f*x
]^2*(a*Sin[e + f*x])^m)/(f*(2 + m))
```

Maple [F] time = 0.546, size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)\right)^m \tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^m*tan(f*x+e),x)
```

```
[Out] int((a*sin(f*x+e))^m*tan(f*x+e),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)\right)^m \tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^m*tan(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sin\left(fx+e\right)\right)^{m}\tan\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e))^m*tan(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(e+fx\right)\right)^{m}\tan\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**m*tan(f*x+e),x)
```

```
[Out] Integral((a*sin(e + f*x))**m*tan(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)\right)^{m}\tan\left(fx+e\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^m*tan(f*x + e), x)
```

3.163 $\int \cot(e+fx)(a\sin(e+fx))^m\,dx$

Optimal. Leaf size=17

$$\frac{(a\sin(e+fx))^m}{fm}$$

```
[Out] (a*Sin[e + f*x])^m/(f*m)
```

Rubi [A] time = 0.0286763, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2592, 30}

$$\frac{(a\sin(e+fx))^m}{fm}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a*Sin[e + f*x])^m,x]
```

```
[Out] (a*Sin[e + f*x])^m/(f*m)
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cot(e+fx)(a\sin(e+fx))^m\,dx &= \frac{\operatorname{Subst}\left(\int x^{-1+m}\,dx, x, a\sin(e+fx)\right)}{f}\\ &= \frac{(a\sin(e+fx))^m}{fm}\end{aligned}$$

Mathematica [A] time = 0.0093166, size = 17, normalized size = 1.

$$\frac{(a\sin(e+fx))^m}{fm}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a*Sin[e + f*x])^m,x]
```

```
[Out] (a*Sin[e + f*x])^m/(f*m)
```

Maple [A] time = 0.011, size = 18, normalized size = 1.1

$$\frac{\left(a\sin\left(fx+e\right)\right)^m}{fm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a*sin(f*x+e))^m,x)
```

```
[Out] (a*sin(f*x+e))^m/f/m
```

Maxima [A] time = 0.945853, size = 24, normalized size = 1.41

$$\frac{a^m\sin\left(fx+e\right)^m}{fm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] a^m*sin(f*x + e)^m/(f*m)
```

Fricas [A] time = 1.58187, size = 35, normalized size = 2.06

$$\frac{\left(a\sin\left(fx+e\right)\right)^m}{fm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] (a*sin(f*x + e))^m/(f*m)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(e+fx\right)\right)^{m}\cot\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*sin(e + f*x))**m*cot(e + f*x), x)
```

Giac [A] time = 1.17099, size = 24, normalized size = 1.41

$$\frac{\left(a\sin\left(fx+e\right)\right)^{m}}{fm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] (a*sin(f*x + e))^m/(f*m)
```

3.164 $\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx$

Optimal. Leaf size=46

$$-\frac{a^2(a\sin(e+fx))^{m-2}}{f(2-m)}-\frac{(a\sin(e+fx))^m}{fm}$$

```
[Out] -((a^2*(a*Sin[e + f*x])^(-2 + m))/(f*(2 - m))) - (a*Sin[e + f*x])^m/(f*m)
```

Rubi [A] time = 0.0500821, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2592, 14}

$$-\frac{a^2(a\sin(e+fx))^{m-2}}{f(2-m)}-\frac{(a\sin(e+fx))^m}{fm}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a*Sin[e + f*x])^m,x]
```

```
[Out] -((a^2*(a*Sin[e + f*x])^(-2 + m))/(f*(2 - m))) - (a*Sin[e + f*x])^m/(f*m)
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx = \frac{\text{Subst}\left(\int x^{-3+m}\left(a^2-x^2\right)dx, x, a\sin(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(a^2x^{-3+m}-x^{-1+m}\right)dx, x, a\sin(e+fx)\right)}{f}$$
$$= -\frac{a^2(a\sin(e+fx))^{-2+m}}{f(2-m)} - \frac{(a\sin(e+fx))^m}{fm}$$

Mathematica [A] time = 0.0563261, size = 37, normalized size = 0.8

$$\frac{\left(m\csc^2(e+fx)-m+2\right)(a\sin(e+fx))^m}{f(m-2)m}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a*Sin[e + f*x])^m,x]
```

```
[Out] ((2 - m + m*Csc[e + f*x]^2)*(a*Sin[e + f*x])^m)/(f*(-2 + m)*m)
```

Maple [C] time = 1.074, size = 3161, normalized size = 68.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a*sin(f*x+e))^m,x)
```

```
[Out] -1/(-2+m)/f/(exp(2*I*(f*x+e))-1)^2/m*(1/(exp(I*(Re(f*x)+Re(e)))^m)*(exp(I*(
f*x+e))-1)^m*(exp(I*(f*x+e))+1)^m/(2^m)*a^m*m*exp(m*Im(f*x)+m*Im(e))*exp(1/
2*I*m*csgn(I*exp(2*I*(f*x+e))-I)^2*csgn(I*exp(I*(f*x+e))-I)*Pi)*exp(1/2*I*m
*csgn(sin(f*x+e))^2*csgn(I*exp(-I*(f*x+e)))*Pi)*exp(-1/2*I*m*csgn(I*exp(2*I
*(f*x+e))-I)*csgn(I*exp(I*(f*x+e))-I)*csgn(I*exp(I*(f*x+e))+I)*Pi)*exp(1/2*
I*m*csgn(I*a*sin(f*x+e))^2*Pi)*exp(1/2*I*m*csgn(I*exp(2*I*(f*x+e))-I)*csgn(
sin(f*x+e))^2*Pi)*exp(1/2*I*m*Pi*csgn(sin(f*x+e))^3)*exp(1/2*I*m*csgn(I*exp
(2*I*(f*x+e))-I)^2*csgn(I*exp(I*(f*x+e))+I)*Pi)*exp(-1/2*I*m*csgn(a*sin(f*x
+e))^2*Pi*csgn(sin(f*x+e)))*exp(-1/2*I*m*Pi*csgn(I*a*sin(f*x+e))^3)*exp(1/2
*I*m*Pi*csgn(a*sin(f*x+e))^3)*exp(1/2*I*m*csgn(a*sin(f*x+e))^2*csgn(I*a)*Pi
)*exp(1/2*I*m*Pi*csgn(a*sin(f*x+e))*csgn(I*a*sin(f*x+e)))*exp(1/2*I*m*csgn(
```

```
I*exp(2*I*(f*x+e))-I)*csgn(I*exp(-I*(f*x+e)))*Pi*csgn(sin(f*x+e)))*exp(-1/2
*I*m*csgn(I*exp(2*I*(f*x+e))-I)^3*Pi)*exp(-1/2*I*m*csgn(I*a*sin(f*x+e))^2*P
i*csgn(a*sin(f*x+e)))*exp(-1/2*I*Pi*m)*exp(-1/2*I*m*csgn(I*a)*Pi*csgn(sin(f
*x+e))*csgn(a*sin(f*x+e)))*exp(4*I*f*x)*exp(4*I*e)-2/(exp(I*(Re(f*x)+Re(e))
)^m)*(exp(I*(f*x+e))-1)^m*(exp(I*(f*x+e))+1)^m/(2^m)*a^m*exp(m*Im(f*x)+m*Im
(e))*exp(1/2*I*m*csgn(I*exp(2*I*(f*x+e))-I)^2*csgn(I*exp(I*(f*x+e))-I)*Pi)*
exp(1/2*I*m*csgn(sin(f*x+e))^2*csgn(I*exp(-I*(f*x+e)))*Pi)*exp(-1/2*I*m*csg
n(I*exp(2*I*(f*x+e))-I)*csgn(I*exp(I*(f*x+e))-I)*csgn(I*exp(I*(f*x+e))+I)*P
i)*exp(1/2*I*m*csgn(I*a*sin(f*x+e))^2*Pi)*exp(1/2*I*m*csgn(I*exp(2*I*(f*x+e
))-I)*csgn(sin(f*x+e))^2*Pi)*exp(1/2*I*m*Pi*csgn(sin(f*x+e))^3)*exp(1/2*I*m
*csgn(I*exp(2*I*(f*x+e))-I)^2*csgn(I*exp(I*(f*x+e))+I)*Pi)*exp(-1/2*I*m*csg
n(a*sin(f*x+e))^2*Pi*csgn(sin(f*x+e)))*exp(-1/2*I*m*Pi*csgn(I*a*sin(f*x+e))
^3)*exp(1/2*I*m*Pi*csgn(a*sin(f*x+e))^3)*exp(1/2*I*m*csgn(a*sin(f*x+e))^2*c
sgn(I*a)*Pi)*exp(1/2*I*m*Pi*csgn(a*sin(f*x+e))*csgn(I*a*sin(f*x+e)))*exp(1/
2*I*m*csgn(I*exp(2*I*(f*x+e))-I)*csgn(I*exp(-I*(f*x+e)))*Pi*csgn(sin(f*x+e)
))*exp(-1/2*I*m*csgn(I*exp(2*I*(f*x+e))-I)^3*Pi)*exp(-1/2*I*m*csgn(I*a*sin(
f*x+e))^2*Pi*csgn(a*sin(f*x+e)))*exp(-1/2*I*Pi*m)*exp(-1/2*I*m*csgn(I*a)*Pi
*csgn(sin(f*x+e))*csgn(a*sin(f*x+e)))*exp(4*I*f*x)*exp(4*I*e)+2/(exp(I*(Re(
f*x)+Re(e)))^m)*(exp(I*(f*x+e))-1)^m*(exp(I*(f*x+e))+1)^m/(2^m)*a^m*m*exp(m
*Im(f*x)+m*Im(e))*exp(1/2*I*m*csgn(I*exp(2*I*(f*x+e))-I)^2*csgn(I*exp(I*(f*
x+e))-I)*Pi)*exp(1/2*I*m*csgn(sin(f*x+e))^2*csgn(I*exp(-I*(f*x+e)))*Pi)*exp
(-1/2*I*m*csgn(I*exp(2*I*(f*x+e))-I)*csgn(I*exp(I*(f*x+e))-I)*csgn(I*exp(I*
(f*x+e))+I)*Pi)*exp(1/2*I*m*csgn(I*a*sin(f*x+e))^2*Pi)*exp(1/2*I*m*csgn(I*e
xp(2*I*(f*x+e))-I)*csgn(sin(f*x+e))^2*Pi)*exp(1/2*I*m*Pi*csgn(sin(f*x+e))^3
)*exp(1/2*I*m*csgn(I*exp(2*I*(f*x+e))-I)^2*csgn(I*exp(I*(f*x+e))+I)*Pi)*exp
(-1/2*I*m*csgn(a*sin(f*x+e))^2*Pi*csgn(sin(f*x+e)))*exp(-1/2*I*m*Pi*csgn(I*
a*sin(f*x+e))^3)*exp(1/2*I*m*Pi*csgn(a*sin(f*x+e))^3)*exp(1/2*I*m*csgn(a*si
n(f*x+e))^2*csgn(I*a)*Pi)*exp(1/2*I*m*Pi*csgn(a*sin(f*x+e))*csgn(I*a*sin(f*
x+e)))*exp(1/2*I*m*csgn(I*exp(2*I*(f*x+e))-I)*csgn(I*exp(-I*(f*x+e)))*Pi*cs
gn(sin(f*x+e)))*exp(-1/2*I*m*csgn(I*exp(2*I*(f*x+e))-I)^3*Pi)*exp(-1/2*I*m*
csgn(I*a*sin(f*x+e))^2*Pi*csgn(a*sin(f*x+e)))*exp(-1/2*I*Pi*m)*exp(-1/2*I*m
*csgn(I*a)*Pi*csgn(sin(f*x+e))*csgn(a*sin(f*x+e)))*exp(2*I*f*x)*exp(2*I*e)+
4/(exp(I*(Re(f*x)+Re(e)))^m)*(exp(I*(f*x+e))-1)^m*(exp(I*(f*x+e))+1)^m/(2^m
)*a^m*exp(m*Im(f*x)+m*Im(e))*exp(1/2*I*m*csgn(I*exp(2*I*(f*x+e))-I)^2*csgn(
I*exp(I*(f*x+e))-I)*Pi)*exp(1/2*I*m*csgn(sin(f*x+e))^2*csgn(I*exp(-I*(f*x+e
)))*Pi)*exp(-1/2*I*m*csgn(I*exp(2*I*(f*x+e))-I)*csgn(I*exp(I*(f*x+e))-I)*cs
gn(I*exp(I*(f*x+e))+I)*Pi)*exp(1/2*I*m*csgn(I*a*sin(f*x+e))^2*Pi)*exp(1/2*I
*m*csgn(I*exp(2*I*(f*x+e))-I)*csgn(sin(f*x+e))^2*Pi)*exp(1/2*I*m*Pi*csgn(si
n(f*x+e))^3)*exp(1/2*I*m*csgn(I*exp(2*I*(f*x+e))-I)^2*csgn(I*exp(I*(f*x+e))
+I)*Pi)*exp(-1/2*I*m*csgn(a*sin(f*x+e))^2*Pi*csgn(sin(f*x+e)))*exp(-1/2*I*m
*Pi*csgn(I*a*sin(f*x+e))^3)*exp(1/2*I*m*Pi*csgn(a*sin(f*x+e))^3)*exp(1/2*I*
m*csgn(a*sin(f*x+e))^2*csgn(I*a)*Pi)*exp(1/2*I*m*Pi*csgn(a*sin(f*x+e))*csgn
(I*a*sin(f*x+e)))*exp(1/2*I*m*csgn(I*exp(2*I*(f*x+e))-I)*csgn(I*exp(-I*(f*x
+e)))*Pi*csgn(sin(f*x+e)))*exp(-1/2*I*m*csgn(I*exp(2*I*(f*x+e))-I)^3*Pi)*ex
p(-1/2*I*m*csgn(I*a*sin(f*x+e))^2*Pi*csgn(a*sin(f*x+e)))*exp(-1/2*I*Pi*m)*e
```

```
xp(-1/2*I*m*csgn(I*a)*Pi*csgn(sin(f*x+e))*csgn(a*sin(f*x+e)))*exp(2*I*f*x)*
exp(2*I*e)+1/(exp(I*(Re(f*x)+Re(e)))^m)*(exp(I*(f*x+e))-1)^m*(exp(I*(f*x+e)
)+1)^m/(2^m)*a^m*m*exp(1/2*m*(I*Pi*csgn(a*sin(f*x+e))*csgn(I*a*sin(f*x+e))+
I*csgn(I*a*sin(f*x+e))^2*Pi+I*csgn(sin(f*x+e))^2*csgn(I*exp(-I*(f*x+e)))*Pi
+I*csgn(I*exp(2*I*(f*x+e))-I)^2*csgn(I*exp(I*(f*x+e))-I)*Pi-I*csgn(I*a*sin(
f*x+e))^2*Pi*csgn(a*sin(f*x+e))+I*csgn(I*exp(2*I*(f*x+e))-I)*csgn(sin(f*x+e
))^2*Pi-I*csgn(a*sin(f*x+e))^2*Pi*csgn(sin(f*x+e))+I*csgn(I*exp(2*I*(f*x+e)
)-I)*csgn(I*exp(-I*(f*x+e)))*Pi*csgn(sin(f*x+e))+I*Pi*csgn(sin(f*x+e))^3-I*
csgn(I*exp(2*I*(f*x+e))-I)^3*Pi-I*Pi*csgn(I*a*sin(f*x+e))^3+I*csgn(I*exp(2*
I*(f*x+e))-I)^2*csgn(I*exp(I*(f*x+e))+I)*Pi-I*csgn(I*a)*Pi*csgn(sin(f*x+e))
*csgn(a*sin(f*x+e))+I*csgn(a*sin(f*x+e))^2*csgn(I*a)*Pi+I*Pi*csgn(a*sin(f*x
+e))^3-I*csgn(I*exp(2*I*(f*x+e))-I)*csgn(I*exp(I*(f*x+e))-I)*csgn(I*exp(I*(
f*x+e))+I)*Pi-I*Pi+2*Im(f*x)+2*Im(e)))-2/(exp(I*(Re(f*x)+Re(e)))^m)*(exp(I*
(f*x+e))-1)^m*(exp(I*(f*x+e))+1)^m/(2^m)*a^m*exp(1/2*m*(I*Pi*csgn(a*sin(f*x
+e))*csgn(I*a*sin(f*x+e))+I*csgn(I*a*sin(f*x+e))^2*Pi+I*csgn(sin(f*x+e))^2*
csgn(I*exp(-I*(f*x+e)))*Pi+I*csgn(I*exp(2*I*(f*x+e))-I)^2*csgn(I*exp(I*(f*x
+e))-I)*Pi-I*csgn(I*a*sin(f*x+e))^2*Pi*csgn(a*sin(f*x+e))+I*csgn(I*exp(2*I*
(f*x+e))-I)*csgn(sin(f*x+e))^2*Pi-I*csgn(a*sin(f*x+e))^2*Pi*csgn(sin(f*x+e)
)+I*csgn(I*exp(2*I*(f*x+e))-I)*csgn(I*exp(-I*(f*x+e)))*Pi*csgn(sin(f*x+e))+
I*Pi*csgn(sin(f*x+e))^3-I*csgn(I*exp(2*I*(f*x+e))-I)^3*Pi-I*Pi*csgn(I*a*sin
(f*x+e))^3+I*csgn(I*exp(2*I*(f*x+e))-I)^2*csgn(I*exp(I*(f*x+e))+I)*Pi-I*csg
n(I*a)*Pi*csgn(sin(f*x+e))*csgn(a*sin(f*x+e))+I*csgn(a*sin(f*x+e))^2*csgn(I
*a)*Pi+I*Pi*csgn(a*sin(f*x+e))^3-I*csgn(I*exp(2*I*(f*x+e))-I)*csgn(I*exp(I*
(f*x+e))-I)*csgn(I*exp(I*(f*x+e))+I)*Pi-I*Pi+2*Im(f*x)+2*Im(e))))
```

Maxima [A] time = 0.991215, size = 63, normalized size = 1.37

$$-\frac{\frac{a^m\sin\left(fx+e\right)^m}{m}-\frac{a^m\sin\left(fx+e\right)^m}{(m-2)\sin\left(fx+e\right)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] -(a^m*sin(f*x + e)^m/m - a^m*sin(f*x + e)^m/((m - 2)*sin(f*x + e)^2))/f
```

Fricas [A] time = 1.59485, size = 132, normalized size = 2.87

$$\frac{\left((m-2)\cos\left(fx+e\right)^{2}+2\right)\left(a\sin\left(fx+e\right)\right)^{m}}{fm^{2}-\left(fm^{2}-2\,fm\right)\cos\left(fx+e\right)^{2}-2\,fm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] ((m - 2)*cos(f*x + e)^2 + 2)*(a*sin(f*x + e))^m/(f*m^2 - (f*m^2 - 2*f*m)*co
s(f*x + e)^2 - 2*f*m)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(e+fx\right)\right)^{m}\cot^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*sin(e + f*x))**m*cot(e + f*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)\right)^{m}\cot\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^m*cot(f*x + e)^3, x)
```

3.165 $\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx$

Optimal. Leaf size=72

$$-\frac{a^4(a\sin(e+fx))^{m-4}}{f(4-m)}+\frac{2a^2(a\sin(e+fx))^{m-2}}{f(2-m)}+\frac{(a\sin(e+fx))^m}{fm}$$

```
[Out] -((a^4*(a*Sin[e + f*x])^(-4 + m))/(f*(4 - m))) + (2*a^2*(a*Sin[e + f*x])^(-
2 + m))/(f*(2 - m)) + (a*Sin[e + f*x])^m/(f*m)
```

Rubi [A] time = 0.0609768, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2592, 270}

$$-\frac{a^4(a\sin(e+fx))^{m-4}}{f(4-m)}+\frac{2a^2(a\sin(e+fx))^{m-2}}{f(2-m)}+\frac{(a\sin(e+fx))^m}{fm}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*(a*Sin[e + f*x])^m,x]
```

```
[Out] -((a^4*(a*Sin[e + f*x])^(-4 + m))/(f*(4 - m))) + (2*a^2*(a*Sin[e + f*x])^(-
2 + m))/(f*(2 - m)) + (a*Sin[e + f*x])^m/(f*m)
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx = \frac{\text{Subst}\left(\int x^{-5+m}\left(a^2-x^2\right)^2\,dx,x,a\sin(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(a^4x^{-5+m}-2a^2x^{-3+m}+x^{-1+m}\right)\,dx,x,a\sin(e+fx)\right)}{f}$$
$$= -\frac{a^4(a\sin(e+fx))^{-4+m}}{f(4-m)}+\frac{2a^2(a\sin(e+fx))^{-2+m}}{f(2-m)}+\frac{(a\sin(e+fx))^m}{fm}$$

Mathematica [A] time = 0.324903, size = 62, normalized size = 0.86

$$\frac{\left((m-2)m\csc^4(e+fx)-2(m-4)m\csc^2(e+fx)+m^2-6m+8\right)(a\sin(e+fx))^m}{f(m-4)(m-2)m}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*(a*Sin[e + f*x])^m,x]
```

```
[Out] ((8 - 6*m + m^2 - 2*(-4 + m)*m*Csc[e + f*x]^2 + (-2 + m)*m*Csc[e + f*x]^4)*
(a*Sin[e + f*x])^m)/(f*(-4 + m)*(-2 + m)*m)
```

Maple [C] time = 0.691, size = 7964, normalized size = 110.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a*sin(f*x+e))^m,x)
```

```
[Out] result too large to display
```

Maxima [A] time = 0.959883, size = 96, normalized size = 1.33

$$\frac{\frac{a^m\sin\left(fx+e\right)^m}{m}-\frac{2\,a^m\sin\left(fx+e\right)^m}{(m-2)\sin\left(fx+e\right)^2}+\frac{a^m\sin\left(fx+e\right)^m}{(m-4)\sin\left(fx+e\right)^4}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] (a^m*sin(f*x + e)^m/m - 2*a^m*sin(f*x + e)^m/((m - 2)*sin(f*x + e)^2) + a^m
*sin(f*x + e)^m/((m - 4)*sin(f*x + e)^4))/f
```

Fricas [A] time = 1.65627, size = 267, normalized size = 3.71

$$\frac{\left(\left(m^2-6\,m+8\right)\cos\left(fx+e\right)^4+4\,(m-4)\cos\left(fx+e\right)^2+8\right)\left(a\sin\left(fx+e\right)\right)^m}{\left(fm^3-6\,fm^2+8\,fm\right)\cos\left(fx+e\right)^4+fm^3-6\,fm^2-2\left(fm^3-6\,fm^2+8\,fm\right)\cos\left(fx+e\right)^2+8\,fm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] ((m^2 - 6*m + 8)*cos(f*x + e)^4 + 4*(m - 4)*cos(f*x + e)^2 + 8)*(a*sin(f*x
+ e))^m/((f*m^3 - 6*f*m^2 + 8*f*m)*cos(f*x + e)^4 + f*m^3 - 6*f*m^2 - 2*(f*
m^3 - 6*f*m^2 + 8*f*m)*cos(f*x + e)^2 + 8*f*m)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)\right)^m\cot\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^m*cot(f*x + e)^5, x)
```

3.166 $\int (a\sin(e+fx))^m \tan^4(e+fx)\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)(a\sin(e+fx))^{m+5}\,{}_2F_1\left(\frac{5}{2},\frac{m+5}{2};\frac{m+7}{2};\sin^2(e+fx)\right)}{a^5f(m+5)}$$

```
[Out] (Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[5/2, (5 + m)/2, (7 + m)/2, Sin[e +
f*x]^2]*Sec[e + f*x]*(a*Sin[e + f*x])^(5 + m))/(a^5*f*(5 + m))
```

Rubi [A] time = 0.0897263, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2600, 2577}

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)(a\sin(e+fx))^{m+5}\,{}_2F_1\left(\frac{5}{2},\frac{m+5}{2};\frac{m+7}{2};\sin^2(e+fx)\right)}{a^5f(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^m*Tan[e + f*x]^4,x]
```

```
[Out] (Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[5/2, (5 + m)/2, (7 + m)/2, Sin[e +
f*x]^2]*Sec[e + f*x]*(a*Sin[e + f*x])^(5 + m))/(a^5*f*(5 + m))
```

Rule 2600

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(n_), x_Sy
mbol] :> Dist[1/a^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /;
 FreeQ[{a, e, f, m}, x] && IntegerQ[n] &&  !IntegerQ[m]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (a\sin(e+fx))^m \tan^4(e+fx)\,dx = \frac{\int \sec^4(e+fx)(a\sin(e+fx))^{4+m}\,dx}{a^4}$$
$$= \frac{\sqrt{\cos^2(e+fx)}\,{}_2F_1\left(\frac{5}{2},\frac{5+m}{2};\frac{7+m}{2};\sin^2(e+fx)\right)\sec(e+fx)(a\sin(e+fx))^{5+m}}{a^5 f(5+m)}$$

Mathematica [A] time = 0.141436, size = 71, normalized size = 1.04

$$\frac{\sin^4(e+fx)\sqrt{\cos^2(e+fx)}\tan(e+fx)(a\sin(e+fx))^m\,{}_2F_1\left(\frac{5}{2},\frac{m+5}{2};\frac{m+7}{2};\sin^2(e+fx)\right)}{f(m+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^m*Tan[e + f*x]^4,x]
```

```
[Out] (Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[5/2, (5 + m)/2, (7 + m)/2, Sin[e +
f*x]^2]*Sin[e + f*x]^4*(a*Sin[e + f*x])^m*Tan[e + f*x])/(f*(5 + m))
```

Maple [F] time = 0.211, size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)\right)^m \left(\tan\left(fx+e\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^m*tan(f*x+e)^4,x)
```

```
[Out] int((a*sin(f*x+e))^m*tan(f*x+e)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)\right)^m \tan\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^m*tan(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sin\left(fx+e\right)\right)^m\tan\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e))^m*tan(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**m*tan(f*x+e)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)\right)^m\tan\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^m*tan(f*x + e)^4, x)
```

3.167 $\int (a\sin(e+fx))^m\tan^2(e+fx)\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)(a\sin(e+fx))^{m+3}\,{}_2F_1\left(\frac{3}{2},\frac{m+3}{2};\frac{m+5}{2};\sin^2(e+fx)\right)}{a^3f(m+3)}$$

```
[Out] (Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[3/2, (3 + m)/2, (5 + m)/2, Sin[e +
f*x]^2]*Sec[e + f*x]*(a*Sin[e + f*x])^(3 + m))/(a^3*f*(3 + m))
```

Rubi [A] time = 0.0888997, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2600, 2577}

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)(a\sin(e+fx))^{m+3}\,{}_2F_1\left(\frac{3}{2},\frac{m+3}{2};\frac{m+5}{2};\sin^2(e+fx)\right)}{a^3f(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^m*Tan[e + f*x]^2,x]
```

```
[Out] (Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[3/2, (3 + m)/2, (5 + m)/2, Sin[e +
f*x]^2]*Sec[e + f*x]*(a*Sin[e + f*x])^(3 + m))/(a^3*f*(3 + m))
```

Rule 2600

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(n_), x_Sy
mbol] :> Dist[1/a^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /;
 FreeQ[{a, e, f, m}, x] && IntegerQ[n] &&  !IntegerQ[m]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (a\sin(e+fx))^m \tan^2(e+fx)\,dx = \frac{\int \sec^2(e+fx)(a\sin(e+fx))^{2+m}\,dx}{a^2}$$
$$= \frac{\sqrt{\cos^2(e+fx)}\,{}_2F_1\left(\frac{3}{2},\frac{3+m}{2};\frac{5+m}{2};\sin^2(e+fx)\right)\sec(e+fx)(a\sin(e+fx))^{3+m}}{a^3f(3+m)}$$

Mathematica [A] time = 0.0844246, size = 71, normalized size = 1.04

$$\frac{\sin^2(e+fx)\sqrt{\cos^2(e+fx)}\tan(e+fx)(a\sin(e+fx))^m\,{}_2F_1\left(\frac{3}{2},\frac{m+3}{2};\frac{m+5}{2};\sin^2(e+fx)\right)}{f(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^m*Tan[e + f*x]^2,x]
```

```
[Out] (Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[3/2, (3 + m)/2, (5 + m)/2, Sin[e +
f*x]^2]*Sin[e + f*x]^2*(a*Sin[e + f*x])^m*Tan[e + f*x])/(f*(3 + m))
```

Maple [F] time = 0.194, size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)\right)^m\left(\tan\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^m*tan(f*x+e)^2,x)
```

```
[Out] int((a*sin(f*x+e))^m*tan(f*x+e)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)\right)^m\tan\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^m*tan(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sin\left(fx+e\right)\right)^{m}\tan\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e))^m*tan(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(e+fx\right)\right)^{m}\tan^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**m*tan(f*x+e)**2,x)
```

```
[Out] Integral((a*sin(e + f*x))**m*tan(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)\right)^{m}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^m*tan(f*x + e)^2, x)
```

3.168 $\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx$

Optimal. Leaf size=69

$$-\frac{a\cos(e+fx)(a\sin(e+fx))^{m-1}\,{}_2F_1\left(-\frac{1}{2},\frac{m-1}{2};\frac{m+1}{2};\sin^2(e+fx)\right)}{f(1-m)\sqrt{\cos^2(e+fx)}}$$

```
[Out] -((a*Cos[e + f*x]*Hypergeometric2F1[-1/2, (-1 + m)/2, (1 + m)/2, Sin[e + f*
x]^2]*(a*Sin[e + f*x])^(-1 + m))/(f*(1 - m)*Sqrt[Cos[e + f*x]^2]))
```

Rubi [A] time = 0.0921402, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2600, 2577}

$$-\frac{a\cos(e+fx)(a\sin(e+fx))^{m-1}\,{}_2F_1\left(-\frac{1}{2},\frac{m-1}{2};\frac{m+1}{2};\sin^2(e+fx)\right)}{f(1-m)\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a*Sin[e + f*x])^m,x]
```

```
[Out] -((a*Cos[e + f*x]*Hypergeometric2F1[-1/2, (-1 + m)/2, (1 + m)/2, Sin[e + f*
x]^2]*(a*Sin[e + f*x])^(-1 + m))/(f*(1 - m)*Sqrt[Cos[e + f*x]^2]))
```

Rule 2600

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(n_), x_Sy
mbol] :> Dist[1/a^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /;
 FreeQ[{a, e, f, m}, x] && IntegerQ[n] &&  !IntegerQ[m]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx = a^2\int \cos^2(e+fx)(a\sin(e+fx))^{-2+m}\,dx$$
$$= -\frac{a\cos(e+fx)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(-1+m);\frac{1+m}{2};\sin^2(e+fx)\right)(a\sin(e+fx))^{-1+m}}{f(1-m)\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 0.0797154, size = 66, normalized size = 0.96

$$\frac{a\sqrt{\cos^2(e+fx)}\sec(e+fx)(a\sin(e+fx))^{m-1}\,{}_2F_1\left(-\frac{1}{2},\frac{m-1}{2};\frac{m+1}{2};\sin^2(e+fx)\right)}{f(m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(a*Sin[e + f*x])^m,x]
```

```
[Out] (a*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/2, (-1 + m)/2, (1 + m)/2, Sin[
e + f*x]^2]*Sec[e + f*x]*(a*Sin[e + f*x])^(-1 + m))/(f*(-1 + m))
```

Maple [F] time = 0.206, size = 0, normalized size = 0.

$$\int \left(\cot\left(fx+e\right)\right)^2\left(a\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a*sin(f*x+e))^m,x)
```

```
[Out] int(cot(f*x+e)^2*(a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)\right)^m\cot\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^m*cot(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sin\left(fx+e\right)\right)^{m}\cot\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e))^m*cot(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(e+fx\right)\right)^{m}\cot^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*sin(e + f*x))**m*cot(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)\right)^{m}\cot\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^m*cot(f*x + e)^2, x)
```

3.169 $\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx$

Optimal. Leaf size=71

$$-\frac{a^3\cos(e+fx)(a\sin(e+fx))^{m-3}\,{}_2F_1\left(-\frac{3}{2},\frac{m-3}{2};\frac{m-1}{2};\sin^2(e+fx)\right)}{f(3-m)\sqrt{\cos^2(e+fx)}}$$

```
[Out] -((a^3*Cos[e + f*x]*Hypergeometric2F1[-3/2, (-3 + m)/2, (-1 + m)/2, Sin[e +
 f*x]^2]*(a*Sin[e + f*x])^(-3 + m))/(f*(3 - m)*Sqrt[Cos[e + f*x]^2]))
```

Rubi [A] time = 0.0882018, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2600, 2577}

$$-\frac{a^3\cos(e+fx)(a\sin(e+fx))^{m-3}\,{}_2F_1\left(-\frac{3}{2},\frac{m-3}{2};\frac{m-1}{2};\sin^2(e+fx)\right)}{f(3-m)\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a*Sin[e + f*x])^m,x]
```

```
[Out] -((a^3*Cos[e + f*x]*Hypergeometric2F1[-3/2, (-3 + m)/2, (-1 + m)/2, Sin[e +
 f*x]^2]*(a*Sin[e + f*x])^(-3 + m))/(f*(3 - m)*Sqrt[Cos[e + f*x]^2]))
```

Rule 2600

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(n_), x_Sy
mbol] :> Dist[1/a^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /;
 FreeQ[{a, e, f, m}, x] && IntegerQ[n] &&  !IntegerQ[m]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx = a^4\int \cos^4(e+fx)(a\sin(e+fx))^{-4+m}\,dx$$
$$= -\frac{a^3\cos(e+fx)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}(-3+m);\frac{1}{2}(-1+m);\sin^2(e+fx)\right)(a\sin(e+fx))^{-3+m}}{f(3-m)\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 0.065022, size = 71, normalized size = 1.

$$\frac{\sqrt{\cos^2(e+fx)}\csc^3(e+fx)\sec(e+fx)(a\sin(e+fx))^m\,{}_2F_1\left(-\frac{3}{2},\frac{m-3}{2};\frac{m-1}{2};\sin^2(e+fx)\right)}{f(m-3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(a*Sin[e + f*x])^m,x]
```

```
[Out] (Sqrt[Cos[e + f*x]^2]*Csc[e + f*x]^3*Hypergeometric2F1[-3/2, (-3 + m)/2, (-
1 + m)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(a*Sin[e + f*x])^m)/(f*(-3 + m))
```

Maple [F] time = 0.238, size = 0, normalized size = 0.

$$\int \left(\cot\left(fx+e\right)\right)^4\left(a\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a*sin(f*x+e))^m,x)
```

```
[Out] int(cot(f*x+e)^4*(a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)\right)^m\cot\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^m*cot(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sin\left(fx+e\right)\right)^{m}\cot\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e))^m*cot(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)\right)^{m}\cot\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^m*cot(f*x + e)^4, x)
```

3.170 $\int (a\sin(e+fx))^m (b\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=79

$$\frac{2\cos^2(e+fx)^{5/4}(b\tan(e+fx))^{5/2}(a\sin(e+fx))^m\,{}_2F_1\left(\frac{5}{4},\frac{1}{4}(2m+5);\frac{1}{4}(2m+9);\sin^2(e+fx)\right)}{bf(2m+5)}$$

```
[Out] (2*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[5/4, (5 + 2*m)/4, (9 + 2*m)/4,
Sin[e + f*x]^2]*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(5/2))/(b*f*(5 + 2*m))
```

Rubi [A] time = 0.115703, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2602, 2577}

$$\frac{2\cos^2(e+fx)^{5/4}(b\tan(e+fx))^{5/2}(a\sin(e+fx))^m\,{}_2F_1\left(\frac{5}{4},\frac{1}{4}(2m+5);\frac{1}{4}(2m+9);\sin^2(e+fx)\right)}{bf(2m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[5/4, (5 + 2*m)/4, (9 + 2*m)/4,
Sin[e + f*x]^2]*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(5/2))/(b*f*(5 + 2*m))
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^{3/2}\,dx = \frac{\left(a\cos^{\frac{5}{2}}(e+fx)(b\tan(e+fx))^{5/2}\right)\int \frac{(a\sin(e+fx))^{\frac{3}{2}+m}}{\cos^{\frac{3}{2}}(e+fx)}\,dx}{b(a\sin(e+fx))^{5/2}}$$
$$= \frac{2\cos^2(e+fx)^{5/4}\,{}_2F_1\left(\frac{5}{4},\frac{1}{4}(5+2m);\frac{1}{4}(9+2m);\sin^2(e+fx)\right)(a\sin(e+fx))^m}{bf(5+2m)}$$

Mathematica [A] time = 8.10383, size = 87, normalized size = 1.1

$$\frac{2(b\tan(e+fx))^{5/2}\sec^2(e+fx)^{m/2}(a\sin(e+fx))^m\,{}_2F_1\left(\frac{m+2}{2},\frac{1}{4}(2m+5);\frac{1}{4}(2m+9);-\tan^2(e+fx)\right)}{bf(2m+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*Hypergeometric2F1[(2 + m)/2, (5 + 2*m)/4, (9 + 2*m)/4, -Tan[e + f*x]^2]*
(Sec[e + f*x]^2)^(m/2)*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(5/2))/(b*f*(5 +
 2*m))
```

Maple [F] time = 0.14, size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)\right)^m\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^m*(b*tan(f*x+e))^(3/2),x)
```

```
[Out] int((a*sin(f*x+e))^m*(b*tan(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^(3/2)*(a*sin(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\tan\left(fx+e\right)}\left(a\sin\left(fx+e\right)\right)^{m}b\tan\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*tan(f*x + e))*(a*sin(f*x + e))^m*b*tan(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**m*(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.171 $\int (a \sin(e+fx))^m \sqrt{b \tan(e+fx)} \, dx$

Optimal. Leaf size=79

$$\frac{2 \cos^2(e+fx)^{3/4} (b \tan(e+fx))^{3/2} (a \sin(e+fx))^m \, {}_2F_1\left(\frac{3}{4}, \frac{1}{4}(2m+3); \frac{1}{4}(2m+7); \sin^2(e+fx)\right)}{bf(2m+3)}$$

```
[Out] (2*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[3/4, (3 + 2*m)/4, (7 + 2*m)/4,
Sin[e + f*x]^2]*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(3/2))/(b*f*(3 + 2*m))
```

Rubi [A] time = 0.103751, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2602, 2577}

$$\frac{2 \cos^2(e+fx)^{3/4} (b \tan(e+fx))^{3/2} (a \sin(e+fx))^m \, {}_2F_1\left(\frac{3}{4}, \frac{1}{4}(2m+3); \frac{1}{4}(2m+7); \sin^2(e+fx)\right)}{bf(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^m*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[3/4, (3 + 2*m)/4, (7 + 2*m)/4,
Sin[e + f*x]^2]*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(3/2))/(b*f*(3 + 2*m))
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (a\sin(e+fx))^m\sqrt{b\tan(e+fx)}\,dx = \frac{\left(a\cos^{\frac{3}{2}}(e+fx)(b\tan(e+fx))^{3/2}\right)\int \frac{(a\sin(e+fx))^{\frac{1}{2}+m}}{\sqrt{\cos(e+fx)}}\,dx}{b(a\sin(e+fx))^{3/2}}$$

$$= \frac{2\cos^2(e+fx)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{1}{4}(3+2m);\frac{1}{4}(7+2m);\sin^2(e+fx)\right)(a\sin(e+fx))^m(b\,\mathrm{t}}{bf(3+2m)}$$

Mathematica [A] time = 3.17363, size = 87, normalized size = 1.1

$$\frac{2(b\tan(e+fx))^{3/2}\sec^2(e+fx)^{m/2}(a\sin(e+fx))^m\,{}_2F_1\left(\frac{m+2}{2},\frac{1}{4}(2m+3);\frac{1}{4}(2m+7);-\tan^2(e+fx)\right)}{bf(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^m*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (2*Hypergeometric2F1[(2 + m)/2, (3 + 2*m)/4, (7 + 2*m)/4, -Tan[e + f*x]^2]*
(Sec[e + f*x]^2)^(m/2)*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(3/2))/(b*f*(3 +
 2*m))
```

Maple [F] time = 0.132, size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)\right)^m\sqrt{b\tan\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^m*(b*tan(f*x+e))^(1/2),x)
```

```
[Out] int((a*sin(f*x+e))^m*(b*tan(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan\left(fx+e\right)}\left(a\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))*(a*sin(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\tan\left(fx+e\right)}\left(a\sin\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*tan(f*x + e))*(a*sin(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**m*(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.172 $\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal. Leaf size=79

$$\frac{2\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}(a\sin(e+fx))^m\,{}_2F_1\left(\frac{1}{4},\frac{1}{4}(2m+1);\frac{1}{4}(2m+5);\sin^2(e+fx)\right)}{bf(2m+1)}$$

```
[Out] (2*(Cos[e + f*x]^2)^(1/4)*Hypergeometric2F1[1/4, (1 + 2*m)/4, (5 + 2*m)/4,
Sin[e + f*x]^2]*(a*Sin[e + f*x])^m*Sqrt[b*Tan[e + f*x]])/(b*f*(1 + 2*m))
```

Rubi [A] time = 0.102909, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2602, 2577}

$$\frac{2\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}(a\sin(e+fx))^m\,{}_2F_1\left(\frac{1}{4},\frac{1}{4}(2m+1);\frac{1}{4}(2m+5);\sin^2(e+fx)\right)}{bf(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^m/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(1/4)*Hypergeometric2F1[1/4, (1 + 2*m)/4, (5 + 2*m)/4,
Sin[e + f*x]^2]*(a*Sin[e + f*x])^m*Sqrt[b*Tan[e + f*x]])/(b*f*(1 + 2*m))
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx = \frac{\left(a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int \sqrt{\cos(e+fx)}(a\sin(e+fx))^{-\frac{1}{2}+m}\,dx}{b\sqrt{a\sin(e+fx)}}$$
$$= \frac{2\sqrt[4]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4}(1+2m);\frac{1}{4}(5+2m);\sin^2(e+fx)\right)(a\sin(e+fx))^m\sqrt{b\tan(e+fx)}}{bf(1+2m)}$$

Mathematica [A] time = 2.81801, size = 87, normalized size = 1.1

$$\frac{2\sqrt{b\tan(e+fx)}\sec^2(e+fx)^{m/2}(a\sin(e+fx))^m\,{}_2F_1\left(\frac{m+2}{2},\frac{1}{4}(2m+1);\frac{1}{4}(2m+5);-\tan^2(e+fx)\right)}{bf(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^m/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (2*Hypergeometric2F1[(2 + m)/2, (1 + 2*m)/4, (5 + 2*m)/4, -Tan[e + f*x]^2]*
(Sec[e + f*x]^2)^(m/2)*(a*Sin[e + f*x])^m*Sqrt[b*Tan[e + f*x]])/(b*f*(1 + 2
*m))
```

Maple [F] time = 0.132, size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)\right)^m \frac{1}{\sqrt{b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^m/(b*tan(f*x+e))^(1/2),x)
```

```
[Out] int((a*sin(f*x+e))^m/(b*tan(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(fx+e\right)\right)^m}{\sqrt{b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^m/sqrt(b*tan(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\tan\left(fx+e\right)}\left(a\sin\left(fx+e\right)\right)^{m}}{b\tan\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*tan(f*x + e))*(a*sin(f*x + e))^m/(b*tan(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\sin\left(e+fx\right)\right)^{m}}{\sqrt{b\tan\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**m/(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*sin(e + f*x))**m/sqrt(b*tan(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\sin\left(fx+e\right)\right)^{m}}{\sqrt{b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^m/sqrt(b*tan(f*x + e)), x)
```

3.173 $\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=79

$$-\frac{2(a\sin(e+fx))^m\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4}(2m-1);\frac{1}{4}(2m+3);\sin^2(e+fx)\right)}{bf(1-2m)\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[Out] (-2*Hypergeometric2F1[-1/4, (-1 + 2*m)/4, (3 + 2*m)/4, Sin[e + f*x]^2]*(a*S
in[e + f*x])^m)/(b*f*(1 - 2*m)*(Cos[e + f*x]^2)^(1/4)*Sqrt[b*Tan[e + f*x]])
```

Rubi [A] time = 0.119777, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2602, 2577}

$$-\frac{2(a\sin(e+fx))^m\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4}(2m-1);\frac{1}{4}(2m+3);\sin^2(e+fx)\right)}{bf(1-2m)\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^m/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, (-1 + 2*m)/4, (3 + 2*m)/4, Sin[e + f*x]^2]*(a*S
in[e + f*x])^m)/(b*f*(1 - 2*m)*(Cos[e + f*x]^2)^(1/4)*Sqrt[b*Tan[e + f*x]])
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx = \frac{\left(a\sqrt{a\sin(e+fx)}\right)\int \cos^{\frac{3}{2}}(e+fx)(a\sin(e+fx))^{-\frac{3}{2}+m}\,dx}{b\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$
$$= -\frac{2\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4}(-1+2m);\frac{1}{4}(3+2m);\sin^2(e+fx)\right)(a\sin(e+fx))^m}{bf(1-2m)\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

Mathematica [B] time = 5.1392, size = 225, normalized size = 2.85

$$\frac{\sec^4(e+fx)\sec^2(e+fx)^{\frac{m-4}{2}}(a\sin(e+fx))^m\left({}_2F_1\left(\frac{m}{2},\frac{1}{4}(2m-1);\frac{1}{4}(2m+3);-\tan^2(e+fx)\right)+\frac{\cos(2(e+fx))\sec^2(e+fx)\left((2m-\right.}{}\right.}{bf(2m-1)\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^m/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]^4*(Sec[e + f*x]^2)^((-4 + m)/2)*(a*Sin[e + f*x])^m*(Hypergeom
etric2F1[m/2, (-1 + 2*m)/4, (3 + 2*m)/4, -Tan[e + f*x]^2] + (Cos[2*(e + f*x
)]*Sec[e + f*x]^2*(-((3 + 2*m)*Hypergeometric2F1[(2 + m)/2, (-1 + 2*m)/4, (
3 + 2*m)/4, -Tan[e + f*x]^2]) + (-1 + 2*m)*Hypergeometric2F1[(2 + m)/2, (3
+ 2*m)/4, (7 + 2*m)/4, -Tan[e + f*x]^2]*Tan[e + f*x]^2))/((3 + 2*m)*(-1 + T
an[e + f*x]^2))))/(b*f*(-1 + 2*m)*Sqrt[b*Tan[e + f*x]])
```

Maple [F] time = 0.125, size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)\right)^m \left(b\tan\left(fx+e\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^m/(b*tan(f*x+e))^(3/2),x)
```

```
[Out] int((a*sin(f*x+e))^m/(b*tan(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(fx+e))^m}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^m/(b*tan(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\tan(fx+e)}\,(a\sin(fx+e))^m}{b^2\tan(fx+e)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*tan(f*x + e))*(a*sin(f*x + e))^m/(b^2*tan(f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**m/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(fx+e))^m}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^m/(b*tan(f*x + e))^(3/2), x)
```

3.174 $\int (a\sin(e+fx))^m(b\tan(e+fx))^n\,dx$

Optimal. Leaf size=83

$$\frac{\cos^2(e+fx)^{\frac{n+1}{2}}(a\sin(e+fx))^m(b\tan(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);\sin^2(e+fx)\right)}{bf(m+n+1)}$$

```
[Out] ((Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (1 + m + n)/2, (
3 + m + n)/2, Sin[e + f*x]^2]*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(1 + n))/
(b*f*(1 + m + n))
```

Rubi [A] time = 0.102836, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2602, 2577}

$$\frac{\cos^2(e+fx)^{\frac{n+1}{2}}(a\sin(e+fx))^m(b\tan(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);\sin^2(e+fx)\right)}{bf(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (1 + m + n)/2, (
3 + m + n)/2, Sin[e + f*x]^2]*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(1 + n))/
(b*f*(1 + m + n))
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^n\,dx = \frac{\left(a\cos^{1+n}(e+fx)(a\sin(e+fx))^{-1-n}(b\tan(e+fx))^{1+n}\right)\int \cos^{-n}(e+fx)(a\sin(}{b}$$

$$= \frac{\cos^2(e+fx)^{\frac{1+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{2}(1+m+n);\frac{1}{2}(3+m+n);\sin^2(e+fx)\right)(a\sin(e+}{bf(1+m+n)}$$

Mathematica [C] time = 1.90652, size = 260, normalized size = 3.13

$$\frac{(m+n+3)\sin(e+fx)(a\sin(e+}{f(m+n+1)\left((m+n+3)F_1\left(\frac{1}{2}(m+n+1);n,m+1;\frac{1}{2}(m+n+3);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2\tan^2\left(\frac{1}{2}(\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] ((3 + m + n)*AppellF1[(1 + m + n)/2, n, 1 + m, (3 + m + n)/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]*Sin[e + f*x]*(a*Sin[e + f*x])^m*(b*Tan[e + f*x]
)^n)/(f*(1 + m + n)*((3 + m + n)*AppellF1[(1 + m + n)/2, n, 1 + m, (3 + m +
 n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*((1 + m)*AppellF1[(3 +
m + n)/2, n, 2 + m, (5 + m + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 - n*AppellF1[(3 + m + n)/2, 1 + n, 1 + m, (5 + m + n)/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))
```

Maple [F] time = 0.915, size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)\right)^m \left(b\tan\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

```
[Out] int((a*sin(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)\right)^m \left(b\tan\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sin\left(fx+e\right)\right)^m \left(b\tan\left(fx+e\right)\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**m*(b*tan(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)\right)^m \left(b\tan\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

3.175 $\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx$

Optimal. Leaf size=50

$$\frac{(b\tan(e+fx))^{n+5}\,{}_2F_1\left(3,\frac{n+5}{2};\frac{n+7}{2};-\tan^2(e+fx)\right)}{b^5f(n+5)}$$

```
[Out] (Hypergeometric2F1[3, (5 + n)/2, (7 + n)/2, -Tan[e + f*x]^2]*(b*Tan[e + f*x
])^(5 + n))/(b^5*f*(5 + n))
```

Rubi [A] time = 0.0572467, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2591, 364}

$$\frac{(b\tan(e+fx))^{n+5}\,{}_2F_1\left(3,\frac{n+5}{2};\frac{n+7}{2};-\tan^2(e+fx)\right)}{b^5f(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4*(b*Tan[e + f*x])^n,x]
```


```
[Out] (Hypergeometric2F1[3, (5 + n)/2, (7 + n)/2, -Tan[e + f*x]^2]*(b*Tan[e + f*x
])^(5 + n))/(b^5*f*(5 + n))
```


Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx = \frac{b\,\mathrm{Subst}\left(\int \frac{x^{4+n}}{(b^2+x^2)^3}\,dx, x, b\tan(e+fx)\right)}{f}$$
$$= \frac{{}_2F_1\left(3,\frac{5+n}{2};\frac{7+n}{2};-\tan^2(e+fx)\right)(b\tan(e+fx))^{5+n}}{b^5f(5+n)}$$

Mathematica [C] time = 4.74523, size = 916, normalized size = 18.32

$$\frac{}{f(n+1)\left((n+3)F_1\left(\frac{n+1}{2};n,3;\frac{n+3}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)(\cos(e+fx)+1)+(n+3)F_1\left(\frac{n+1}{2};n,5;\frac{n+3}{2};\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^4*(b*Tan[e + f*x])^n,x]
```

```
[Out] (64*(3 + n)*(AppellF1[(1 + n)/2, n, 3, (3 + n)/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] - 2*AppellF1[(1 + n)/2, n, 4, (3 + n)/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2] + AppellF1[(1 + n)/2, n, 5, (3 + n)/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2])*Cos[(e + f*x)/2]^7*Sin[(e + f*x)/2]^5*(b*Tan[e
 + f*x])^n)/(f*(1 + n)*((3 + n)*AppellF1[(1 + n)/2, n, 3, (3 + n)/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x]) + (3 + n)*AppellF1[(1
 + n)/2, n, 5, (3 + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos
[e + f*x]) + 2*(-5*AppellF1[(3 + n)/2, n, 6, (5 + n)/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] + n*AppellF1[(3 + n)/2, 1 + n, 3, (5 + n)/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*n*AppellF1[(3 + n)/2, 1 + n, 4, (5 + n
)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*AppellF1[(3 + n)/2, 1 + n
, 5, (5 + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 6*AppellF1[(1 +
n)/2, n, 4, (3 + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*
x)/2]^2 - 2*n*AppellF1[(1 + n)/2, n, 4, (3 + n)/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + 3*AppellF1[(3 + n)/2, n, 4, (5 + n)/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(-1 + Cos[e + f*x]) - 8*AppellF1
[(3 + n)/2, n, 5, (5 + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(-1 +
 Cos[e + f*x]) + 5*AppellF1[(3 + n)/2, n, 6, (5 + n)/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Cos[e + f*x] - n*AppellF1[(3 + n)/2, 1 + n, 3, (5 + n
)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x] + 2*n*AppellF1[(
3 + n)/2, 1 + n, 4, (5 + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos
[e + f*x] - n*AppellF1[(3 + n)/2, 1 + n, 5, (5 + n)/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2]*Cos[e + f*x]))
```

Maple [F] time = 0.826, size = 0, normalized size = 0.

$$\int \left(\sin\left(fx+e\right)\right)^{4} \left(b\tan\left(fx+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(b*tan(f*x+e))^n,x)
```

```
[Out] int(sin(f*x+e)^4*(b*tan(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)\right)^{n} \sin\left(fx+e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^n*sin(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\cos\left(fx+e\right)^{4} - 2\,\cos\left(fx+e\right)^{2} + 1\right)\left(b\tan\left(fx+e\right)\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(b*tan(f*x + e))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(b*tan(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)\right)^{n}\sin\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(b*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^n*sin(f*x + e)^4, x)
```

3.176 $\int \sin^2(e + fx)(b\tan(e + fx))^n\,dx$

Optimal. Leaf size=50

$$\frac{(b\tan(e+fx))^{n+3}\,{}_2F_1\left(2,\frac{n+3}{2};\frac{n+5}{2};-\tan^2(e+fx)\right)}{b^3f(n+3)}$$

```
[Out] (Hypergeometric2F1[2, (3 + n)/2, (5 + n)/2, -Tan[e + f*x]^2]*(b*Tan[e + f*x
])^(3 + n))/(b^3*f*(3 + n))
```

Rubi [A] time = 0.0531362, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2591, 364}

$$\frac{(b\tan(e+fx))^{n+3}\,{}_2F_1\left(2,\frac{n+3}{2};\frac{n+5}{2};-\tan^2(e+fx)\right)}{b^3f(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*(b*Tan[e + f*x])^n,x]
```

```
[Out] (Hypergeometric2F1[2, (3 + n)/2, (5 + n)/2, -Tan[e + f*x]^2]*(b*Tan[e + f*x
])^(3 + n))/(b^3*f*(3 + n))
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx = \frac{b\,\mathrm{Subst}\left(\int \frac{x^{2+n}}{(b^2+x^2)^2}\,dx, x, b\tan(e+fx)\right)}{f}$$
$$= \frac{{}_2F_1\left(2, \frac{3+n}{2}; \frac{5+n}{2}; -\tan^2(e+fx)\right)(b\tan(e+fx))^{3+n}}{b^3 f(3+n)}$$

Mathematica [C] time = 2.11771, size = 450, normalized size = 9.

$$\overline{f(n+1)\left(-2(n+3)\cos^2\left(\frac{1}{2}(e+fx)\right)F_1\left(\frac{n+1}{2};n,3;\frac{n+3}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+2(\cos(e+fx)-1)\left(2F_1\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^2*(b*Tan[e + f*x])^n,x]
```

```
[Out] (16*(3 + n)*(AppellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] - AppellF1[(1 + n)/2, n, 3, (3 + n)/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2])*Cos[(e + f*x)/2]^5*Sin[(e + f*x)/2]^3*(b*Tan[e + f*x]
)^n)/(f*(1 + n)*(-2*(3 + n)*AppellF1[(1 + n)/2, n, 3, (3 + n)/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + 2*(2*AppellF1[(3 + n)/2
, n, 3, (5 + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 3*AppellF1[(3
 + n)/2, n, 4, (5 + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*(-Ap
pellF1[(3 + n)/2, 1 + n, 2, (5 + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] + AppellF1[(3 + n)/2, 1 + n, 3, (5 + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]))*(-1 + Cos[e + f*x]) + (3 + n)*AppellF1[(1 + n)/2, n, 2, (3
+ n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x])))
```

Maple [F] time = 0.71, size = 0, normalized size = 0.

$$\int (\sin(fx+e))^2 (b\tan(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(b*tan(f*x+e))^n,x)
```

```
[Out] int(sin(f*x+e)^2*(b*tan(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)\right)^{n}\sin\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^n*sin(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(\cos\left(fx+e\right)^{2}-1\right)\left(b\tan\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*(b*tan(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(e+fx\right)\right)^{n}\sin^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(b*tan(f*x+e))**n,x)
```

```
[Out] Integral((b*tan(e + f*x))**n*sin(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)\right)^n \sin\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(b*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^n*sin(f*x + e)^2, x)
```

3.177 $\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx$

Optimal. Leaf size=25

$$-\frac{b(b\tan(e+fx))^{n-1}}{f(1-n)}$$

```
[Out] -((b*(b*Tan[e + f*x])^(-1 + n))/(f*(1 - n)))
```

Rubi [A] time = 0.0424143, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2591, 30}

$$-\frac{b(b\tan(e+fx))^{n-1}}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(b*Tan[e + f*x])^n,x]
```

```
[Out] -((b*(b*Tan[e + f*x])^(-1 + n))/(f*(1 - n)))
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx &= \frac{b\,\text{Subst}\left(\int x^{-2+n}\,dx, x, b\tan(e+fx)\right)}{f}\\ &= -\frac{b(b\tan(e+fx))^{-1+n}}{f(1-n)}\end{aligned}$$

Mathematica [A] time = 0.0661165, size = 22, normalized size = 0.88

$$\frac{b(b\tan(e+fx))^{n-1}}{f(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(b*Tan[e + f*x])^n,x]
```

```
[Out] (b*(b*Tan[e + f*x])^(-1 + n))/(f*(-1 + n))
```

Maple [C] time = 1.787, size = 4284, normalized size = 171.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(b*tan(f*x+e))^n,x)
```

```
[Out] I/(-1+n)/f/(exp(2*I*(f*x+e))-1)*(b^n/((exp(I*(f*x+e))+I)^n)*(exp(I*(f*x+e))
-1)^n*(exp(I*(f*x+e))+1)^n/((exp(I*(f*x+e))-I)^n)*exp(-1/2*I*Pi*n)*exp(-1/2
*I*n*Pi*csgn(I*b/(exp(I*(f*x+e))+I)/(exp(I*(f*x+e))-I)*exp(2*I*(f*x+e))-I*b
/(exp(I*(f*x+e))+I)/(exp(I*(f*x+e))-I))^3)*exp(1/2*I*n*Pi*csgn(b/(exp(I*(f*
x+e))+I)/(exp(I*(f*x+e))-I)*exp(2*I*(f*x+e))-b/(exp(I*(f*x+e))+I)/(exp(I*(f
*x+e))-I))^2)*exp(-1/2*I*n*Pi*csgn(b/(exp(I*(f*x+e))+I)/(exp(I*(f*x+e))-I)*
exp(2*I*(f*x+e))-b/(exp(I*(f*x+e))+I)/(exp(I*(f*x+e))-I))^3)*exp(-1/2*I*n*P
i*csgn(I/(exp(I*(f*x+e))-I)/(exp(I*(f*x+e))+I)*exp(2*I*(f*x+e))-I/(exp(I*(f
*x+e))-I)/(exp(I*(f*x+e))+I))*csgn(I*b/(exp(I*(f*x+e))+I)/(exp(I*(f*x+e))-I
)*exp(2*I*(f*x+e))-I*b/(exp(I*(f*x+e))+I)/(exp(I*(f*x+e))-I))*csgn(I*b))*ex
p(-1/2*I*n*Pi*csgn(I/(exp(I*(f*x+e))-I)/(exp(I*(f*x+e))+I)*exp(I*(f*x+e))+I
/(exp(I*(f*x+e))-I)/(exp(I*(f*x+e))+I))^3)*exp(-1/2*I*n*Pi*csgn(I/(exp(I*(f
*x+e))-I)/(exp(I*(f*x+e))+I)*exp(2*I*(f*x+e))-I/(exp(I*(f*x+e))-I)/(exp(I*(
f*x+e))+I))^3)*exp(-1/2*I*n*Pi*csgn(I/(exp(I*(f*x+e))-I)/(exp(I*(f*x+e))+I)
*exp(I*(f*x+e))+I/(exp(I*(f*x+e))-I)/(exp(I*(f*x+e))+I))*csgn(I*exp(I*(f*x+
e))+I)*csgn(I/(exp(I*(f*x+e))-I)/(exp(I*(f*x+e))+I)))*exp(-1/2*I*csgn(I/(ex
p(I*(f*x+e))-I)/(exp(I*(f*x+e))+I))*Pi*csgn(I/(exp(I*(f*x+e))-I))*csgn(I/(e
xp(I*(f*x+e))+I))*n)*exp(-1/2*I*csgn(I/(exp(I*(f*x+e))-I)/(exp(I*(f*x+e))+I
))^3*Pi*n)*exp(-1/2*I*n*Pi*csgn(I/(exp(I*(f*x+e))-I)/(exp(I*(f*x+e))+I)*exp
(I*(f*x+e))+I/(exp(I*(f*x+e))-I)/(exp(I*(f*x+e))+I))*csgn(I/(exp(I*(f*x+e))
-I)/(exp(I*(f*x+e))+I)*exp(2*I*(f*x+e))-I/(exp(I*(f*x+e))-I)/(exp(I*(f*x+e)
)+I))*csgn(I*exp(I*(f*x+e))-I))*exp(1/2*I*n*Pi*csgn(I/(exp(I*(f*x+e))-I)/(e
xp(I*(f*x+e))+I)*exp(I*(f*x+e))+I/(exp(I*(f*x+e))-I)/(exp(I*(f*x+e))+I))*cs
```

gn(I/(exp(I*(f*x+e))-I)/(exp(I*(f*x+e))+I)*exp(2*I*(f*x+e))-I/(exp(I*(f*x+e
))-I)/(exp(I*(f*x+e))+I))^2)*exp(1/2*I*n*Pi*csgn(I/(exp(I*(f*x+e))-I)/(exp(
I*(f*x+e))+I)*exp(2*I*(f*x+e))-I/(exp(I*(f*x+e))-I)/(exp(I*(f*x+e))+I))^2*c
sgn(I*exp(I*(f*x+e))-I))*exp(1/2*I*n*Pi*csgn(I/(exp(I*(f*x+e))-I)/(exp(I*(f
*x+e))+I)*exp(2*I*(f*x+e))-I/(exp(I*(f*x+e))-I)/(exp(I*(f*x+e))+I))*csgn(I*
b/(exp(I*(f*x+e))+I)/(exp(I*(f*x+e))-I)*exp(2*I*(f*x+e))-I*b/(exp(I*(f*x+e)
)+I)/(exp(I*(f*x+e))-I))^2)*exp(1/2*I*csgn(I/(exp(I*(f*x+e))-I)/(exp(I*(f*x
+e))+I))^2*Pi*csgn(I/(exp(I*(f*x+e))+I))*n)*exp(1/2*I*csgn(I/(exp(I*(f*x+e)
)-I)/(exp(I*(f*x+e))+I))^2*Pi*csgn(I/(exp(I*(f*x+e))-I))*n)*exp(1/2*I*n*Pi*
csgn(I/(exp(I*(f*x+e))-I)/(exp(I*(f*x+e))+I)*exp(I*(f*x+e))+I/(exp(I*(f*x+e
))-I)/(exp(I*(f*x+e))+I))^2*csgn(I*exp(I*(f*x+e))+I))*exp(-1/2*I*n*Pi*csgn(
I*b/(exp(I*(f*x+e))+I)/(exp(I*(f*x+e))-I)*exp(2*I*(f*x+e))-I*b/(exp(I*(f*x+
e))+I)/(exp(I*(f*x+e))-I))*csgn(b/(exp(I*(f*x+e))+I)/(exp(I*(f*x+e))-I)*exp
(2*I*(f*x+e))-b/(exp(I*(f*x+e))+I)/(exp(I*(f*x+e))-I)))*exp(1/2*I*n*Pi*csgn
(I*b/(exp(I*(f*x+e))+I)/(exp(I*(f*x+e))-I)*exp(2*I*(f*x+e))-I*b/(exp(I*(f*x
+e))+I)/(exp(I*(f*x+e))-I))^2*csgn(I*b))*exp(1/2*I*n*Pi*csgn(I/(exp(I*(f*x+
e))-I)/(exp(I*(f*x+e))+I)*exp(I*(f*x+e))+I/(exp(I*(f*x+e))-I)/(exp(I*(f*x+e
))+I))^2*csgn(I/(exp(I*(f*x+e))-I)/(exp(I*(f*x+e))+I)))*exp(1/2*I*n*Pi*csgn
(I*b/(exp(I*(f*x+e))+I)/(exp(I*(f*x+e))-I)*exp(2*I*(f*x+e))-I*b/(exp(I*(f*x
+e))+I)/(exp(I*(f*x+e))-I))*csgn(b/(exp(I*(f*x+e))+I)/(exp(I*(f*x+e))-I)*ex
p(2*I*(f*x+e))-b/(exp(I*(f*x+e))+I)/(exp(I*(f*x+e))-I))^2)*exp(2*I*f*x)*exp
(2*I*e)+b^n/((exp(I*(f*x+e))+I)^n)*(exp(I*(f*x+e))-1)^n*(exp(I*(f*x+e))+1)^
n/((exp(I*(f*x+e))-I)^n)*exp(-1/2*I*Pi*n*(csgn(I/(exp(I*(f*x+e))-I)/(exp(I*
(f*x+e))+I)*exp(I*(f*x+e))+I/(exp(I*(f*x+e))-I)/(exp(I*(f*x+e))+I))*csgn(I/
(exp(I*(f*x+e))-I)/(exp(I*(f*x+e))+I)*exp(2*I*(f*x+e))-I/(exp(I*(f*x+e))-I)
/(exp(I*(f*x+e))+I))*csgn(I*exp(I*(f*x+e))-I)-csgn(I/(exp(I*(f*x+e))-I)/(ex
p(I*(f*x+e))+I))^2*csgn(I/(exp(I*(f*x+e))+I))-csgn(I/(exp(I*(f*x+e))-I)/(ex
p(I*(f*x+e))+I)*exp(2*I*(f*x+e))-I/(exp(I*(f*x+e))-I)/(exp(I*(f*x+e))+I))^2
*csgn(I*exp(I*(f*x+e))-I)-csgn(I/(exp(I*(f*x+e))-I)/(exp(I*(f*x+e))+I)*exp(
I*(f*x+e))+I/(exp(I*(f*x+e))-I)/(exp(I*(f*x+e))+I))^2*csgn(I*exp(I*(f*x+e))
+I)-csgn(I/(exp(I*(f*x+e))-I)/(exp(I*(f*x+e))+I)*exp(I*(f*x+e))+I/(exp(I*(f
x+e))-I)/(exp(I(f*x+e))+I))*csgn(I/(exp(I*(f*x+e))-I)/(exp(I*(f*x+e))+I)*
exp(2*I*(f*x+e))-I/(exp(I*(f*x+e))-I)/(exp(I*(f*x+e))+I))^2+csgn(I/(exp(I*(
f*x+e))-I)/(exp(I*(f*x+e))+I)*exp(2*I*(f*x+e))-I/(exp(I*(f*x+e))-I)/(exp(I*
(f*x+e))+I))^3+csgn(I/(exp(I*(f*x+e))-I)/(exp(I*(f*x+e))+I)*exp(I*(f*x+e))+
I/(exp(I*(f*x+e))-I)/(exp(I*(f*x+e))+I))*csgn(I*exp(I*(f*x+e))+I)*csgn(I/(e
xp(I*(f*x+e))-I)/(exp(I*(f*x+e))+I))+csgn(I/(exp(I*(f*x+e))-I)/(exp(I*(f*x+
e))+I)*exp(I*(f*x+e))+I/(exp(I*(f*x+e))-I)/(exp(I*(f*x+e))+I))^3+csgn(I/(ex
p(I*(f*x+e))-I)/(exp(I*(f*x+e))+I))^3-csgn(I*b/(exp(I*(f*x+e))+I)/(exp(I*(f
*x+e))-I)*exp(2*I*(f*x+e))-I*b/(exp(I*(f*x+e))+I)/(exp(I*(f*x+e))-I))*csgn(
b/(exp(I*(f*x+e))+I)/(exp(I*(f*x+e))-I)*exp(2*I*(f*x+e))-b/(exp(I*(f*x+e))+
I)/(exp(I*(f*x+e))-I))^2+csgn(I*b/(exp(I*(f*x+e))+I)/(exp(I*(f*x+e))-I)*exp
(2*I*(f*x+e))-I*b/(exp(I*(f*x+e))+I)/(exp(I*(f*x+e))-I))*csgn(b/(exp(I*(f*x
+e))+I)/(exp(I*(f*x+e))-I)*exp(2*I*(f*x+e))-b/(exp(I*(f*x+e))+I)/(exp(I*(f*
x+e))-I))-csgn(I/(exp(I*(f*x+e))-I)/(exp(I*(f*x+e))+I))^2*csgn(I/(exp(I*(f*

```
x+e))-I))+csgn(b/(exp(I*(f*x+e))+I)/(exp(I*(f*x+e))-I)*exp(2*I*(f*x+e))-b/(
exp(I*(f*x+e))+I)/(exp(I*(f*x+e))-I))^3+csgn(I/(exp(I*(f*x+e))-I)/(exp(I*(f
*x+e))+I))*csgn(I/(exp(I*(f*x+e))-I))*csgn(I/(exp(I*(f*x+e))+I))-csgn(I/(ex
p(I*(f*x+e))-I)/(exp(I*(f*x+e))+I)*exp(I*(f*x+e))+I/(exp(I*(f*x+e))-I)/(exp
(I*(f*x+e))+I))^2*csgn(I/(exp(I*(f*x+e))-I)/(exp(I*(f*x+e))+I))+csgn(I/(exp
(I*(f*x+e))-I)/(exp(I*(f*x+e))+I)*exp(2*I*(f*x+e))-I/(exp(I*(f*x+e))-I)/(ex
p(I*(f*x+e))+I))*csgn(I*b/(exp(I*(f*x+e))+I)/(exp(I*(f*x+e))-I)*exp(2*I*(f*
x+e))-I*b/(exp(I*(f*x+e))+I)/(exp(I*(f*x+e))-I))*csgn(I*b)+csgn(I*b/(exp(I*
(f*x+e))+I)/(exp(I*(f*x+e))-I)*exp(2*I*(f*x+e))-I*b/(exp(I*(f*x+e))+I)/(exp
(I*(f*x+e))-I))^3-csgn(I*b/(exp(I*(f*x+e))+I)/(exp(I*(f*x+e))-I)*exp(2*I*(f
*x+e))-I*b/(exp(I*(f*x+e))+I)/(exp(I*(f*x+e))-I))^2*csgn(I*b)-csgn(I/(exp(I
*(f*x+e))-I)/(exp(I*(f*x+e))+I)*exp(2*I*(f*x+e))-I/(exp(I*(f*x+e))-I)/(exp(
I*(f*x+e))+I))*csgn(I*b/(exp(I*(f*x+e))+I)/(exp(I*(f*x+e))-I)*exp(2*I*(f*x+
e))-I*b/(exp(I*(f*x+e))+I)/(exp(I*(f*x+e))-I))^2-csgn(b/(exp(I*(f*x+e))+I)/
(exp(I*(f*x+e))-I)*exp(2*I*(f*x+e))-b/(exp(I*(f*x+e))+I)/(exp(I*(f*x+e))-I)
)^2+1)))
```

Maxima [A] time = 0.963509, size = 38, normalized size = 1.52

$$\frac{b^n \tan\left(fx+e\right)^n}{f(n-1)\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] b^n*tan(f*x + e)^n/(f*(n - 1)*tan(f*x + e))
```

Fricas [A] time = 1.63448, size = 96, normalized size = 3.84

$$\frac{\left(\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\right)^n \cos\left(fx+e\right)}{\left(fn-f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] (b*sin(f*x + e)/cos(f*x + e))^n*cos(f*x + e)/((f*n - f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(e+fx\right)\right)^n \csc^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(b*tan(f*x+e))**n,x)
```

```
[Out] Integral((b*tan(e + f*x))**n*csc(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)\right)^n \csc\left(fx+e\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^n*csc(f*x + e)^2, x)
```

3.178 $\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx$

Optimal. Leaf size=53

$$-\frac{b^3(b\tan(e+fx))^{n-3}}{f(3-n)}-\frac{b(b\tan(e+fx))^{n-1}}{f(1-n)}$$

```
[Out] -((b^3*(b*Tan[e + f*x])^(-3 + n))/(f*(3 - n))) - (b*(b*Tan[e + f*x])^(-1 +
n))/(f*(1 - n))
```

Rubi [A] time = 0.0550273, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2591, 14}

$$-\frac{b^3(b\tan(e+fx))^{n-3}}{f(3-n)}-\frac{b(b\tan(e+fx))^{n-1}}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(b*Tan[e + f*x])^n,x]
```

```
[Out] -((b^3*(b*Tan[e + f*x])^(-3 + n))/(f*(3 - n))) - (b*(b*Tan[e + f*x])^(-1 +
n))/(f*(1 - n))
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx = \frac{b\,\mathrm{Subst}\left(\int x^{-4+n}\left(b^2+x^2\right)\,dx, x, b\tan(e+fx)\right)}{f}$$
$$= \frac{b\,\mathrm{Subst}\left(\int \left(b^2x^{-4+n}+x^{-2+n}\right)\,dx, x, b\tan(e+fx)\right)}{f}$$
$$= -\frac{b^3(b\tan(e+fx))^{-3+n}}{f(3-n)} - \frac{b(b\tan(e+fx))^{-1+n}}{f(1-n)}$$

Mathematica [A] time = 0.147351, size = 46, normalized size = 0.87

$$\frac{b\csc^2(e+fx)(\cos(2(e+fx))+n-2)(b\tan(e+fx))^{n-1}}{f(n-3)(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(b*Tan[e + f*x])^n,x]
```

```
[Out] (b*(-2 + n + Cos[2*(e + f*x)])*Csc[e + f*x]^2*(b*Tan[e + f*x])^(-1 + n))/(f
*(-3 + n)*(-1 + n))
```

Maple [C] time = 0.641, size = 13019, normalized size = 245.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(b*tan(f*x+e))^n,x)
```

```
[Out] result too large to display
```

Maxima [A] time = 0.97252, size = 74, normalized size = 1.4

$$\frac{\frac{b^n\tan\left(fx+e\right)^n}{(n-1)\tan\left(fx+e\right)} + \frac{b^n\tan\left(fx+e\right)^n}{(n-3)\tan\left(fx+e\right)^3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] (b^n*tan(f*x + e)^n/((n - 1)*tan(f*x + e)) + b^n*tan(f*x + e)^n/((n - 3)*ta
n(f*x + e)^3))/f
```

Fricas [A] time = 1.62222, size = 204, normalized size = 3.85

$$\frac{\left(2\cos\left(fx+e\right)^{3}+(n-3)\cos\left(fx+e\right)\right)\left(\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\right)^{n}}{\left(fn^{2}-\left(fn^{2}-4\,fn+3\,f\right)\cos\left(fx+e\right)^{2}-4\,fn+3\,f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] (2*cos(f*x + e)^3 + (n - 3)*cos(f*x + e))*(b*sin(f*x + e)/cos(f*x + e))^n/(
(f*n^2 - (f*n^2 - 4*f*n + 3*f)*cos(f*x + e)^2 - 4*f*n + 3*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(b*tan(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan\left(fx+e\right)\right)^{n}\csc\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^n*csc(f*x + e)^4, x)
```

3.179 $\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx$

Optimal. Leaf size=80

$$-\frac{b^5(b\tan(e+fx))^{n-5}}{f(5-n)}-\frac{2b^3(b\tan(e+fx))^{n-3}}{f(3-n)}-\frac{b(b\tan(e+fx))^{n-1}}{f(1-n)}$$

```
[Out] -((b^5*(b*Tan[e + f*x])^(-5 + n))/(f*(5 - n))) - (2*b^3*(b*Tan[e + f*x])^(-
3 + n))/(f*(3 - n)) - (b*(b*Tan[e + f*x])^(-1 + n))/(f*(1 - n))
```

Rubi [A] time = 0.063032, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2591, 270}

$$-\frac{b^5(b\tan(e+fx))^{n-5}}{f(5-n)}-\frac{2b^3(b\tan(e+fx))^{n-3}}{f(3-n)}-\frac{b(b\tan(e+fx))^{n-1}}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*(b*Tan[e + f*x])^n,x]
```

```
[Out] -((b^5*(b*Tan[e + f*x])^(-5 + n))/(f*(5 - n))) - (2*b^3*(b*Tan[e + f*x])^(-
3 + n))/(f*(3 - n)) - (b*(b*Tan[e + f*x])^(-1 + n))/(f*(1 - n))
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx = \frac{b\,\mathrm{Subst}\left(\int x^{-6+n}\left(b^2+x^2\right)^2\,dx,x,b\tan(e+fx)\right)}{f}$$
$$= \frac{b\,\mathrm{Subst}\left(\int \left(b^4x^{-6+n}+2b^2x^{-4+n}+x^{-2+n}\right)dx,x,b\tan(e+fx)\right)}{f}$$
$$= -\frac{b^5(b\tan(e+fx))^{-5+n}}{f(5-n)} - \frac{2b^3(b\tan(e+fx))^{-3+n}}{f(3-n)} - \frac{b(b\tan(e+fx))^{-1+n}}{f(1-n)}$$

Mathematica [A] time = 0.264945, size = 69, normalized size = 0.86

$$\frac{b\csc^4(e+fx)\left(2(n-3)\cos(2(e+fx))+\cos(4(e+fx))+n^2-6n+8\right)(b\tan(e+fx))^{n-1}}{f(n-5)(n-3)(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*(b*Tan[e + f*x])^n,x]
```

```
[Out] (b*(8 - 6*n + n^2 + 2*(-3 + n)*Cos[2*(e + f*x)] + Cos[4*(e + f*x)])*Csc[e +
 f*x]^4*(b*Tan[e + f*x])^(-1 + n))/(f*(-5 + n)*(-3 + n)*(-1 + n))
```

Maple [C] time = 1.119, size = 26124, normalized size = 326.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(b*tan(f*x+e))^n,x)
```

```
[Out] result too large to display
```

Maxima [A] time = 0.993214, size = 109, normalized size = 1.36

$$\frac{\frac{b^n\tan\left(fx+e\right)^n}{(n-1)\tan\left(fx+e\right)}+\frac{2\,b^n\tan\left(fx+e\right)^n}{(n-3)\tan\left(fx+e\right)^3}+\frac{b^n\tan\left(fx+e\right)^n}{(n-5)\tan\left(fx+e\right)^5}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] (b^n*tan(f*x + e)^n/((n - 1)*tan(f*x + e)) + 2*b^n*tan(f*x + e)^n/((n - 3)*
tan(f*x + e)^3) + b^n*tan(f*x + e)^n/((n - 5)*tan(f*x + e)^5))/f
```

Fricas [A] time = 1.87641, size = 356, normalized size = 4.45

$$\frac{\left(8\cos\left(fx+e\right)^{5}+4\left(n-5\right)\cos\left(fx+e\right)^{3}+\left(n^{2}-8n+15\right)\cos\left(fx+e\right)\right)\left(\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\right)^{n}}{\left(\left(fn^{3}-9fn^{2}+23fn-15f\right)\cos\left(fx+e\right)^{4}+fn^{3}-9fn^{2}-2\left(fn^{3}-9fn^{2}+23fn-15f\right)\cos\left(fx+e\right)^{2}+23fn-15\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] (8*cos(f*x + e)^5 + 4*(n - 5)*cos(f*x + e)^3 + (n^2 - 8*n + 15)*cos(f*x + e
))*(b*sin(f*x + e)/cos(f*x + e))^n/(((f*n^3 - 9*f*n^2 + 23*f*n - 15*f)*cos(
f*x + e)^4 + f*n^3 - 9*f*n^2 - 2*(f*n^3 - 9*f*n^2 + 23*f*n - 15*f)*cos(f*x
+ e)^2 + 23*f*n - 15*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(b*tan(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)\right)^{n}\csc\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(b*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^n*csc(f*x + e)^6, x)
```

3.180 $\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx$

Optimal. Leaf size=78

$$\frac{\sin^3(e+fx)\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{n+4}{2};\frac{n+6}{2};\sin^2(e+fx)\right)}{bf(n+4)}$$

```
[Out] ((Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (4 + n)/2, (6 +
n)/2, Sin[e + f*x]^2]*Sin[e + f*x]^3*(b*Tan[e + f*x])^(1 + n))/(b*f*(4 + n)
)
```

Rubi [A] time = 0.084357, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2602, 2577}

$$\frac{\sin^3(e+fx)\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{n+4}{2};\frac{n+6}{2};\sin^2(e+fx)\right)}{bf(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(b*Tan[e + f*x])^n,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (4 + n)/2, (6 +
n)/2, Sin[e + f*x]^2]*Sin[e + f*x]^3*(b*Tan[e + f*x])^(1 + n))/(b*f*(4 + n)
)
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx = \frac{\left(\cos^{1+n}(e+fx)\sin^{-1-n}(e+fx)(b\tan(e+fx))^{1+n}\right)\int \cos^{-n}(e+fx)\sin^{3+n}(e+fx)\,d}{b}$$
$$= \frac{\cos^2(e+fx)^{\frac{1+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{4+n}{2};\frac{6+n}{2};\sin^2(e+fx)\right)\sin^3(e+fx)(b\tan(e+fx))^{1+n}}{bf(4+n)}$$

Mathematica [C] time = 2.85598, size = 456, normalized size = 5.85

$$\frac{}{f(n+2)\left(-2(n+4)\cos^2\left(\frac{1}{2}(e+fx)\right)F_1\left(\frac{n}{2}+1;n,4;\frac{n}{2}+2;\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+2(\cos(e+fx)-1)\left(3\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^3*(b*Tan[e + f*x])^n,x]
```

```
[Out] (4*(4 + n)*(AppellF1[1 + n/2, n, 3, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2] - AppellF1[1 + n/2, n, 4, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2])*Cos[(e + f*x)/2]^3*Sin[(e + f*x)/2]*Sin[e + f*x]^3*(b*Tan[e +
f*x])^n)/(f*(2 + n)*(-2*(4 + n)*AppellF1[1 + n/2, n, 4, 2 + n/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + 2*(3*AppellF1[2 + n/2,
n, 4, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 4*AppellF1[2 + n/
2, n, 5, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*(-AppellF1[2
 + n/2, 1 + n, 3, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + Appel
lF1[2 + n/2, 1 + n, 4, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*
(-1 + Cos[e + f*x]) + (4 + n)*AppellF1[1 + n/2, n, 3, 2 + n/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x])))
```

Maple [F] time = 0.778, size = 0, normalized size = 0.

$$\int \left(\sin\left(fx+e\right)\right)^3\left(b\tan\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(b*tan(f*x+e))^n,x)
```

```
[Out] int(sin(f*x+e)^3*(b*tan(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)\right)^{n}\sin\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^n*sin(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(\cos\left(fx+e\right)^{2}-1\right)\left(b\tan\left(fx+e\right)\right)^{n}\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*(b*tan(f*x + e))^n*sin(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(b*tan(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)\right)^{n}\sin\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(b*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^n*sin(f*x + e)^3, x)
```

3.181 $\int \sin(e+fx)(b\tan(e+fx))^n\,dx$

Optimal. Leaf size=76

$$\frac{\sin(e+fx)\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{bf(n+2)}$$

```
[Out] ((Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (2 + n)/2, (4 +
n)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(b*Tan[e + f*x])^(1 + n))/(b*f*(2 + n))
```

Rubi [A] time = 0.0702338, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2602, 2577}

$$\frac{\sin(e+fx)\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{bf(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*(b*Tan[e + f*x])^n,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (2 + n)/2, (4 +
n)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(b*Tan[e + f*x])^(1 + n))/(b*f*(2 + n))
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sin(e+fx)(b\tan(e+fx))^n\,dx = \frac{\left(\cos^{1+n}(e+fx)\sin^{-1-n}(e+fx)(b\tan(e+fx))^{1+n}\right)\int \cos^{-n}(e+fx)\sin^{1+n}(e+fx)\,d}{b}$$

$$= \frac{\cos^2(e+fx)^{\frac{1+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{2+n}{2};\frac{4+n}{2};\sin^2(e+fx)\right)\sin(e+fx)(b\tan(e+fx))^{1+n}}{bf(2+n)}$$

Mathematica [C] time = 1.04791, size = 252, normalized size = 3.32

$$\frac{8(n+4)\sin^2\left(\frac{1}{2}(e+fx)\right)\cos^4\left(\frac{1}{2}(e+fx)\right)F_1\left(\frac{n}{2}+1;n,2\right.}{f(n+2)\left(2(\cos(e+fx)-1)\left(2F_1\left(\frac{n}{2}+2;n,3;\frac{n}{2}+3;\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-nF_1\left(\frac{n}{2}+2;n+1,2;\frac{n}{2}+3;\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]*(b*Tan[e + f*x])^n,x]
```

```
[Out] (8*(4 + n)*AppellF1[1 + n/2, n, 2, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2]*Cos[(e + f*x)/2]^4*Sin[(e + f*x)/2]^2*(b*Tan[e + f*x])^n)/(f*(2 +
 n)*(2*(2*AppellF1[2 + n/2, n, 3, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2] - n*AppellF1[2 + n/2, 1 + n, 2, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2])*(-1 + Cos[e + f*x]) + (4 + n)*AppellF1[1 + n/2, n, 2, 2 +
n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x])))
```

Maple [F] time = 0.664, size = 0, normalized size = 0.

$$\int \sin(fx+e)\left(b\tan(fx+e)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(b*tan(f*x+e))^n,x)
```

```
[Out] int(sin(f*x+e)*(b*tan(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan(fx+e)\right)^n\sin(fx+e)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^n*sin(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan\left(fx+e\right)\right)^{n}\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e))^n*sin(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan\left(e+fx\right)\right)^{n}\sin\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(b*tan(f*x+e))**n,x)
```

```
[Out] Integral((b*tan(e + f*x))**n*sin(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan\left(fx+e\right)\right)^{n}\sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(b*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^n*sin(f*x + e), x)
```

3.182 $\int \csc(e+fx)(b\tan(e+fx))^n\,dx$

Optimal. Leaf size=78

$$-\frac{\cos(e+fx)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n\,{}_2F_1\left(\frac{1-n}{2},\frac{2-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[(1 - n)/2, (2 - n)/2, (3 - n)/2, Cos[e +
f*x]^2]*(b*Tan[e + f*x])^n)/(f*(1 - n)*(Sin[e + f*x]^2)^(n/2)))
```

Rubi [A] time = 0.0782896, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2601, 2576}

$$-\frac{\cos(e+fx)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n\,{}_2F_1\left(\frac{1-n}{2},\frac{2-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(b*Tan[e + f*x])^n,x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[(1 - n)/2, (2 - n)/2, (3 - n)/2, Cos[e +
f*x]^2]*(b*Tan[e + f*x])^n)/(f*(1 - n)*(Sin[e + f*x]^2)^(n/2)))
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int \csc(e+fx)(b\tan(e+fx))^n\,dx = \left(\cos^n(e+fx)\sin^{-n}(e+fx)(b\tan(e+fx))^n\right)\int \cos^{-n}(e+fx)\sin^{-1+n}(e+fx)\,dx$$
$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1-n}{2},\frac{2-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n}{f(1-n)}$$

Mathematica [A] time = 0.201303, size = 64, normalized size = 0.82

$$\frac{(b\tan(e+fx))^n\,{}_2F_1\left(\frac{n}{2},n;\frac{n}{2}+1;\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\right)^n}{fn}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]*(b*Tan[e + f*x])^n,x]
```

```
[Out] (Hypergeometric2F1[n/2, n, 1 + n/2, Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(
e + f*x)/2]^2)^n*(b*Tan[e + f*x])^n)/(f*n)
```

Maple [F] time = 0.232, size = 0, normalized size = 0.

$$\int \csc\left(fx+e\right)\left(b\tan\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(b*tan(f*x+e))^n,x)
```

```
[Out] int(csc(f*x+e)*(b*tan(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)\right)^n\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^n*csc(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan\left(fx+e\right)\right)^{n}\csc\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e))^n*csc(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan\left(e+fx\right)\right)^{n}\csc\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*tan(f*x+e))**n,x)
```

```
[Out] Integral((b*tan(e + f*x))**n*csc(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan\left(fx+e\right)\right)^{n}\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^n*csc(f*x + e), x)
```

3.183 $\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx$

Optimal. Leaf size=78

$$-\frac{\cos(e+fx)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n\,{}_2F_1\left(\frac{1-n}{2},\frac{4-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[(1 - n)/2, (4 - n)/2, (3 - n)/2, Cos[e +
f*x]^2]*(b*Tan[e + f*x])^n)/(f*(1 - n)*(Sin[e + f*x]^2)^(n/2)))
```

Rubi [A] time = 0.0818814, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2601, 2576}

$$-\frac{\cos(e+fx)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n\,{}_2F_1\left(\frac{1-n}{2},\frac{4-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(b*Tan[e + f*x])^n,x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[(1 - n)/2, (4 - n)/2, (3 - n)/2, Cos[e +
f*x]^2]*(b*Tan[e + f*x])^n)/(f*(1 - n)*(Sin[e + f*x]^2)^(n/2)))
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx = \left(\cos^n(e+fx)\sin^{-n}(e+fx)(b\tan(e+fx))^n\right)\int \cos^{-n}(e+fx)\sin^{-3+n}(e+fx)\,dx$$
$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1-n}{2},\frac{4-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n}{f(1-n)}$$

Mathematica [B] time = 7.39898, size = 182, normalized size = 2.33

$$\frac{\tan^2\left(\frac{1}{2}(e+fx)\right)(b\tan(e+fx))^n\left(\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\right)^n\left(n(n+2)\cot^4\left(\frac{1}{2}(e+fx)\right){}_2F_1\left(\frac{n}{2}-1,n;\frac{n}{2};\tan^2\left(\frac{1}{2}(e+\right.\right.\right.}{4fn(}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^3*(b*Tan[e + f*x])^n,x]
```

```
[Out] ((n*(2 + n)*Cot[(e + f*x)/2]^4*Hypergeometric2F1[-1 + n/2, n, n/2, Tan[(e +
 f*x)/2]^2] + (-2 + n)*(n*Hypergeometric2F1[1 + n/2, n, 2 + n/2, Tan[(e + f
*x)/2]^2] + 2*(2 + n)*Cot[(e + f*x)/2]^2*Hypergeometric2F1[n/2, n, 1 + n/2,
 Tan[(e + f*x)/2]^2]))*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*Tan[(e + f*x)/2]
^2*(b*Tan[e + f*x])^n)/(4*f*n*(-4 + n^2))
```

Maple [F] time = 0.278, size = 0, normalized size = 0.

$$\int \left(\csc\left(fx+e\right)\right)^3\left(b\tan\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(b*tan(f*x+e))^n,x)
```

```
[Out] int(csc(f*x+e)^3*(b*tan(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)\right)^n\csc\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^n*csc(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan\left(fx+e\right)\right)^{n}\csc\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e))^n*csc(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(b*tan(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan\left(fx+e\right)\right)^{n}\csc\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^n*csc(f*x + e)^3, x)
```

3.184 $\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx$

Optimal. Leaf size=78

$$-\frac{\cos(e+fx)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n\,{}_2F_1\left(\frac{1-n}{2},\frac{6-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[(1 - n)/2, (6 - n)/2, (3 - n)/2, Cos[e +
f*x]^2]*(b*Tan[e + f*x])^n)/(f*(1 - n)*(Sin[e + f*x]^2)^(n/2)))
```

Rubi [A] time = 0.082808, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2601, 2576}

$$-\frac{\cos(e+fx)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n\,{}_2F_1\left(\frac{1-n}{2},\frac{6-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(b*Tan[e + f*x])^n,x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[(1 - n)/2, (6 - n)/2, (3 - n)/2, Cos[e +
f*x]^2]*(b*Tan[e + f*x])^n)/(f*(1 - n)*(Sin[e + f*x]^2)^(n/2)))
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx = \left(\cos^n(e+fx)\sin^{-n}(e+fx)(b\tan(e+fx))^n\right)\int \cos^{-n}(e+fx)\sin^{-5+n}(e+fx)\,dx$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1-n}{2},\frac{6-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n}{f(1-n)}$$

Mathematica [B] time = 7.63341, size = 254, normalized size = 3.26

$$\frac{(b\tan(e+fx))^n\left(\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\right)^n\left(\frac{\tan^4\left(\frac{1}{2}(e+fx)\right){}_2F_1\left(\frac{n}{2}+2,n;\frac{n}{2}+3;\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}{n+4}+\frac{4\tan^2\left(\frac{1}{2}(e+fx)\right){}_2F_1\left(\frac{n}{2}+1,n;\frac{n}{2}+2;\tan\right.}{n+2}\right.}{16}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^5*(b*Tan[e + f*x])^n,x]
```

```
[Out] ((Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*((Cot[(e + f*x)/2]^4*Hypergeometric2F1
[-2 + n/2, n, -1 + n/2, Tan[(e + f*x)/2]^2])/(-4 + n) + (4*Cot[(e + f*x)/2]
^2*Hypergeometric2F1[-1 + n/2, n, n/2, Tan[(e + f*x)/2]^2])/(-2 + n) + (6*H
ypergeometric2F1[n/2, n, 1 + n/2, Tan[(e + f*x)/2]^2])/n + (4*Hypergeometri
c2F1[1 + n/2, n, 2 + n/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)/(2 + n) +
 (Hypergeometric2F1[2 + n/2, n, 3 + n/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/
2]^4)/(4 + n))*(b*Tan[e + f*x])^n)/(16*f)
```

Maple [F] time = 0.337, size = 0, normalized size = 0.

$$\int \left(\csc\left(fx+e\right)\right)^5\left(b\tan\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(b*tan(f*x+e))^n,x)
```

```
[Out] int(csc(f*x+e)^5*(b*tan(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)\right)^{n}\csc\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^n*csc(f*x + e)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan\left(fx+e\right)\right)^{n}\csc\left(fx+e\right)^{5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e))^n*csc(f*x + e)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(b*tan(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)\right)^{n}\csc\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(b*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^n*csc(f*x + e)^5, x)
```

3.185 $\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx$

Optimal. Leaf size=89

$$\frac{2(a\sin(e+fx))^{3/2}\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{1}{4}(2n+5);\frac{1}{4}(2n+9);\sin^2(e+fx)\right)}{bf(2n+5)}$$

```
[Out] (2*(Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (5 + 2*n)/4, (
9 + 2*n)/4, Sin[e + f*x]^2]*(a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^(1 + n)
)/(b*f*(5 + 2*n))
```

Rubi [A] time = 0.120763, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2602, 2577}

$$\frac{2(a\sin(e+fx))^{3/2}\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{1}{4}(2n+5);\frac{1}{4}(2n+9);\sin^2(e+fx)\right)}{bf(2n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^n,x]
```

```
[Out] (2*(Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (5 + 2*n)/4, (
9 + 2*n)/4, Sin[e + f*x]^2]*(a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^(1 + n)
)/(b*f*(5 + 2*n))
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx = \frac{\left(a\cos^{1+n}(e+fx)(a\sin(e+fx))^{-1-n}(b\tan(e+fx))^{1+n}\right)\int \cos^{-n}(e+fx)(a\sin}{b}$$
$$= \frac{2\cos^2(e+fx)^{\frac{1+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{4}(5+2n);\frac{1}{4}(9+2n);\sin^2(e+fx)\right)(a\sin(e+fx))}{bf(5+2n)}$$

Mathematica [C] time = 2.44902, size = 297, normalized size = 3.34

$$\frac{8(2n+9)\sin\left(\frac{1}{2}(e+fx)\right)\cos^3\left(\frac{1}{2}(e+fx)\right)(a\sin(e+fx))^{3/2}}{f(2n+5)\left(2(2n+9)\cos^2\left(\frac{1}{2}(e+fx)\right)F_1\left(\frac{n}{2}+\frac{5}{4};n,\frac{5}{2};\frac{n}{2}+\frac{9}{4};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+2(\cos(e+fx)-1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^n,x]
```

```
[Out] (8*(9 + 2*n)*AppellF1[5/4 + n/2, n, 5/2, 9/4 + n/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*Sin[(e + f*x)/2]*(a*Sin[e + f*x])^(3/2
)*(b*Tan[e + f*x])^n)/(f*(5 + 2*n)*(2*(9 + 2*n)*AppellF1[5/4 + n/2, n, 5/2,
 9/4 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + 2
*(5*AppellF1[9/4 + n/2, n, 7/2, 13/4 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2] - 2*n*AppellF1[9/4 + n/2, 1 + n, 5/2, 13/4 + n/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2])*(-1 + Cos[e + f*x])))
```

Maple [F] time = 0.144, size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\tan\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^n,x)
```

```
[Out] int((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\tan\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(3/2)*(b*tan(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a\sin\left(fx+e\right)}\left(b\tan\left(fx+e\right)\right)^{n}a\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^n*a*sin(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(3/2)*(b*tan(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\tan\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^(3/2)*(b*tan(f*x + e))^n, x)
```

3.186 $\int \sqrt{a \sin(e+fx)}(b \tan(e+fx))^n \, dx$

Optimal. Leaf size=89

$$\frac{2\sqrt{a \sin(e+fx)} \cos^2(e+fx)^{\frac{n+1}{2}} (b \tan(e+fx))^{n+1} {}_2F_1\left(\frac{n+1}{2}, \frac{1}{4}(2n+3); \frac{1}{4}(2n+7); \sin^2(e+fx)\right)}{bf(2n+3)}$$

```
[Out] (2*(Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (3 + 2*n)/4, (
7 + 2*n)/4, Sin[e + f*x]^2]*Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(1 + n))/
(b*f*(3 + 2*n))
```

Rubi [A] time = 0.106581, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2602, 2577}

$$\frac{2\sqrt{a \sin(e+fx)} \cos^2(e+fx)^{\frac{n+1}{2}} (b \tan(e+fx))^{n+1} {}_2F_1\left(\frac{n+1}{2}, \frac{1}{4}(2n+3); \frac{1}{4}(2n+7); \sin^2(e+fx)\right)}{bf(2n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^n,x]
```

```
[Out] (2*(Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (3 + 2*n)/4, (
7 + 2*n)/4, Sin[e + f*x]^2]*Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(1 + n))/
(b*f*(3 + 2*n))
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^n\,dx = \frac{\left(a\cos^{1+n}(e+fx)(a\sin(e+fx))^{-1-n}(b\tan(e+fx))^{1+n}\right)\int \cos^{-n}(e+fx)(a\sin(e$$

$$b$$

$$= \frac{2\cos^2(e+fx)^{\frac{1+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{4}(3+2n);\frac{1}{4}(7+2n);\sin^2(e+fx)\right)\sqrt{a\sin(e+fx)}(b}{bf(3+2n)}$$

Mathematica [A] time = 1.57254, size = 91, normalized size = 1.02

$$\frac{\sin(2(e+fx))\sqrt{a\sin(e+fx)}\cos^2(e+fx)^{\frac{n-1}{2}}(b\tan(e+fx))^n\,{}_2F_1\left(\frac{n+1}{2},\frac{1}{4}(2n+3);\frac{1}{4}(2n+7);\sin^2(e+fx)\right)}{f(2n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^n,x]
```

```
[Out] ((Cos[e + f*x]^2)^((-1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (3 + 2*n)/4, (7
 + 2*n)/4, Sin[e + f*x]^2]*Sqrt[a*Sin[e + f*x]]*Sin[2*(e + f*x)]*(b*Tan[e +
 f*x])^n)/(f*(3 + 2*n))
```

Maple [F] time = 0.136, size = 0, normalized size = 0.

$$\int \sqrt{a\sin\left(fx+e\right)}\left(b\tan\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^n,x)
```

```
[Out] int((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin\left(fx+e\right)}\left(b\tan\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a\sin\left(fx+e\right)}\left(b\tan\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(1/2)*(b*tan(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\sin\left(fx+e\right)}\left(b\tan\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^n, x)
```

3.187 $\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx$

Optimal. Leaf size=89

$$\frac{2\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{1}{4}(2n+1);\frac{1}{4}(2n+5);\sin^2(e+fx)\right)}{bf(2n+1)\sqrt{a\sin(e+fx)}}$$

```
[Out] (2*(Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (1 + 2*n)/4, (
5 + 2*n)/4, Sin[e + f*x]^2]*(b*Tan[e + f*x])^(1 + n))/(b*f*(1 + 2*n)*Sqrt[a
*Sin[e + f*x]])
```

Rubi [A] time = 0.106694, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2602, 2577}

$$\frac{2\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{1}{4}(2n+1);\frac{1}{4}(2n+5);\sin^2(e+fx)\right)}{bf(2n+1)\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x])^n/Sqrt[a*Sin[e + f*x]],x]
```

```
[Out] (2*(Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (1 + 2*n)/4, (
5 + 2*n)/4, Sin[e + f*x]^2]*(b*Tan[e + f*x])^(1 + n))/(b*f*(1 + 2*n)*Sqrt[a
*Sin[e + f*x]])
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
```

```
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx = \frac{\left(a\cos^{1+n}(e+fx)(a\sin(e+fx))^{-1-n}(b\tan(e+fx))^{1+n}\right)\int \cos^{-n}(e+fx)(a\sin(e+fx))^{-\frac{1}{2}+n}\,dx}{b}$$
$$= \frac{2\cos^2(e+fx)^{\frac{1+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{4}(1+2n);\frac{1}{4}(5+2n);\sin^2(e+fx)\right)(b\tan(e+fx))^{1+n}}{bf(1+2n)\sqrt{a\sin(e+fx)}}$$

Mathematica [A] time = 1.25083, size = 89, normalized size = 1.

$$\frac{\sin(2(e+fx))\cos^2(e+fx)^{\frac{n-1}{2}}(b\tan(e+fx))^n\,{}_2F_1\left(\frac{n+1}{2},\frac{1}{4}(2n+1);\frac{1}{4}(2n+5);\sin^2(e+fx)\right)}{(2fn+f)\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x])^n/Sqrt[a*Sin[e + f*x]],x]
```

```
[Out] ((Cos[e + f*x]^2)^((-1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (1 + 2*n)/4, (5
 + 2*n)/4, Sin[e + f*x]^2]*Sin[2*(e + f*x)]*(b*Tan[e + f*x])^n)/((f + 2*f*n
)*Sqrt[a*Sin[e + f*x]])
```

Maple [F] time = 0.135, size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)\right)^n \frac{1}{\sqrt{a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^n/(a*sin(f*x+e))^(1/2),x)
```

```
[Out] int((b*tan(f*x+e))^n/(a*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)\right)^{n}}{\sqrt{a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^n/(a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^n/sqrt(a*sin(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a\sin\left(fx+e\right)}\left(b\tan\left(fx+e\right)\right)^{n}}{a\sin\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^n/(a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^n/(a*sin(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(e+fx\right)\right)^{n}}{\sqrt{a\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**n/(a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((b*tan(e + f*x))**n/sqrt(a*sin(e + f*x)), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)\right)^{n}}{\sqrt{a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^n/(a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^n/sqrt(a*sin(f*x + e)), x)
```

3.188 $\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=89

$$-\frac{2\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{1}{4}(2n-1);\frac{1}{4}(2n+3);\sin^2(e+fx)\right)}{bf(1-2n)(a\sin(e+fx))^{3/2}}$$

```
[Out] (-2*(Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (-1 + 2*n)/4,
 (3 + 2*n)/4, Sin[e + f*x]^2]*(b*Tan[e + f*x])^(1 + n))/(b*f*(1 - 2*n)*(a*S
in[e + f*x])^(3/2))
```

Rubi [A] time = 0.124773, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2602, 2577}

$$-\frac{2\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{1}{4}(2n-1);\frac{1}{4}(2n+3);\sin^2(e+fx)\right)}{bf(1-2n)(a\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x])^n/(a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (-1 + 2*n)/4,
 (3 + 2*n)/4, Sin[e + f*x]^2]*(b*Tan[e + f*x])^(1 + n))/(b*f*(1 - 2*n)*(a*S
in[e + f*x])^(3/2))
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
```

```
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx = \frac{\left(a\cos^{1+n}(e+fx)(a\sin(e+fx))^{-1-n}(b\tan(e+fx))^{1+n}\right)\int \cos^{-n}(e+fx)(a\sin(e+fx))^{-\frac{3}{2}+n}\,dx}{b}$$
$$= -\frac{2\cos^2(e+fx)^{\frac{1+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{4}(-1+2n);\frac{1}{4}(3+2n);\sin^2(e+fx)\right)(b\tan(e+fx))^{1+n}}{bf(1-2n)(a\sin(e+fx))^{3/2}}$$

Mathematica [A] time = 1.85927, size = 90, normalized size = 1.01

$$\frac{2b\sqrt{a\sin(e+fx)}\cos^2(e+fx)^{\frac{n-1}{2}}(b\tan(e+fx))^{n-1}\,{}_2F_1\left(\frac{n+1}{2},\frac{1}{4}(2n-1);\frac{1}{4}(2n+3);\sin^2(e+fx)\right)}{a^2f(2n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x])^n/(a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*b*(Cos[e + f*x]^2)^((-1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (-1 + 2*n)/
4, (3 + 2*n)/4, Sin[e + f*x]^2]*Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(-1 +
 n))/(a^2*f*(-1 + 2*n))
```

Maple [F] time = 0.13, size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)\right)^n\left(a\sin\left(fx+e\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^n/(a*sin(f*x+e))^(3/2),x)
```

```
[Out] int((b*tan(f*x+e))^n/(a*sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)\right)^{n}}{\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^n/(a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^n/(a*sin(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{a\sin\left(fx+e\right)}\left(b\tan\left(fx+e\right)\right)^{n}}{a^{2}\cos\left(fx+e\right)^{2}-a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^n/(a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^n/(a^2*cos(f*x + e)^2 - a^2
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**n/(a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)\right)^{n}}{\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^n/(a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^n/(a*sin(f*x + e))^(3/2), x)
```

3.189 $\int (a\cos(e+fx))^m (b\tan(e+fx))^n\,dx$

Optimal. Leaf size=86

$$\frac{(a\cos(e+fx))^m (b\tan(e+fx))^{n+1}\cos^2(e+fx)^{\frac{1}{2}(-m+n+1)}\,{}_2F_1\left(\frac{n+1}{2},\frac{1}{2}(-m+n+1);\frac{n+3}{2};\sin^2(e+fx)\right)}{bf(n+1)}$$

```
[Out] ((a*Cos[e + f*x])^m*(Cos[e + f*x]^2)^((1 - m + n)/2)*Hypergeometric2F1[(1 +
 n)/2, (1 - m + n)/2, (3 + n)/2, Sin[e + f*x]^2]*(b*Tan[e + f*x])^(1 + n))/
(b*f*(1 + n))
```

Rubi [A] time = 0.0954729, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2603, 2617}

$$\frac{(a\cos(e+fx))^m (b\tan(e+fx))^{n+1}\cos^2(e+fx)^{\frac{1}{2}(-m+n+1)}\,{}_2F_1\left(\frac{n+1}{2},\frac{1}{2}(-m+n+1);\frac{n+3}{2};\sin^2(e+fx)\right)}{bf(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] ((a*Cos[e + f*x])^m*(Cos[e + f*x]^2)^((1 - m + n)/2)*Hypergeometric2F1[(1 +
 n)/2, (1 - m + n)/2, (3 + n)/2, Sin[e + f*x]^2]*(b*Tan[e + f*x])^(1 + n))/
(b*f*(1 + n))
```

Rule 2603

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Cos[e + f*x])^FracPart[m]*(Sec[e + f*x]/a)^FracPar
t[m], Int[(b*Tan[e + f*x])^n/(Sec[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (a\cos(e+fx))^m (b\tan(e+fx))^n\,dx = \left((a\cos(e+fx))^m \left(\frac{\sec(e+fx)}{a}\right)^m\right)\int \left(\frac{\sec(e+fx)}{a}\right)^{-m} (b\tan(e+fx))^n\,dx$$
$$= \frac{(a\cos(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(1-m+n)}\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{2}(1-m+n);\frac{3+n}{2};\sin^2(e+fx)\right)}{bf(1+n)}$$

Mathematica [A] time = 0.447265, size = 81, normalized size = 0.94

$$\frac{\tan(e+fx)\sec^2(e+fx)^{m/2}(a\cos(e+fx))^m(b\tan(e+fx))^n\,{}_2F_1\left(\frac{m+2}{2},\frac{n+1}{2};\frac{n+3}{2};-\tan^2(e+fx)\right)}{f(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] ((a*Cos[e + f*x])^m*Hypergeometric2F1[(2 + m)/2, (1 + n)/2, (3 + n)/2, -Tan
[e + f*x]^2]*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]*(b*Tan[e + f*x])^n)/(f*(1
+ n))
```

Maple [F] time = 0.784, size = 0, normalized size = 0.

$$\int \left(a\cos\left(fx+e\right)\right)^m \left(b\tan\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

```
[Out] int((a*cos(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cos\left(fx+e\right)\right)^m \left(b\tan\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\cos\left(fx+e\right)\right)^{m}\left(b\tan\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*cos(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))**m*(b*tan(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\cos\left(fx+e\right)\right)^{m}\left(b\tan\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*cos(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

3.190 $\int (a\tan(e+fx))^m (b\tan(e+fx))^n\,dx$

Optimal. Leaf size=63

$$\frac{(a\tan(e+fx))^{m+1}(b\tan(e+fx))^n\,{}_2F_1\left(1,\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);-\tan^2(e+fx)\right)}{af(m+n+1)}$$

```
[Out] (Hypergeometric2F1[1, (1 + m + n)/2, (3 + m + n)/2, -Tan[e + f*x]^2]*(a*Tan
[e + f*x])^(1 + m)*(b*Tan[e + f*x])^n)/(a*f*(1 + m + n))
```

Rubi [A] time = 0.0362079, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {20, 3476, 364}

$$\frac{(a\tan(e+fx))^{m+1}(b\tan(e+fx))^n\,{}_2F_1\left(1,\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);-\tan^2(e+fx)\right)}{af(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Tan[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + m + n)/2, (3 + m + n)/2, -Tan[e + f*x]^2]*(a*Tan
[e + f*x])^(1 + m)*(b*Tan[e + f*x])^n)/(a*f*(1 + m + n))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
```

```
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (a\tan(e+fx))^m (b\tan(e+fx))^n\,dx = \left((a\tan(e+fx))^{-n}(b\tan(e+fx))^n\right)\int (a\tan(e+fx))^{m+n}\,dx$$
$$= \frac{\left(a(a\tan(e+fx))^{-n}(b\tan(e+fx))^n\right)\text{Subst}\left(\int \frac{x^{m+n}}{a^2+x^2}\,dx, x, a\tan(e+fx)\right)}{f}$$
$$= \frac{{}_2F_1\left(1,\frac{1}{2}(1+m+n);\frac{1}{2}(3+m+n);-\tan^2(e+fx)\right)(a\tan(e+fx))^{1+m}(b\tan(}{af(1+m+n)}$$

Mathematica [A] time = 0.0715552, size = 66, normalized size = 1.05

$$\frac{\tan(e+fx)(a\tan(e+fx))^m(b\tan(e+fx))^n\,{}_2F_1\left(1,\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+1)+1;-\tan^2(e+fx)\right)}{f(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Tan[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + m + n)/2, 1 + (1 + m + n)/2, -Tan[e + f*x]^2]*Ta
n[e + f*x]*(a*Tan[e + f*x])^m*(b*Tan[e + f*x])^n)/(f*(1 + m + n))
```

Maple [F] time = 0.393, size = 0, normalized size = 0.

$$\int \left(a\tan\left(fx+e\right)\right)^m \left(b\tan\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*tan(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

```
[Out] int((a*tan(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\tan\left(fx+e\right)\right)^m \left(b\tan\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tan(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*tan(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\tan\left(fx+e\right)\right)^m \left(b\tan\left(fx+e\right)\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tan(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*tan(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\tan\left(e+fx\right)\right)^m \left(b\tan\left(e+fx\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tan(f*x+e))**m*(b*tan(f*x+e))**n,x)
```

```
[Out] Integral((a*tan(e + f*x))**m*(b*tan(e + f*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\tan\left(fx+e\right)\right)^m \left(b\tan\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tan(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="giac")

[Out] integrate((a*tan(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

3.191 $\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx$

Optimal. Leaf size=232

$$\frac{2d^3}{5f(d\cot(e+fx))^{5/2}}-\frac{2d}{f\sqrt{d\cot(e+fx)}}-\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)\right.}{2}$$

```
[Out] (Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) +
(2*d^3)/(5*f*(d*Cot[e + f*x])^(5/2)) - (2*d)/(f*Sqrt[d*Cot[e + f*x]]) - (Sq
rt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(
2*Sqrt[2]*f) + (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d
*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rubi [A] time = 0.227133, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {16, 3474, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{2d^3}{5f(d\cot(e+fx))^{5/2}}-\frac{2d}{f\sqrt{d\cot(e+fx)}}-\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x]^4,x]
```

```
[Out] (Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) +
(2*d^3)/(5*f*(d*Cot[e + f*x])^(5/2)) - (2*d)/(f*Sqrt[d*Cot[e + f*x]]) - (Sq
rt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(
2*Sqrt[2]*f) + (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d
*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx = d^4\int \frac{1}{(d\cot(e+fx))^{7/2}}\,dx$$

$$= \frac{2d^3}{5f(d\cot(e+fx))^{5/2}} - d^2\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx$$

$$= \frac{2d^3}{5f(d\cot(e+fx))^{5/2}} - \frac{2d}{f\sqrt{d\cot(e+fx)}} + \int \sqrt{d\cot(e+fx)}\,dx$$

$$= \frac{2d^3}{5f(d\cot(e+fx))^{5/2}} - \frac{2d}{f\sqrt{d\cot(e+fx)}} - \frac{d\,\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= \frac{2d^3}{5f(d\cot(e+fx))^{5/2}} - \frac{2d}{f\sqrt{d\cot(e+fx)}} - \frac{(2d)\,\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx}\right.}{f}$$

$$= \frac{2d^3}{5f(d\cot(e+fx))^{5/2}} - \frac{2d}{f\sqrt{d\cot(e+fx)}} + \frac{d\,\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= \frac{2d^3}{5f(d\cot(e+fx))^{5/2}} - \frac{2d}{f\sqrt{d\cot(e+fx)}} - \frac{\sqrt{d}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e}\right.}{2\sqrt{2}f}$$

$$= \frac{2d^3}{5f(d\cot(e+fx))^{5/2}} - \frac{2d}{f\sqrt{d\cot(e+fx)}} - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\,\text{co}}\right.}{2\sqrt{2}f}$$

$$= \frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{2d^3}{5f(d\cot(e+fx))^{5/}}$$

Mathematica [C] time = 0.0789621, size = 45, normalized size = 0.19

$$\frac{2\tan^3(e+fx)\sqrt{d\cot(e+fx)}\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\cot^2(e+fx)\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x]^4,x]
```

```
[Out] (2*Sqrt[d*Cot[e + f*x]]*Hypergeometric2F1[-5/4, 1, -1/4, -Cot[e + f*x]^2]*T
an[e + f*x]^3)/(5*f)
```

Maple [C] time = 0.258, size = 728, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^(1/2)*tan(f*x+e)^4,x)
```

```
[Out] 1/10/f*2^(1/2)*(cos(f*x+e)-1)*(5*I*cos(f*x+e)^2*sin(f*x+e)*((cos(f*x+e)-1)/
sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(cos(f*x+e
)-1-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(cos(f*x+e)-1-sin(f*x+e))/si
n(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-5*I*cos(f*x+e)^2*sin(f*x+e)*((cos(f*
x+e)-1)/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(c
os(f*x+e)-1-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(cos(f*x+e)-1-sin(f*
x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*cos(f*x+e)^2*sin(f*x+e)*((
cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2
)*(-(cos(f*x+e)-1-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(cos(f*x+e)-1-
sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))+10*cos(f*x+e)^2*sin(f*
x+e)*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e
))^(1/2)*(-(cos(f*x+e)-1-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(cos(f*x
+e)-1-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-5*cos(f*x+e)^2*sin(f*x+e)*
((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1
/2)*(-(cos(f*x+e)-1-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(cos(f*x+e)-
1-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-12*cos(f*x+e)^3*2^(1
/2)+12*cos(f*x+e)^2*2^(1/2)+2*cos(f*x+e)*2^(1/2)-2*2^(1/2))*(cos(f*x+e)+1)^
2*(d*cos(f*x+e)/sin(f*x+e))^(1/2)/cos(f*x+e)^3/sin(f*x+e)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.99986, size = 1511, normalized size = 6.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] 1/20*(20*sqrt(2)*f*(d^2/f^4)^(1/4)*arctan(-(sqrt(2)*d*f*sqrt(d*cos(f*x + e)
/sin(f*x + e))*(d^2/f^4)^(1/4) - sqrt(2)*f*sqrt((sqrt(2)*d*f^3*sqrt(d*cos(f
*x + e)/sin(f*x + e))*(d^2/f^4)^(3/4)*sin(f*x + e) + d^2*f^2*sqrt(d^2/f^4)*
sin(f*x + e) + d^3*cos(f*x + e))/sin(f*x + e))*(d^2/f^4)^(1/4) + d^2)/d^2)*
cos(f*x + e)^3 + 20*sqrt(2)*f*(d^2/f^4)^(1/4)*arctan(-(sqrt(2)*d*f*sqrt(d*c
os(f*x + e)/sin(f*x + e))*(d^2/f^4)^(1/4) - sqrt(2)*f*sqrt(-(sqrt(2)*d*f^3*
sqrt(d*cos(f*x + e)/sin(f*x + e))*(d^2/f^4)^(3/4)*sin(f*x + e) - d^2*f^2*sq
rt(d^2/f^4)*sin(f*x + e) - d^3*cos(f*x + e))/sin(f*x + e))*(d^2/f^4)^(1/4)
- d^2)/d^2)*cos(f*x + e)^3 + 5*sqrt(2)*f*(d^2/f^4)^(1/4)*cos(f*x + e)^3*log
((sqrt(2)*d*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*(d^2/f^4)^(3/4)*sin(f*x +
 e) + d^2*f^2*sqrt(d^2/f^4)*sin(f*x + e) + d^3*cos(f*x + e))/sin(f*x + e))
- 5*sqrt(2)*f*(d^2/f^4)^(1/4)*cos(f*x + e)^3*log(-(sqrt(2)*d*f^3*sqrt(d*cos
(f*x + e)/sin(f*x + e))*(d^2/f^4)^(3/4)*sin(f*x + e) - d^2*f^2*sqrt(d^2/f^4
)*sin(f*x + e) - d^3*cos(f*x + e))/sin(f*x + e)) - 8*(6*cos(f*x + e)^2 - 1)
*sqrt(d*cos(f*x + e)/sin(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \cot{\left(e + f x \right)}} \tan^{4}{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**(1/2)*tan(f*x+e)**4,x)
```

```
[Out] Integral(sqrt(d*cot(e + f*x))*tan(e + f*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \cot \left(fx + e\right)} \tan \left(fx + e\right)^{4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cot(f*x + e))*tan(f*x + e)^4, x)
```

3.192 $\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx$

Optimal. Leaf size=214

$$\frac{2d^2}{3f(d\cot(e+fx))^{3/2}}-\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+f}\right.}{2\sqrt{2}f}$$

```
[Out] -((Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 + (2*d^2)/(3*f*(d*Cot[e + f*x])^(3/2)) - (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Co
t[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) + (Sqrt[d]*Log[Sq
rt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rubi [A] time = 0.181517, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {16, 3474, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{2d^2}{3f(d\cot(e+fx))^{3/2}}-\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+f}\right.}{2\sqrt{2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x]^3,x]
```

```
[Out] -((Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 + (2*d^2)/(3*f*(d*Cot[e + f*x])^(3/2)) - (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Co
t[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) + (Sqrt[d]*Log[Sq
rt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx = d^3\int \frac{1}{(d\cot(e+fx))^{5/2}}\,dx$$
$$= \frac{2d^2}{3f(d\cot(e+fx))^{3/2}} - d\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx$$
$$= \frac{2d^2}{3f(d\cot(e+fx))^{3/2}} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)}\,dx, x, d\cot(e+fx)\right)}{f}$$
$$= \frac{2d^2}{3f(d\cot(e+fx))^{3/2}} + \frac{(2d^2)\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$
$$= \frac{2d^2}{3f(d\cot(e+fx))^{3/2}} + \frac{d\,\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} + \frac{d\,\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\right.}{}$$
$$= \frac{2d^2}{3f(d\cot(e+fx))^{3/2}} - \frac{\sqrt{d}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{\sqrt{d}\,\text{Subs}}{}$$
$$= \frac{2d^2}{3f(d\cot(e+fx))^{3/2}} - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{\sqrt{d}\,\text{lo}}{}$$
$$= -\frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{2d^2}{3f(d\cot(e+fx)}$$

Mathematica [C] time = 0.0439815, size = 45, normalized size = 0.21

$$\frac{2\tan^2(e+fx)\sqrt{d\cot(e+fx)}\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\cot^2(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x]^3,x]
```

```
[Out] (2*Sqrt[d*Cot[e + f*x]]*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[e + f*x]^2]*Ta
n[e + f*x]^2)/(3*f)
```

Maple [C] time = 0.185, size = 540, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^(1/2)*tan(f*x+e)^3,x)
```

```
[Out] -1/6/f*2^(1/2)*(cos(f*x+e)-1)*(3*I*cos(f*x+e)*EllipticPi(((1-cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos(f*x+e)-1)/sin(f*x+e
))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*
x+e))/sin(f*x+e))^(1/2)-3*I*cos(f*x+e)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((cos(f*x+e)-1)/sin(f*x+e))^(1/2
)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/s
in(f*x+e))^(1/2)+3*cos(f*x+e)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((1-cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e)
)^(1/2)+3*cos(f*x+e)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2
),1/2-1/2*I,1/2*2^(1/2))*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)-2
*cos(f*x+e)*2^(1/2)+2*2^(1/2))*(cos(f*x+e)+1)^2*(d*cos(f*x+e)/sin(f*x+e))^(
1/2)/sin(f*x+e)^2/cos(f*x+e)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.0551, size = 1440, normalized size = 6.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(12*sqrt(2)*f*(d^2/f^4)^(1/4)*arctan(-(sqrt(2)*f^3*sqrt(d*cos(f*x + e
)/sin(f*x + e))*(d^2/f^4)^(3/4) - sqrt(2)*f^3*sqrt((f^2*sqrt(d^2/f^4)*sin(f
*x + e) + sqrt(2)*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*(d^2/f^4)^(1/4)*sin(f
*x + e) + d*cos(f*x + e))/sin(f*x + e))*(d^2/f^4)^(3/4) + d^2)/d^2)*cos(f*x
 + e)^2 + 12*sqrt(2)*f*(d^2/f^4)^(1/4)*arctan(-(sqrt(2)*f^3*sqrt(d*cos(f*x
+ e)/sin(f*x + e))*(d^2/f^4)^(3/4) - sqrt(2)*f^3*sqrt((f^2*sqrt(d^2/f^4)*si
n(f*x + e) - sqrt(2)*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*(d^2/f^4)^(1/4)*si
n(f*x + e) + d*cos(f*x + e))/sin(f*x + e))*(d^2/f^4)^(3/4) - d^2)/d^2)*cos(
f*x + e)^2 - 3*sqrt(2)*f*(d^2/f^4)^(1/4)*cos(f*x + e)^2*log((f^2*sqrt(d^2/f
^4)*sin(f*x + e) + sqrt(2)*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*(d^2/f^4)^(1
/4)*sin(f*x + e) + d*cos(f*x + e))/sin(f*x + e)) + 3*sqrt(2)*f*(d^2/f^4)^(1
/4)*cos(f*x + e)^2*log((f^2*sqrt(d^2/f^4)*sin(f*x + e) - sqrt(2)*f*sqrt(d*c
os(f*x + e)/sin(f*x + e))*(d^2/f^4)^(1/4)*sin(f*x + e) + d*cos(f*x + e))/si
n(f*x + e)) + 8*(cos(f*x + e)^2 - 1)*sqrt(d*cos(f*x + e)/sin(f*x + e)))/(f*
cos(f*x + e)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\cot\left(e+fx\right)}\tan^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**(1/2)*tan(f*x+e)**3,x)
```

```
[Out] Integral(sqrt(d*cot(e + f*x))*tan(e + f*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\cot\left(fx+e\right)}\tan\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cot(f*x + e))*tan(f*x + e)^3, x)
```

3.193 $\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx$

Optimal. Leaf size=210

$$\frac{2d}{f\sqrt{d\cot(e+fx)}}+\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}-\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\right.}{2\sqrt{2}f}$$

```
[Out] -((Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 + (2*d)/(f*Sqrt[d*Cot[e + f*x]]) + (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e +
f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) - (Sqrt[d]*Log[Sqrt[d]
+ Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rubi [A] time = 0.178992, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {16, 3474, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{2d}{f\sqrt{d\cot(e+fx)}}+\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}-\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\right.}{2\sqrt{2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x]^2,x]
```

```
[Out] -((Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 + (2*d)/(f*Sqrt[d*Cot[e + f*x]]) + (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e +
f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) - (Sqrt[d]*Log[Sqrt[d]
+ Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx &= d^2\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx\\
&= \frac{2d}{f\sqrt{d\cot(e+fx)}} - \int \sqrt{d\cot(e+fx)}\,dx\\
&= \frac{2d}{f\sqrt{d\cot(e+fx)}} + \frac{d\,\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f}\\
&= \frac{2d}{f\sqrt{d\cot(e+fx)}} + \frac{(2d)\,\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}\\
&= \frac{2d}{f\sqrt{d\cot(e+fx)}} - \frac{d\,\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} + \frac{d\,\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, \right.}{f}\\
&= \frac{2d}{f\sqrt{d\cot(e+fx)}} + \frac{\sqrt{d}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{\sqrt{d}\,\text{Subst}\left(\right.}{}\\
&= \frac{2d}{f\sqrt{d\cot(e+fx)}} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{\sqrt{d}\log\left(\right.}{}\\
&= -\frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{2d}{f\sqrt{d\cot(e+fx)}}
\end{aligned}$$

Mathematica [C] time = 0.0542135, size = 36, normalized size = 0.17

$$\frac{2d\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(e+fx)\right)}{f\sqrt{d\cot(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x]^2,x]
```

```
[Out] (2*d*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[e + f*x]^2])/(f*Sqrt[d*Cot[e + f*
x]])
```

Maple [C] time = 0.167, size = 650, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^(1/2)*tan(f*x+e)^2,x)
```

```
[Out] 1/2/f*2^(1/2)*(d*cos(f*x+e)/sin(f*x+e))^(1/2)*(cos(f*x+e)+1)^2*(cos(f*x+e)-
1)*(I*sin(f*x+e)*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticP
i(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*sin
(f*x+e)*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))+sin(f*x+e)*((c
os(f*x+e)-1)/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)
*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))+sin(f*x+e)*((cos(f*x+e)-1
)/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+
e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-2*sin(f*x+e)*((cos(f*x+e)-1)/sin(f*x
+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(
f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^
(1/2),1/2*2^(1/2))+2*cos(f*x+e)*2^(1/2)-2*2^(1/2))/cos(f*x+e)/sin(f*x+e)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.91088, size = 1458, normalized size = 6.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*f*(d^2/f^4)^(1/4)*arctan(-(sqrt(2)*d*f*sqrt(d*cos(f*x + e)/
sin(f*x + e))*(d^2/f^4)^(1/4) - sqrt(2)*f*sqrt((sqrt(2)*d*f^3*sqrt(d*cos(f*
x + e)/sin(f*x + e))*(d^2/f^4)^(3/4)*sin(f*x + e) + d^2*f^2*sqrt(d^2/f^4)*s
in(f*x + e) + d^3*cos(f*x + e))/sin(f*x + e))*(d^2/f^4)^(1/4) + d^2)/d^2)*c
os(f*x + e) + 4*sqrt(2)*f*(d^2/f^4)^(1/4)*arctan(-(sqrt(2)*d*f*sqrt(d*cos(f
*x + e)/sin(f*x + e))*(d^2/f^4)^(1/4) - sqrt(2)*f*sqrt(-(sqrt(2)*d*f^3*sqrt
(d*cos(f*x + e)/sin(f*x + e))*(d^2/f^4)^(3/4)*sin(f*x + e) - d^2*f^2*sqrt(d
^2/f^4)*sin(f*x + e) - d^3*cos(f*x + e))/sin(f*x + e))*(d^2/f^4)^(1/4) - d^
2)/d^2)*cos(f*x + e) + sqrt(2)*f*(d^2/f^4)^(1/4)*cos(f*x + e)*log((sqrt(2)*
d*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*(d^2/f^4)^(3/4)*sin(f*x + e) + d^2*
f^2*sqrt(d^2/f^4)*sin(f*x + e) + d^3*cos(f*x + e))/sin(f*x + e)) - sqrt(2)*
f*(d^2/f^4)^(1/4)*cos(f*x + e)*log(-(sqrt(2)*d*f^3*sqrt(d*cos(f*x + e)/sin(
f*x + e))*(d^2/f^4)^(3/4)*sin(f*x + e) - d^2*f^2*sqrt(d^2/f^4)*sin(f*x + e)
 - d^3*cos(f*x + e))/sin(f*x + e)) - 8*sqrt(d*cos(f*x + e)/sin(f*x + e))*si
n(f*x + e))/(f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\cot\left(e+fx\right)}\tan^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**(1/2)*tan(f*x+e)**2,x)
```

```
[Out] Integral(sqrt(d*cot(e + f*x))*tan(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\cot\left(fx+e\right)}\tan\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cot(f*x + e))*tan(f*x + e)^2, x)
```

3.194 $\int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx$

Optimal. Leaf size=192

$$\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}-\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{\sqrt{d}\tan^{-1}\left(1\right.}{}$$

```
[Out] (Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) +
(Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]
)/(2*Sqrt[2]*f) - (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqr
t[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rubi [A] time = 0.137801, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {16, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}-\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{\sqrt{d}\tan^{-1}\left(1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x],x]
```



```
[Out] (Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) +
(Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]
)/(2*Sqrt[2]*f) - (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqr
t[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

IntegerQ[n]

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx = d\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx$$
$$= -\frac{d^2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)}\,dx, x, d\cot(e+fx)\right)}{f}$$
$$= -\frac{(2d^2)\,\mathrm{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$
$$= -\frac{d\,\mathrm{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} - \frac{d\,\mathrm{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$
$$= \frac{\sqrt{d}\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{\sqrt{d}\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{\right.}{2\sqrt{2}f}$$
$$= \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)\right.}{2\sqrt{2}f}$$
$$= \frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\,\mathrm{co}\right.}{}$$

Mathematica [A] time = 0.183199, size = 132, normalized size = 0.69

$$\frac{d\sqrt{\cot(e+fx)}\left(\log\left(\cot(e+fx)-\sqrt{2}\sqrt{\cot(e+fx)}+1\right)-\log\left(\cot(e+fx)+\sqrt{2}\sqrt{\cot(e+fx)}+1\right)+2\tan^{-1}\left(1-\sqrt{2}\sqrt{\right.\right.}{2\sqrt{2}f\sqrt{d\cot(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x],x]
```

```
[Out] (d*Sqrt[Cot[e + f*x]]*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]] - 2*ArcTan[
1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[
```

```
e + f*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/(2*Sqrt[2]
*f*Sqrt[d*Cot[e + f*x]])
```

Maple [C] time = 0.124, size = 287, normalized size = 1.5

$$-\frac{\sqrt{2}\left(\cos\left(fx+e\right)-1\right)\left(\cos\left(fx+e\right)+1\right)^{2}}{2f\left(\sin\left(fx+e\right)\right)^{2}\cos\left(fx+e\right)}\sqrt{\frac{d\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{\cos\left(fx+e\right)-1+\sin}{\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^(1/2)*tan(f*x+e),x)
```

```
[Out] -1/2/f*2^(1/2)*(d*cos(f*x+e)/sin(f*x+e))^(1/2)*(cos(f*x+e)-1)*((1-cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2
)*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*(I*EllipticPi(((1-cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi(((1-cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-EllipticPi(((1-cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-EllipticPi(((1-cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2)))/sin(f*x+e)^2/cos(
f*x+e)*(cos(f*x+e)+1)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.88023, size = 1230, normalized size = 6.41

$$\sqrt{2}\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}f^3\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{3}{4}}-\sqrt{2}f^3\sqrt{\frac{f^2\sqrt{\frac{d^2}{f^4}}\sin(fx+e)+\sqrt{2}f\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}\sin(fx+e)+d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{3}{4}}+d^2}{d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] sqrt(2)*(d^2/f^4)^(1/4)*arctan(-(sqrt(2)*f^3*sqrt(d*cos(f*x + e)/sin(f*x +
e))*(d^2/f^4)^(3/4) - sqrt(2)*f^3*sqrt((f^2*sqrt(d^2/f^4)*sin(f*x + e) + sq
rt(2)*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*(d^2/f^4)^(1/4)*sin(f*x + e) + d*
cos(f*x + e))/sin(f*x + e))*(d^2/f^4)^(3/4) + d^2)/d^2) + sqrt(2)*(d^2/f^4)
^(1/4)*arctan(-(sqrt(2)*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*(d^2/f^4)^(3/
4) - sqrt(2)*f^3*sqrt((f^2*sqrt(d^2/f^4)*sin(f*x + e) - sqrt(2)*f*sqrt(d*co
s(f*x + e)/sin(f*x + e))*(d^2/f^4)^(1/4)*sin(f*x + e) + d*cos(f*x + e))/sin
(f*x + e))*(d^2/f^4)^(3/4) - d^2)/d^2) - 1/4*sqrt(2)*(d^2/f^4)^(1/4)*log((f
^2*sqrt(d^2/f^4)*sin(f*x + e) + sqrt(2)*f*sqrt(d*cos(f*x + e)/sin(f*x + e))
*(d^2/f^4)^(1/4)*sin(f*x + e) + d*cos(f*x + e))/sin(f*x + e)) + 1/4*sqrt(2)
*(d^2/f^4)^(1/4)*log((f^2*sqrt(d^2/f^4)*sin(f*x + e) - sqrt(2)*f*sqrt(d*cos
(f*x + e)/sin(f*x + e))*(d^2/f^4)^(1/4)*sin(f*x + e) + d*cos(f*x + e))/sin(
f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{d\cot(e+fx)}\tan(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**(1/2)*tan(f*x+e),x)
```

```
[Out] Integral(sqrt(d*cot(e + f*x))*tan(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\cot\left(fx+e\right)}\tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cot(f*x + e))*tan(f*x + e), x)
```

3.195 $\int \sqrt{d\cot(e+fx)}\,dx$

Optimal. Leaf size=192

$$-\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{\sqrt{d}\tan^{-1}}{}$$

```
[Out] (Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]
)/(2*Sqrt[2]*f) + (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqr
t[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rubi [A] time = 0.117145, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3476, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{\sqrt{d}\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] (Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]
)/(2*Sqrt[2]*f) + (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqr
t[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \sqrt{d\cot(e+fx)}\,dx = -\frac{d\,\mathrm{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= -\frac{(2d)\,\mathrm{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= \frac{d\,\mathrm{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} - \frac{d\,\mathrm{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= -\frac{\sqrt{d}\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{\sqrt{d}\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+}\right.}{2\sqrt{2}f}$$

$$= -\frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\,\mathrm{c}}\right.}{2\sqrt{2}f}$$

$$= \frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\right.}{2\sqrt{2}f}$$

Mathematica [C] time = 0.0446374, size = 40, normalized size = 0.21

$$-\frac{2(d\cot(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(e+fx)\right)}{3df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] (-2*(d*Cot[e + f*x])^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[e + f*x]^2])
/(3*d*f)
```

Maple [A] time = 0.025, size = 160, normalized size = 0.8

$$-\frac{d\sqrt{2}}{4f}\ln\left(\left(d\cot(fx+e)-\sqrt[4]{d^2}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}\right)\left(d\cot(fx+e)+\sqrt[4]{d^2}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}\right)^{-1}\right)\frac{1}{\sqrt[4]{d^2}} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^(1/2),x)
```

```
[Out] -1/4/f*d/(d^2)^(1/4)*2^(1/2)*ln((d*cot(f*x+e)-(d^2)^(1/4)*(d*cot(f*x+e))^(1
/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)+(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(
1/2)+(d^2)^(1/2)))-1/2/f*d/(d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2)^(1/4)*(
d*cot(f*x+e))^(1/2)+1)+1/2/f*d/(d^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(d^2)^(1
/4)*(d*cot(f*x+e))^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \cot \left(e + f x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*cot(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\cot(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cot(f*x + e)), x)
```

3.196 $\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx$

Optimal. Leaf size=209

$$-\frac{2\sqrt{d\cot(e+fx)}}{f}-\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx}\right.}{2\sqrt{2}f}$$

```
[Out] -((Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 - (2*Sqrt[d*Cot[e + f*x]])/f - (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x]
 - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) + (Sqrt[d]*Log[Sqrt[d] + Sq
rt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rubi [A] time = 0.165036, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {16, 3473, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{2\sqrt{d\cot(e+fx)}}{f}-\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx}\right.}{2\sqrt{2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] -((Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 - (2*Sqrt[d*Cot[e + f*x]])/f - (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x]
 - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) + (Sqrt[d]*Log[Sqrt[d] + Sq
rt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx &= \frac{\int (d\cot(e+fx))^{3/2}\,dx}{d} \\
&= -\frac{2\sqrt{d\cot(e+fx)}}{f} - d\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx \\
&= -\frac{2\sqrt{d\cot(e+fx)}}{f} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)}\,dx, x, d\cot(e+fx)\right)}{f} \\
&= -\frac{2\sqrt{d\cot(e+fx)}}{f} + \frac{(2d^2)\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} \\
&= -\frac{2\sqrt{d\cot(e+fx)}}{f} + \frac{d\,\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} + \frac{d\,\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx \right.}{\ldots} \\
&= -\frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{\sqrt{d}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{\sqrt{d}\,\text{Subst}\left(\ldots\right.}{\ldots} \\
&= -\frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{\sqrt{d}\log\left(\ldots\right.}{\ldots} \\
&= -\frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{2\sqrt{d\cot(e+fx)}}{f}
\end{aligned}$$

Mathematica [A] time = 0.227703, size = 162, normalized size = 0.78

$$-\frac{(d\cot(e+fx))^{3/2}\left(8\sqrt{\cot(e+fx)}+\sqrt{2}\log\left(\cot(e+fx)-\sqrt{2}\sqrt{\cot(e+fx)}+1\right)-\sqrt{2}\log\left(\cot(e+fx)+\sqrt{2}\sqrt{\cot(e+\ldots}\right.\right.}{4df\cot^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] -((d*Cot[e + f*x])^(3/2)*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]]
- 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] + 8*Sqrt[Cot[e + f*x]] +
 Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]] - Sqrt[2]*Log[1
 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/(4*d*f*Cot[e + f*x]^(3/2))
```

Maple [A] time = 0.02, size = 172, normalized size = 0.8

$$-2\,\frac{\sqrt{d\cot\left(fx+e\right)}}{f}+\frac{\sqrt{2}}{2\,f}\sqrt[4]{d^2}\arctan\left(\sqrt{2}\sqrt{d\cot\left(fx+e\right)}\frac{1}{\sqrt[4]{d^2}}+1\right)-\frac{\sqrt{2}}{2\,f}\sqrt[4]{d^2}\arctan\left(-\sqrt{2}\sqrt{d\cot\left(fx+e\right)}\frac{1}{\sqrt[4]{d^2}}+1\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(d*cot(f*x+e))^(1/2),x)
```

```
[Out] -2*(d*cot(f*x+e))^(1/2)/f+1/2/f*(d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2)^(1
/4)*(d*cot(f*x+e))^(1/2)+1)-1/2/f*(d^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(d^2)
^(1/4)*(d*cot(f*x+e))^(1/2)+1)+1/4/f*(d^2)^(1/4)*2^(1/2)*ln((d*cot(f*x+e)+(
d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)-(d^2)^(1
/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\cot\left(e+fx\right)}\cot\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(d*cot(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*cot(e + f*x))*cot(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\cot\left(fx+e\right)}\cot\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cot(f*x + e))*cot(f*x + e), x)
```

3.197 $\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx$

Optimal. Leaf size=214

$$-\frac{2(d\cot(e+fx))^{3/2}}{3df}+\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}-\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+f}\right.}{2\sqrt{2}f}$$

```
[Out] -((Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 - (2*(d*Cot[e + f*x])^(3/2))/(3*d*f) + (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[
e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) - (Sqrt[d]*Log[Sqrt
[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rubi [A] time = 0.161636, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {16, 3473, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2(d\cot(e+fx))^{3/2}}{3df}+\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}-\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+f}\right.}{2\sqrt{2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] -((Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 - (2*(d*Cot[e + f*x])^(3/2))/(3*d*f) + (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[
e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) - (Sqrt[d]*Log[Sqrt
[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx = \frac{\int (d\cot(e+fx))^{5/2}\,dx}{d^2}$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3df} - \int \sqrt{d\cot(e+fx)}\,dx$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3df} + \frac{d\,\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3df} + \frac{(2d)\,\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3df} - \frac{d\,\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} + \frac{d\,\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\right.}{}$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3df} + \frac{\sqrt{d}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{\sqrt{d}\,\text{Subs}}{}$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3df} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{\sqrt{d}\log}{}$$

$$= -\frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{2(d\cot(e+fx))^{3/}}{3df}$$

Mathematica [C] time = 0.04719, size = 42, normalized size = 0.2

$$\frac{2(d\cot(e+fx))^{3/2}\left({}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(e+fx)\right)-1\right)}{3df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] (2*(d*Cot[e + f*x])^(3/2)*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Cot[e + f*x
]^2]))/(3*d*f)
```

Maple [A] time = 0.027, size = 178, normalized size = 0.8

$$-\frac{2}{3df}\left(d\cot\left(fx+e\right)\right)^{\frac{3}{2}}+\frac{d\sqrt{2}}{4f}\ln\left(\left(d\cot\left(fx+e\right)-\sqrt[4]{d^2}\sqrt{d\cot\left(fx+e\right)}\sqrt{2}+\sqrt{d^2}\right)\left(d\cot\left(fx+e\right)+\sqrt[4]{d^2}\sqrt{d\cot\left(fx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(d*cot(f*x+e))^(1/2),x)
```

```
[Out] -2/3*(d*cot(f*x+e))^(3/2)/d/f+1/4/f*d/(d^2)^(1/4)*2^(1/2)*ln((d*cot(f*x+e)-
(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)+(d^2)^(
1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+1/2/f*d/(d^2)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)-1/2/f*d/(d^2)^(1/4)*2^(
1/2)*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \cot\left(e + fx\right)} \cot^2\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(d*cot(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*cot(e + f*x))*cot(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \cot\left(fx + e\right)} \cot\left(fx + e\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cot(f*x + e))*cot(f*x + e)^2, x)
```

3.198 $\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx$

Optimal. Leaf size=231

$$-\frac{2(d\cot(e+fx))^{5/2}}{5d^2f}+\frac{2\sqrt{d\cot(e+fx)}}{f}+\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}-\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+f\right.}{}$$

```
[Out] (Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) +
(2*Sqrt[d*Cot[e + f*x]])/f - (2*(d*Cot[e + f*x])^(5/2))/(5*d^2*f) + (Sqrt[d
]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sq
rt[2]*f) - (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot
[e + f*x]]])/(2*Sqrt[2]*f)
```

Rubi [A] time = 0.197684, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {16, 3473, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{2(d\cot(e+fx))^{5/2}}{5d^2f}+\frac{2\sqrt{d\cot(e+fx)}}{f}+\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}-\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+f\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*Sqrt[d*Cot[e + f*x]],x]
```


```
[Out] (Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) +
(2*Sqrt[d*Cot[e + f*x]])/f - (2*(d*Cot[e + f*x])^(5/2))/(5*d^2*f) + (Sqrt[d
]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sq
rt[2]*f) - (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot
[e + f*x]]])/(2*Sqrt[2]*f)
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx = \frac{\int (d\cot(e+fx))^{7/2}\,dx}{d^3}$$

$$= -\frac{2(d\cot(e+fx))^{5/2}}{5d^2f} - \frac{\int (d\cot(e+fx))^{3/2}\,dx}{d}$$

$$= \frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^2f} + d\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx$$

$$= \frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^2f} - \frac{d^2\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)}\,dx, x, d\cot(e+fx)\right.}{f}$$

$$= \frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^2f} - \frac{(2d^2)\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+f}\right.}{f}$$

$$= \frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^2f} - \frac{d\,\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} -$$

$$= \frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^2f} + \frac{\sqrt{d}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e}\right.}{2\sqrt{2}f}$$

$$= \frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^2f} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\,\text{co}}\right.}{2\sqrt{2}f}$$

$$= \frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{2\sqrt{d\cot(e+fx)}}{f} -$$

Mathematica [A] time = 0.43853, size = 172, normalized size = 0.74

$$\frac{\sqrt{d\cot(e+fx)}\left(-8\cot^{\frac{5}{2}}(e+fx)+40\sqrt{\cot(e+fx)}+5\sqrt{2}\log\left(\cot(e+fx)-\sqrt{2}\sqrt{\cot(e+fx)}+1\right)-5\sqrt{2}\log\left(\cot(e+f\right.\right.}{20f\sqrt{\cot(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] (Sqrt[d*Cot[e + f*x]]*(10*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]] -
10*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] + 40*Sqrt[Cot[e + f*x]] -
 8*Cot[e + f*x]^(5/2) + 5*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[
e + f*x]] - 5*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/
(20*f*Sqrt[Cot[e + f*x]])
```

Maple [A] time = 0.029, size = 190, normalized size = 0.8

$$-\frac{2}{5d^2f}\left(d\cot\left(fx+e\right)\right)^{\frac{5}{2}}+2\frac{\sqrt{d\cot\left(fx+e\right)}}{f}+\frac{\sqrt{2}}{2f}\sqrt[4]{d^2}\arctan\left(-\sqrt{2}\sqrt{d\cot\left(fx+e\right)}\frac{1}{\sqrt[4]{d^2}}+1\right)-\frac{\sqrt{2}}{4f}\sqrt[4]{d^2}\ln\left(\left(d\cot\left(f\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(d*cot(f*x+e))^(1/2),x)
```

```
[Out] -2/5*(d*cot(f*x+e))^(5/2)/d^2/f+2*(d*cot(f*x+e))^(1/2)/f+1/2/f*(d^2)^(1/4)*
2^(1/2)*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)-1/4/f*(d^2)^(1/
4)*2^(1/2)*ln((d*cot(f*x+e)+(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^
(1/2))/(d*cot(f*x+e)-(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))
-1/2/f*(d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+
1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\cot\left(e+fx\right)}\cot^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(d*cot(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*cot(e + f*x))*cot(e + f*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\cot\left(fx+e\right)}\cot\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cot(f*x + e))*cot(f*x + e)^3, x)
```

3.199 $\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx$

Optimal. Leaf size=234

$$\frac{2d^4}{5f(d\cot(e+fx))^{5/2}}-\frac{2d^2}{f\sqrt{d\cot(e+fx)}}-\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx\right.}{}$$

```
[Out] (d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) +
(2*d^4)/(5*f*(d*Cot[e + f*x])^(5/2)) - (2*d^2)/(f*Sqrt[d*Cot[e + f*x]]) - (
d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])
/(2*Sqrt[2]*f) + (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt
[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rubi [A] time = 0.214465, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {16, 3474, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{2d^4}{5f(d\cot(e+fx))^{5/2}}-\frac{2d^2}{f\sqrt{d\cot(e+fx)}}-\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^5,x]
```

```
[Out] (d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) +
(2*d^4)/(5*f*(d*Cot[e + f*x])^(5/2)) - (2*d^2)/(f*Sqrt[d*Cot[e + f*x]]) - (
d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])
/(2*Sqrt[2]*f) + (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt
[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx = d^5\int \frac{1}{(d\cot(e+fx))^{7/2}}\,dx$$

$$= \frac{2d^4}{5f(d\cot(e+fx))^{5/2}} - d^3\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx$$

$$= \frac{2d^4}{5f(d\cot(e+fx))^{5/2}} - \frac{2d^2}{f\sqrt{d\cot(e+fx)}} + d\int \sqrt{d\cot(e+fx)}\,dx$$

$$= \frac{2d^4}{5f(d\cot(e+fx))^{5/2}} - \frac{2d^2}{f\sqrt{d\cot(e+fx)}} - \frac{d^2\,\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= \frac{2d^4}{5f(d\cot(e+fx))^{5/2}} - \frac{2d^2}{f\sqrt{d\cot(e+fx)}} - \frac{\left(2d^2\right)\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+ \ldots}{f}$$

$$= \frac{2d^4}{5f(d\cot(e+fx))^{5/2}} - \frac{2d^2}{f\sqrt{d\cot(e+fx)}} + \frac{d^2\,\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx \ldots}{f}$$

$$= \frac{2d^4}{5f(d\cot(e+fx))^{5/2}} - \frac{2d^2}{f\sqrt{d\cot(e+fx)}} - \frac{d^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot \ldots}{2\sqrt{2}f}$$

$$= \frac{2d^4}{5f(d\cot(e+fx))^{5/2}} - \frac{2d^2}{f\sqrt{d\cot(e+fx)}} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d \ldots}{2\sqrt{2}f}$$

$$= \frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{2d^4}{5f(d\cot(e+fx \ldots}$$

Mathematica [C] time = 0.0606452, size = 45, normalized size = 0.19

$$\frac{2\tan^4(e+fx)(d\cot(e+fx))^{3/2}\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\cot^2(e+fx)\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^5,x]
```

```
[Out] (2*(d*Cot[e + f*x])^(3/2)*Hypergeometric2F1[-5/4, 1, -1/4, -Cot[e + f*x]^2]
*Tan[e + f*x]^4)/(5*f)
```

Maple [C] time = 0.175, size = 718, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^(3/2)*tan(f*x+e)^5,x)
```

```
[Out] 1/10/f*2^(1/2)*(cos(f*x+e)-1)*(5*I*sin(f*x+e)*((cos(f*x+e)-1)/sin(f*x+e))^(
1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e)
)/sin(f*x+e))^(1/2)*cos(f*x+e)^2*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-5*I*sin(f*x+e)*((cos(f*x+e)-1)/sin(f*x
+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(
f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)^2*EllipticPi(((1-cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*sin(f*x+e)*((cos(f*x+e)-1)/sin
(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+
sin(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)^2*EllipticPi(((1-cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))+10*sin(f*x+e)*((cos(f*x+e)-1
)/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+
e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)^2*EllipticF(((1-cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-5*sin(f*x+e)*((cos(f*x+e)-1)/sin(f
*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+si
n(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)^2*EllipticPi(((1-cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-12*cos(f*x+e)^3*2^(1/2)+12*cos
(f*x+e)^2*2^(1/2)+2*cos(f*x+e)*2^(1/2)-2*2^(1/2))*(cos(f*x+e)+1)^2*(d*cos(f
*x+e)/sin(f*x+e))^(3/2)/sin(f*x+e)^2/cos(f*x+e)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.21869, size = 1527, normalized size = 6.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] 1/20*(20*sqrt(2)*(d^6/f^4)^(1/4)*f*arctan(-(sqrt(2)*(d^6/f^4)^(1/4)*d^4*f*s
qrt(d*cos(f*x + e)/sin(f*x + e)) + d^6 - sqrt(2)*(d^6/f^4)^(1/4)*f*sqrt((d^
9*cos(f*x + e) + sqrt(2)*(d^6/f^4)^(3/4)*d^4*f^3*sqrt(d*cos(f*x + e)/sin(f*
x + e))*sin(f*x + e) + sqrt(d^6/f^4)*d^6*f^2*sin(f*x + e))/sin(f*x + e)))/d
^6)*cos(f*x + e)^3 + 20*sqrt(2)*(d^6/f^4)^(1/4)*f*arctan(-(sqrt(2)*(d^6/f^4
)^(1/4)*d^4*f*sqrt(d*cos(f*x + e)/sin(f*x + e)) - d^6 - sqrt(2)*(d^6/f^4)^(
1/4)*f*sqrt((d^9*cos(f*x + e) - sqrt(2)*(d^6/f^4)^(3/4)*d^4*f^3*sqrt(d*cos(
f*x + e)/sin(f*x + e))*sin(f*x + e) + sqrt(d^6/f^4)*d^6*f^2*sin(f*x + e))/s
in(f*x + e)))/d^6)*cos(f*x + e)^3 + 5*sqrt(2)*(d^6/f^4)^(1/4)*f*cos(f*x + e
)^3*log((d^9*cos(f*x + e) + sqrt(2)*(d^6/f^4)^(3/4)*d^4*f^3*sqrt(d*cos(f*x
+ e)/sin(f*x + e))*sin(f*x + e) + sqrt(d^6/f^4)*d^6*f^2*sin(f*x + e))/sin(f
*x + e)) - 5*sqrt(2)*(d^6/f^4)^(1/4)*f*cos(f*x + e)^3*log((d^9*cos(f*x + e)
 - sqrt(2)*(d^6/f^4)^(3/4)*d^4*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*sin(f*
x + e) + sqrt(d^6/f^4)*d^6*f^2*sin(f*x + e))/sin(f*x + e)) - 8*(6*d*cos(f*x
 + e)^2 - d)*sqrt(d*cos(f*x + e)/sin(f*x + e))*sin(f*x + e))/(f*cos(f*x + e
)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**(3/2)*tan(f*x+e)**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cot\left(fx+e\right)\right)^{\frac{3}{2}}\tan\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^(3/2)*tan(f*x + e)^5, x)
```

3.200 $\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx$

Optimal. Leaf size=214

$$\frac{2d^3}{3f(d\cot(e+fx))^{3/2}}-\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+f}\right.}{2\sqrt{2}f}$$

```
[Out] -((d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 + (2*d^3)/(3*f*(d*Cot[e + f*x])^(3/2)) - (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Co
t[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) + (d^(3/2)*Log[Sq
rt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rubi [A] time = 0.17876, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {16, 3474, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{2d^3}{3f(d\cot(e+fx))^{3/2}}-\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+f}\right.}{2\sqrt{2}f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^4,x]
```

```
[Out] -((d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 + (2*d^3)/(3*f*(d*Cot[e + f*x])^(3/2)) - (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Co
t[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) + (d^(3/2)*Log[Sq
rt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx &= d^4\int \frac{1}{(d\cot(e+fx))^{5/2}}\,dx\\
&= \frac{2d^3}{3f(d\cot(e+fx))^{3/2}} - d^2\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx\\
&= \frac{2d^3}{3f(d\cot(e+fx))^{3/2}} + \frac{d^3\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)}\,dx, x, d\cot(e+fx)\right)}{f}\\
&= \frac{2d^3}{3f(d\cot(e+fx))^{3/2}} + \frac{(2d^3)\,\mathrm{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}\\
&= \frac{2d^3}{3f(d\cot(e+fx))^{3/2}} + \frac{d^2\,\mathrm{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} + \frac{d^2\,\mathrm{Subst}\left(\int \frac{d}{d^2}\right.}{}\\
&= \frac{2d^3}{3f(d\cot(e+fx))^{3/2}} - \frac{d^{3/2}\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{d^{3/2}\,\mathrm{Su}}{}\\
&= \frac{2d^3}{3f(d\cot(e+fx))^{3/2}} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{d^{3/2}}{}\\
&= -\frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{2d^3}{3f(d\cot(e+f}\end{aligned}$$

Mathematica [C] time = 0.0452526, size = 45, normalized size = 0.21

$$\frac{2\tan^3(e+fx)(d\cot(e+fx))^{3/2}\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\cot^2(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^4,x]
```

```
[Out] (2*(d*Cot[e + f*x])^(3/2)*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[e + f*x]^2]*
Tan[e + f*x]^3)/(3*f)
```

Maple [C] time = 0.184, size = 540, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^(3/2)*tan(f*x+e)^4,x)
```

```
[Out] 1/6/f*2^(1/2)*(cos(f*x+e)-1)*(3*I*cos(f*x+e)*((cos(f*x+e)-1)/sin(f*x+e))^(1
/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))
/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),
1/2-1/2*I,1/2*2^(1/2))-3*I*cos(f*x+e)*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((1
-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*
x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2
*I,1/2*2^(1/2))-3*cos(f*x+e)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((1-cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))
^(1/2)-3*cos(f*x+e)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)
,1/2+1/2*I,1/2*2^(1/2))*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)+2*
cos(f*x+e)*2^(1/2)-2*2^(1/2))*(cos(f*x+e)+1)^2*(d*cos(f*x+e)/sin(f*x+e))^(3
/2)/sin(f*x+e)/cos(f*x+e)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.15299, size = 1472, normalized size = 6.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] -1/12*(12*sqrt(2)*(d^6/f^4)^(1/4)*f*arctan(-(d^6 + sqrt(2)*(d^6/f^4)^(3/4)*
d*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e)) - sqrt(2)*(d^6/f^4)^(3/4)*f^3*sqrt(
(sqrt(2)*(d^6/f^4)^(1/4)*d*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*sin(f*x + e)
 + d^3*cos(f*x + e) + sqrt(d^6/f^4)*f^2*sin(f*x + e))/sin(f*x + e)))/d^6)*c
os(f*x + e)^2 + 12*sqrt(2)*(d^6/f^4)^(1/4)*f*arctan((d^6 - sqrt(2)*(d^6/f^4
)^(3/4)*d*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e)) + sqrt(2)*(d^6/f^4)^(3/4)*f
^3*sqrt(-(sqrt(2)*(d^6/f^4)^(1/4)*d*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*sin
(f*x + e) - d^3*cos(f*x + e) - sqrt(d^6/f^4)*f^2*sin(f*x + e))/sin(f*x + e)
))/d^6)*cos(f*x + e)^2 - 3*sqrt(2)*(d^6/f^4)^(1/4)*f*cos(f*x + e)^2*log((sq
rt(2)*(d^6/f^4)^(1/4)*d*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*sin(f*x + e) +
d^3*cos(f*x + e) + sqrt(d^6/f^4)*f^2*sin(f*x + e))/sin(f*x + e)) + 3*sqrt(2
)*(d^6/f^4)^(1/4)*f*cos(f*x + e)^2*log(-(sqrt(2)*(d^6/f^4)^(1/4)*d*f*sqrt(d
*cos(f*x + e)/sin(f*x + e))*sin(f*x + e) - d^3*cos(f*x + e) - sqrt(d^6/f^4)
*f^2*sin(f*x + e))/sin(f*x + e)) + 8*(d*cos(f*x + e)^2 - d)*sqrt(d*cos(f*x
+ e)/sin(f*x + e)))/(f*cos(f*x + e)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**(3/2)*tan(f*x+e)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cot\left(fx+e\right)\right)^{\frac{3}{2}}\tan\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^(3/2)*tan(f*x + e)^4, x)
```

3.201 $\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx$

Optimal. Leaf size=212

$$\frac{2d^2}{f\sqrt{d\cot(e+fx)}}+\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}-\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right.}{2\sqrt{2}f}$$

```
[Out] -((d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 + (2*d^2)/(f*Sqrt[d*Cot[e + f*x]]) + (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e
+ f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) - (d^(3/2)*Log[Sqrt[d
] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rubi [A] time = 0.173838, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {16, 3474, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{2d^2}{f\sqrt{d\cot(e+fx)}}+\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}-\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right.}{2\sqrt{2}f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^3,x]
```

```
[Out] -((d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 + (2*d^2)/(f*Sqrt[d*Cot[e + f*x]]) + (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e
+ f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) - (d^(3/2)*Log[Sqrt[d
] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx &= d^3\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx\\
&= \frac{2d^2}{f\sqrt{d\cot(e+fx)}} - d\int \sqrt{d\cot(e+fx)}\,dx\\
&= \frac{2d^2}{f\sqrt{d\cot(e+fx)}} + \frac{d^2\,\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f}\\
&= \frac{2d^2}{f\sqrt{d\cot(e+fx)}} + \frac{\left(2d^2\right)\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}\\
&= \frac{2d^2}{f\sqrt{d\cot(e+fx)}} - \frac{d^2\,\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} + \frac{d^2\,\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\right.}{}\\
&= \frac{2d^2}{f\sqrt{d\cot(e+fx)}} + \frac{d^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{d^{3/2}\,\text{Subs}}{}\\
&= \frac{2d^2}{f\sqrt{d\cot(e+fx)}} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{d^{3/2}\log}{}\\
&= -\frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{2d^2}{f\sqrt{d\cot(e+f}}
\end{aligned}$$

Mathematica [C] time = 0.0466503, size = 38, normalized size = 0.18

$$\frac{2d^2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(e+fx)\right)}{f\sqrt{d\cot(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^3,x]
```

```
[Out] (2*d^2*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[e + f*x]^2])/(f*Sqrt[d*Cot[e +
f*x]])
```

Maple [C] time = 0.17, size = 650, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^(3/2)*tan(f*x+e)^3,x)
```

```
[Out] 1/2/f*2^(1/2)*(I*sin(f*x+e)*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((1-cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2
)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(
1/2))-I*sin(f*x+e)*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)*Ellipti
cPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))+sin
(f*x+e)*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))+sin(f*x+e)*((c
os(f*x+e)-1)/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)
*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-2*sin(f*x+e)*((cos(f*x+e)
-1)/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*
x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))^(1/2),1/2*2^(1/2))+2*cos(f*x+e)*2^(1/2)-2*2^(1/2))*(cos(f*x+e)+1
)^2*(cos(f*x+e)-1)*(d*cos(f*x+e)/sin(f*x+e))^(3/2)/sin(f*x+e)^2/cos(f*x+e)^
2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.12071, size = 1474, normalized size = 6.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*(d^6/f^4)^(1/4)*f*arctan(-(sqrt(2)*(d^6/f^4)^(1/4)*d^4*f*sq
rt(d*cos(f*x + e)/sin(f*x + e)) + d^6 - sqrt(2)*(d^6/f^4)^(1/4)*f*sqrt((d^9
*cos(f*x + e) + sqrt(2)*(d^6/f^4)^(3/4)*d^4*f^3*sqrt(d*cos(f*x + e)/sin(f*x
 + e))*sin(f*x + e) + sqrt(d^6/f^4)*d^6*f^2*sin(f*x + e))/sin(f*x + e)))/d^
6)*cos(f*x + e) + 4*sqrt(2)*(d^6/f^4)^(1/4)*f*arctan(-(sqrt(2)*(d^6/f^4)^(1
/4)*d^4*f*sqrt(d*cos(f*x + e)/sin(f*x + e)) - d^6 - sqrt(2)*(d^6/f^4)^(1/4)
*f*sqrt((d^9*cos(f*x + e) - sqrt(2)*(d^6/f^4)^(3/4)*d^4*f^3*sqrt(d*cos(f*x
+ e)/sin(f*x + e))*sin(f*x + e) + sqrt(d^6/f^4)*d^6*f^2*sin(f*x + e))/sin(f
*x + e)))/d^6)*cos(f*x + e) + sqrt(2)*(d^6/f^4)^(1/4)*f*cos(f*x + e)*log((d
^9*cos(f*x + e) + sqrt(2)*(d^6/f^4)^(3/4)*d^4*f^3*sqrt(d*cos(f*x + e)/sin(f
*x + e))*sin(f*x + e) + sqrt(d^6/f^4)*d^6*f^2*sin(f*x + e))/sin(f*x + e)) -
 sqrt(2)*(d^6/f^4)^(1/4)*f*cos(f*x + e)*log((d^9*cos(f*x + e) - sqrt(2)*(d^
6/f^4)^(3/4)*d^4*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*sin(f*x + e) + sqrt(
d^6/f^4)*d^6*f^2*sin(f*x + e))/sin(f*x + e)) - 8*d*sqrt(d*cos(f*x + e)/sin(
f*x + e))*sin(f*x + e))/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**(3/2)*tan(f*x+e)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cot\left(fx+e\right)\right)^{\frac{3}{2}}\tan\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^(3/2)*tan(f*x + e)^3, x)
```

3.202 $\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx$

Optimal. Leaf size=192

$$\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}-\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{d^{3/2}\tan^{-1}}{}$$

```
[Out] (d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) +
(d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]
)/(2*Sqrt[2]*f) - (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqr
t[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rubi [A] time = 0.146697, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {16, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}-\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{d^{3/2}\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^2,x]
```

```
[Out] (d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) +
(d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]
)/(2*Sqrt[2]*f) - (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqr
t[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx = d^2\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx$$

$$= -\frac{d^3\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\left(d^2+x^2\right)}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= -\frac{\left(2d^3\right)\mathrm{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= -\frac{d^2\,\mathrm{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} - \frac{d^2\,\mathrm{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+ \ldots\right)}{f}$$

$$= \frac{d^{3/2}\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{d^{3/2}\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, \ldots\right)}{2\sqrt{2}f}$$

$$= \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+ \ldots\right)}{2\sqrt{\ldots}}$$

$$= \frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{d^{3/2}\log\left(\sqrt{d}+ \ldots\right.}{}$$

Mathematica [A] time = 0.0235415, size = 134, normalized size = 0.7

$$\frac{d^2\sqrt{\cot(e+fx)}\left(\log\left(\cot(e+fx)-\sqrt{2}\sqrt{\cot(e+fx)}+1\right)-\log\left(\cot(e+fx)+\sqrt{2}\sqrt{\cot(e+fx)}+1\right)+2\tan^{-1}\left(1-\sqrt{2}\ldots\right.\right.}{2\sqrt{2}f\sqrt{d\cot(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^2,x]
```

```
[Out] (d^2*Sqrt[Cot[e + f*x]]*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]] - 2*ArcTa
n[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Co
```

```
t[e + f*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]])/(2*Sqrt[
2]*f*Sqrt[d*Cot[e + f*x]])
```

Maple [C] time = 0.152, size = 287, normalized size = 1.5

$$-\frac{\sqrt{2}\left(\cos\left(fx+e\right)+1\right)^{2}\left(\cos\left(fx+e\right)-1\right)}{2f\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{2}}\left(i\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)-i\mathrm{EllipticPi}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^(3/2)*tan(f*x+e)^2,x)
```

```
[Out] -1/2/f*2^(1/2)*(cos(f*x+e)+1)^2*(I*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi(((1-cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-EllipticPi(((1-cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-EllipticPi(((1-cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2)))*((cos(f*x+e)-1)/sin(f
*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*(cos(f*x+e)-1)*(d*cos(f*x+e)/sin(f*x+e))^(3/2)/
sin(f*x+e)/cos(f*x+e)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.25654, size = 1258, normalized size = 6.55

$$\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{d^6+\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{3}{4}}df^3\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}-\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{3}{4}}f^3\sqrt{\frac{\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}}df\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\sin(fx+e)+d^3\cos(fx+e)+\sqrt{\frac{d^6}{f^4}}f^2\sin(fx}{\sin(fx+e)}}}{d^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] sqrt(2)*(d^6/f^4)^(1/4)*arctan(-(d^6 + sqrt(2)*(d^6/f^4)^(3/4)*d*f^3*sqrt(d
*cos(f*x + e)/sin(f*x + e)) - sqrt(2)*(d^6/f^4)^(3/4)*f^3*sqrt((sqrt(2)*(d^
6/f^4)^(1/4)*d*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*sin(f*x + e) + d^3*cos(f
*x + e) + sqrt(d^6/f^4)*f^2*sin(f*x + e))/sin(f*x + e)))/d^6) + sqrt(2)*(d^
6/f^4)^(1/4)*arctan((d^6 - sqrt(2)*(d^6/f^4)^(3/4)*d*f^3*sqrt(d*cos(f*x + e
)/sin(f*x + e)) + sqrt(2)*(d^6/f^4)^(3/4)*f^3*sqrt(-(sqrt(2)*(d^6/f^4)^(1/4
)*d*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*sin(f*x + e) - d^3*cos(f*x + e) - s
qrt(d^6/f^4)*f^2*sin(f*x + e))/sin(f*x + e)))/d^6) - 1/4*sqrt(2)*(d^6/f^4)^
(1/4)*log((sqrt(2)*(d^6/f^4)^(1/4)*d*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*si
n(f*x + e) + d^3*cos(f*x + e) + sqrt(d^6/f^4)*f^2*sin(f*x + e))/sin(f*x + e
)) + 1/4*sqrt(2)*(d^6/f^4)^(1/4)*log(-(sqrt(2)*(d^6/f^4)^(1/4)*d*f*sqrt(d*c
os(f*x + e)/sin(f*x + e))*sin(f*x + e) - d^3*cos(f*x + e) - sqrt(d^6/f^4)*f
^2*sin(f*x + e))/sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**(3/2)*tan(f*x+e)**2,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cot\left(fx+e\right)\right)^{\frac{3}{2}}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^(3/2)*tan(f*x + e)^2, x)
```

3.203 $\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx$

Optimal. Leaf size=192

$$-\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{d^{3/2}\tan^{-}}{}$$

```
[Out] (d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]
)/(2*Sqrt[2]*f) + (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqr
t[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rubi [A] time = 0.137959, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {16, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{d^{3/2}\tan^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x],x]
```

```
[Out] (d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]
)/(2*Sqrt[2]*f) + (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqr
t[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx = d\int\sqrt{d\cot(e+fx)}\,dx$$
$$= -\frac{d^2\,\text{Subst}\left(\int\frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f}$$
$$= -\frac{\left(2d^2\right)\text{Subst}\left(\int\frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$
$$= \frac{d^2\,\text{Subst}\left(\int\frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} - \frac{d^2\,\text{Subst}\left(\int\frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$
$$= -\frac{d^{3/2}\,\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{d^{3/2}\,\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx\right.}{2\sqrt{2}f}$$
$$= -\frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+\right.}{2\sqrt{}}$$
$$= \frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{}\right.}{}$$

Mathematica [C] time = 0.0287894, size = 37, normalized size = 0.19

$$-\frac{2(d\cot(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x],x]
```

```
[Out] (-2*(d*Cot[e + f*x])^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[e + f*x]^2])
/(3*f)
```

Maple [C] time = 0.136, size = 318, normalized size = 1.7

$$-\frac{\sqrt{2}\left(\cos\left(fx+e\right)+1\right)^{2}\left(\cos\left(fx+e\right)-1\right)}{2f\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{2}}\left(i\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)-i\mathrm{EllipticPi}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^(3/2)*tan(f*x+e),x)
```

```
[Out] -1/2/f*2^(1/2)*(cos(f*x+e)+1)^2*(I*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi(((1-cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-2*EllipticF(((1-cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+EllipticPi(((1-cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))+EllipticPi(((1-cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2)))*((cos(f*x+e)-1)/sin(f*x+e)
)^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))^(1/2)*(cos(f*x+e)-1)*(d*cos(f*x+e)/sin(f*x+e))^(3/2)/sin(f
*x+e)/cos(f*x+e)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.89237, size = 1295, normalized size = 6.74

$$\sqrt{2}\left(\frac{d^{6}}{f^{4}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\frac{d^{6}}{f^{4}}\right)^{\frac{1}{4}}d^{4}f\sqrt{\frac{d\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}+d^{6}-\sqrt{2}\left(\frac{d^{6}}{f^{4}}\right)^{\frac{1}{4}}f\sqrt{\frac{d^{9}\cos\left(fx+e\right)+\sqrt{2}\left(\frac{d^{6}}{f^{4}}\right)^{\frac{3}{4}}d^{4}f^{3}\sqrt{\frac{d\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}\sin\left(fx+e\right)+\sqrt{\frac{d^{6}}{f^{4}}}d^{6}f^{2}\sin}{\sin\left(fx+e\right)}}}{d^{6}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] sqrt(2)*(d^6/f^4)^(1/4)*arctan(-(sqrt(2)*(d^6/f^4)^(1/4)*d^4*f*sqrt(d*cos(f
*x + e)/sin(f*x + e)) + d^6 - sqrt(2)*(d^6/f^4)^(1/4)*f*sqrt((d^9*cos(f*x +
 e) + sqrt(2)*(d^6/f^4)^(3/4)*d^4*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*sin
(f*x + e) + sqrt(d^6/f^4)*d^6*f^2*sin(f*x + e))/sin(f*x + e)))/d^6) + sqrt(
2)*(d^6/f^4)^(1/4)*arctan(-(sqrt(2)*(d^6/f^4)^(1/4)*d^4*f*sqrt(d*cos(f*x +
e)/sin(f*x + e)) - d^6 - sqrt(2)*(d^6/f^4)^(1/4)*f*sqrt((d^9*cos(f*x + e) -
 sqrt(2)*(d^6/f^4)^(3/4)*d^4*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*sin(f*x
+ e) + sqrt(d^6/f^4)*d^6*f^2*sin(f*x + e))/sin(f*x + e)))/d^6) + 1/4*sqrt(2
)*(d^6/f^4)^(1/4)*log((d^9*cos(f*x + e) + sqrt(2)*(d^6/f^4)^(3/4)*d^4*f^3*s
qrt(d*cos(f*x + e)/sin(f*x + e))*sin(f*x + e) + sqrt(d^6/f^4)*d^6*f^2*sin(f
*x + e))/sin(f*x + e)) - 1/4*sqrt(2)*(d^6/f^4)^(1/4)*log((d^9*cos(f*x + e)
- sqrt(2)*(d^6/f^4)^(3/4)*d^4*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*sin(f*x
 + e) + sqrt(d^6/f^4)*d^6*f^2*sin(f*x + e))/sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cot\left(e+fx\right)\right)^{\frac{3}{2}}\tan\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**(3/2)*tan(f*x+e),x)
```

```
[Out] Integral((d*cot(e + f*x))**(3/2)*tan(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cot\left(fx+e\right)\right)^{\frac{3}{2}}\tan\left(fx+e\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^(3/2)*tan(f*x + e), x)
```

3.204 $\int (d\cot(e+fx))^{3/2}\,dx$

Optimal. Leaf size=210

$$-\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}-\frac{d^{3/2}\tan}{}$$

```
[Out] -((d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 - (2*d*Sqrt[d*Cot[e + f*x]])/f - (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*
x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) + (d^(3/2)*Log[Sqrt[d] +
Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rubi [A] time = 0.144309, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3473, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}-\frac{d^{3/2}\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] -((d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 - (2*d*Sqrt[d*Cot[e + f*x]])/f - (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*
x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) + (d^(3/2)*Log[Sqrt[d] +
Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d\cot(e+fx))^{3/2}\,dx = -\frac{2d\sqrt{d\cot(e+fx)}}{f} - d^2\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx$$

$$= -\frac{2d\sqrt{d\cot(e+fx)}}{f} + \frac{d^3\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= -\frac{2d\sqrt{d\cot(e+fx)}}{f} + \frac{(2d^3)\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= -\frac{2d\sqrt{d\cot(e+fx)}}{f} + \frac{d^2\,\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} + \frac{d^2\,\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d}\right.}{f}$$

$$= -\frac{2d\sqrt{d\cot(e+fx)}}{f} - \frac{d^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{d^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{2}}{-d+\sqrt{}}\right.}{}$$

$$= -\frac{2d\sqrt{d\cot(e+fx)}}{f} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{d^{3/2}\log\left(\sqrt{d}+\right.}{}$$

$$= -\frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{2d\sqrt{d\cot(e+fx)}}{f} - \frac{d^{3/2}\,\text{l}}{}$$

Mathematica [A] time = 0.131001, size = 159, normalized size = 0.76

$$-\frac{(d\cot(e+fx))^{3/2}\left(8\sqrt{\cot(e+fx)}+\sqrt{2}\log\left(\cot(e+fx)-\sqrt{2}\sqrt{\cot(e+fx)}+1\right)-\sqrt{2}\log\left(\cot(e+fx)+\sqrt{2}\sqrt{\cot(e+\right.\right.}{4f\cot^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] -((d*Cot[e + f*x])^(3/2)*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]]
- 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] + 8*Sqrt[Cot[e + f*x]] +
```

```
Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]] - Sqrt[2]*Log[1
+ Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/(4*f*Cot[e + f*x]^(3/2))
```

Maple [A] time = 0.013, size = 176, normalized size = 0.8

$$-2\,{\frac{d\sqrt{d\cot\left(fx+e\right)}}{f}}-{\frac{d\sqrt{2}}{2\,f}}\sqrt[4]{{d}^{2}}\arctan\left(-\sqrt{2}\sqrt{d\cot\left(fx+e\right)}{\frac{1}{\sqrt[4]{{d}^{2}}}}+1\right)+{\frac{d\sqrt{2}}{4\,f}}\sqrt[4]{{d}^{2}}\ln\left(\left(d\cot\left(fx+e\right)+\sqrt[4]{{d}^{2}}\sqrt{d\cot\left(f\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^(3/2),x)
```

```
[Out] -2*d*(d*cot(f*x+e))^(1/2)/f-1/2/f*d*(d^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(d^
2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)+1/4/f*d*(d^2)^(1/4)*2^(1/2)*ln((d*cot(f*x+
e)+(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)-(d^2
)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+1/2/f*d*(d^2)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cot\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*cot(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cot\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^(3/2), x)
```

3.205 $\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx$

Optimal. Leaf size=211

$$\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}-\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}-\frac{d^{3/2}\tan^{-1}}{}$$

```
[Out] -((d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 - (2*(d*Cot[e + f*x])^(3/2))/(3*f) + (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e
+ f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) - (d^(3/2)*Log[Sqrt[d
] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rubi [A] time = 0.157328, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {16, 3473, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}-\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}-\frac{d^{3/2}\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] -((d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 - (2*(d*Cot[e + f*x])^(3/2))/(3*f) + (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e
+ f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) - (d^(3/2)*Log[Sqrt[d
] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx = \frac{\int (d\cot(e+fx))^{5/2}\,dx}{d}$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3f} - d\int \sqrt{d\cot(e+fx)}\,dx$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3f} + \frac{d^2\,\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3f} + \frac{\left(2d^2\right)\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3f} - \frac{d^2\,\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} + \frac{d^2\,\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4} \cdots\right.}{\cdots}$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3f} + \frac{d^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{d^{3/2}\,\text{Sub}\cdots}{\cdots}$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3f} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{d^{3/2}\cdots}{\cdots}$$

$$= -\frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{2(d\cot(e+fx))\cdots}{3f}$$

Mathematica [C] time = 0.0573496, size = 39, normalized size = 0.18

$$\frac{2(d\cot(e+fx))^{3/2}\left({}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(e+fx)\right)-1\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] (2*(d*Cot[e + f*x])^(3/2)*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Cot[e + f*x
]^2]))/(3*f)
```

Maple [A] time = 0.012, size = 181, normalized size = 0.9

$$-\frac{2}{3f}\left(d\cot\left(fx+e\right)\right)^{\frac{3}{2}}+\frac{d^{2}\sqrt{2}}{4f}\ln\left(\left(d\cot\left(fx+e\right)-\sqrt[4]{d^{2}}\sqrt{d\cot\left(fx+e\right)}\sqrt{2}+\sqrt{d^{2}}\right)\left(d\cot\left(fx+e\right)+\sqrt[4]{d^{2}}\sqrt{d\cot\left(fx\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(d*cot(f*x+e))^(3/2),x)
```

```
[Out] -2/3*(d*cot(f*x+e))^(3/2)/f+1/4/f*d^2/(d^2)^(1/4)*2^(1/2)*ln((d*cot(f*x+e)-
(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)+(d^2)^(
1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+1/2/f*d^2/(d^2)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)-1/2/f*d^2/(d^2)^(1/4)
*2^(1/2)*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cot\left(e+fx\right)\right)^{\frac{3}{2}}\cot\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(d*cot(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*cot(e + f*x))**(3/2)*cot(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cot\left(fx+e\right)\right)^{\frac{3}{2}}\cot\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^(3/2)*cot(f*x + e), x)
```

3.206 $\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx$

Optimal. Leaf size=232

$$\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}-\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{d^{3/2}\tan^{-}}{}$$

```
[Out] (d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) +
(2*d*Sqrt[d*Cot[e + f*x]])/f - (2*(d*Cot[e + f*x])^(5/2))/(5*d*f) + (d^(3/2
)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sq
rt[2]*f) - (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot
[e + f*x]]])/(2*Sqrt[2]*f)
```

Rubi [A] time = 0.196081, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {16, 3473, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}-\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{d^{3/2}\tan^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] (d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) +
(2*d*Sqrt[d*Cot[e + f*x]])/f - (2*(d*Cot[e + f*x])^(5/2))/(5*d*f) + (d^(3/2
)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sq
rt[2]*f) - (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot
[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx = \frac{\int (d\cot(e+fx))^{7/2}\,dx}{d^2}$$

$$= -\frac{2(d\cot(e+fx))^{5/2}}{5df} - \int (d\cot(e+fx))^{3/2}\,dx$$

$$= \frac{2d\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5df} + d^2\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx$$

$$= \frac{2d\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5df} - \frac{d^3\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)}\,dx, x, d\cot(e+\cdots\right.}{f}$$

$$= \frac{2d\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5df} - \frac{(2d^3)\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e\cdots}\right.}{f}$$

$$= \frac{2d\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5df} - \frac{d^2\,\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+f\cdots}\right.}{f}$$

$$= \frac{2d\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5df} + \frac{d^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\,\text{co}\cdots}\right.}{2\sqrt{2}f}$$

$$= \frac{2d\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5df} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{\cdots}\right.}{2\sqrt{2}f}$$

$$= \frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{2d\sqrt{d\cot(e+\cdots}}{f}$$

Mathematica [A] time = 0.27557, size = 172, normalized size = 0.74

$$\frac{(d\cot(e+fx))^{3/2}\left(-8\cot^{\frac{5}{2}}(e+fx)+40\sqrt{\cot(e+fx)}+5\sqrt{2}\log\left(\cot(e+fx)-\sqrt{2}\sqrt{\cot(e+fx)}+1\right)-5\sqrt{2}\log\left(\cot(e+\right.}{20f\cot^{\frac{3}{2}}(e+fx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] ((d*Cot[e + f*x])^(3/2)*(10*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]]
- 10*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] + 40*Sqrt[Cot[e + f*x]]
 - 8*Cot[e + f*x]^(5/2) + 5*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Co
t[e + f*x]] - 5*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]])
)/(20*f*Cot[e + f*x]^(3/2))
```

Maple [A] time = 0.019, size = 194, normalized size = 0.8

$$-\frac{2}{5df}\left(d\cot\left(fx+e\right)\right)^{\frac{5}{2}}+2\,\frac{d\sqrt{d\cot\left(fx+e\right)}}{f}-\frac{d\sqrt{2}}{2f}\sqrt[4]{d^2}\arctan\left(\sqrt{2}\sqrt{d\cot\left(fx+e\right)}\frac{1}{\sqrt[4]{d^2}}+1\right)+\frac{d\sqrt{2}}{2f}\sqrt[4]{d^2}\arctan\left(-\sqrt{\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(d*cot(f*x+e))^(3/2),x)
```

```
[Out] -2/5*(d*cot(f*x+e))^(5/2)/d/f+2*d*(d*cot(f*x+e))^(1/2)/f-1/2/f*d*(d^2)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)+1/2/f*d*(d^2)^
(1/4)*2^(1/2)*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)-1/4/f*d*(
d^2)^(1/4)*2^(1/2)*ln((d*cot(f*x+e)+(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2
)+(d^2)^(1/2))/(d*cot(f*x+e)-(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)
^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cot \left(e+fx\right)\right)^{\frac{3}{2}}\cot ^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(d*cot(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*cot(e + f*x))**(3/2)*cot(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cot \left(fx+e\right)\right)^{\frac{3}{2}}\cot \left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^(3/2)*cot(f*x + e)^2, x)
```

3.207 $\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

Optimal. Leaf size=231

$$\frac{2d^2}{5f(d\cot(e+fx))^{5/2}}-\frac{2}{f\sqrt{d\cot(e+fx)}}-\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}+\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}$$

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) + (2*d
^2)/(5*f*(d*Cot[e + f*x])^(5/2)) - 2/(f*Sqrt[d*Cot[e + f*x]]) - Log[Sqrt[d]
 + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*
f) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*
Sqrt[2]*Sqrt[d]*f)
```

Rubi [A] time = 0.206992, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {16, 3474, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{2d^2}{5f(d\cot(e+fx))^{5/2}}-\frac{2}{f\sqrt{d\cot(e+fx)}}-\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}+\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) + (2*d
^2)/(5*f*(d*Cot[e + f*x])^(5/2)) - 2/(f*Sqrt[d*Cot[e + f*x]]) - Log[Sqrt[d]
 + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*
f) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*
Sqrt[2]*Sqrt[d]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = d^3\int \frac{1}{(d\cot(e+fx))^{7/2}}\,dx$$

$$= \frac{2d^2}{5f(d\cot(e+fx))^{5/2}} - d\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx$$

$$= \frac{2d^2}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{f\sqrt{d\cot(e+fx)}} + \frac{\int \sqrt{d\cot(e+fx)}\,dx}{d}$$

$$= \frac{2d^2}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{f\sqrt{d\cot(e+fx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= \frac{2d^2}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{f\sqrt{d\cot(e+fx)}} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= \frac{2d^2}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{f\sqrt{d\cot(e+fx)}} + \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} - \frac{\text{Subst}\left(\int \frac{d-}{d^2}\right.}{}$$

$$= \frac{2d^2}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{f\sqrt{d\cot(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} - \frac{\text{Subst}}{}$$

$$= \frac{2d^2}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{f\sqrt{d\cot(e+fx)}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} + \frac{\text{lo}}{}$$

$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{2d^2}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{f\sqrt{d\cot(e+fx}}$$

Mathematica [C] time = 0.120259, size = 40, normalized size = 0.17

$$\frac{2d^2\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\cot^2(e+fx)\right)}{5f(d\cot(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] (2*d^2*Hypergeometric2F1[-5/4, 1, -1/4, -Cot[e + f*x]^2])/(5*f*(d*Cot[e + f
*x])^(5/2))
```

Maple [C] time = 0.194, size = 718, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(d*cot(f*x+e))^(1/2),x)
```

```
[Out] 1/10/f*2^(1/2)*(cos(f*x+e)-1)*(5*I*sin(f*x+e)*((cos(f*x+e)-1)/sin(f*x+e))^(
1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e)
)/sin(f*x+e))^(1/2)*cos(f*x+e)^2*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-5*I*sin(f*x+e)*((cos(f*x+e)-1)/sin(f*x
+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(
f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)^2*EllipticPi(((1-cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*sin(f*x+e)*((cos(f*x+e)-1)/sin
(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+
sin(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)^2*EllipticPi(((1-cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))+10*sin(f*x+e)*((cos(f*x+e)-1
)/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+
e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)^2*EllipticF(((1-cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-5*sin(f*x+e)*((cos(f*x+e)-1)/sin(f
*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+si
n(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)^2*EllipticPi(((1-cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-12*cos(f*x+e)^3*2^(1/2)+12*cos
(f*x+e)^2*2^(1/2)+2*cos(f*x+e)*2^(1/2)-2*2^(1/2))*(cos(f*x+e)+1)^2/sin(f*x+
e)^4/cos(f*x+e)^2/(d*cos(f*x+e)/sin(f*x+e))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.80521, size = 1584, normalized size = 6.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/20*(20*sqrt(2)*d*f*(1/(d^2*f^4))^(1/4)*arctan(-sqrt(2)*f*sqrt(d*cos(f*x +
 e)/sin(f*x + e))*(1/(d^2*f^4))^(1/4) + sqrt(2)*f*sqrt((sqrt(2)*d^2*f^3*sqr
t(d*cos(f*x + e)/sin(f*x + e))*(1/(d^2*f^4))^(3/4)*sin(f*x + e) + d^2*f^2*s
qrt(1/(d^2*f^4))*sin(f*x + e) + d*cos(f*x + e))/sin(f*x + e))*(1/(d^2*f^4))
^(1/4) - 1)*cos(f*x + e)^3 + 20*sqrt(2)*d*f*(1/(d^2*f^4))^(1/4)*arctan(-sqr
t(2)*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/(d^2*f^4))^(1/4) + sqrt(2)*f*sq
rt(-(sqrt(2)*d^2*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/(d^2*f^4))^(3/4)*
sin(f*x + e) - d^2*f^2*sqrt(1/(d^2*f^4))*sin(f*x + e) - d*cos(f*x + e))/sin
(f*x + e))*(1/(d^2*f^4))^(1/4) + 1)*cos(f*x + e)^3 + 5*sqrt(2)*d*f*(1/(d^2*
f^4))^(1/4)*cos(f*x + e)^3*log((sqrt(2)*d^2*f^3*sqrt(d*cos(f*x + e)/sin(f*x
 + e))*(1/(d^2*f^4))^(3/4)*sin(f*x + e) + d^2*f^2*sqrt(1/(d^2*f^4))*sin(f*x
 + e) + d*cos(f*x + e))/sin(f*x + e)) - 5*sqrt(2)*d*f*(1/(d^2*f^4))^(1/4)*c
os(f*x + e)^3*log(-(sqrt(2)*d^2*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/(d
^2*f^4))^(3/4)*sin(f*x + e) - d^2*f^2*sqrt(1/(d^2*f^4))*sin(f*x + e) - d*co
s(f*x + e))/sin(f*x + e)) - 8*(6*cos(f*x + e)^2 - 1)*sqrt(d*cos(f*x + e)/si
n(f*x + e))*sin(f*x + e))/(d*f*cos(f*x + e)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3\left(e + fx\right)}{\sqrt{d\cot\left(e + fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(d*cot(f*x+e))**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/sqrt(d*cot(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^3}{\sqrt{d\cot\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^3/sqrt(d*cot(f*x + e)), x)
```

3.208 $\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

Optimal. Leaf size=212

$$\frac{2d}{3f(d\cot(e+fx))^{3/2}}-\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}+\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}$$

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) + (
2*d)/(3*f*(d*Cot[e + f*x])^(3/2)) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sq
rt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f) + Log[Sqrt[d] + Sqrt[d]*C
ot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f)
```

Rubi [A] time = 0.17054, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {16, 3474, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{2d}{3f(d\cot(e+fx))^{3/2}}-\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}+\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) + (
2*d)/(3*f*(d*Cot[e + f*x])^(3/2)) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sq
rt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f) + Log[Sqrt[d] + Sqrt[d]*C
ot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = d^2\int \frac{1}{(d\cot(e+fx))^{5/2}}\,dx$$

$$= \frac{2d}{3f(d\cot(e+fx))^{3/2}} - \int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx$$

$$= \frac{2d}{3f(d\cot(e+fx))^{3/2}} + \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= \frac{2d}{3f(d\cot(e+fx))^{3/2}} + \frac{(2d)\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= \frac{2d}{3f(d\cot(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} + \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e-\right.}{f}$$

$$= \frac{2d}{3f(d\cot(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} + \frac{\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x\right.}{2f}$$

$$= \frac{2d}{3f(d\cot(e+fx))^{3/2}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+\right.}{2\sqrt{}}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{2d}{3f(d\cot(e+fx))^{3/2}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\right.}{}$$

Mathematica [C] time = 0.0946855, size = 38, normalized size = 0.18

$$\frac{2d\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\cot^2(e+fx)\right)}{3f(d\cot(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] (2*d*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[e + f*x]^2])/(3*f*(d*Cot[e + f*x]
)^(3/2))
```

Maple [C] time = 0.179, size = 540, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(d*cot(f*x+e))^(1/2),x)
```

```
[Out] -1/6/f*2^(1/2)*(cos(f*x+e)+1)^2*(cos(f*x+e)-1)*(3*I*cos(f*x+e)*EllipticPi((
(1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos(f*
x+e)-1)/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((co
s(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)-3*I*cos(f*x+e)*EllipticPi(((1-cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((cos(f*x+e)-1)
/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e
)-1+sin(f*x+e))/sin(f*x+e))^(1/2)+3*cos(f*x+e)*EllipticPi(((1-cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos(f*x+e)-1)/sin(f*x+
e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f
*x+e))/sin(f*x+e))^(1/2)+3*cos(f*x+e)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)
*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/si
n(f*x+e))^(1/2)-2*cos(f*x+e)*2^(1/2)+2*2^(1/2))/sin(f*x+e)^3/(d*cos(f*x+e)/
sin(f*x+e))^(1/2)/cos(f*x+e)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.81648, size = 1562, normalized size = 7.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/12*(12*sqrt(2)*d*f*(1/(d^2*f^4))^(1/4)*arctan(-sqrt(2)*d*f^3*sqrt(d*cos(
f*x + e)/sin(f*x + e))*(1/(d^2*f^4))^(3/4) + sqrt(2)*d*f^3*sqrt((d^2*f^2*sq
rt(1/(d^2*f^4))*sin(f*x + e) + sqrt(2)*d*f*sqrt(d*cos(f*x + e)/sin(f*x + e)
)*(1/(d^2*f^4))^(1/4)*sin(f*x + e) + d*cos(f*x + e))/sin(f*x + e))*(1/(d^2*
f^4))^(3/4) - 1)*cos(f*x + e)^2 + 12*sqrt(2)*d*f*(1/(d^2*f^4))^(1/4)*arctan
(-sqrt(2)*d*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/(d^2*f^4))^(3/4) + sqr
t(2)*d*f^3*sqrt((d^2*f^2*sqrt(1/(d^2*f^4))*sin(f*x + e) - sqrt(2)*d*f*sqrt(
d*cos(f*x + e)/sin(f*x + e))*(1/(d^2*f^4))^(1/4)*sin(f*x + e) + d*cos(f*x +
 e))/sin(f*x + e))*(1/(d^2*f^4))^(3/4) + 1)*cos(f*x + e)^2 - 3*sqrt(2)*d*f*
(1/(d^2*f^4))^(1/4)*cos(f*x + e)^2*log((d^2*f^2*sqrt(1/(d^2*f^4))*sin(f*x +
 e) + sqrt(2)*d*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/(d^2*f^4))^(1/4)*sin
(f*x + e) + d*cos(f*x + e))/sin(f*x + e)) + 3*sqrt(2)*d*f*(1/(d^2*f^4))^(1/
4)*cos(f*x + e)^2*log((d^2*f^2*sqrt(1/(d^2*f^4))*sin(f*x + e) - sqrt(2)*d*f
*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/(d^2*f^4))^(1/4)*sin(f*x + e) + d*cos
(f*x + e))/sin(f*x + e)) + 8*(cos(f*x + e)^2 - 1)*sqrt(d*cos(f*x + e)/sin(f
*x + e)))/(d*f*cos(f*x + e)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2\left(e+fx\right)}{\sqrt{d\cot\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(d*cot(f*x+e))**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/sqrt(d*cot(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^2}{\sqrt{d\cot\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/sqrt(d*cot(f*x + e)), x)
```

3.209 $\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

Optimal. Leaf size=209

$$\frac{2}{f\sqrt{d\cot(e+fx)}}+\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}-\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}-$$

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) + 2
/(f*Sqrt[d*Cot[e + f*x]]) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sq
rt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f
*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f)
```

Rubi [A] time = 0.164406, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {16, 3474, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{2}{f\sqrt{d\cot(e+fx)}}+\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}-\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}-$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/Sqrt[d*Cot[e + f*x]],x]
```


```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) + 2
/(f*Sqrt[d*Cot[e + f*x]]) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sq
rt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f
*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f)
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = d\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx$$

$$= \frac{2}{f\sqrt{d\cot(e+fx)}} - \frac{\int \sqrt{d\cot(e+fx)}\,dx}{d}$$

$$= \frac{2}{f\sqrt{d\cot(e+fx)}} + \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= \frac{2}{f\sqrt{d\cot(e+fx)}} + \frac{2\,\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= \frac{2}{f\sqrt{d\cot(e+fx)}} - \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} + \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= \frac{2}{f\sqrt{d\cot(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} + \frac{\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{\right.}{2f}$$

$$= \frac{2}{f\sqrt{d\cot(e+fx)}} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx\right.}{2\sqrt{2}\sqrt{}}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{2}{f\sqrt{d\cot(e+fx)}} + \log\left(\sqrt{d}+\sqrt{d}\,\text{co}\right.$$

Mathematica [C] time = 0.0653089, size = 35, normalized size = 0.17

$$\frac{2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(e+fx)\right)}{f\sqrt{d\cot(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] (2*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[e + f*x]^2])/(f*Sqrt[d*Cot[e + f*x]
])
```

Maple [C] time = 0.174, size = 642, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(d*cot(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*2^(1/2)*(cos(f*x+e)+1)^2*(cos(f*x+e)-1)*(I*sin(f*x+e)*((cos(f*x+e)-1)
/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e
)-1+sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*sin(f*x+e)*((cos(f*x+e)-1)/sin(f*x+
e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f
*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^
(1/2),1/2+1/2*I,1/2*2^(1/2))+sin(f*x+e)*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*(
(1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(
f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1
/2*I,1/2*2^(1/2))+sin(f*x+e)*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((1-cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/
2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^
(1/2))-2*sin(f*x+e)*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*cos(f*x+e)*
2^(1/2)-2*2^(1/2))/sin(f*x+e)^4/(d*cos(f*x+e)/sin(f*x+e))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.7455, size = 1531, normalized size = 7.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*d*f*(1/(d^2*f^4))^(1/4)*arctan(-sqrt(2)*f*sqrt(d*cos(f*x +
e)/sin(f*x + e))*(1/(d^2*f^4))^(1/4) + sqrt(2)*f*sqrt((sqrt(2)*d^2*f^3*sqrt
(d*cos(f*x + e)/sin(f*x + e))*(1/(d^2*f^4))^(3/4)*sin(f*x + e) + d^2*f^2*sq
rt(1/(d^2*f^4))*sin(f*x + e) + d*cos(f*x + e))/sin(f*x + e))*(1/(d^2*f^4))^
(1/4) - 1)*cos(f*x + e) + 4*sqrt(2)*d*f*(1/(d^2*f^4))^(1/4)*arctan(-sqrt(2)
*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/(d^2*f^4))^(1/4) + sqrt(2)*f*sqrt(-
(sqrt(2)*d^2*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/(d^2*f^4))^(3/4)*sin(
f*x + e) - d^2*f^2*sqrt(1/(d^2*f^4))*sin(f*x + e) - d*cos(f*x + e))/sin(f*x
 + e))*(1/(d^2*f^4))^(1/4) + 1)*cos(f*x + e) + sqrt(2)*d*f*(1/(d^2*f^4))^(1
/4)*cos(f*x + e)*log((sqrt(2)*d^2*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/
(d^2*f^4))^(3/4)*sin(f*x + e) + d^2*f^2*sqrt(1/(d^2*f^4))*sin(f*x + e) + d*
cos(f*x + e))/sin(f*x + e)) - sqrt(2)*d*f*(1/(d^2*f^4))^(1/4)*cos(f*x + e)*
log(-(sqrt(2)*d^2*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/(d^2*f^4))^(3/4)
*sin(f*x + e) - d^2*f^2*sqrt(1/(d^2*f^4))*sin(f*x + e) - d*cos(f*x + e))/si
n(f*x + e)) - 8*sqrt(d*cos(f*x + e)/sin(f*x + e))*sin(f*x + e))/(d*f*cos(f*
x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(e + fx\right)}{\sqrt{d\cot\left(e + fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(d*cot(f*x+e))**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)/sqrt(d*cot(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)}{\sqrt{d\cot\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)/sqrt(d*cot(f*x + e)), x)
```

3.210 $\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx$

Optimal. Leaf size=192

$$\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}-\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot}}{\sqrt{d}}\right.}{\sqrt{2}\sqrt{d}f}$$

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) + Log[
Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*S
qrt[d]*f) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x
]]]/(2*Sqrt[2]*Sqrt[d]*f)
```

Rubi [A] time = 0.111457, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}-\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot}}{\sqrt{d}}\right.}{\sqrt{2}\sqrt{d}f}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) + Log[
Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*S
qrt[d]*f) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x
]]]/(2*Sqrt[2]*Sqrt[d]*f)
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(d^2+x^2\right)}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= -\frac{(2d)\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} - \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} - \frac{\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} + \cdots$$

$$= \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+f\cdots}\right.}{2\sqrt{2}\sqrt{d}f}$$

$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot\cdots}\right.}{2\sqrt{2}\sqrt{d}f}$$

Mathematica [A] time = 0.0146639, size = 131, normalized size = 0.68

$$\frac{\sqrt{\cot(e+fx)}\left(\log\left(\cot(e+fx)-\sqrt{2}\sqrt{\cot(e+fx)}+1\right)-\log\left(\cot(e+fx)+\sqrt{2}\sqrt{\cot(e+fx)}+1\right)+2\tan^{-1}\left(1-\sqrt{2}\sqrt{\cdots}\right.\right.}{2\sqrt{2}f\sqrt{d\cot(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] (Sqrt[Cot[e + f*x]]*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]] - 2*ArcTan[1
+ Sqrt[2]*Sqrt[Cot[e + f*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e
+ f*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/(2*Sqrt[2]*f
*Sqrt[d*Cot[e + f*x]])
```

Maple [A] time = 0.023, size = 166, normalized size = 0.9

$$-\frac{\sqrt{2}}{4fd}\sqrt[4]{d^2}\ln\left(\left(d\cot\left(fx+e\right)+\sqrt[4]{d^2}\sqrt{d\cot\left(fx+e\right)}\sqrt{2}+\sqrt{d^2}\right)\left(d\cot\left(fx+e\right)-\sqrt[4]{d^2}\sqrt{d\cot\left(fx+e\right)}\sqrt{2}+\sqrt{d^2}\right)^{-1}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*cot(f*x+e))^(1/2),x)
```

```
[Out] -1/4/f/d*(d^2)^(1/4)*2^(1/2)*ln((d*cot(f*x+e)+(d^2)^(1/4)*(d*cot(f*x+e))^(1
/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)-(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(
1/2)+(d^2)^(1/2)))-1/2/f/d*(d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2)^(1/4)*(
d*cot(f*x+e))^(1/2)+1)+1/2/f/d*(d^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(d^2)^(1
/4)*(d*cot(f*x+e))^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d\cot\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cot(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/sqrt(d*cot(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d\cot\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(d*cot(f*x + e)), x)
```

3.211 $\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

Optimal. Leaf size=192

$$-\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}+\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\cdot}}{}\right.}{\sqrt{2}\sqrt{d}}$$

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) - Log[
Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*S
qrt[d]*f) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x
]]]/(2*Sqrt[2]*Sqrt[d]*f)
```

Rubi [A] time = 0.129909, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {16, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}+\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\cdot}}{}\right.}{\sqrt{2}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) - Log[
Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*S
qrt[d]*f) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x
]]]/(2*Sqrt[2]*Sqrt[d]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{\int \sqrt{d\cot(e+fx)}\,dx}{d}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= -\frac{2\operatorname{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} - \frac{\operatorname{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} - \frac{\operatorname{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} -$$

$$= -\frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e-}\right.}{2\sqrt{2}\sqrt{d}f}$$

$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\,\text{co}}\right.}{2\sqrt{2}\sqrt{d}f}$$

Mathematica [C] time = 0.0276018, size = 40, normalized size = 0.21

$$-\frac{2(d\cot(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(e+fx)\right)}{3d^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] (-2*(d*Cot[e + f*x])^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[e + f*x]^2])
/(3*d^2*f)
```

Maple [A] time = 0.021, size = 157, normalized size = 0.8

$$-\frac{\sqrt{2}}{4f}\ln\left(\left(d\cot\left(fx+e\right)-\sqrt[4]{d^{2}}\sqrt{d\cot\left(fx+e\right)}\sqrt{2}+\sqrt{d^{2}}\right)\left(d\cot\left(fx+e\right)+\sqrt[4]{d^{2}}\sqrt{d\cot\left(fx+e\right)}\sqrt{2}+\sqrt{d^{2}}\right)^{-1}\right)\frac{1}{\sqrt[4]{d^{2}}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(d*cot(f*x+e))^(1/2),x)
```

```
[Out] -1/4/f/(d^2)^(1/4)*2^(1/2)*ln((d*cot(f*x+e)-(d^2)^(1/4)*(d*cot(f*x+e))^(1/2
)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)+(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/
2)+(d^2)^(1/2)))-1/2/f/(d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2)^(1/4)*(d*co
t(f*x+e))^(1/2)+1)+1/2/f/(d^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(d^2)^(1/4)*(d
*cot(f*x+e))^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(e+fx\right)}{\sqrt{d\cot\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(d*cot(f*x+e))**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)/sqrt(d*cot(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)}{\sqrt{d\cot\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)/sqrt(d*cot(f*x + e)), x)
```

3.212 $\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

Optimal. Leaf size=212

$$-\frac{2\sqrt{d\cot(e+fx)}}{df}-\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}+\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}$$

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) - (
2*Sqrt[d*Cot[e + f*x]])/(d*f) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2
]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f) + Log[Sqrt[d] + Sqrt[d]*Cot[e
 + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f)
```

Rubi [A] time = 0.162575, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {16, 3473, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{2\sqrt{d\cot(e+fx)}}{df}-\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}+\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/Sqrt[d*Cot[e + f*x]],x]
```


```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) - (
2*Sqrt[d*Cot[e + f*x]])/(d*f) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2
]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f) + Log[Sqrt[d] + Sqrt[d]*Cot[e
 + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f)
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx &= \frac{\int (d\cot(e+fx))^{3/2}\,dx}{d^2}\\
&= -\frac{2\sqrt{d\cot(e+fx)}}{df} - \int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx\\
&= -\frac{2\sqrt{d\cot(e+fx)}}{df} + \frac{d\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\left(d^2+x^2\right)}\,dx, x, d\cot(e+fx)\right)}{f}\\
&= -\frac{2\sqrt{d\cot(e+fx)}}{df} + \frac{(2d)\,\mathrm{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}\\
&= -\frac{2\sqrt{d\cot(e+fx)}}{df} + \frac{\mathrm{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} + \frac{\mathrm{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+
\end{aligned}
$$

$$
= -\frac{2\sqrt{d\cot(e+fx)}}{df} + \frac{\mathrm{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} + \frac{\mathrm{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx, x,
$$

$$
= -\frac{2\sqrt{d\cot(e+fx)}}{df} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+f
$$

$$
= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{2\sqrt{d\cot(e+fx)}}{df} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot
$$

Mathematica [A] time = 0.159294, size = 159, normalized size = 0.75

$$
-\frac{\sqrt{\cot(e+fx)}\left(8\sqrt{\cot(e+fx)}+\sqrt{2}\log\left(\cot(e+fx)-\sqrt{2}\sqrt{\cot(e+fx)}+1\right)-\sqrt{2}\log\left(\cot(e+fx)+\sqrt{2}\sqrt{\cot(e+fx)}\right.\right.}{4f\sqrt{d\cot(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] -(Sqrt[Cot[e + f*x]]*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]] - 2*
Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] + 8*Sqrt[Cot[e + f*x]] + Sqr
t[2]*Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]] - Sqrt[2]*Log[1 + S
qrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/(4*f*Sqrt[d*Cot[e + f*x]])
```

Maple [A] time = 0.027, size = 184, normalized size = 0.9

$$-2\,\frac{\sqrt{d\cot\left(fx+e\right)}}{df}-\frac{\sqrt{2}}{2\,df}\sqrt[4]{d^2}\arctan\left(-\sqrt{2}\sqrt{d\cot\left(fx+e\right)}\frac{1}{\sqrt[4]{d^2}}+1\right)+\frac{\sqrt{2}}{4\,df}\sqrt[4]{d^2}\ln\left(\left(d\cot\left(fx+e\right)+\sqrt[4]{d^2}\sqrt{d\cot\left(f\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(d*cot(f*x+e))^(1/2),x)
```

```
[Out] -2*(d*cot(f*x+e))^(1/2)/d/f-1/2/f/d*(d^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(d^
2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)+1/4/f/d*(d^2)^(1/4)*2^(1/2)*ln((d*cot(f*x+
e)+(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)-(d^2
)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+1/2/f/d*(d^2)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2\left(e+fx\right)}{\sqrt{d\cot\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(d*cot(f*x+e))**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/sqrt(d*cot(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^2}{\sqrt{d\cot\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^2/sqrt(d*cot(f*x + e)), x)
```

3.213 $\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

Optimal. Leaf size=214

$$-\frac{2(d\cot(e+fx))^{3/2}}{3d^2f}+\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}-\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\right.}{2\sqrt{2}\sqrt{d}f}$$

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) - (
2*(d*Cot[e + f*x])^(3/2))/(3*d^2*f) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] -
Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f) - Log[Sqrt[d] + Sqrt[d]
*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f)
```

Rubi [A] time = 0.159854, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {16, 3473, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2(d\cot(e+fx))^{3/2}}{3d^2f}+\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}-\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\right.}{2\sqrt{2}\sqrt{d}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/Sqrt[d*Cot[e + f*x]],x]
```


```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) - (
2*(d*Cot[e + f*x])^(3/2))/(3*d^2*f) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] -
Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f) - Log[Sqrt[d] + Sqrt[d]
*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f)
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx &= \frac{\int (d\cot(e+fx))^{5/2}\,dx}{d^3} \\
&= -\frac{2(d\cot(e+fx))^{3/2}}{3d^2f} - \frac{\int \sqrt{d\cot(e+fx)}\,dx}{d} \\
&= -\frac{2(d\cot(e+fx))^{3/2}}{3d^2f} + \frac{\operatorname{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f} \\
&= -\frac{2(d\cot(e+fx))^{3/2}}{3d^2f} + \frac{2\operatorname{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} \\
&= -\frac{2(d\cot(e+fx))^{3/2}}{3d^2f} - \frac{\operatorname{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} + \frac{\operatorname{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e
\end{aligned}
$$

$$
= -\frac{2(d\cot(e+fx))^{3/2}}{3d^2f} + \frac{\operatorname{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} + \frac{\operatorname{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}x+x^2}\,dx
$$

$$
= -\frac{2(d\cot(e+fx))^{3/2}}{3d^2f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e
$$

$$
= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{2(d\cot(e+fx))^{3/2}}{3d^2f} + \frac{\log\left(\sqrt{d}+\sqrt{d}
$$

Mathematica [C] time = 0.0611562, size = 47, normalized size = 0.22

$$
\frac{2\cot^2(e+fx)\left({}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(e+fx)\right)-1\right)}{3f\sqrt{d\cot(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] (2*Cot[e + f*x]^2*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Cot[e + f*x]^2]))/(
3*f*Sqrt[d*Cot[e + f*x]])
```

Maple [A] time = 0.028, size = 175, normalized size = 0.8

$$-\frac{2}{3\,d^2 f}\left(d\cot\left(fx+e\right)\right)^{\frac{3}{2}}+\frac{\sqrt{2}}{4\,f}\ln\left(\left(d\cot\left(fx+e\right)-\sqrt[4]{d^2}\sqrt{d\cot\left(fx+e\right)}\sqrt{2}+\sqrt{d^2}\right)\left(d\cot\left(fx+e\right)+\sqrt[4]{d^2}\sqrt{d\cot\left(fx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(d*cot(f*x+e))^(1/2),x)
```

```
[Out] -2/3*(d*cot(f*x+e))^(3/2)/d^2/f+1/4/f/(d^2)^(1/4)*2^(1/2)*ln((d*cot(f*x+e)-
(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)+(d^2)^(
1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+1/2/f/(d^2)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)-1/2/f/(d^2)^(1/4)*2^(1/2)
*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^3\left(e+fx\right)}{\sqrt{d\cot\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(d*cot(f*x+e))**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/sqrt(d*cot(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^3}{\sqrt{d\cot\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^3/sqrt(d*cot(f*x + e)), x)
```

3.214 $\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=232

$$-\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f}+\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f}$$

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) + (2*d
)/(5*f*(d*Cot[e + f*x])^(5/2)) - 2/(d*f*Sqrt[d*Cot[e + f*x]]) - Log[Sqrt[d]
 + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*
f) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*
Sqrt[2]*d^(3/2)*f)
```

Rubi [A] time = 0.209284, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {16, 3474, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f}+\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(d*Cot[e + f*x])^(3/2),x]
```


```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) + (2*d
)/(5*f*(d*Cot[e + f*x])^(5/2)) - 2/(d*f*Sqrt[d*Cot[e + f*x]]) - Log[Sqrt[d]
 + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*
f) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*
Sqrt[2]*d^(3/2)*f)
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = d^2\int \frac{1}{(d\cot(e+fx))^{7/2}}\,dx$$
$$= \frac{2d}{5f(d\cot(e+fx))^{5/2}} - \int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx$$
$$= \frac{2d}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{df\sqrt{d\cot(e+fx)}} + \frac{\int \sqrt{d\cot(e+fx)}\,dx}{d^2}$$
$$= \frac{2d}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{df\sqrt{d\cot(e+fx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{df}$$
$$= \frac{2d}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{df\sqrt{d\cot(e+fx)}} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df}$$
$$= \frac{2d}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{df\sqrt{d\cot(e+fx)}} + \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df} - \frac{\text{Subst}\left(\int\right.}{}$$
$$= \frac{2d}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{df\sqrt{d\cot(e+fx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\text{Su}}{}$$
$$= \frac{2d}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{df\sqrt{d\cot(e+fx)}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} +$$
$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{2d}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{df\sqrt{d\cot(e+}}$$

Mathematica [C] time = 0.164548, size = 38, normalized size = 0.16

$$\frac{2d\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\cot^2(e+fx)\right)}{5f(d\cot(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] (2*d*Hypergeometric2F1[-5/4, 1, -1/4, -Cot[e + f*x]^2])/(5*f*(d*Cot[e + f*x
])^(5/2))
```

Maple [C] time = 0.176, size = 718, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(d*cot(f*x+e))^(3/2),x)
```

```
[Out] 1/10/f*2^(1/2)*(cos(f*x+e)-1)*(5*I*sin(f*x+e)*((cos(f*x+e)-1)/sin(f*x+e))^(
1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e)
)/sin(f*x+e))^(1/2)*cos(f*x+e)^2*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-5*I*sin(f*x+e)*((cos(f*x+e)-1)/sin(f*x
+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(
f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)^2*EllipticPi(((1-cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*sin(f*x+e)*((cos(f*x+e)-1)/sin
(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+
sin(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)^2*EllipticPi(((1-cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))+10*sin(f*x+e)*((cos(f*x+e)-1
)/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+
e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)^2*EllipticF(((1-cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-5*sin(f*x+e)*((cos(f*x+e)-1)/sin(f
*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+si
n(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)^2*EllipticPi(((1-cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-12*cos(f*x+e)^3*2^(1/2)+12*cos
(f*x+e)^2*2^(1/2)+2*cos(f*x+e)*2^(1/2)-2*2^(1/2))*(cos(f*x+e)+1)^2/(d*cos(f
*x+e)/sin(f*x+e))^(3/2)/sin(f*x+e)^5/cos(f*x+e)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.78413, size = 1608, normalized size = 6.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/20*(20*sqrt(2)*d^2*f*(1/(d^6*f^4))^(1/4)*arctan(-sqrt(2)*d*f*sqrt(d*cos(f
*x + e)/sin(f*x + e))*(1/(d^6*f^4))^(1/4) + sqrt(2)*d*f*sqrt((sqrt(2)*d^5*f
^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/(d^6*f^4))^(3/4)*sin(f*x + e) + d^4
*f^2*sqrt(1/(d^6*f^4))*sin(f*x + e) + d*cos(f*x + e))/sin(f*x + e))*(1/(d^6
*f^4))^(1/4) - 1)*cos(f*x + e)^3 + 20*sqrt(2)*d^2*f*(1/(d^6*f^4))^(1/4)*arc
tan(-sqrt(2)*d*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/(d^6*f^4))^(1/4) + sq
rt(2)*d*f*sqrt(-(sqrt(2)*d^5*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/(d^6*
f^4))^(3/4)*sin(f*x + e) - d^4*f^2*sqrt(1/(d^6*f^4))*sin(f*x + e) - d*cos(f
*x + e))/sin(f*x + e))*(1/(d^6*f^4))^(1/4) + 1)*cos(f*x + e)^3 + 5*sqrt(2)*
d^2*f*(1/(d^6*f^4))^(1/4)*cos(f*x + e)^3*log((sqrt(2)*d^5*f^3*sqrt(d*cos(f*
x + e)/sin(f*x + e))*(1/(d^6*f^4))^(3/4)*sin(f*x + e) + d^4*f^2*sqrt(1/(d^6
*f^4))*sin(f*x + e) + d*cos(f*x + e))/sin(f*x + e)) - 5*sqrt(2)*d^2*f*(1/(d
^6*f^4))^(1/4)*cos(f*x + e)^3*log(-(sqrt(2)*d^5*f^3*sqrt(d*cos(f*x + e)/sin
(f*x + e))*(1/(d^6*f^4))^(3/4)*sin(f*x + e) - d^4*f^2*sqrt(1/(d^6*f^4))*sin
(f*x + e) - d*cos(f*x + e))/sin(f*x + e)) - 8*(6*cos(f*x + e)^2 - 1)*sqrt(d
*cos(f*x + e)/sin(f*x + e))*sin(f*x + e))/(d^2*f*cos(f*x + e)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2\left(e+fx\right)}{\left(d\cot\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(d*cot(f*x+e))**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/(d*cot(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^{2}}{\left(d\cot\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/(d*cot(f*x + e))^(3/2), x)
```

3.215 $\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=211

$$-\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f}+\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot}}{}\right.}{\sqrt{2}d^{3/2}f}$$

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) + 2
/(3*f*(d*Cot[e + f*x])^(3/2)) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2
]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f) + Log[Sqrt[d] + Sqrt[d]*Cot[e
 + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f)
```

Rubi [A] time = 0.173529, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {16, 3474, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f}+\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot}}{}\right.}{\sqrt{2}d^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(d*Cot[e + f*x])^(3/2),x]
```


```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) + 2
/(3*f*(d*Cot[e + f*x])^(3/2)) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2
]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f) + Log[Sqrt[d] + Sqrt[d]*Cot[e
 + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f)
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = d\int \frac{1}{(d\cot(e+fx))^{5/2}}\,dx$$

$$= \frac{2}{3f(d\cot(e+fx))^{3/2}} - \frac{\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx}{d}$$

$$= \frac{2}{3f(d\cot(e+fx))^{3/2}} + \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{x}\left(d^2+x^2\right)}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= \frac{2}{3f(d\cot(e+fx))^{3/2}} + \frac{2\operatorname{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= \frac{2}{3f(d\cot(e+fx))^{3/2}} + \frac{\operatorname{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df} + \frac{\operatorname{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(\right.}{df}$$

$$= \frac{2}{3f(d\cot(e+fx))^{3/2}} - \frac{\operatorname{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\operatorname{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\right.}{2\sqrt{2}d}$$

$$= \frac{2}{3f(d\cot(e+fx))^{3/2}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e\right.}{2}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{2}{3f(d\cot(e+fx))^{3/2}} - \frac{\log\left(\sqrt{d}+\right.}{}$$

Mathematica [C] time = 0.0288565, size = 37, normalized size = 0.18

$$\frac{2\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\cot^2(e+fx)\right)}{3f(d\cot(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] (2*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[e + f*x]^2])/(3*f*(d*Cot[e + f*x])^
(3/2))
```

Maple [C] time = 0.155, size = 532, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(d*cot(f*x+e))^(3/2),x)
```

```
[Out] 1/6/f*2^(1/2)*(cos(f*x+e)-1)*(3*I*cos(f*x+e)*((cos(f*x+e)-1)/sin(f*x+e))^(1
/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))
/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),
1/2-1/2*I,1/2*2^(1/2))-3*I*cos(f*x+e)*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((1
-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*
x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2
*I,1/2*2^(1/2))-3*cos(f*x+e)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((1-cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))
^(1/2)-3*cos(f*x+e)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)
,1/2+1/2*I,1/2*2^(1/2))*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)+2*
cos(f*x+e)*2^(1/2)-2*2^(1/2))*(cos(f*x+e)+1)^2/sin(f*x+e)^4/(d*cos(f*x+e)/s
in(f*x+e))^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.76665, size = 1597, normalized size = 7.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/12*(12*sqrt(2)*d^2*f*(1/(d^6*f^4))^(1/4)*arctan(-sqrt(2)*d^4*f^3*sqrt(d*
cos(f*x + e)/sin(f*x + e))*(1/(d^6*f^4))^(3/4) + sqrt(2)*d^4*f^3*sqrt((d^4*
f^2*sqrt(1/(d^6*f^4))*sin(f*x + e) + sqrt(2)*d^2*f*sqrt(d*cos(f*x + e)/sin(
f*x + e))*(1/(d^6*f^4))^(1/4)*sin(f*x + e) + d*cos(f*x + e))/sin(f*x + e))*
(1/(d^6*f^4))^(3/4) - 1)*cos(f*x + e)^2 + 12*sqrt(2)*d^2*f*(1/(d^6*f^4))^(1
/4)*arctan(-sqrt(2)*d^4*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/(d^6*f^4))
^(3/4) + sqrt(2)*d^4*f^3*sqrt((d^4*f^2*sqrt(1/(d^6*f^4))*sin(f*x + e) - sqr
t(2)*d^2*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/(d^6*f^4))^(1/4)*sin(f*x +
e) + d*cos(f*x + e))/sin(f*x + e))*(1/(d^6*f^4))^(3/4) + 1)*cos(f*x + e)^2
- 3*sqrt(2)*d^2*f*(1/(d^6*f^4))^(1/4)*cos(f*x + e)^2*log((d^4*f^2*sqrt(1/(d
^6*f^4))*sin(f*x + e) + sqrt(2)*d^2*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/
(d^6*f^4))^(1/4)*sin(f*x + e) + d*cos(f*x + e))/sin(f*x + e)) + 3*sqrt(2)*d
^2*f*(1/(d^6*f^4))^(1/4)*cos(f*x + e)^2*log((d^4*f^2*sqrt(1/(d^6*f^4))*sin(
f*x + e) - sqrt(2)*d^2*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/(d^6*f^4))^(1
/4)*sin(f*x + e) + d*cos(f*x + e))/sin(f*x + e)) + 8*(cos(f*x + e)^2 - 1)*s
qrt(d*cos(f*x + e)/sin(f*x + e)))/(d^2*f*cos(f*x + e)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(e+fx\right)}{\left(d\cot\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(d*cot(f*x+e))**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)/(d*cot(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)}{\left(d\cot\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)/(d*cot(f*x + e))^(3/2), x)
```

3.216 $\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=212

$$\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f}-\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot}}{\sqrt{d}}\right.}{\sqrt{2}d^{3/2}f}$$

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) + 2
/(d*f*Sqrt[d*Cot[e + f*x]]) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*
Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f) - Log[Sqrt[d] + Sqrt[d]*Cot[e +
 f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f)
```

Rubi [A] time = 0.142489, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3474, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f}-\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot}}{\sqrt{d}}\right.}{\sqrt{2}d^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^(-3/2),x]
```


```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) + 2
/(d*f*Sqrt[d*Cot[e + f*x]]) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*
Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f) - Log[Sqrt[d] + Sqrt[d]*Cot[e +
 f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f)
```


Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx = \frac{2}{df\sqrt{d\cot(e+fx)}} - \frac{\int \sqrt{d\cot(e+fx)}\,dx}{d^2}$$

$$= \frac{2}{df\sqrt{d\cot(e+fx)}} + \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{df}$$

$$= \frac{2}{df\sqrt{d\cot(e+fx)}} + \frac{2\,\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df}$$

$$= \frac{2}{df\sqrt{d\cot(e+fx)}} - \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df} + \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+}\right.}{df}$$

$$= \frac{2}{df\sqrt{d\cot(e+fx)}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx\right.}{2\sqrt{2}d^{3/}}$$

$$= \frac{2}{df\sqrt{d\cot(e+fx)}} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+\right.}{2\sqrt{}}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{2}{df\sqrt{d\cot(e+fx)}} + \frac{\log\left(\sqrt{d}+\sqrt{d}\right.}{}$$

Mathematica [C] time = 0.0519924, size = 38, normalized size = 0.18

$$\frac{2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(e+fx)\right)}{df\sqrt{d\cot(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cot[e + f*x])^(-3/2),x]
```

```
[Out] (2*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[e + f*x]^2])/(d*f*Sqrt[d*Cot[e + f*
x]])
```

Maple [A] time = 0.017, size = 184, normalized size = 0.9

$$2\,\frac{1}{df\sqrt{d\cot\left(fx+e\right)}}+\frac{\sqrt{2}}{4\,df}\ln\left(\left(d\cot\left(fx+e\right)-\sqrt[4]{d^2}\sqrt{d\cot\left(fx+e\right)}\sqrt{2}+\sqrt{d^2}\right)\left(d\cot\left(fx+e\right)+\sqrt[4]{d^2}\sqrt{d\cot\left(fx+e\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*cot(f*x+e))^(3/2),x)
```

```
[Out] 2/d/f/(d*cot(f*x+e))^(1/2)+1/4/f/d/(d^2)^(1/4)*2^(1/2)*ln((d*cot(f*x+e)-(d^
2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)+(d^2)^(1/4
)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+1/2/f/d/(d^2)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)-1/2/f/d/(d^2)^(1/4)*2^(1/2
)*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\cot\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cot(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*cot(e + f*x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\cot\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^(-3/2), x)
```

3.217 $\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=192

$$\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f}-\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\,c}}{}\right.}{\sqrt{2}d^{3/2}f}$$

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) + Log[
Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d
^(3/2)*f) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x
]]]/(2*Sqrt[2]*d^(3/2)*f)
```

Rubi [A] time = 0.13238, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {16, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f}-\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\,c}}{}\right.}{\sqrt{2}d^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) + Log[
Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d
^(3/2)*f) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x
]]]/(2*Sqrt[2]*d^(3/2)*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

IntegerQ[n]

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(d^2+x^2\right)}\,dx, x, d\cot(e+fx)\right)}{f}$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df} - \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df}$$
$$= \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f}$$
$$= \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e}{2\sqrt{2}d^{3/2}f}$$
$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d}}{2\sqrt{2}d^{3/2}f}$$

Mathematica [A] time = 0.0289426, size = 134, normalized size = 0.7

$$\frac{\sqrt{\cot(e+fx)}\left(\log\left(\cot(e+fx)-\sqrt{2}\sqrt{\cot(e+fx)}+1\right)-\log\left(\cot(e+fx)+\sqrt{2}\sqrt{\cot(e+fx)}+1\right)+2\tan^{-1}\left(1-\sqrt{2}\sqrt{}\right.\right.}{2\sqrt{2}df\sqrt{d\cot(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] (Sqrt[Cot[e + f*x]]*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]] - 2*ArcTan[1
+ Sqrt[2]*Sqrt[Cot[e + f*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e
```

```
+ f*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/(2*Sqrt[2]*d
*f*Sqrt[d*Cot[e + f*x]])
```

Maple [A] time = 0.014, size = 166, normalized size = 0.9

$$-\frac{\sqrt{2}}{4\,fd^2}\sqrt[4]{d^2}\ln\left(\left(d\cot\left(fx+e\right)+\sqrt[4]{d^2}\sqrt{d\cot\left(fx+e\right)}\sqrt{2}+\sqrt{d^2}\right)\left(d\cot\left(fx+e\right)-\sqrt[4]{d^2}\sqrt{d\cot\left(fx+e\right)}\sqrt{2}+\sqrt{d^2}\right)^{-1}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(d*cot(f*x+e))^(3/2),x)
```

```
[Out] -1/4/f*(d^2)^(1/4)/d^2*2^(1/2)*ln((d*cot(f*x+e)+(d^2)^(1/4)*(d*cot(f*x+e))^
(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)-(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2
^(1/2)+(d^2)^(1/2)))-1/2/f*(d^2)^(1/4)/d^2*2^(1/2)*arctan(2^(1/2)/(d^2)^(1/
4)*(d*cot(f*x+e))^(1/2)+1)+1/2/f*(d^2)^(1/4)/d^2*2^(1/2)*arctan(-2^(1/2)/(d
^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(e + fx)}{(d\cot(e + fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(d*cot(f*x+e))**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)/(d*cot(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(fx + e)}{(d\cot(fx + e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)/(d*cot(f*x + e))^(3/2), x)
```

3.218 $\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=192

$$-\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f}+\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cdots}}{}\right.}{\sqrt{2}d^{3/2}f}$$

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) - Log[
Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d
^(3/2)*f) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x
]]]/(2*Sqrt[2]*d^(3/2)*f)
```

Rubi [A] time = 0.13169, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {16, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f}+\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cdots}}{}\right.}{\sqrt{2}d^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) - Log[
Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d
^(3/2)*f) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x
]]]/(2*Sqrt[2]*d^(3/2)*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx &= \frac{\int \sqrt{d\cot(e+fx)}\,dx}{d^2}\\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{df}\\
&= -\frac{2\,\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df}\\
&= \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df} - \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df}\\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f}\\
&= -\frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e\right.}{2\sqrt{2}d^{3/2}f}\\
&= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\,c\right.}{2\sqrt{2}d^{3/2}f}
\end{aligned}$$

Mathematica [C] time = 0.0094438, size = 40, normalized size = 0.21

$$-\frac{2(d\cot(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(e+fx)\right)}{3d^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] (-2*(d*Cot[e + f*x])^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[e + f*x]^2])
/(3*d^3*f)
```

Maple [A] time = 0.02, size = 166, normalized size = 0.9

$$-\frac{\sqrt{2}}{4fd}\ln\left(\left(d\cot\left(fx+e\right)-\sqrt[4]{d^2}\sqrt{d\cot\left(fx+e\right)}\sqrt{2}+\sqrt{d^2}\right)\left(d\cot\left(fx+e\right)+\sqrt[4]{d^2}\sqrt{d\cot\left(fx+e\right)}\sqrt{2}+\sqrt{d^2}\right)^{-1}\right)\frac{1}{\sqrt[4]{d^2}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(d*cot(f*x+e))^(3/2),x)
```

```
[Out] -1/4/f/d/(d^2)^(1/4)*2^(1/2)*ln((d*cot(f*x+e)-(d^2)^(1/4)*(d*cot(f*x+e))^(1
/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)+(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(
1/2)+(d^2)^(1/2)))-1/2/f/d/(d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2)^(1/4)*(
d*cot(f*x+e))^(1/2)+1)+1/2/f/d/(d^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(d^2)^(1
/4)*(d*cot(f*x+e))^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2\left(e+fx\right)}{\left(d\cot\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(d*cot(f*x+e))**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/(d*cot(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^2}{\left(d\cot\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^2/(d*cot(f*x + e))^(3/2), x)
```

3.219 $\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=212

$$-\frac{2\sqrt{d\cot(e+fx)}}{d^2f}-\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f}+\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right.}{2\sqrt{2}d^{3/2}f}$$

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) - (
2*Sqrt[d*Cot[e + f*x]])/(d^2*f) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt
[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f) + Log[Sqrt[d] + Sqrt[d]*Cot
[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f)
```

Rubi [A] time = 0.161977, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {16, 3473, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{2\sqrt{d\cot(e+fx)}}{d^2f}-\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f}+\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right.}{2\sqrt{2}d^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(d*Cot[e + f*x])^(3/2),x]
```


```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) - (
2*Sqrt[d*Cot[e + f*x]])/(d^2*f) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt
[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f) + Log[Sqrt[d] + Sqrt[d]*Cot
[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f)
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\int (d\cot(e+fx))^{3/2}\,dx}{d^3}$$

$$= -\frac{2\sqrt{d\cot(e+fx)}}{d^2 f} - \frac{\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx}{d}$$

$$= -\frac{2\sqrt{d\cot(e+fx)}}{d^2 f} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}(d^2+x^2)}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= -\frac{2\sqrt{d\cot(e+fx)}}{d^2 f} + \frac{2\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= -\frac{2\sqrt{d\cot(e+fx)}}{d^2 f} + \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df} + \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e}\right.}{df}$$

$$= -\frac{2\sqrt{d\cot(e+fx)}}{d^2 f} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,d\right.}{2\sqrt{2}d^{3}}$$

$$= -\frac{2\sqrt{d\cot(e+fx)}}{d^2 f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e\right.}{2}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{2\sqrt{d\cot(e+fx)}}{d^2 f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\right.}{}$$

Mathematica [A] time = 0.151401, size = 159, normalized size = 0.75

$$-\frac{\cot^{\frac{3}{2}}(e+fx)\left(8\sqrt{\cot(e+fx)}+\sqrt{2}\log\left(\cot(e+fx)-\sqrt{2}\sqrt{\cot(e+fx)}+1\right)-\sqrt{2}\log\left(\cot(e+fx)+\sqrt{2}\sqrt{\cot(e+fx)}\right.\right.}{4f(d\cot(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] -(Cot[e + f*x]^(3/2)*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]] - 2*
Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] + 8*Sqrt[Cot[e + f*x]] + Sqr
t[2]*Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]] - Sqrt[2]*Log[1 + S
qrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/(4*f*(d*Cot[e + f*x])^(3/2))
```

Maple [A] time = 0.02, size = 184, normalized size = 0.9

$$-2\frac{\sqrt{d\cot\left(fx+e\right)}}{d^2f}-\frac{\sqrt{2}}{2\,d^2f}\sqrt[4]{d^2}\arctan\left(-\sqrt{2}\sqrt{d\cot\left(fx+e\right)}\frac{1}{\sqrt[4]{d^2}}+1\right)+\frac{\sqrt{2}}{4\,d^2f}\sqrt[4]{d^2}\ln\left(\left(d\cot\left(fx+e\right)+\sqrt[4]{d^2}\sqrt{d\cot\left(f\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(d*cot(f*x+e))^(3/2),x)
```

```
[Out] -2*(d*cot(f*x+e))^(1/2)/d^2/f-1/2/f*(d^2)^(1/4)/d^2*2^(1/2)*arctan(-2^(1/2)
/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)+1/4/f*(d^2)^(1/4)/d^2*2^(1/2)*ln((d*co
t(f*x+e)+(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e
)-(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+1/2/f*(d^2)^(1/4)/
d^2*2^(1/2)*arctan(2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^3\left(e+fx\right)}{\left(d\cot\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(d*cot(f*x+e))**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/(d*cot(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^3}{\left(d\cot\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^3/(d*cot(f*x + e))^(3/2), x)
```

3.220 $\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=214

$$-\frac{2(d\cot(e+fx))^{3/2}}{3d^3f}+\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f}-\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right.}{2\sqrt{2}d^{3/2}f}$$

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) - (
2*(d*Cot[e + f*x])^(3/2))/(3*d^3*f) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] -
Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f) - Log[Sqrt[d] + Sqrt[d]
*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f)
```

Rubi [A] time = 0.161316, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {16, 3473, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2(d\cot(e+fx))^{3/2}}{3d^3f}+\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f}-\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right.}{2\sqrt{2}d^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) - (
2*(d*Cot[e + f*x])^(3/2))/(3*d^3*f) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] -
Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f) - Log[Sqrt[d] + Sqrt[d]
*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\int (d\cot(e+fx))^{5/2}\,dx}{d^4}$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3d^3f} - \frac{\int \sqrt{d\cot(e+fx)}\,dx}{d^2}$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3d^3f} + \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{df}$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3d^3f} + \frac{2\,\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df}$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3d^3f} - \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df} + \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(}\right.}{df}$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3d^3f} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\right.}{2\sqrt{2}d}$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3d^3f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e\right.}{2}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{2(d\cot(e+fx))^{3/2}}{3d^3f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\right.}{}$$

Mathematica [C] time = 0.0825041, size = 47, normalized size = 0.22

$$\frac{2\cot^3(e+fx)\left({}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(e+fx)\right)-1\right)}{3f(d\cot(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] (2*Cot[e + f*x]^3*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Cot[e + f*x]^2]))/(
3*f*(d*Cot[e + f*x])^(3/2))
```

Maple [A] time = 0.017, size = 184, normalized size = 0.9

$$-\frac{2}{3\,d^{3}f}\left(d\cot\left(fx+e\right)\right)^{\frac{3}{2}}+\frac{\sqrt{2}}{4\,fd}\ln\left(\left(d\cot\left(fx+e\right)-\sqrt[4]{d^{2}}\sqrt{d\cot\left(fx+e\right)}\sqrt{2}+\sqrt{d^{2}}\right)\left(d\cot\left(fx+e\right)+\sqrt[4]{d^{2}}\sqrt{d\cot\left(fx\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(d*cot(f*x+e))^(3/2),x)
```

```
[Out] -2/3*(d*cot(f*x+e))^(3/2)/d^3/f+1/4/f/d/(d^2)^(1/4)*2^(1/2)*ln((d*cot(f*x+e
)-(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)+(d^2)
^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+1/2/f/d/(d^2)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)-1/2/f/d/(d^2)^(1/4)*2
^(1/2)*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^4\left(e+fx\right)}{\left(d\cot\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(d*cot(f*x+e))**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/(d*cot(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^4}{\left(d\cot\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^4/(d*cot(f*x + e))^(3/2), x)
```

3.221 $\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=234

$$-\frac{2(d\cot(e+fx))^{5/2}}{5d^4f}+\frac{2\sqrt{d\cot(e+fx)}}{d^2f}+\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f}-\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\right.}{2\sqrt{2}d}$$

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) + (2*S
qrt[d*Cot[e + f*x]])/(d^2*f) - (2*(d*Cot[e + f*x])^(5/2))/(5*d^4*f) + Log[S
qrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^
(3/2)*f) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]
]]/(2*Sqrt[2]*d^(3/2)*f)
```

Rubi [A] time = 0.188422, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {16, 3473, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{2(d\cot(e+fx))^{5/2}}{5d^4f}+\frac{2\sqrt{d\cot(e+fx)}}{d^2f}+\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}d^{3/2}f}-\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\right.}{2\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(d*Cot[e + f*x])^(3/2),x]
```


```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) + (2*S
qrt[d*Cot[e + f*x]])/(d^2*f) - (2*(d*Cot[e + f*x])^(5/2))/(5*d^4*f) + Log[S
qrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^
(3/2)*f) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]
]]/(2*Sqrt[2]*d^(3/2)*f)
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\int (d\cot(e+fx))^{7/2}\,dx}{d^5}$$

$$= -\frac{2(d\cot(e+fx))^{5/2}}{5d^4f} - \frac{\int (d\cot(e+fx))^{3/2}\,dx}{d^3}$$

$$= \frac{2\sqrt{d\cot(e+fx)}}{d^2f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^4f} + \frac{\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx}{d}$$

$$= \frac{2\sqrt{d\cot(e+fx)}}{d^2f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^4f} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(d^2+x^2\right)}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= \frac{2\sqrt{d\cot(e+fx)}}{d^2f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^4f} - \frac{2\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= \frac{2\sqrt{d\cot(e+fx)}}{d^2f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^4f} - \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{2\sqrt{d\cot(e+fx)}}{d^2f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^4f} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\text{Su}}{}$$

$$= \frac{2\sqrt{d\cot(e+fx)}}{d^2f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^4f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} -$$

$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{2\sqrt{d\cot(e+fx)}}{d^2f} - \frac{2(d\cot(e+fx)}{5d^4f}$$

Mathematica [A] time = 0.292425, size = 172, normalized size = 0.74

$$\frac{\cot^{\frac{3}{2}}(e+fx)\left(-8\cot^{\frac{5}{2}}(e+fx)+40\sqrt{\cot(e+fx)}+5\sqrt{2}\log\left(\cot(e+fx)-\sqrt{2}\sqrt{\cot(e+fx)}+1\right)-5\sqrt{2}\log\left(\cot(e+fx\right.\right.}{20f(d\cot(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] (Cot[e + f*x]^(3/2)*(10*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]] - 10
*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] + 40*Sqrt[Cot[e + f*x]] - 8
*Cot[e + f*x]^(5/2) + 5*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e
+ f*x]] - 5*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/(2
0*f*(d*Cot[e + f*x])^(3/2))
```

Maple [A] time = 0.016, size = 202, normalized size = 0.9

$$-\frac{2}{5d^4f}\left(d\cot\left(fx+e\right)\right)^{\frac{5}{2}}+2\frac{\sqrt{d\cot\left(fx+e\right)}}{d^2f}+\frac{\sqrt{2}}{2d^2f}\sqrt[4]{d^2}\arctan\left(-\sqrt{2}\sqrt{d\cot\left(fx+e\right)}\frac{1}{\sqrt[4]{d^2}}+1\right)-\frac{\sqrt{2}}{4d^2f}\sqrt[4]{d^2}\ln\left(\left(d\cot\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(d*cot(f*x+e))^(3/2),x)
```

```
[Out] -2/5*(d*cot(f*x+e))^(5/2)/d^4/f+2*(d*cot(f*x+e))^(1/2)/d^2/f+1/2/f*(d^2)^(1
/4)/d^2*2^(1/2)*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)-1/4/f*(
d^2)^(1/4)/d^2*2^(1/2)*ln((d*cot(f*x+e)+(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^
(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)-(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(
d^2)^(1/2)))-1/2/f*(d^2)^(1/4)/d^2*2^(1/2)*arctan(2^(1/2)/(d^2)^(1/4)*(d*co
t(f*x+e))^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^{5}\left(e+fx\right)}{\left(d\cot\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(d*cot(f*x+e))**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**5/(d*cot(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^{5}}{\left(d\cot\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^5/(d*cot(f*x + e))^(3/2), x)
```

3.222 $\int \cot^m(e+fx)\tan^n(e+fx)\,dx$

Optimal. Leaf size=62

$$\frac{\cot^m(e+fx)\tan^{n+1}(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(-m+n+1);\frac{1}{2}(-m+n+3);-\tan^2(e+fx)\right)}{f(-m+n+1)}$$

```
[Out] (Cot[e + f*x]^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Tan[e +
 f*x]^2]*Tan[e + f*x]^(1 + n))/(f*(1 - m + n))
```

Rubi [A] time = 0.0582763, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2604, 3476, 364}

$$\frac{\cot^m(e+fx)\tan^{n+1}(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(-m+n+1);\frac{1}{2}(-m+n+3);-\tan^2(e+fx)\right)}{f(-m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^m*Tan[e + f*x]^n,x]
```

```
[Out] (Cot[e + f*x]^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Tan[e +
 f*x]^2]*Tan[e + f*x]^(1 + n))/(f*(1 - m + n))
```

Rule 2604

```
Int[(cot[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^m, Int[(b*Tan[e +
 f*x])^(n - m), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[m] &&
 !IntegerQ[n]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
```

```
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \cot^m(e+fx)\tan^n(e+fx)\,dx &= \left(\cot^m(e+fx)\tan^m(e+fx)\right)\int \tan^{-m+n}(e+fx)\,dx\\ &= \frac{\left(\cot^m(e+fx)\tan^m(e+fx)\right)\operatorname{Subst}\left(\int \frac{x^{-m+n}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\cot^m(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(1-m+n);\frac{1}{2}(3-m+n);-\tan^2(e+fx)\right)\tan^{1+n}(e+fx)}{f(1-m+n)}\end{aligned}$$

Mathematica [A] time = 0.0743462, size = 62, normalized size = 1.

$$\frac{\cot^m(e+fx)\tan^{n+1}(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(-m+n+1);\frac{1}{2}(-m+n+3);-\tan^2(e+fx)\right)}{f(-m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^m*Tan[e + f*x]^n,x]
```

```
[Out] (Cot[e + f*x]^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Tan[e +
 f*x]^2]*Tan[e + f*x]^(1 + n))/(f*(1 - m + n))
```

Maple [F] time = 0.444, size = 0, normalized size = 0.

$$\int \left(\cot\left(fx+e\right)\right)^m\left(\tan\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^m*tan(f*x+e)^n,x)
```

```
[Out] int(cot(f*x+e)^m*tan(f*x+e)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cot \left(fx+e\right)^{m} \tan \left(fx+e\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^m*tan(f*x+e)^n,x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^m*tan(f*x + e)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\cot \left(fx+e\right)^{m} \tan \left(fx+e\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^m*tan(f*x+e)^n,x, algorithm="fricas")
```

```
[Out] integral(cot(f*x + e)^m*tan(f*x + e)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \tan^{n}\left(e+fx\right) \cot^{m}\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**m*tan(f*x+e)**n,x)
```

```
[Out] Integral(tan(e + f*x)**n*cot(e + f*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cot \left(fx+e\right)^{m} \tan \left(fx+e\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^m*tan(f*x+e)^n,x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^m*tan(f*x + e)^n, x)
```

3.223 $\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx$

Optimal. Leaf size=67

$$\frac{\cot^m(e+fx)(b\tan(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{1}{2}(-m+n+1);\frac{1}{2}(-m+n+3);-\tan^2(e+fx)\right)}{bf(-m+n+1)}$$

```
[Out] (Cot[e + f*x]^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Tan[e +
 f*x]^2]*(b*Tan[e + f*x])^(1 + n))/(b*f*(1 - m + n))
```

Rubi [A] time = 0.067043, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2604, 3476, 364}

$$\frac{\cot^m(e+fx)(b\tan(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{1}{2}(-m+n+1);\frac{1}{2}(-m+n+3);-\tan^2(e+fx)\right)}{bf(-m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] (Cot[e + f*x]^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Tan[e +
 f*x]^2]*(b*Tan[e + f*x])^(1 + n))/(b*f*(1 - m + n))
```

Rule 2604

```
Int[(cot[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^m, Int[(b*Tan[e +
 f*x])^(n - m), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[m] &&
 !IntegerQ[n]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
```

```
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx = \left(\cot^m(e+fx)(b\tan(e+fx))^m\right)\int (b\tan(e+fx))^{-m+n}\,dx$$
$$= \frac{\left(b\cot^m(e+fx)(b\tan(e+fx))^m\right)\operatorname{Subst}\left(\int \frac{x^{-m+n}}{b^2+x^2}\,dx, x, b\tan(e+fx)\right)}{f}$$
$$= \frac{\cot^m(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(1-m+n);\frac{1}{2}(3-m+n);-\tan^2(e+fx)\right)(b\tan(e+fx))^{1}}{bf(1-m+n)}$$

Mathematica [A] time = 0.0663491, size = 64, normalized size = 0.96

$$\frac{\cot^{m-1}(e+fx)(b\tan(e+fx))^n\,{}_2F_1\left(1,\frac{1}{2}(-m+n+1);\frac{1}{2}(-m+n+3);-\tan^2(e+fx)\right)}{f(-m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] (Cot[e + f*x]^(-1 + m)*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -
Tan[e + f*x]^2]*(b*Tan[e + f*x])^n)/(f*(1 - m + n))
```

Maple [F] time = 0.307, size = 0, normalized size = 0.

$$\int \left(\cot\left(fx+e\right)\right)^m \left(b\tan\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^m*(b*tan(f*x+e))^n,x)
```

```
[Out] int(cot(f*x+e)^m*(b*tan(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)\right)^{n}\cot\left(fx+e\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^n*cot(f*x + e)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\tan\left(fx+e\right)\right)^{n}\cot\left(fx+e\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e))^n*cot(f*x + e)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(e+fx\right)\right)^{n}\cot^{m}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**m*(b*tan(f*x+e))**n,x)
```

```
[Out] Integral((b*tan(e + f*x))**n*cot(e + f*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)\right)^{n}\cot\left(fx+e\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^m*(b*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^n*cot(f*x + e)^m, x)
```

3.224 $\int (a\cot(e+fx))^m \tan^n(e+fx)\,dx$

Optimal. Leaf size=64

$$\frac{\tan^{n+1}(e+fx)(a\cot(e+fx))^m\,{}_2F_1\left(1,\frac{1}{2}(-m+n+1);\frac{1}{2}(-m+n+3);-\tan^2(e+fx)\right)}{f(-m+n+1)}$$

```
[Out] ((a*Cot[e + f*x])^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Tan
[e + f*x]^2]*Tan[e + f*x]^(1 + n))/(f*(1 - m + n))
```

Rubi [A] time = 0.059477, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2604, 3476, 364}

$$\frac{\tan^{n+1}(e+fx)(a\cot(e+fx))^m\,{}_2F_1\left(1,\frac{1}{2}(-m+n+1);\frac{1}{2}(-m+n+3);-\tan^2(e+fx)\right)}{f(-m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[e + f*x])^m*Tan[e + f*x]^n,x]
```

```
[Out] ((a*Cot[e + f*x])^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Tan
[e + f*x]^2]*Tan[e + f*x]^(1 + n))/(f*(1 - m + n))
```

Rule 2604

```
Int[(cot[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^m, Int[(b*Tan[e +
 f*x])^(n - m), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
```

```
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (a\cot(e+fx))^m \tan^n(e+fx)\,dx = \left((a\cot(e+fx))^m \tan^m(e+fx)\right)\int \tan^{-m+n}(e+fx)\,dx$$
$$= \frac{\left((a\cot(e+fx))^m \tan^m(e+fx)\right)\operatorname{Subst}\left(\int \frac{x^{-m+n}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{(a\cot(e+fx))^m\, {}_2F_1\left(1, \frac{1}{2}(1-m+n); \frac{1}{2}(3-m+n); -\tan^2(e+fx)\right)\tan^{1+n}(e+f}{f(1-m+n)}$$

Mathematica [A] time = 0.0621316, size = 64, normalized size = 1.

$$\frac{\tan^{n+1}(e+fx)(a\cot(e+fx))^m\, {}_2F_1\left(1, \frac{1}{2}(-m+n+1); \frac{1}{2}(-m+n+3); -\tan^2(e+fx)\right)}{f(-m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[e + f*x])^m*Tan[e + f*x]^n,x]
```

```
[Out] ((a*Cot[e + f*x])^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Tan
[e + f*x]^2]*Tan[e + f*x]^(1 + n))/(f*(1 - m + n))
```

Maple [F] time = 0.372, size = 0, normalized size = 0.

$$\int \left(a\cot\left(fx+e\right)\right)^m \left(\tan\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cot(f*x+e))^m*tan(f*x+e)^n,x)
```

```
[Out] int((a*cot(f*x+e))^m*tan(f*x+e)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cot\left(fx+e\right)\right)^{m}\tan\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(f*x+e))^m*tan(f*x+e)^n,x, algorithm="maxima")
```

```
[Out] integrate((a*cot(f*x + e))^m*tan(f*x + e)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\cot\left(fx+e\right)\right)^{m}\tan\left(fx+e\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(f*x+e))^m*tan(f*x+e)^n,x, algorithm="fricas")
```

```
[Out] integral((a*cot(f*x + e))^m*tan(f*x + e)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cot\left(e+fx\right)\right)^{m}\tan^{n}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(f*x+e))**m*tan(f*x+e)**n,x)
```

```
[Out] Integral((a*cot(e + f*x))**m*tan(e + f*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cot\left(fx+e\right)\right)^{m}\tan\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(f*x+e))^m*tan(f*x+e)^n,x, algorithm="giac")
```

```
[Out] integrate((a*cot(f*x + e))^m*tan(f*x + e)^n, x)
```

3.225 $\int (a\cot(e+fx))^m (b\tan(e+fx))^n\,dx$

Optimal. Leaf size=69

$$\frac{(a\cot(e+fx))^m(b\tan(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{1}{2}(-m+n+1);\frac{1}{2}(-m+n+3);-\tan^2(e+fx)\right)}{bf(-m+n+1)}$$

```
[Out] ((a*Cot[e + f*x])^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Tan
[e + f*x]^2]*(b*Tan[e + f*x])^(1 + n))/(b*f*(1 - m + n))
```

Rubi [A] time = 0.0687408, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2604, 3476, 364}

$$\frac{(a\cot(e+fx))^m(b\tan(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{1}{2}(-m+n+1);\frac{1}{2}(-m+n+3);-\tan^2(e+fx)\right)}{bf(-m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] ((a*Cot[e + f*x])^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Tan
[e + f*x]^2]*(b*Tan[e + f*x])^(1 + n))/(b*f*(1 - m + n))
```

Rule 2604

```
Int[(cot[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^m, Int[(b*Tan[e +
 f*x])^(n - m), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[m] &&
 !IntegerQ[n]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
```

```
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (a\cot(e+fx))^m(b\tan(e+fx))^n\,dx = \left((a\cot(e+fx))^m(b\tan(e+fx))^m\right)\int (b\tan(e+fx))^{-m+n}\,dx$$
$$= \frac{\left(b(a\cot(e+fx))^m(b\tan(e+fx))^m\right)\text{Subst}\left(\int \frac{x^{-m+n}}{b^2+x^2}\,dx,x,b\tan(e+fx)\right)}{f}$$
$$= \frac{(a\cot(e+fx))^m\,{}_2F_1\left(1,\frac{1}{2}(1-m+n);\frac{1}{2}(3-m+n);-\tan^2(e+fx)\right)(b\tan(e+$$
$$bf(1-m+n)$$

Mathematica [A] time = 0.0900358, size = 67, normalized size = 0.97

$$\frac{a(a\cot(e+fx))^{m-1}(b\tan(e+fx))^n\,{}_2F_1\left(1,\frac{1}{2}(-m+n+1);\frac{1}{2}(-m+n+3);-\tan^2(e+fx)\right)}{f(-m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] (a*(a*Cot[e + f*x])^(-1 + m)*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n
)/2, -Tan[e + f*x]^2]*(b*Tan[e + f*x])^n)/(f*(1 - m + n))
```

Maple [F] time = 0.326, size = 0, normalized size = 0.

$$\int \left(a\cot\left(fx+e\right)\right)^m\left(b\tan\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cot(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

```
[Out] int((a*cot(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cot\left(fx+e\right)\right)^m \left(b\tan\left(fx+e\right)\right)^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*cot(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\cot\left(fx+e\right)\right)^m \left(b\tan\left(fx+e\right)\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*cot(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cot\left(e+fx\right)\right)^m \left(b\tan\left(e+fx\right)\right)^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(f*x+e))**m*(b*tan(f*x+e))**n,x)
```

```
[Out] Integral((a*cot(e + f*x))**m*(b*tan(e + f*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cot\left(fx+e\right)\right)^m \left(b\tan\left(fx+e\right)\right)^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="giac")

[Out] integrate((a*cot(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

3.226 $\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal. Leaf size=67

$$\frac{2(d\tan(e+fx))^{11/2}}{11d^5f}+\frac{4(d\tan(e+fx))^{7/2}}{7d^3f}+\frac{2(d\tan(e+fx))^{3/2}}{3df}$$

```
[Out] (2*(d*Tan[e + f*x])^(3/2))/(3*d*f) + (4*(d*Tan[e + f*x])^(7/2))/(7*d^3*f) +
 (2*(d*Tan[e + f*x])^(11/2))/(11*d^5*f)
```

Rubi [A] time = 0.0538728, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2607, 270}

$$\frac{2(d\tan(e+fx))^{11/2}}{11d^5f}+\frac{4(d\tan(e+fx))^{7/2}}{7d^3f}+\frac{2(d\tan(e+fx))^{3/2}}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*(d*Tan[e + f*x])^(3/2))/(3*d*f) + (4*(d*Tan[e + f*x])^(7/2))/(7*d^3*f) +
 (2*(d*Tan[e + f*x])^(11/2))/(11*d^5*f)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{\text{Subst}\left(\int \sqrt{dx}\left(1+x^2\right)^2\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(\sqrt{dx}+\frac{2(dx)^{5/2}}{d^2}+\frac{(dx)^{9/2}}{d^4}\right)dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{2(d\tan(e+fx))^{3/2}}{3df}+\frac{4(d\tan(e+fx))^{7/2}}{7d^3f}+\frac{2(d\tan(e+fx))^{11/2}}{11d^5f}$$

Mathematica [A] time = 0.187972, size = 52, normalized size = 0.78

$$\frac{2(28\cos(2(e+fx))+4\cos(4(e+fx))+45)\sec^4(e+fx)(d\tan(e+fx))^{3/2}}{231df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*(45 + 28*Cos[2*(e + f*x)] + 4*Cos[4*(e + f*x)])*Sec[e + f*x]^4*(d*Tan[e
+ f*x])^(3/2))/(231*d*f)
```

Maple [A] time = 0.256, size = 60, normalized size = 0.9

$$\frac{\left(64\left(\cos\left(fx+e\right)\right)^4+48\left(\cos\left(fx+e\right)\right)^2+42\right)\sin\left(fx+e\right)}{231\,f\left(\cos\left(fx+e\right)\right)^5}\sqrt{\frac{d\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(d*tan(f*x+e))^(1/2),x)
```

```
[Out] 2/231/f*(32*cos(f*x+e)^4+24*cos(f*x+e)^2+21)*(d*sin(f*x+e)/cos(f*x+e))^(1/2
)*sin(f*x+e)/cos(f*x+e)^5
```

Maxima [A] time = 1.11092, size = 69, normalized size = 1.03

$$\frac{2\left(21\left(d\tan\left(fx+e\right)\right)^{\frac{11}{2}}+66\left(d\tan\left(fx+e\right)\right)^{\frac{7}{2}}d^2+77\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}d^4\right)}{231\,d^5f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/231*(21*(d*tan(f*x + e))^(11/2) + 66*(d*tan(f*x + e))^(7/2)*d^2 + 77*(d*t
an(f*x + e))^(3/2)*d^4)/(d^5*f)
```

Fricas [A] time = 1.86404, size = 159, normalized size = 2.37

$$\frac{2\left(32\cos\left(fx+e\right)^{4}+24\cos\left(fx+e\right)^{2}+21\right)\sqrt{\frac{d\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)}{231\,f\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/231*(32*cos(f*x + e)^4 + 24*cos(f*x + e)^2 + 21)*sqrt(d*sin(f*x + e)/cos(
f*x + e))*sin(f*x + e)/(f*cos(f*x + e)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29671, size = 111, normalized size = 1.66

$$\frac{2\left(21\sqrt{d\tan\left(fx+e\right)}d^{5}\tan\left(fx+e\right)^{5}+66\sqrt{d\tan\left(fx+e\right)}d^{5}\tan\left(fx+e\right)^{3}+77\sqrt{d\tan\left(fx+e\right)}d^{5}\tan\left(fx+e\right)\right)}{231\,d^{5}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 2/231*(21*sqrt(d*tan(f*x + e))*d^5*tan(f*x + e)^5 + 66*sqrt(d*tan(f*x + e))
*d^5*tan(f*x + e)^3 + 77*sqrt(d*tan(f*x + e))*d^5*tan(f*x + e))/(d^5*f)
```

3.227 $\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal. Leaf size=45

$$\frac{2(d\tan(e+fx))^{7/2}}{7d^3f}+\frac{2(d\tan(e+fx))^{3/2}}{3df}$$

```
[Out] (2*(d*Tan[e + f*x])^(3/2))/(3*d*f) + (2*(d*Tan[e + f*x])^(7/2))/(7*d^3*f)
```

Rubi [A] time = 0.0449009, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2607, 14}

$$\frac{2(d\tan(e+fx))^{7/2}}{7d^3f}+\frac{2(d\tan(e+fx))^{3/2}}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*(d*Tan[e + f*x])^(3/2))/(3*d*f) + (2*(d*Tan[e + f*x])^(7/2))/(7*d^3*f)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{\text{Subst}\left(\int \sqrt{dx}\left(1+x^2\right)\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(\sqrt{dx}+\frac{(dx)^{5/2}}{d^2}\right)\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{2(d\tan(e+fx))^{3/2}}{3df} + \frac{2(d\tan(e+fx))^{7/2}}{7d^3f}$$

Mathematica [A] time = 0.141476, size = 34, normalized size = 0.76

$$\frac{2\left(3\sec^2(e+fx)+4\right)(d\tan(e+fx))^{3/2}}{21df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*(4 + 3*Sec[e + f*x]^2)*(d*Tan[e + f*x])^(3/2))/(21*d*f)
```

Maple [A] time = 0.163, size = 50, normalized size = 1.1

$$\frac{\left(8\left(\cos\left(fx+e\right)\right)^2+6\right)\sin\left(fx+e\right)}{21\,f\left(\cos\left(fx+e\right)\right)^3}\sqrt{\frac{d\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(d*tan(f*x+e))^(1/2),x)
```

```
[Out] 2/21/f*(4*cos(f*x+e)^2+3)*(d*sin(f*x+e)/cos(f*x+e))^(1/2)*sin(f*x+e)/cos(f*
x+e)^3
```

Maxima [A] time = 0.973216, size = 49, normalized size = 1.09

$$\frac{2\left(3\left(d\tan\left(fx+e\right)\right)^{\frac{7}{2}}+7\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}d^2\right)}{21\,d^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/21*(3*(d*tan(f*x + e))^(7/2) + 7*(d*tan(f*x + e))^(3/2)*d^2)/(d^3*f)
```

Fricas [A] time = 1.71276, size = 128, normalized size = 2.84

$$\frac{2\left(4\cos\left(fx+e\right)^{2}+3\right)\sqrt{\frac{d\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)}{21\,f\cos\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*(4*cos(f*x + e)^2 + 3)*sqrt(d*sin(f*x + e)/cos(f*x + e))*sin(f*x + e)/
(f*cos(f*x + e)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{d\tan\left(e+fx\right)}\sec^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*tan(e + f*x))*sec(e + f*x)**4, x)
```

Giac [A] time = 1.14318, size = 77, normalized size = 1.71

$$\frac{2\left(3\sqrt{d\tan\left(fx+e\right)}d^{3}\tan\left(fx+e\right)^{3}+7\sqrt{d\tan\left(fx+e\right)}d^{3}\tan\left(fx+e\right)\right)}{21\,d^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 2/21*(3*sqrt(d*tan(f*x + e))*d^3*tan(f*x + e)^3 + 7*sqrt(d*tan(f*x + e))*d^
3*tan(f*x + e))/(d^3*f)
```

3.228 $\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal. Leaf size=22

$$\frac{2(d\tan(e+fx))^{3/2}}{3df}$$

```
[Out] (2*(d*Tan[e + f*x])^(3/2))/(3*d*f)
```

Rubi [A] time = 0.0371223, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2607, 32}

$$\frac{2(d\tan(e+fx))^{3/2}}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*(d*Tan[e + f*x])^(3/2))/(3*d*f)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \sqrt{dx}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{2(d\tan(e+fx))^{3/2}}{3df}\end{aligned}$$

Mathematica [A] time = 0.0355374, size = 22, normalized size = 1.

$$\frac{2(d\tan(e+fx))^{3/2}}{3df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*(d*Tan[e + f*x])^(3/2))/(3*d*f)
```

Maple [A] time = 0.03, size = 19, normalized size = 0.9

$$\frac{2}{3\,df}\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(d*tan(f*x+e))^(1/2),x)
```

```
[Out] 2/3*(d*tan(f*x+e))^(3/2)/d/f
```

Maxima [A] time = 0.93431, size = 24, normalized size = 1.09

$$\frac{2\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}}{3\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(d*tan(f*x + e))^(3/2)/(d*f)
```

Fricas [B] time = 1.61369, size = 93, normalized size = 4.23

$$\frac{2\sqrt{\frac{d\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(d*sin(f*x + e)/cos(f*x + e))*sin(f*x + e)/(f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \tan \left(e + f x\right)} \sec^2 \left(e + f x\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*tan(e + f*x))*sec(e + f*x)**2, x)
```

Giac [A] time = 1.18061, size = 31, normalized size = 1.41

$$\frac{2 \sqrt{d \tan \left(f x + e\right)} \tan \left(f x + e\right)}{3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(d*tan(f*x + e))*tan(f*x + e)/f
```

3.229 $\int \sqrt{d \tan(e+fx)}\,dx$

Optimal. Leaf size=192

$$-\frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}f}+\frac{\sqrt{d}\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}$$

```
[Out] -((Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 + (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*x
]]])/(2*Sqrt[2]*f) - (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt[2]*
Sqrt[d*Tan[e + f*x]]])/(2*Sqrt[2]*f)
```

Rubi [A] time = 0.110625, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3476, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}f}+\frac{\sqrt{d}\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] -((Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 + (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*x
]]])/(2*Sqrt[2]*f) - (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt[2]*
Sqrt[d*Tan[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \sqrt{d\tan(e+fx)}\,dx = \frac{d\,\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\tan(e+fx)\right)}{f}$$

$$= \frac{(2d)\,\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{f}$$

$$= -\frac{d\,\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{f} + \frac{d\,\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{f}$$

$$= \frac{\sqrt{d}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{2\sqrt{2}f} + \frac{\sqrt{d}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(e+\right.}{2\sqrt{2}f}$$

$$= \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{2\sqrt{2}f} - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\,t\right.}{2\sqrt{2}f}$$

$$= -\frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(e+f\right.}{2\sqrt{2}}$$

Mathematica [C] time = 0.0371811, size = 40, normalized size = 0.21

$$\frac{2(d\tan(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(e+fx)\right)}{3df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*Hypergeometric2F1[3/4, 1, 7/4, -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(3/2))/
(3*d*f)
```

Maple [A] time = 0.019, size = 160, normalized size = 0.8

$$\frac{d\sqrt{2}}{4f}\ln\left(\left(d\tan(fx+e)-\sqrt[4]{d^2}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}\right)\left(d\tan(fx+e)+\sqrt[4]{d^2}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}\right)^{-1}\right)\frac{1}{\sqrt[4]{d^2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(1/2),x)
```

```
[Out] 1/4/f*d/(d^2)^(1/4)*2^(1/2)*ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/
2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1
/2)+(d^2)^(1/2)))+1/2/f*d/(d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2)^(1/4)*(d
*tan(f*x+e))^(1/2)+1)-1/2/f*d/(d^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(d^2)^(1/
4)*(d*tan(f*x+e))^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.70532, size = 1283, normalized size = 6.68

$$-\sqrt{2}\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}} \arctan\left(-\frac{\sqrt{2}df\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}} - \sqrt{2}f\sqrt{\frac{\sqrt{2}df^3\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{3}{4}}\cos(fx+e)+d^2f^2\sqrt{\frac{d^2}{f^4}}\cos(fx+e)+d^3\sin(fx+e)}{\cos(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}} -}{d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*(d^2/f^4)^(1/4)*arctan(-(sqrt(2)*d*f*sqrt(d*sin(f*x + e)/cos(f*x +
 e))*(d^2/f^4)^(1/4) - sqrt(2)*f*sqrt((sqrt(2)*d*f^3*sqrt(d*sin(f*x + e)/co
s(f*x + e))*(d^2/f^4)^(3/4)*cos(f*x + e) + d^2*f^2*sqrt(d^2/f^4)*cos(f*x +
e) + d^3*sin(f*x + e))/cos(f*x + e))*(d^2/f^4)^(1/4) + d^2)/d^2) - sqrt(2)*
(d^2/f^4)^(1/4)*arctan(-(sqrt(2)*d*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*(d^2
/f^4)^(1/4) - sqrt(2)*f*sqrt(-(sqrt(2)*d*f^3*sqrt(d*sin(f*x + e)/cos(f*x +
e))*(d^2/f^4)^(3/4)*cos(f*x + e) - d^2*f^2*sqrt(d^2/f^4)*cos(f*x + e) - d^3
```

```
*sin(f*x + e))/cos(f*x + e))*(d^2/f^4)^(1/4) - d^2)/d^2) - 1/4*sqrt(2)*(d^2
/f^4)^(1/4)*log((sqrt(2)*d*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*(d^2/f^4)^
(3/4)*cos(f*x + e) + d^2*f^2*sqrt(d^2/f^4)*cos(f*x + e) + d^3*sin(f*x + e))
/cos(f*x + e)) + 1/4*sqrt(2)*(d^2/f^4)^(1/4)*log(-(sqrt(2)*d*f^3*sqrt(d*sin
(f*x + e)/cos(f*x + e))*(d^2/f^4)^(3/4)*cos(f*x + e) - d^2*f^2*sqrt(d^2/f^4
)*cos(f*x + e) - d^3*sin(f*x + e))/cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\tan\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*tan(e + f*x)), x)
```

Giac [A] time = 1.13979, size = 259, normalized size = 1.35

$$\frac{1}{4}d\left(\frac{2\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{d^2f}+\frac{2\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{d^2f}-\frac{\sqrt{2}|d|^{\frac{3}{2}}\log\left(d\tan\left(fx\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*d*(2*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*
sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(d^2*f) + 2*sqrt(2)*abs(d)^(3/2)*arctan
(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))
/(d^2*f) - sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x
 + e))*sqrt(abs(d)) + abs(d))/(d^2*f) + sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x
+ e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(d^2*f))
```

3.230 $\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal. Leaf size=227

$$-\frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f}+\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}f}+\frac{\sqrt{d}\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{8\sqrt{2}f}-$$

```
[Out] -(Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(4*Sqrt[2]*f)
 + (Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(4*Sqrt[2]*
f) + (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f
*x]]])/(8*Sqrt[2]*f) - (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt[2
]*Sqrt[d*Tan[e + f*x]]])/(8*Sqrt[2]*f) + (Cos[e + f*x]^2*(d*Tan[e + f*x])^(
3/2))/(2*d*f)
```

Rubi [A] time = 0.169722, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2607, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f}+\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}f}+\frac{\sqrt{d}\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{8\sqrt{2}f}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] -(Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(4*Sqrt[2]*f)
 + (Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(4*Sqrt[2]*
f) + (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f
*x]]])/(8*Sqrt[2]*f) - (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt[2
]*Sqrt[d*Tan[e + f*x]]])/(8*Sqrt[2]*f) + (Cos[e + f*x]^2*(d*Tan[e + f*x])^(
3/2))/(2*d*f)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{dx}}{\left(1+x^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2df} + \frac{\text{Subst}\left(\int \frac{\sqrt{dx}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{4f}$$

$$= \frac{\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2df} + \frac{\text{Subst}\left(\int \frac{x^2}{1+\frac{x^4}{d^2}}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{2df}$$

$$= \frac{\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2df} - \frac{\text{Subst}\left(\int \frac{d-x^2}{1+\frac{x^4}{d^2}}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{4df} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2df} + \frac{\sqrt{d}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{8\sqrt{2}f} +$$

$$= \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{8\sqrt{2}f} - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx\right.}{8\sqrt{2}f}$$

$$= -\frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f} + \frac{\sqrt{d}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\right.}{}$$

Mathematica [A] time = 0.187389, size = 102, normalized size = 0.45

$$-\frac{\sqrt{\sin(2(e+fx))}\sqrt{d\tan(e+fx)}\left(-2\sqrt{\sin(2(e+fx))}+\csc(e+fx)\sin^{-1}(\cos(e+fx)-\sin(e+fx))+\csc(e+fx)\log\left(\text{s}\right.\right.}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] -((ArcSin[Cos[e + f*x] - Sin[e + f*x]]*Csc[e + f*x] + Csc[e + f*x]*Log[Cos[
e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]]] - 2*Sqrt[Sin[2*(e + f*x)]
])*Sqrt[Sin[2*(e + f*x)]]*Sqrt[d*Tan[e + f*x]])/(8*f)
```

Maple [C] time = 0.183, size = 516, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(d*tan(f*x+e))^(1/2),x)
```

```
[Out] -1/8/f*2^(1/2)*(cos(f*x+e)-1)*(I*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((1-cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))
^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/
2*2^(1/2))-I*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((
1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-((cos(f*x
+e)-1)/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos
(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-((cos(f*x+e)-1)/sin(f*x+e))^(1/
2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/
sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1
/2+1/2*I,1/2*2^(1/2))-2*cos(f*x+e)^2*2^(1/2)+2*cos(f*x+e)*2^(1/2))*(cos(f*x
+e)+1)^2*(d*sin(f*x+e)/cos(f*x+e))^(1/2)/sin(f*x+e)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \tan \left(e + f x\right)} \cos^{2}\left(e + f x\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*tan(e + f*x))*cos(e + f*x)**2, x)
```

Giac [A] time = 1.2925, size = 323, normalized size = 1.42

$$\frac{1}{16}\,d^{3}\left(\frac{8\,\sqrt{d\tan\left(fx+e\right)}\tan\left(fx+e\right)}{\left(d^{2}\tan\left(fx+e\right)^{2}+d^{2}\right)df}+\frac{2\,\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\,\sqrt{d\tan\left(fx+e\right)}\right)}{2\,\sqrt{|d|}}\right)}{d^{4}f}+\frac{2\,\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\,\sqrt{d\tan\right.}\right.}{2\,\sqrt{|d|}}\right.}{d^{4}f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*d^3*(8*sqrt(d*tan(f*x + e))*tan(f*x + e)/((d^2*tan(f*x + e)^2 + d^2)*d
*f) + 2*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*s
qrt(d*tan(f*x + e)))/sqrt(abs(d)))/(d^4*f) + 2*sqrt(2)*abs(d)^(3/2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/
(d^4*f) - sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x
```

```
+ e))*sqrt(abs(d)) + abs(d))/(d^4*f) + sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x +
 e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(d^4*f))
```

3.231 $\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal. Leaf size=107

$$\frac{4\cos(e+fx)(d\tan(e+fx))^{3/2}}{5df}+\frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}-\frac{4\cos(e+fx)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(e+fx)}}{5f\sqrt{\sin(2e+2fx)}}$$

```
[Out] (-4*Cos[e + f*x]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Tan[e + f*x]])/(5*f*Sq
rt[Sin[2*e + 2*f*x]]) + (4*Cos[e + f*x]*(d*Tan[e + f*x])^(3/2))/(5*d*f) + (
2*Sec[e + f*x]*(d*Tan[e + f*x])^(3/2))/(5*d*f)
```

Rubi [A] time = 0.129166, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2613, 2615, 2572, 2639}

$$\frac{4\cos(e+fx)(d\tan(e+fx))^{3/2}}{5df}+\frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}-\frac{4\cos(e+fx)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(e+fx)}}{5f\sqrt{\sin(2e+2fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (-4*Cos[e + f*x]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Tan[e + f*x]])/(5*f*Sq
rt[Sin[2*e + 2*f*x]]) + (4*Cos[e + f*x]*(d*Tan[e + f*x])^(3/2))/(5*d*f) + (
2*Sec[e + f*x]*(d*Tan[e + f*x])^(3/2))/(5*d*f)
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}+\frac{2}{5}\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx$$

$$= \frac{4\cos(e+fx)(d\tan(e+fx))^{3/2}}{5df}+\frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}-\frac{4}{5}\int \cos(e+f$$

$$= \frac{4\cos(e+fx)(d\tan(e+fx))^{3/2}}{5df}+\frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}-\frac{\left(4\sqrt{\cos(e+f\right.}{}$$

$$= \frac{4\cos(e+fx)(d\tan(e+fx))^{3/2}}{5df}+\frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}-\frac{\left(4\cos(e+fx)\right.}{}$$

$$= -\frac{4\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{5f\sqrt{\sin(2e+2fx)}}+\frac{4\cos(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

Mathematica [C] time = 0.42256, size = 102, normalized size = 0.95

$$\frac{2\sqrt{d\tan(e+fx)}\left(3\sqrt{\sec^2(e+fx)}(2\sin(e+fx)+\tan(e+fx)\sec(e+fx))-4\tan(e+fx)\sec(e+fx)\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\text{ta}\right.\right.}{15f\sqrt{\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*Sqrt[d*Tan[e + f*x]]*(-4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[e + f*x]^
2]*Sec[e + f*x]*Tan[e + f*x] + 3*Sqrt[Sec[e + f*x]^2]*(2*Sin[e + f*x] + Sec
[e + f*x]*Tan[e + f*x])))/(15*f*Sqrt[Sec[e + f*x]^2])
```

Maple [B] time = 0.16, size = 551, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(d*tan(f*x+e))^(1/2),x)
```

```
[Out] -1/5/f*2^(1/2)*(cos(f*x+e)-1)^2*(2*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((1-co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e
))^(1/2)*cos(f*x+e)^3*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2
),1/2*2^(1/2))-4*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e
)^3*EllipticE(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*(
(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/
2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)^2*EllipticF(((1-
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-4*((cos(f*x+e)-1)/sin
(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+
sin(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)^2*EllipticE(((1-cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*cos(f*x+e)^3*2^(1/2)-cos(f*x+e)^2*2^(
1/2)-2^(1/2))*(cos(f*x+e)+1)^2*(d*sin(f*x+e)/cos(f*x+e))^(1/2)/sin(f*x+e)^5
/cos(f*x+e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \tan\left(fx+e\right)} \sec\left(fx+e\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*tan(f*x + e))*sec(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d \tan\left(fx+e\right)} \sec\left(fx+e\right)^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(f*x + e))*sec(f*x + e)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\tan\left(e+fx\right)}\sec^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*tan(e + f*x))*sec(e + f*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\tan\left(fx+e\right)}\sec\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*tan(f*x + e))*sec(f*x + e)^3, x)
```

3.232 $\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal. Leaf size=75

$$\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}-\frac{2\cos(e+fx)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(e+fx)}}{f\sqrt{\sin(2e+2fx)}}$$

```
[Out] (-2*Cos[e + f*x]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Tan[e + f*x]])/(f*Sqrt
[Sin[2*e + 2*f*x]]) + (2*Cos[e + f*x]*(d*Tan[e + f*x])^(3/2))/(d*f)
```

Rubi [A] time = 0.0899136, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2613, 2615, 2572, 2639}

$$\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}-\frac{2\cos(e+fx)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(e+fx)}}{f\sqrt{\sin(2e+2fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (-2*Cos[e + f*x]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Tan[e + f*x]])/(f*Sqrt
[Sin[2*e + 2*f*x]]) + (2*Cos[e + f*x]*(d*Tan[e + f*x])^(3/2))/(d*f)
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df} - 2\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx$$

$$= \frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df} - \frac{\left(2\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\right)\int \sqrt{\cos(e+fx)}\sqrt{}}{\sqrt{\sin(e+fx)}}$$

$$= \frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df} - \frac{\left(2\cos(e+fx)\sqrt{d\tan(e+fx)}\right)\int \sqrt{\sin(2e+2fx)}}{\sqrt{\sin(2e+2fx)}}$$

$$= -\frac{2\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{f\sqrt{\sin(2e+2fx)}} + \frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}$$

Mathematica [C] time = 0.266433, size = 61, normalized size = 0.81

$$-\frac{2\sin(e+fx)\sqrt{d\tan(e+fx)}\left(2\sqrt{\sec^2(e+fx)}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(e+fx)\right)-3\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (-2*(-3 + 2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[e + f*x]^2]*Sqrt[Sec[e +
f*x]^2])*Sin[e + f*x]*Sqrt[d*Tan[e + f*x]])/(3*f)
```

Maple [B] time = 0.15, size = 505, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(d*tan(f*x+e))^(1/2),x)
```

```
[Out] 1/f*2^(1/2)*(d*sin(f*x+e)/cos(f*x+e))^(1/2)*(cos(f*x+e)+1)^2*(cos(f*x+e)-1)
^2*(2*EllipticE(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*c
os(f*x+e)*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)-EllipticF(((1-co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)*((cos(f*x+e)
-1)/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*
x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)+2*EllipticE(((1-cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))^(1/2),1/2*2^(1/2))*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((1-cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^
(1/2)-EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(
(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/
2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)-cos(f*x+e)*2^(1/2)+2^(1/2))
/sin(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \tan \left(fx+e\right)} \sec \left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*tan(f*x + e))*sec(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d \tan \left(fx+e\right)} \sec \left(fx+e\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(f*x + e))*sec(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\tan\left(e+fx\right)}\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*tan(e + f*x))*sec(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\tan\left(fx+e\right)}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*tan(f*x + e))*sec(f*x + e), x)
```

3.233 $\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal. Leaf size=47

$$\frac{\cos(e+fx)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(e+fx)}}{f\sqrt{\sin(2e+2fx)}}$$

```
[Out] (Cos[e + f*x]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Tan[e + f*x]])/(f*Sqrt[Si
n[2*e + 2*f*x]])
```

Rubi [A] time = 0.0649298, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2615, 2572, 2639}

$$\frac{\cos(e+fx)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(e+fx)}}{f\sqrt{\sin(2e+2fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Tan[e + f*x]])/(f*Sqrt[Si
n[2*e + 2*f*x]])
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{\left(\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\right)\int\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}\,dx}{\sqrt{\sin(e+fx)}}$$
$$= \frac{\left(\cos(e+fx)\sqrt{d\tan(e+fx)}\right)\int\sqrt{\sin(2e+2fx)}\,dx}{\sqrt{\sin(2e+2fx)}}$$
$$= \frac{\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{f\sqrt{\sin(2e+2fx)}}$$

Mathematica [C] time = 0.094254, size = 57, normalized size = 1.21

$$\frac{2\sin(e+fx)\sqrt{\sec^2(e+fx)}\sqrt{d\tan(e+fx)}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[e + f*x]^2]*Sqrt[Sec[e + f*x]^2]*S
in[e + f*x]*Sqrt[d*Tan[e + f*x]])/(3*f)
```

Maple [B] time = 0.157, size = 523, normalized size = 11.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(d*tan(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*2^(1/2)*(cos(f*x+e)-1)^2*(2*cos(f*x+e)*EllipticE((-(cos(f*x+e)-1-sin
(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*(
(cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(cos(f*x+e)-1-sin(f*x+e))/sin
(f*x+e))^(1/2)-cos(f*x+e)*EllipticF((-(cos(f*x+e)-1-sin(f*x+e))/sin(f*x+e))
^(1/2),1/2*2^(1/2))*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*
x+e))/sin(f*x+e))^(1/2)*(-(cos(f*x+e)-1-sin(f*x+e))/sin(f*x+e))^(1/2)+2*Ell
ipticE((-(cos(f*x+e)-1-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*((cos(f*x
```

```
+e)-1)/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(co
s(f*x+e)-1-sin(f*x+e))/sin(f*x+e))^(1/2)-EllipticF((-(cos(f*x+e)-1-sin(f*x+
e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((cos(
f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(cos(f*x+e)-1-sin(f*x+e))/sin(f*x+
e))^(1/2)+cos(f*x+e)^2*2^(1/2)-cos(f*x+e)*2^(1/2))*(cos(f*x+e)+1)^2*(d*sin(
f*x+e)/cos(f*x+e))^(1/2)/sin(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\tan\left(fx+e\right)}\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*tan(f*x + e))*cos(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\tan\left(fx+e\right)}\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(f*x + e))*cos(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\tan\left(e+fx\right)}\cos\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*tan(e + f*x))*cos(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\tan\left(fx+e\right)}\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*tan(f*x + e))*cos(f*x + e), x)
```

3.234 $\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal. Leaf size=81

$$\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}+\frac{\cos(e+fx)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(e+fx)}}{2f\sqrt{\sin(2e+2fx)}}$$

```
[Out] (Cos[e + f*x]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Tan[e + f*x]])/(2*f*Sqrt[
Sin[2*e + 2*f*x]]) + (Cos[e + f*x]^3*(d*Tan[e + f*x])^(3/2))/(3*d*f)
```

Rubi [A] time = 0.101796, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2612, 2615, 2572, 2639}

$$\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}+\frac{\cos(e+fx)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(e+fx)}}{2f\sqrt{\sin(2e+2fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Tan[e + f*x]])/(2*f*Sqrt[
Sin[2*e + 2*f*x]]) + (Cos[e + f*x]^3*(d*Tan[e + f*x])^(3/2))/(3*d*f)
```

Rule 2612

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*m
), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e + f
*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1] &
& EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df} + \frac{1}{2}\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx$$

$$= \frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df} + \frac{\left(\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\right)\int \sqrt{\cos(e+fx)}\sqrt{}}{2\sqrt{\sin(e+fx)}}$$

$$= \frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df} + \frac{\left(\cos(e+fx)\sqrt{d\tan(e+fx)}\right)\int \sqrt{\sin(2e+2fx)}\,d}{2\sqrt{\sin(2e+2fx)}}$$

$$= \frac{\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{2f\sqrt{\sin(2e+2fx)}} + \frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}$$

Mathematica [C] time = 0.4308, size = 94, normalized size = 1.16

$$\frac{\sqrt{d\tan(e+fx)}\left(4\tan(e+fx)\sec(e+fx)\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(e+fx)\right)+(\sin(e+fx)+\sin(3(e+fx)))\sqrt{\sec^2(e+fx)}\right)}{12f\sqrt{\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (Sqrt[d*Tan[e + f*x]]*(Sqrt[Sec[e + f*x]^2]*(Sin[e + f*x] + Sin[3*(e + f*x)
]) + 4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[e + f*x]^2]*Sec[e + f*x]*Tan[e
 + f*x]))/(12*f*Sqrt[Sec[e + f*x]^2])
```

Maple [B] time = 0.192, size = 536, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(d*tan(f*x+e))^(1/2),x)
```

```
[Out] -1/12/f*2^(1/2)*(cos(f*x+e)-1)^2*(2*cos(f*x+e)^4*2^(1/2)-3*cos(f*x+e)*Ellip
ticF((-(cos(f*x+e)-1-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*((cos(f*x+e
)-1)/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(cos(
f*x+e)-1-sin(f*x+e))/sin(f*x+e))^(1/2)+6*cos(f*x+e)*EllipticE((-(cos(f*x+e)
-1-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*((cos(f*x+e)-1)/sin(f*x+e))^(
1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(cos(f*x+e)-1-sin(f*x+e
))/sin(f*x+e))^(1/2)-3*EllipticF((-(cos(f*x+e)-1-sin(f*x+e))/sin(f*x+e))^(1
/2),1/2*2^(1/2))*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e
))/sin(f*x+e))^(1/2)*(-(cos(f*x+e)-1-sin(f*x+e))/sin(f*x+e))^(1/2)+6*Ellipt
icE((-(cos(f*x+e)-1-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*((cos(f*x+e)
-1)/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(cos(f
*x+e)-1-sin(f*x+e))/sin(f*x+e))^(1/2)+cos(f*x+e)^2*2^(1/2)-3*cos(f*x+e)*2^(
1/2))*(cos(f*x+e)+1)^2*(d*sin(f*x+e)/cos(f*x+e))^(1/2)/sin(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \tan\left(fx+e\right)} \cos\left(fx+e\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*tan(f*x + e))*cos(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{d \tan\left(fx+e\right)} \cos\left(fx+e\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(f*x + e))*cos(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.235 $\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal. Leaf size=111

$$\frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df}+\frac{7\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{30df}+\frac{7\cos(e+fx)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(e+fx)}}{20f\sqrt{\sin(2e+2fx)}}$$

```
[Out] (7*Cos[e + f*x]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Tan[e + f*x]])/(20*f*Sq
rt[Sin[2*e + 2*f*x]]) + (7*Cos[e + f*x]^3*(d*Tan[e + f*x])^(3/2))/(30*d*f)
+ (Cos[e + f*x]^5*(d*Tan[e + f*x])^(3/2))/(5*d*f)
```

Rubi [A] time = 0.139029, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2612, 2615, 2572, 2639}

$$\frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df}+\frac{7\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{30df}+\frac{7\cos(e+fx)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(e+fx)}}{20f\sqrt{\sin(2e+2fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5*Sqrt[d*Tan[e + f*x]],x]
```


```
[Out] (7*Cos[e + f*x]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Tan[e + f*x]])/(20*f*Sq
rt[Sin[2*e + 2*f*x]]) + (7*Cos[e + f*x]^3*(d*Tan[e + f*x])^(3/2))/(30*d*f)
+ (Cos[e + f*x]^5*(d*Tan[e + f*x])^(3/2))/(5*d*f)
```


Rule 2612

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*m
), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e + f
*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1] &
& EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df} + \frac{7}{10}\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx$$

$$= \frac{7\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{30df} + \frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df} + \frac{7}{20}\int \cos(e+$$

$$= \frac{7\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{30df} + \frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df} + \frac{\left(7\sqrt{\cos(e+f\right.}}{}$$

$$= \frac{7\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{30df} + \frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df} + \frac{\left(7\cos(e+fx\right.}{}$$

$$= \frac{7\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{20f\sqrt{\sin(2e+2fx)}} + \frac{7\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{30df}$$

Mathematica [C] time = 0.749883, size = 86, normalized size = 0.77

$$\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}\left(28\tan(e+fx)\sqrt{\sec^2(e+fx)}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(e+fx)\right)+20\sin(2(e+fx))+3\sin(4(e-\right.}{120f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^5*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[d*Tan[e + f*x]]*(20*Sin[2*(e + f*x)] + 3*Sin[4*(e + f*x)
] + 28*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[e + f*x]^2]*Sqrt[Sec[e + f*x]^
2]*Tan[e + f*x]))/(120*f)
```

Maple [B] time = 0.171, size = 542, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5*(d*tan(f*x+e))^(1/2),x)
```

```
[Out] -1/120/f*2^(1/2)*(cos(f*x+e)-1)^2*(12*2^(1/2)*cos(f*x+e)^6+2*cos(f*x+e)^4*2
^(1/2)+42*EllipticE(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2
))*cos(f*x+e)*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/
sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)-21*EllipticF
(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)*((cos
(f*x+e)-1)/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(
(cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)+42*EllipticE(((1-cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*(
(1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(
f*x+e))^(1/2)-21*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2
*2^(1/2))*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)+7*cos(f*x+e)^2*2
^(1/2)-21*cos(f*x+e)*2^(1/2))*(cos(f*x+e)+1)^2*(d*sin(f*x+e)/cos(f*x+e))^(1
/2)/sin(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \tan\left(fx+e\right)} \cos\left(fx+e\right)^{5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*tan(f*x + e))*cos(f*x + e)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{d \tan\left(fx+e\right)} \cos\left(fx+e\right)^{5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(f*x + e))*cos(f*x + e)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5*(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\tan\left(fx+e\right)}\cos\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*tan(f*x + e))*cos(f*x + e)^5, x)
```

3.236 $\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=67

$$\frac{2(d\tan(a+bx))^{13/2}}{13bd^5}+\frac{4(d\tan(a+bx))^{9/2}}{9bd^3}+\frac{2(d\tan(a+bx))^{5/2}}{5bd}$$

```
[Out] (2*(d*Tan[a + b*x])^(5/2))/(5*b*d) + (4*(d*Tan[a + b*x])^(9/2))/(9*b*d^3) +
 (2*(d*Tan[a + b*x])^(13/2))/(13*b*d^5)
```

Rubi [A] time = 0.0579711, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2607, 270}

$$\frac{2(d\tan(a+bx))^{13/2}}{13bd^5}+\frac{4(d\tan(a+bx))^{9/2}}{9bd^3}+\frac{2(d\tan(a+bx))^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^6*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*(d*Tan[a + b*x])^(5/2))/(5*b*d) + (4*(d*Tan[a + b*x])^(9/2))/(9*b*d^3) +
 (2*(d*Tan[a + b*x])^(13/2))/(13*b*d^5)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{\text{Subst}\left(\int (dx)^{3/2}\left(1+x^2\right)^2\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left((dx)^{3/2}+\frac{2(dx)^{7/2}}{d^2}+\frac{(dx)^{11/2}}{d^4}\right)dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{2(d\tan(a+bx))^{5/2}}{5bd}+\frac{4(d\tan(a+bx))^{9/2}}{9bd^3}+\frac{2(d\tan(a+bx))^{13/2}}{13bd^5}$$

Mathematica [A] time = 0.137308, size = 52, normalized size = 0.78

$$\frac{2d\left(45\sec^6(a+bx)-5\sec^4(a+bx)-8\sec^2(a+bx)-32\right)\sqrt{d\tan(a+bx)}}{585b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^6*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*d*(-32 - 8*Sec[a + b*x]^2 - 5*Sec[a + b*x]^4 + 45*Sec[a + b*x]^6)*Sqrt[d
*Tan[a + b*x]])/(585*b)
```

Maple [A] time = 0.189, size = 60, normalized size = 0.9

$$\frac{\left(64\,(\cos(bx+a))^4+80\,(\cos(bx+a))^2+90\right)\sin(bx+a)}{585\,b\,(\cos(bx+a))^5}\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^6*(d*tan(b*x+a))^(3/2),x)
```

```
[Out] 2/585/b*(32*cos(b*x+a)^4+40*cos(b*x+a)^2+45)*(d*sin(b*x+a)/cos(b*x+a))^(3/2
)*sin(b*x+a)/cos(b*x+a)^5
```

Maxima [A] time = 0.937855, size = 69, normalized size = 1.03

$$\frac{2\left(45\,(d\tan(bx+a))^{\frac{13}{2}}+130\,(d\tan(bx+a))^{\frac{9}{2}}d^2+117\,(d\tan(bx+a))^{\frac{5}{2}}d^4\right)}{585\,bd^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/585*(45*(d*tan(b*x + a))^(13/2) + 130*(d*tan(b*x + a))^(9/2)*d^2 + 117*(d
*tan(b*x + a))^(5/2)*d^4)/(b*d^5)
```

Fricas [A] time = 2.17808, size = 178, normalized size = 2.66

$$-\frac{2\left(32\,d\cos\left(bx+a\right)^{6}+8\,d\cos\left(bx+a\right)^{4}+5\,d\cos\left(bx+a\right)^{2}-45\,d\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{585\,b\cos\left(bx+a\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/585*(32*d*cos(b*x + a)^6 + 8*d*cos(b*x + a)^4 + 5*d*cos(b*x + a)^2 - 45*
d)*sqrt(d*sin(b*x + a)/cos(b*x + a))/(b*cos(b*x + a)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**6*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}\sec\left(bx+a\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*(d*tan(b*x+a))^(3/2),x, algorithm="giac")

[Out] integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a)^6, x)
```

3.237 $\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=45

$$\frac{2(d\tan(a+bx))^{9/2}}{9bd^3}+\frac{2(d\tan(a+bx))^{5/2}}{5bd}$$

```
[Out] (2*(d*Tan[a + b*x])^(5/2))/(5*b*d) + (2*(d*Tan[a + b*x])^(9/2))/(9*b*d^3)
```

Rubi [A] time = 0.0526398, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2607, 14}

$$\frac{2(d\tan(a+bx))^{9/2}}{9bd^3}+\frac{2(d\tan(a+bx))^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^4*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*(d*Tan[a + b*x])^(5/2))/(5*b*d) + (2*(d*Tan[a + b*x])^(9/2))/(9*b*d^3)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{\text{Subst}\left(\int (dx)^{3/2}\left(1+x^2\right)\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left((dx)^{3/2}+\frac{(dx)^{7/2}}{d^2}\right)\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{2(d\tan(a+bx))^{5/2}}{5bd} + \frac{2(d\tan(a+bx))^{9/2}}{9bd^3}$$

Mathematica [A] time = 0.133345, size = 42, normalized size = 0.93

$$\frac{2d\left(5\sec^4(a+bx)-\sec^2(a+bx)-4\right)\sqrt{d\tan(a+bx)}}{45b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^4*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*d*(-4 - Sec[a + b*x]^2 + 5*Sec[a + b*x]^4)*Sqrt[d*Tan[a + b*x]])/(45*b)
```

Maple [A] time = 0.24, size = 50, normalized size = 1.1

$$\frac{\left(8\,(\cos(bx+a))^2+10\right)\sin(bx+a)}{45\,b\,(\cos(bx+a))^3}\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4*(d*tan(b*x+a))^(3/2),x)
```

```
[Out] 2/45/b*(4*cos(b*x+a)^2+5)*(d*sin(b*x+a)/cos(b*x+a))^(3/2)*sin(b*x+a)/cos(b*
x+a)^3
```

Maxima [A] time = 0.941261, size = 49, normalized size = 1.09

$$\frac{2\left(5\,(d\tan(bx+a))^{\frac{9}{2}}+9\,(d\tan(bx+a))^{\frac{5}{2}}\,d^2\right)}{45\,bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/45*(5*(d*tan(b*x + a))^(9/2) + 9*(d*tan(b*x + a))^(5/2)*d^2)/(b*d^3)
```

Fricas [A] time = 1.89385, size = 143, normalized size = 3.18

$$-\frac{2\left(4\,d\cos\left(bx+a\right)^{4}+d\cos\left(bx+a\right)^{2}-5\,d\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{45\,b\cos\left(bx+a\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/45*(4*d*cos(b*x + a)^4 + d*cos(b*x + a)^2 - 5*d)*sqrt(d*sin(b*x + a)/cos
(b*x + a))/(b*cos(b*x + a)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}\sec\left(bx+a\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a)^4, x)
```

3.238 $\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=22

$$\frac{2(d\tan(a+bx))^{5/2}}{5bd}$$

```
[Out] (2*(d*Tan[a + b*x])^(5/2))/(5*b*d)
```

Rubi [A] time = 0.0432917, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2607, 32}

$$\frac{2(d\tan(a+bx))^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*(d*Tan[a + b*x])^(5/2))/(5*b*d)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx &= \frac{\text{Subst}\left(\int (dx)^{3/2}\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{2(d\tan(a+bx))^{5/2}}{5bd}\end{aligned}$$

Mathematica [A] time = 0.0529561, size = 22, normalized size = 1.

$$\frac{2(d\tan(a+bx))^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*(d*Tan[a + b*x])^(5/2))/(5*b*d)
```

Maple [A] time = 0.02, size = 19, normalized size = 0.9

$$\frac{2}{5\,bd}\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*(d*tan(b*x+a))^(3/2),x)
```

```
[Out] 2/5*(d*tan(b*x+a))^(5/2)/b/d
```

Maxima [A] time = 0.941616, size = 24, normalized size = 1.09

$$\frac{2\,\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}}{5\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(d*tan(b*x + a))^(5/2)/(b*d)
```

Fricas [B] time = 1.71208, size = 111, normalized size = 5.05

$$-\frac{2\left(d\cos\left(bx+a\right)^{2}-d\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{5\,b\cos\left(bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(d*cos(b*x + a)^2 - d)*sqrt(d*sin(b*x + a)/cos(b*x + a))/(b*cos(b*x +
a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d \tan (bx+a))^{\frac{3}{2}} \sec (bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a)^2, x)
```

3.239 $\int (d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=210

$$\frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}b}-\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{\sqrt{2}b}+\frac{d^{3/2}\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{2\sqrt{2}b}$$

```
[Out] (d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(Sqrt[2]*b) -
(d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(Sqrt[2]*b) +
(d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b*x]]]
)/(2*Sqrt[2]*b) - (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqrt[2]*Sqr
t[d*Tan[a + b*x]]])/(2*Sqrt[2]*b) + (2*d*Sqrt[d*Tan[a + b*x]])/b
```

Rubi [A] time = 0.141573, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3473, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}b}-\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{\sqrt{2}b}+\frac{d^{3/2}\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{2\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(Sqrt[2]*b) -
(d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(Sqrt[2]*b) +
(d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b*x]]]
)/(2*Sqrt[2]*b) - (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqrt[2]*Sqr
t[d*Tan[a + b*x]]])/(2*Sqrt[2]*b) + (2*d*Sqrt[d*Tan[a + b*x]])/b
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int (d\tan(a+bx))^{3/2}\,dx &= \frac{2d\sqrt{d\tan(a+bx)}}{b} - d^2\int \frac{1}{\sqrt{d\tan(a+bx)}}\,dx \\
&= \frac{2d\sqrt{d\tan(a+bx)}}{b} - \frac{d^3\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(d^2+x^2\right)}\,dx, x, d\tan(a+bx)\right)}{b} \\
&= \frac{2d\sqrt{d\tan(a+bx)}}{b} - \frac{\left(2d^3\right)\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{b} \\
&= \frac{2d\sqrt{d\tan(a+bx)}}{b} - \frac{d^2\,\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{b} - \frac{d^2\,\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\,t}\right.}{b} \\
&= \frac{2d\sqrt{d\tan(a+bx)}}{b} + \frac{d^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{2\sqrt{2}b} + \frac{d^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{2}}{-d+\sqrt{2}}\right.}{} \\
&= \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{2\sqrt{2}b} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\right.}{2\sqrt{2}b} \\
&= \frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}b} - \frac{d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}b} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+\right.}{2\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.25994, size = 159, normalized size = 0.76

$$\frac{(d\tan(a+bx))^{3/2}\left(2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)-2\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)+8\sqrt{\tan(a+bx)}+\sqrt{2}\log\left(\text{t}\right.\right.}{4b\tan^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] ((2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]] - 2*Sqrt[2]*ArcTan[1 + S
qrt[2]*Sqrt[Tan[a + b*x]]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + T
an[a + b*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]] +
```

```
8*Sqrt[Tan[a + b*x]])*(d*Tan[a + b*x])^(3/2))/(4*b*Tan[a + b*x]^(3/2))
```

Maple [A] time = 0.013, size = 176, normalized size = 0.8

$$2\,\frac{d\sqrt{d\tan(bx+a)}}{b}-\frac{d\sqrt{2}}{2\,b}\sqrt[4]{d^2}\arctan\left(\sqrt{2}\sqrt{d\tan(bx+a)}\frac{1}{\sqrt[4]{d^2}}+1\right)+\frac{d\sqrt{2}}{2\,b}\sqrt[4]{d^2}\arctan\left(-\sqrt{2}\sqrt{d\tan(bx+a)}\frac{1}{\sqrt[4]{d^2}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(b*x+a))^(3/2),x)
```

```
[Out] 2*d*(d*tan(b*x+a))^(1/2)/b-1/2/b*d*(d^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2)
^(1/4)*(d*tan(b*x+a))^(1/2)+1)+1/2/b*d*(d^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/
(d^2)^(1/4)*(d*tan(b*x+a))^(1/2)+1)-1/4/b*d*(d^2)^(1/4)*2^(1/2)*ln((d*tan(b
*x+a)+(d^2)^(1/4)*(d*tan(b*x+a))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(b*x+a)-(
d^2)^(1/4)*(d*tan(b*x+a))^(1/2)*2^(1/2)+(d^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.73654, size = 1328, normalized size = 6.32

$$4\,\sqrt{2}\left(\frac{d^6}{b^4}\right)^{\frac{1}{4}}b\arctan\left(-\frac{d^6+\sqrt{2}\left(\frac{d^6}{b^4}\right)^{\frac{3}{4}}b^3d\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}-\sqrt{2}\left(\frac{d^6}{b^4}\right)^{\frac{3}{4}}b^3\sqrt{\frac{\sqrt{2}\left(\frac{d^6}{b^4}\right)^{\frac{1}{4}}bd\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)+d^3\sin(bx+a)+\sqrt{\frac{d^6}{b^4}}b^2\cos(bx+a)}{\cos(bx+a)}}}{d^6}\right)+4\,\sqrt{2}\left(\frac{d^6}{b^4}\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*(d^6/b^4)^(1/4)*b*arctan(-(d^6 + sqrt(2)*(d^6/b^4)^(3/4)*b^3
*d*sqrt(d*sin(b*x + a)/cos(b*x + a)) - sqrt(2)*(d^6/b^4)^(3/4)*b^3*sqrt((sq
rt(2)*(d^6/b^4)^(1/4)*b*d*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a) +
d^3*sin(b*x + a) + sqrt(d^6/b^4)*b^2*cos(b*x + a))/cos(b*x + a)))/d^6) + 4*
sqrt(2)*(d^6/b^4)^(1/4)*b*arctan((d^6 - sqrt(2)*(d^6/b^4)^(3/4)*b^3*d*sqrt(
d*sin(b*x + a)/cos(b*x + a)) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*sqrt(-(sqrt(2)*(
d^6/b^4)^(1/4)*b*d*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a) - d^3*sin
(b*x + a) - sqrt(d^6/b^4)*b^2*cos(b*x + a))/cos(b*x + a)))/d^6) - sqrt(2)*(
d^6/b^4)^(1/4)*b*log((sqrt(2)*(d^6/b^4)^(1/4)*b*d*sqrt(d*sin(b*x + a)/cos(b
*x + a))*cos(b*x + a) + d^3*sin(b*x + a) + sqrt(d^6/b^4)*b^2*cos(b*x + a))/
cos(b*x + a)) + sqrt(2)*(d^6/b^4)^(1/4)*b*log(-(sqrt(2)*(d^6/b^4)^(1/4)*b*d
*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a) - d^3*sin(b*x + a) - sqrt(d
^6/b^4)*b^2*cos(b*x + a))/cos(b*x + a)) + 8*d*sqrt(d*sin(b*x + a)/cos(b*x +
 a)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(a+bx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral((d*tan(a + b*x))**(3/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.240 $\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=225

$$-\frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}b}-\frac{d^{3/2}\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}b}$$

```
[Out] -(d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*b)
 + (d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*
b) - (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b
*x]]])/(8*Sqrt[2]*b) + (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqrt[2
]*Sqrt[d*Tan[a + b*x]]])/(8*Sqrt[2]*b) - (d*Cos[a + b*x]^2*Sqrt[d*Tan[a + b
*x]])/(2*b)
```

Rubi [A] time = 0.162636, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2607, 288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}b}-\frac{d^{3/2}\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```


```
[Out] -(d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*b)
 + (d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*
b) - (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b
*x]]])/(8*Sqrt[2]*b) + (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqrt[2
]*Sqrt[d*Tan[a + b*x]]])/(8*Sqrt[2]*b) - (d*Cos[a + b*x]^2*Sqrt[d*Tan[a + b
*x]])/(2*b)
```


Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{(dx)^{3/2}}{\left(1+x^2\right)^2}\,dx, x, \tan(a+bx)\right)}{b}$$

$$= -\frac{d\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2b} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{\sqrt{dx}\left(1+x^2\right)}\,dx, x, \tan(a+bx)\right)}{4b}$$

$$= -\frac{d\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2b} + \frac{d\,\text{Subst}\left(\int \frac{1}{1+\frac{x^4}{d^2}}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{2b}$$

$$= -\frac{d\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2b} + \frac{\text{Subst}\left(\int \frac{d-x^2}{1+\frac{x^4}{d^2}}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{4b} + \frac{\text{Subst}}{}$$

$$= -\frac{d\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2b} - \frac{d^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right.}{8\sqrt{2}b}$$

$$= -\frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a\right.}{8}$$

$$= -\frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} + \frac{d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} - \frac{d^{3/2}\log\left(\sqrt{d}+\right.}{}$$

Mathematica [A] time = 0.258561, size = 110, normalized size = 0.49

$$-\frac{d\csc(a+bx)\sqrt{d\tan(a+bx)}\left(\sin(a+bx)+\sin(3(a+bx))+\sqrt{\sin(2(a+bx))}\sin^{-1}(\cos(a+bx)-\sin(a+bx))-\sqrt{\sin(2\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -(d*Csc[a + b*x]*(Sin[a + b*x] + ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Sqrt[S
in[2*(a + b*x)]] - Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]]
]*Sqrt[Sin[2*(a + b*x)]] + Sin[3*(a + b*x)])*Sqrt[d*Tan[a + b*x]])/(8*b)
```

Maple [C] time = 0.143, size = 660, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*(d*tan(b*x+a))^(3/2),x)
```

```
[Out] 1/8/b*2^(1/2)*(cos(b*x+a)-1)*(I*sin(b*x+a)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2
)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/s
in(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/
2+1/2*I,1/2*2^(1/2))-I*sin(b*x+a)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos
(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a)
)^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1
/2*2^(1/2))-sin(b*x+a)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1
/2),1/2+1/2*I,1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)
+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)
-sin(b*x+a)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2
*I,1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a)
)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+2*sin(b*x+
a)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))
^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a
)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-2*cos(b*x+a)^3*2^(1/2)+2*cos(b
*x+a)^2*2^(1/2))*cos(b*x+a)*(cos(b*x+a)+1)^2*(d*sin(b*x+a)/cos(b*x+a))^(3/2
)/sin(b*x+a)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan{(bx+a)})^{\frac{3}{2}}\cos{(bx+a)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*cos(b*x + a)^2, x)
```

3.241 $\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=136

$$-\frac{4d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{77b\sqrt{d\tan(a+bx)}}+\frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b}-\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{77b}-$$

```
[Out] (-4*d^2*EllipticF[a - Pi/4 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(
77*b*Sqrt[d*Tan[a + b*x]]) - (4*d*Sec[a + b*x]*Sqrt[d*Tan[a + b*x]])/(77*b)
 - (2*d*Sec[a + b*x]^3*Sqrt[d*Tan[a + b*x]])/(77*b) + (2*d*Sec[a + b*x]^5*S
qrt[d*Tan[a + b*x]])/(11*b)
```

Rubi [A] time = 0.183455, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2611, 2613, 2614, 2573, 2641}

$$-\frac{4d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{77b\sqrt{d\tan(a+bx)}}+\frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b}-\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{77b}-$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^5*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-4*d^2*EllipticF[a - Pi/4 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(
77*b*Sqrt[d*Tan[a + b*x]]) - (4*d*Sec[a + b*x]*Sqrt[d*Tan[a + b*x]])/(77*b)
 - (2*d*Sec[a + b*x]^3*Sqrt[d*Tan[a + b*x]])/(77*b) + (2*d*Sec[a + b*x]^5*S
qrt[d*Tan[a + b*x]])/(11*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
```

```
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b} - \frac{1}{11}d^2\int \frac{\sec^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

$$= -\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{77b} + \frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b} - \frac{1}{77}\left(6d^2\right)\int$$

$$= -\frac{4d\sec(a+bx)\sqrt{d\tan(a+bx)}}{77b} - \frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{77b} + \frac{2d\sec^5(a+b}{}$$

$$= -\frac{4d\sec(a+bx)\sqrt{d\tan(a+bx)}}{77b} - \frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{77b} + \frac{2d\sec^5(a+b}{}$$

$$= -\frac{4d\sec(a+bx)\sqrt{d\tan(a+bx)}}{77b} - \frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{77b} + \frac{2d\sec^5(a+b}{}$$

$$= -\frac{4d^2F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{77b\sqrt{d\tan(a+bx)}} - \frac{4d\sec(a+bx)\sqrt{d\tan(a+bx}}{77b}$$

Mathematica [C] time = 0.799281, size = 90, normalized size = 0.66

$$-\frac{d\sec^5(a+bx)\sqrt{d\tan(a+bx)}\left(16\cos^6(a+bx)\sqrt{\sec^2(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2(a+bx)\right)+6\cos(2(a+bx))+\cos(4(a+bx))-23\right)}{154b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^5*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -(d*Sec[a + b*x]^5*(-23 + 6*Cos[2*(a + b*x)] + Cos[4*(a + b*x)] + 16*Cos[a
+ b*x]^6*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x
]^2])*Sqrt[d*Tan[a + b*x]])/(154*b)
```

Maple [A] time = 0.135, size = 251, normalized size = 1.9

$$\frac{\sqrt{2}\left(\cos(bx+a)-1\right)\left(\cos(bx+a)+1\right)^2}{77\,b\left(\cos(bx+a)\right)^4\left(\sin(bx+a)\right)^5}\left(4\sin(bx+a)\sqrt{\frac{\cos(bx+a)-1}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^5*(d*tan(b*x+a))^(3/2),x)
```

```
[Out] 1/77/b*2^(1/2)*(cos(b*x+a)-1)*(4*sin(b*x+a)*((cos(b*x+a)-1)/sin(b*x+a))^(1/
2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))
/sin(b*x+a))^(1/2)*cos(b*x+a)^5*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b
*x+a))^(1/2),1/2*2^(1/2))-2*2^(1/2)*cos(b*x+a)^5+2*cos(b*x+a)^4*2^(1/2)-cos
(b*x+a)^3*2^(1/2)+cos(b*x+a)^2*2^(1/2)+7*cos(b*x+a)*2^(1/2)-7*2^(1/2))*(cos
(b*x+a)+1)^2*(d*sin(b*x+a)/cos(b*x+a))^(3/2)/cos(b*x+a)^4/sin(b*x+a)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\tan(bx+a)\right)^{\frac{3}{2}}\sec(bx+a)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\tan\left(bx+a\right)}d\sec\left(bx+a\right)^{5}\tan\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*d*sec(b*x + a)^5*tan(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**5*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}\sec\left(bx+a\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a)^5, x)
```

3.242 $\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=108

$$-\frac{2d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{21b\sqrt{d\tan(a+bx)}}+\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b}-\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{21b}$$

```
[Out] (-2*d^2*EllipticF[a - Pi/4 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(
21*b*Sqrt[d*Tan[a + b*x]]) - (2*d*Sec[a + b*x]*Sqrt[d*Tan[a + b*x]])/(21*b)
 + (2*d*Sec[a + b*x]^3*Sqrt[d*Tan[a + b*x]])/(7*b)
```

Rubi [A] time = 0.14484, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2611, 2613, 2614, 2573, 2641}

$$-\frac{2d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{21b\sqrt{d\tan(a+bx)}}+\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b}-\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{21b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*d^2*EllipticF[a - Pi/4 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(
21*b*Sqrt[d*Tan[a + b*x]]) - (2*d*Sec[a + b*x]*Sqrt[d*Tan[a + b*x]])/(21*b)
 + (2*d*Sec[a + b*x]^3*Sqrt[d*Tan[a + b*x]])/(7*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
```

```
*m, 2*n]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b} - \frac{1}{7}d^2\int \frac{\sec^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$
$$= -\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{21b} + \frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b} - \frac{1}{21}\left(2d^2\right)\int \cdots$$
$$= -\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{21b} + \frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b} - \frac{\left(2d^2\sqrt{\sin(a+\cdots}\right.}{21\sqrt{\cos\cdots}}$$
$$= -\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{21b} + \frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b} - \frac{\left(2d^2\sec(a+\cdots\right.}{\cdots}$$
$$= -\frac{2d^2F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{21b\sqrt{d\tan(a+bx)}} - \frac{2d\sec(a+bx)\sqrt{d\tan(a+bx\cdots}}{21b}$$

Mathematica [C] time = 0.456954, size = 80, normalized size = 0.74

$$-\frac{d\sec^3(a+bx)\sqrt{d\tan(a+bx)}\left(4\cos^4(a+bx)\sqrt{\sec^2(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2(a+bx)\right)+\cos(2(a+bx))-5\right)}{21b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -(d*Sec[a + b*x]^3*(-5 + Cos[2*(a + b*x)] + 4*Cos[a + b*x]^4*Hypergeometric
2F1[1/4, 1/2, 5/4, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2])*Sqrt[d*Tan[a + b*
x]])/(21*b)
```

Maple [A] time = 0.159, size = 223, normalized size = 2.1

$$\frac{\sqrt{2}\left(\cos\left(bx+a\right)-1\right)\left(\cos\left(bx+a\right)+1\right)^{2}}{21\,b\left(\sin\left(bx+a\right)\right)^{5}\left(\cos\left(bx+a\right)\right)^{2}}\left(2\,\left(\cos\left(bx+a\right)\right)^{3}\sqrt{\frac{\cos\left(bx+a\right)-1}{\sin\left(bx+a\right)}}\sqrt{\frac{1-\cos\left(bx+a\right)+\sin\left(bx+a\right)}{\sin\left(bx+a\right)}}\sqrt{\frac{\cos\left(b\right.}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3*(d*tan(b*x+a))^(3/2),x)
```

```
[Out] 1/21/b*2^(1/2)*(cos(b*x+a)-1)*(2*cos(b*x+a)^3*((cos(b*x+a)-1)/sin(b*x+a))^(
1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a)
)/sin(b*x+a))^(1/2)*sin(b*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x
+a))^(1/2),1/2*2^(1/2))-cos(b*x+a)^3*2^(1/2)+cos(b*x+a)^2*2^(1/2)+3*cos(b*x
+a)*2^(1/2)-3*2^(1/2))*(cos(b*x+a)+1)^2*(d*sin(b*x+a)/cos(b*x+a))^(3/2)/sin
(b*x+a)^5/cos(b*x+a)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}\sec\left(bx+a\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\sqrt{d\tan\left(bx+a\right)}d\sec\left(bx+a\right)^{3}\tan\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*d*sec(b*x + a)^3*tan(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^{\frac{3}{2}}\sec(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a)^3, x)
```

3.243 $\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=80

$$\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b}-\frac{d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{3b\sqrt{d\tan(a+bx)}}$$

```
[Out] -(d^2*EllipticF[a - Pi/4 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(3*
b*Sqrt[d*Tan[a + b*x]]) + (2*d*Sec[a + b*x]*Sqrt[d*Tan[a + b*x]])/(3*b)
```

Rubi [A] time = 0.0845474, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2611, 2614, 2573, 2641}

$$\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b}-\frac{d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{3b\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -(d^2*EllipticF[a - Pi/4 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(3*
b*Sqrt[d*Tan[a + b*x]]) + (2*d*Sec[a + b*x]*Sqrt[d*Tan[a + b*x]])/(3*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
```

```
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx &= \frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b}-\frac{1}{3}d^2\int \frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx\\ &= \frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b}-\frac{\left(d^2\sqrt{\sin(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}\,dx}{3\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}\\ &= \frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b}-\frac{\left(d^2\sec(a+bx)\sqrt{\sin(2a+2bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{3\sqrt{d\tan(a+bx)}}\\ &= -\frac{d^2F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{3b\sqrt{d\tan(a+bx)}}+\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b}\end{aligned}$$

Mathematica [C] time = 0.299401, size = 69, normalized size = 0.86

$$\frac{2d\cos(a+bx)\sqrt{d\tan(a+bx)}\left(\sec^2(a+bx)-\sqrt{\sec^2(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2(a+bx)\right)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*d*Cos[a + b*x]*(Sec[a + b*x]^2 - Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[a
 + b*x]^2]*Sqrt[Sec[a + b*x]^2])*Sqrt[d*Tan[a + b*x]])/(3*b)
```

Maple [A] time = 0.124, size = 186, normalized size = 2.3

$$\frac{\sqrt{2}\left(\cos(bx+a)-1\right)\left(\cos(bx+a)+1\right)^2}{3\,b\left(\sin(bx+a)\right)^5}\left(\sin(bx+a)\cos(bx+a)\,\mathrm{EllipticF}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sqrt{\mathrm{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*(d*tan(b*x+a))^(3/2),x)
```

```
[Out] 1/3/b*2^(1/2)*(cos(b*x+a)-1)*(sin(b*x+a)*cos(b*x+a)*EllipticF(((1-cos(b*x+a
)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1
/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))
/sin(b*x+a))^(1/2)+cos(b*x+a)*2^(1/2)-2^(1/2))*(cos(b*x+a)+1)^2*(d*sin(b*x+
a)/cos(b*x+a))^(3/2)/sin(b*x+a)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^{\frac{3}{2}}\sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{d\tan(bx+a)}d\sec(bx+a)\tan(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*d*sec(b*x + a)*tan(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(a+bx))^{\frac{3}{2}}\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral((d*tan(a + b*x))**(3/2)*sec(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^{\frac{3}{2}}\sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a), x)
```

3.244 $\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=78

$$\frac{d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b\sqrt{d\tan(a+bx)}}-\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

```
[Out] (d^2*EllipticF[a - Pi/4 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(2*b
*Sqrt[d*Tan[a + b*x]]) - (d*Cos[a + b*x]*Sqrt[d*Tan[a + b*x]])/b
```

Rubi [A] time = 0.0951669, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2610, 2614, 2573, 2641}

$$\frac{d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b\sqrt{d\tan(a+bx)}}-\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (d^2*EllipticF[a - Pi/4 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(2*b
*Sqrt[d*Tan[a + b*x]]) - (d*Cos[a + b*x]*Sqrt[d*Tan[a + b*x]])/b
```

Rule 2610

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m)
, x] - Dist[(b^2*(n - 1))/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1]
 || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
```

```
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx &= -\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b}+\frac{1}{2}d^2\int\frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx\\
&= -\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b}+\frac{\left(d^2\sqrt{\sin(a+bx)}\right)\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}\,dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}\\
&= -\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b}+\frac{\left(d^2\sec(a+bx)\sqrt{\sin(2a+2bx)}\right)\int\frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{2\sqrt{d\tan(a+bx)}}\\
&= \frac{d^2F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{2b\sqrt{d\tan(a+bx)}}-\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b}
\end{aligned}$$

Mathematica [C] time = 0.128121, size = 58, normalized size = 0.74

$$\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}\left(\sqrt{\sec^2(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2(a+bx)\right)-1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (d*Cos[a + b*x]*(-1 + Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[a + b*x]^2]*Sqr
t[Sec[a + b*x]^2])*Sqrt[d*Tan[a + b*x]])/b
```

Maple [B] time = 0.132, size = 196, normalized size = 2.5

$$-\frac{\sqrt{2}\left(\cos(bx+a)-1\right)\cos(bx+a)\left(\cos(bx+a)+1\right)^2}{2\,b\left(\sin(bx+a)\right)^5}\left(\sin(bx+a)\,\mathrm{EllipticF}\left(\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*(d*tan(b*x+a))^(3/2),x)
```

```
[Out] -1/2/b*2^(1/2)*(cos(b*x+a)-1)*(sin(b*x+a)*EllipticF((-(cos(b*x+a)-1-sin(b*x
+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos
(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x
+a))^(1/2)+cos(b*x+a)^2*2^(1/2)-cos(b*x+a)*2^(1/2))*cos(b*x+a)*(cos(b*x+a)+
1)^2*(d*sin(b*x+a)/cos(b*x+a))^(3/2)/sin(b*x+a)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^{\frac{3}{2}}\cos(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*cos(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{d\tan(bx+a)}d\cos(bx+a)\tan(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*d*cos(b*x + a)*tan(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.245 $\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=108

$$\frac{d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{12b\sqrt{d\tan(a+bx)}}-\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}+\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{6b}$$

```
[Out] (d^2*EllipticF[a - Pi/4 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(12*
b*Sqrt[d*Tan[a + b*x]]) + (d*Cos[a + b*x]*Sqrt[d*Tan[a + b*x]])/(6*b) - (d*
Cos[a + b*x]^3*Sqrt[d*Tan[a + b*x]])/(3*b)
```

Rubi [A] time = 0.133509, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2610, 2612, 2614, 2573, 2641}

$$\frac{d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{12b\sqrt{d\tan(a+bx)}}-\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}+\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```


```
[Out] (d^2*EllipticF[a - Pi/4 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(12*
b*Sqrt[d*Tan[a + b*x]]) + (d*Cos[a + b*x]*Sqrt[d*Tan[a + b*x]])/(6*b) - (d*
Cos[a + b*x]^3*Sqrt[d*Tan[a + b*x]])/(3*b)
```


Rule 2610

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m)
, x] - Dist[(b^2*(n - 1))/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1]
 || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

Rule 2612

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*m
), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e + f
*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1] &
```

```
& EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx &= -\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}+\frac{1}{6}d^2\int\frac{\cos(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx\\ &= \frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{6b}-\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}+\frac{1}{12}d^2\int\frac{\sec(a\ldots}{\sqrt{d\tan(\ldots}}\\ &= \frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{6b}-\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}+\frac{\left(d^2\sqrt{\sin(a+bx)}\ldots\right.}{12\sqrt{\cos(a+\ldots}}\\ &= \frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{6b}-\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}+\frac{\left(d^2\sec(a+bx)\sqrt{\ldots}\right.}{12\ldots}\\ &= \frac{d^2F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{12b\sqrt{d\tan(a+bx)}}+\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{6b}-\ldots\end{aligned}$$

Mathematica [C] time = 1.08639, size = 96, normalized size = 0.89

$$-\frac{\cos(a+bx)(d\tan(a+bx))^{3/2}\left(\cos(2(a+bx))\sqrt{\tan(a+bx)}+\sqrt[4]{-1}\sqrt{\sec^2(a+bx)}F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right)\middle|-1\right)\right)}{6b\tan^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -(Cos[a + b*x]*((-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]
]], -1]*Sqrt[Sec[a + b*x]^2] + Cos[2*(a + b*x)]*Sqrt[Tan[a + b*x]])*(d*Tan[
a + b*x])^(3/2))/(6*b*Tan[a + b*x]^(3/2))
```

Maple [A] time = 0.172, size = 220, normalized size = 2.

$$-\frac{\sqrt{2}\left(\cos\left(bx+a\right)-1\right)\cos\left(bx+a\right)\left(\cos\left(bx+a\right)+1\right)^{2}}{12\,b\left(\sin\left(bx+a\right)\right)^{5}}\left(\sin\left(bx+a\right)\sqrt{\frac{\cos\left(bx+a\right)-1}{\sin\left(bx+a\right)}}\sqrt{\frac{1-\cos\left(bx+a\right)+\sin\left(bx+a\right)}{\sin\left(bx+a\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*(d*tan(b*x+a))^(3/2),x)
```

```
[Out] -1/12/b*2^(1/2)*(cos(b*x+a)-1)*(sin(b*x+a)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2
)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/s
in(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2
*2^(1/2))+2*cos(b*x+a)^4*2^(1/2)-2*cos(b*x+a)^3*2^(1/2)-cos(b*x+a)^2*2^(1/2
)+cos(b*x+a)*2^(1/2))*cos(b*x+a)*(cos(b*x+a)+1)^2*(d*sin(b*x+a)/cos(b*x+a))
^(3/2)/sin(b*x+a)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}\cos\left(bx+a\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*cos(b*x + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{d\tan\left(bx+a\right)}d\cos\left(bx+a\right)^{3}\tan\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*d*cos(b*x + a)^3*tan(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.246 $\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=136

$$\frac{d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{24b\sqrt{d\tan(a+bx)}}-\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}+\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{30b}+\frac{d\,\mathrm{co}\cdots}{}$$

```
[Out] (d^2*EllipticF[a - Pi/4 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(24*
b*Sqrt[d*Tan[a + b*x]]) + (d*Cos[a + b*x]*Sqrt[d*Tan[a + b*x]])/(12*b) + (d
*Cos[a + b*x]^3*Sqrt[d*Tan[a + b*x]])/(30*b) - (d*Cos[a + b*x]^5*Sqrt[d*Tan
[a + b*x]])/(5*b)
```

Rubi [A] time = 0.174181, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2610, 2612, 2614, 2573, 2641}

$$\frac{d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{24b\sqrt{d\tan(a+bx)}}-\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}+\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{30b}+\frac{d\,\mathrm{co}\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^5*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (d^2*EllipticF[a - Pi/4 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(24*
b*Sqrt[d*Tan[a + b*x]]) + (d*Cos[a + b*x]*Sqrt[d*Tan[a + b*x]])/(12*b) + (d
*Cos[a + b*x]^3*Sqrt[d*Tan[a + b*x]])/(30*b) - (d*Cos[a + b*x]^5*Sqrt[d*Tan
[a + b*x]])/(5*b)
```

Rule 2610

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m)
, x] - Dist[(b^2*(n - 1))/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1]
 || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

Rule 2612

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*m
), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e + f
```

```
*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1] &
& EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}+\frac{1}{10}d^2\int \frac{\cos^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

$$= \frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{30b}-\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}+\frac{1}{12}d^2\int \frac{\cos(a}{\sqrt{d\tan}}$$

$$= \frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{12b}+\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{30b}-\frac{d\cos^5(a+bx)\sqrt{d}}{5b}$$

$$= \frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{12b}+\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{30b}-\frac{d\cos^5(a+bx)\sqrt{d}}{5b}$$

$$= \frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{12b}+\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{30b}-\frac{d\cos^5(a+bx)\sqrt{d}}{5b}$$

$$= \frac{d^2F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{24b\sqrt{d\tan(a+bx)}}+\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{12b}+$$

Mathematica [C] time = 2.38904, size = 131, normalized size = 0.96

$$\frac{\cos(2(a+bx))\csc(a+bx)(d\tan(a+bx))^{3/2}\left((10\cos(2(a+bx))+3\cos(4(a+bx))-3)\sqrt{\tan(a+bx)}+10\sqrt[4]{-1}\sqrt{\sec^2(a}\right.}{120b\sqrt{\tan(a+bx)}\left(\tan^2(a+bx)-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^5*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (Cos[2*(a + b*x)]*Csc[a + b*x]*(10*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4
)*Sqrt[Tan[a + b*x]]], -1]*Sqrt[Sec[a + b*x]^2] + (-3 + 10*Cos[2*(a + b*x)]
 + 3*Cos[4*(a + b*x)])*Sqrt[Tan[a + b*x]])*(d*Tan[a + b*x])^(3/2))/(120*b*S
qrt[Tan[a + b*x]]*(-1 + Tan[a + b*x]^2))
```

Maple [A] time = 0.16, size = 250, normalized size = 1.8

$$-\frac{\sqrt{2}\left(\cos\left(bx+a\right)-1\right)\cos\left(bx+a\right)\left(\cos\left(bx+a\right)+1\right)^{2}}{120\,b\left(\sin\left(bx+a\right)\right)^{5}}\left(12\,\left(\cos\left(bx+a\right)\right)^{6}\sqrt{2}-12\,\sqrt{2}\left(\cos\left(bx+a\right)\right)^{5}-2\,\left(\cos\left(bx+a\right)\right)^{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^5*(d*tan(b*x+a))^(3/2),x)
```

```
[Out] -1/120/b*2^(1/2)*(cos(b*x+a)-1)*(12*cos(b*x+a)^6*2^(1/2)-12*2^(1/2)*cos(b*x
+a)^5-2*cos(b*x+a)^4*2^(1/2)+5*sin(b*x+a)*EllipticF((-(cos(b*x+a)-1-sin(b*x
+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos
(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x
+a))^(1/2)+2*cos(b*x+a)^3*2^(1/2)-5*cos(b*x+a)^2*2^(1/2)+5*cos(b*x+a)*2^(1/
2))*cos(b*x+a)*(cos(b*x+a)+1)^2*(d*sin(b*x+a)/cos(b*x+a))^(3/2)/sin(b*x+a)^
5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}\cos\left(bx+a\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*cos(b*x + a)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\tan\left(bx+a\right)}d\cos\left(bx+a\right)^{5}\tan\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*d*cos(b*x + a)^5*tan(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**5*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.247 $\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=67

$$\frac{2(d\tan(e+fx))^{15/2}}{15d^5f}+\frac{4(d\tan(e+fx))^{11/2}}{11d^3f}+\frac{2(d\tan(e+fx))^{7/2}}{7df}$$

```
[Out] (2*(d*Tan[e + f*x])^(7/2))/(7*d*f) + (4*(d*Tan[e + f*x])^(11/2))/(11*d^3*f)
 + (2*(d*Tan[e + f*x])^(15/2))/(15*d^5*f)
```

Rubi [A] time = 0.0584089, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2607, 270}

$$\frac{2(d\tan(e+fx))^{15/2}}{15d^5f}+\frac{4(d\tan(e+fx))^{11/2}}{11d^3f}+\frac{2(d\tan(e+fx))^{7/2}}{7df}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (2*(d*Tan[e + f*x])^(7/2))/(7*d*f) + (4*(d*Tan[e + f*x])^(11/2))/(11*d^3*f)
 + (2*(d*Tan[e + f*x])^(15/2))/(15*d^5*f)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{\text{Subst}\left(\int (dx)^{5/2}\left(1+x^2\right)^2 dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left((dx)^{5/2}+\frac{2(dx)^{9/2}}{d^2}+\frac{(dx)^{13/2}}{d^4}\right) dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{2(d\tan(e+fx))^{7/2}}{7df}+\frac{4(d\tan(e+fx))^{11/2}}{11d^3f}+\frac{2(d\tan(e+fx))^{15/2}}{15d^5f}$$

Mathematica [A] time = 0.430815, size = 52, normalized size = 0.78

$$\frac{2(44\cos(2(e+fx))+4\cos(4(e+fx))+117)\sec^4(e+fx)(d\tan(e+fx))^{7/2}}{1155df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6*(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (2*(117 + 44*Cos[2*(e + f*x)] + 4*Cos[4*(e + f*x)])*Sec[e + f*x]^4*(d*Tan[e
 + f*x])^(7/2))/(1155*d*f)
```

Maple [A] time = 0.181, size = 60, normalized size = 0.9

$$\frac{\left(64\left(\cos\left(fx+e\right)\right)^4+112\left(\cos\left(fx+e\right)\right)^2+154\right)\sin\left(fx+e\right)}{1155\,f\left(\cos\left(fx+e\right)\right)^5}\left(\frac{d\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(d*tan(f*x+e))^(5/2),x)
```

```
[Out] 2/1155/f*(32*cos(f*x+e)^4+56*cos(f*x+e)^2+77)*(d*sin(f*x+e)/cos(f*x+e))^(5/
2)*sin(f*x+e)/cos(f*x+e)^5
```

Maxima [A] time = 0.947682, size = 69, normalized size = 1.03

$$\frac{2\left(77\left(d\tan\left(fx+e\right)\right)^{\frac{15}{2}}+210\left(d\tan\left(fx+e\right)\right)^{\frac{11}{2}}d^2+165\left(d\tan\left(fx+e\right)\right)^{\frac{7}{2}}d^4\right)}{1155\,d^5 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(77*(d*tan(f*x + e))^(15/2) + 210*(d*tan(f*x + e))^(11/2)*d^2 + 165*
(d*tan(f*x + e))^(7/2)*d^4)/(d^5*f)
```

Fricas [A] time = 2.35071, size = 211, normalized size = 3.15

$$-\frac{2\left(32\,d^{2}\cos\left(fx+e\right)^{6}+24\,d^{2}\cos\left(fx+e\right)^{4}+21\,d^{2}\cos\left(fx+e\right)^{2}-77\,d^{2}\right)\sqrt{\frac{d\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)}{1155\,f\cos\left(fx+e\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/1155*(32*d^2*cos(f*x + e)^6 + 24*d^2*cos(f*x + e)^4 + 21*d^2*cos(f*x + e
)^2 - 77*d^2)*sqrt(d*sin(f*x + e)/cos(f*x + e))*sin(f*x + e)/(f*cos(f*x + e
)^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(d*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32158, size = 113, normalized size = 1.69

$$\frac{2\left(77\sqrt{d\tan\left(fx+e\right)}d^{7}\tan\left(fx+e\right)^{7}+210\sqrt{d\tan\left(fx+e\right)}d^{7}\tan\left(fx+e\right)^{5}+165\sqrt{d\tan\left(fx+e\right)}d^{7}\tan\left(fx+e\right)^{3}\right)}{1155\,d^{5}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 2/1155*(77*sqrt(d*tan(f*x + e))*d^7*tan(f*x + e)^7 + 210*sqrt(d*tan(f*x + e
))*d^7*tan(f*x + e)^5 + 165*sqrt(d*tan(f*x + e))*d^7*tan(f*x + e)^3)/(d^5*f
)
```

3.248 $\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=45

$$\frac{2(d\tan(e+fx))^{11/2}}{11d^3f}+\frac{2(d\tan(e+fx))^{7/2}}{7df}$$

```
[Out] (2*(d*Tan[e + f*x])^(7/2))/(7*d*f) + (2*(d*Tan[e + f*x])^(11/2))/(11*d^3*f)
```

Rubi [A] time = 0.0503122, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2607, 14}

$$\frac{2(d\tan(e+fx))^{11/2}}{11d^3f}+\frac{2(d\tan(e+fx))^{7/2}}{7df}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (2*(d*Tan[e + f*x])^(7/2))/(7*d*f) + (2*(d*Tan[e + f*x])^(11/2))/(11*d^3*f)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{\text{Subst}\left(\int (dx)^{5/2}\left(1+x^2\right)dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left((dx)^{5/2}+\frac{(dx)^{9/2}}{d^2}\right)dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{2(d\tan(e+fx))^{7/2}}{7df}+\frac{2(d\tan(e+fx))^{11/2}}{11d^3f}$$

Mathematica [A] time = 0.262581, size = 42, normalized size = 0.93

$$\frac{2(2\cos(2(e+fx))+9)\sec^2(e+fx)(d\tan(e+fx))^{7/2}}{77df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (2*(9 + 2*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(d*Tan[e + f*x])^(7/2))/(77*d*f)
```

Maple [A] time = 0.139, size = 50, normalized size = 1.1

$$\frac{\left(8\left(\cos\left(fx+e\right)\right)^2+14\right)\sin\left(fx+e\right)}{77f\left(\cos\left(fx+e\right)\right)^3}\left(\frac{d\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(d*tan(f*x+e))^(5/2),x)
```

```
[Out] 2/77/f*(4*cos(f*x+e)^2+7)*(d*sin(f*x+e)/cos(f*x+e))^(5/2)*sin(f*x+e)/cos(f*
x+e)^3
```

Maxima [A] time = 0.978024, size = 49, normalized size = 1.09

$$\frac{2\left(7\left(d\tan\left(fx+e\right)\right)^{\frac{11}{2}}+11\left(d\tan\left(fx+e\right)\right)^{\frac{7}{2}}d^2\right)}{77d^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/77*(7*(d*tan(f*x + e))^(11/2) + 11*(d*tan(f*x + e))^(7/2)*d^2)/(d^3*f)
```

Fricas [A] time = 1.92287, size = 171, normalized size = 3.8

$$-\frac{2\left(4\,d^2\cos\left(fx+e\right)^4+3\,d^2\cos\left(fx+e\right)^2-7\,d^2\right)\sqrt{\frac{d\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)}{77\,f\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/77*(4*d^2*cos(f*x + e)^4 + 3*d^2*cos(f*x + e)^2 - 7*d^2)*sqrt(d*sin(f*x
+ e)/cos(f*x + e))*sin(f*x + e)/(f*cos(f*x + e)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(d*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21743, size = 80, normalized size = 1.78

$$\frac{2\left(7\sqrt{d\tan\left(fx+e\right)}d^5\tan\left(fx+e\right)^5+11\sqrt{d\tan\left(fx+e\right)}d^5\tan\left(fx+e\right)^3\right)}{77\,d^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 2/77*(7*sqrt(d*tan(f*x + e))*d^5*tan(f*x + e)^5 + 11*sqrt(d*tan(f*x + e))*d
^5*tan(f*x + e)^3)/(d^3*f)
```

3.249 $\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=22

$$\frac{2(d\tan(e+fx))^{7/2}}{7df}$$

```
[Out] (2*(d*Tan[e + f*x])^(7/2))/(7*d*f)
```

Rubi [A] time = 0.0427842, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2607, 32}

$$\frac{2(d\tan(e+fx))^{7/2}}{7df}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (2*(d*Tan[e + f*x])^(7/2))/(7*d*f)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx &= \frac{\text{Subst}\left(\int (dx)^{5/2}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{2(d\tan(e+fx))^{7/2}}{7df}\end{aligned}$$

Mathematica [A] time = 0.0548, size = 22, normalized size = 1.

$$\frac{2(d\tan(e+fx))^{7/2}}{7df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (2*(d*Tan[e + f*x])^(7/2))/(7*d*f)
```

Maple [A] time = 0.02, size = 19, normalized size = 0.9

$$\frac{2}{7df}\left(d\tan\left(fx+e\right)\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(d*tan(f*x+e))^(5/2),x)
```

```
[Out] 2/7*(d*tan(f*x+e))^(7/2)/d/f
```

Maxima [A] time = 0.940318, size = 24, normalized size = 1.09

$$\frac{2\left(d\tan\left(fx+e\right)\right)^{\frac{7}{2}}}{7df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*(d*tan(f*x + e))^(7/2)/(d*f)
```

Fricas [B] time = 1.74948, size = 134, normalized size = 6.09

$$-\frac{2\left(d^2\cos\left(fx+e\right)^2-d^2\right)\sqrt{\frac{d\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)}{7f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/7*(d^2*cos(f*x + e)^2 - d^2)*sqrt(d*sin(f*x + e)/cos(f*x + e))*sin(f*x +
 e)/(f*cos(f*x + e)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(d*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33562, size = 38, normalized size = 1.73

$$\frac{2\sqrt{d\tan\left(fx+e\right)}d^{2}\tan\left(fx+e\right)^{3}}{7\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 2/7*sqrt(d*tan(f*x + e))*d^2*tan(f*x + e)^3/f
```

3.250 $\int (d\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=212

$$\frac{d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}f}-\frac{d^{5/2}\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+$$

```
[Out] (d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*x]]]
)/(2*Sqrt[2]*f) + (d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt[2]*Sqr
t[d*Tan[e + f*x]]])/(2*Sqrt[2]*f) + (2*d*(d*Tan[e + f*x])^(3/2))/(3*f)
```

Rubi [A] time = 0.14225, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3473, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}f}-\frac{d^{5/2}\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(5/2),x]
```


```
[Out] (d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*x]]]
)/(2*Sqrt[2]*f) + (d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt[2]*Sqr
t[d*Tan[e + f*x]]])/(2*Sqrt[2]*f) + (2*d*(d*Tan[e + f*x])^(3/2))/(3*f)
```


Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int (d\tan(e+fx))^{5/2}\,dx = \frac{2d(d\tan(e+fx))^{3/2}}{3f} - d^2\int \sqrt{d\tan(e+fx)}\,dx$$

$$= \frac{2d(d\tan(e+fx))^{3/2}}{3f} - \frac{d^3\,\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\tan(e+fx)\right)}{f}$$

$$= \frac{2d(d\tan(e+fx))^{3/2}}{3f} - \frac{\left(2d^3\right)\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{f}$$

$$= \frac{2d(d\tan(e+fx))^{3/2}}{3f} + \frac{d^3\,\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{f} - \frac{d^3\,\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\right.}{f}$$

$$= \frac{2d(d\tan(e+fx))^{3/2}}{3f} - \frac{d^{5/2}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{2\sqrt{2}f} - \frac{d^{5/2}\,\text{Subst}\left(\int \frac{\sqrt{2}}{-d+\sqrt{}}\right.}{}$$

$$= -\frac{d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{2\sqrt{2}f} + \frac{d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)+\sqrt{2}\right.}{2\sqrt{2}f}$$

$$= \frac{d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+f\right.}{2\sqrt{2}f}$$

Mathematica [C] time = 0.0393839, size = 40, normalized size = 0.19

$$-\frac{2d(d\tan(e+fx))^{3/2}\left({}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(e+fx)\right)-1\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (-2*d*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Tan[e + f*x]^2])*(d*Tan[e + f*x
])^(3/2))/(3*f)
```

Maple [A] time = 0.011, size = 182, normalized size = 0.9

$$\frac{2d}{3f}\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}-\frac{d^3\sqrt{2}}{4f}\ln\left(\left(d\tan\left(fx+e\right)-\sqrt[4]{d^2}\sqrt{d\tan\left(fx+e\right)}\sqrt{2}+\sqrt{d^2}\right)\left(d\tan\left(fx+e\right)+\sqrt[4]{d^2}\sqrt{d\tan\left(fx\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(5/2),x)
```

```
[Out] 2/3*d*(d*tan(f*x+e))^(3/2)/f-1/4/f*d^3/(d^2)^(1/4)*2^(1/2)*ln((d*tan(f*x+e)
-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^
(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))-1/2/f*d^3/(d^2)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)+1/2/f*d^3/(d^2)^(1/4
)*2^(1/2)*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.76278, size = 1521, normalized size = 7.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(12*sqrt(2)*(d^10/f^4)^(1/4)*f*arctan(-(d^10 + sqrt(2)*(d^10/f^4)^(1/4
)*d^7*f*sqrt(d*sin(f*x + e)/cos(f*x + e)) - sqrt(2)*(d^10/f^4)^(1/4)*f*sqrt
((d^15*sin(f*x + e) + sqrt(d^10/f^4)*d^10*f^2*cos(f*x + e) + sqrt(2)*(d^10/
f^4)^(3/4)*d^7*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*cos(f*x + e))/cos(f*x
```

```
+ e)))/d^10)*cos(f*x + e) + 12*sqrt(2)*(d^10/f^4)^(1/4)*f*arctan((d^10 - sq
rt(2)*(d^10/f^4)^(1/4)*d^7*f*sqrt(d*sin(f*x + e)/cos(f*x + e)) + sqrt(2)*(d
^10/f^4)^(1/4)*f*sqrt((d^15*sin(f*x + e) + sqrt(d^10/f^4)*d^10*f^2*cos(f*x
+ e) - sqrt(2)*(d^10/f^4)^(3/4)*d^7*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*c
os(f*x + e))/cos(f*x + e)))/d^10)*cos(f*x + e) + 3*sqrt(2)*(d^10/f^4)^(1/4)
*f*cos(f*x + e)*log((d^15*sin(f*x + e) + sqrt(d^10/f^4)*d^10*f^2*cos(f*x +
e) + sqrt(2)*(d^10/f^4)^(3/4)*d^7*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*cos
(f*x + e))/cos(f*x + e)) - 3*sqrt(2)*(d^10/f^4)^(1/4)*f*cos(f*x + e)*log((d
^15*sin(f*x + e) + sqrt(d^10/f^4)*d^10*f^2*cos(f*x + e) - sqrt(2)*(d^10/f^4
)^(3/4)*d^7*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*cos(f*x + e))/cos(f*x + e
)) + 8*d^2*sqrt(d*sin(f*x + e)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\tan\left(e+fx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral((d*tan(e + f*x))**(5/2), x)
```

Giac [A] time = 1.28874, size = 277, normalized size = 1.31

$$-\frac{1}{12}\left(\frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{f}+\frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{f}-\frac{3\sqrt{2}|d|^{\frac{3}{2}}\log\left(d\tan\left(fx\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] -1/12*(6*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*
sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/f + 6*sqrt(2)*abs(d)^(3/2)*arctan(-1/2*
sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/f - 3
*sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqr
```

```
t(abs(d)) + abs(d))/f + 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) - sqrt(2)
*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/f - 8*sqrt(d*tan(f*x + e))*d*t
an(f*x + e)/f)*d
```

3.251 $\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=225

$$-\frac{3d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f}+\frac{3d^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}f}+\frac{3d^{5/2}\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\right.}{8\sqrt{2}f}$$

```
[Out] (-3*d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(4*Sqrt[2]*
f) + (3*d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(4*Sqrt
[2]*f) + (3*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan
[e + f*x]]])/(8*Sqrt[2]*f) - (3*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x]
+ Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(8*Sqrt[2]*f) - (d*Cos[e + f*x]^2*(d*Tan[e
 + f*x])^(3/2))/(2*f)
```

Rubi [A] time = 0.162825, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2607, 288, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{3d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f}+\frac{3d^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}f}+\frac{3d^{5/2}\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\right.}{8\sqrt{2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (-3*d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(4*Sqrt[2]*
f) + (3*d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(4*Sqrt
[2]*f) + (3*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan
[e + f*x]]])/(8*Sqrt[2]*f) - (3*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x]
+ Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(8*Sqrt[2]*f) - (d*Cos[e + f*x]^2*(d*Tan[e
 + f*x])^(3/2))/(2*f)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{\text{Subst}\left(\int \frac{(dx)^{5/2}}{\left(1+x^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2f} + \frac{\left(3d^2\right)\text{Subst}\left(\int \frac{\sqrt{dx}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{4f}$$

$$= -\frac{d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2f} + \frac{(3d)\,\text{Subst}\left(\int \frac{x^2}{1+\frac{x^4}{d^2}}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{2f}$$

$$= -\frac{d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2f} - \frac{(3d)\,\text{Subst}\left(\int \frac{d-x^2}{1+\frac{x^4}{d^2}}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{4f} +$$

$$= -\frac{d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2f} + \frac{\left(3d^{5/2}\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(e\right.}{8\sqrt{2}f}$$

$$= \frac{3d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{8\sqrt{2}f} - \frac{3d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e\right.}{8}$$

$$= -\frac{3d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f} + \frac{3d^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f} + \frac{3d^{5/2}\log\left(\sqrt{\right.}{}$$

Mathematica [A] time = 0.188438, size = 107, normalized size = 0.48

$$-\frac{d^2\sqrt{\sin(2(e+fx))}\sqrt{d\tan(e+fx)}\left(2\sqrt{\sin(2(e+fx))}+3\csc(e+fx)\sin^{-1}(\cos(e+fx)-\sin(e+fx))+3\csc(e+fx)\,\text{lo}\right.}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] -(d^2*(3*ArcSin[Cos[e + f*x] - Sin[e + f*x]]*Csc[e + f*x] + 3*Csc[e + f*x]*
Log[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]]] + 2*Sqrt[Sin[2*(e
 + f*x)]])*Sqrt[Sin[2*(e + f*x)]]*Sqrt[d*Tan[e + f*x]])/(8*f)
```

Maple [C] time = 0.158, size = 532, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(d*tan(f*x+e))^(5/2),x)
```

```
[Out] -1/8/f*2^(1/2)*(cos(f*x+e)-1)*(3*I*EllipticPi((-(cos(f*x+e)-1-sin(f*x+e))/s
in(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+
e))^(1/2)*(-(cos(f*x+e)-1-sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1)/sin
(f*x+e))^(1/2)-3*I*EllipticPi((-(cos(f*x+e)-1-sin(f*x+e))/sin(f*x+e))^(1/2)
,1/2+1/2*I,1/2*2^(1/2))*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(cos
(f*x+e)-1-sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)-3
*EllipticPi((-(cos(f*x+e)-1-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(
1/2))*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(cos(f*x+e)-1-sin(f*x+
e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)-3*EllipticPi((-(cos
(f*x+e)-1-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos(f*x+e)
-1+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(cos(f*x+e)-1-sin(f*x+e))/sin(f*x+e))^(1
/2)*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)+2*cos(f*x+e)^2*2^(1/2)-2*cos(f*x+e)*2
^(1/2))*cos(f*x+e)^2*(cos(f*x+e)+1)^2*(d*sin(f*x+e)/cos(f*x+e))^(5/2)/sin(f
*x+e)^5
```

Maple [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(d*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27035, size = 321, normalized size = 1.43

$$-\frac{1}{16}\,d^3\left(\frac{8\sqrt{d\tan\left(fx+e\right)}d\tan\left(fx+e\right)}{\left(d^2\tan\left(fx+e\right)^2+d^2\right)f}-\frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{d^2f}-\frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\right.}\right.}{2\sqrt{|d}}\right.}{d^2f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] -1/16*d^3*(8*sqrt(d*tan(f*x + e))*d*tan(f*x + e)/((d^2*tan(f*x + e)^2 + d^2
)*f) - 6*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*
sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(d^2*f) - 6*sqrt(2)*abs(d)^(3/2)*arctan
```

```
(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))
/(d^2*f) + 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f
*x + e))*sqrt(abs(d)) + abs(d))/(d^2*f) - 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(
f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(d^2*f))
```

3.252 $\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=253

$$-\frac{3d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{32\sqrt{2}f}+\frac{3d^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{32\sqrt{2}f}+\frac{3d^{5/2}\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\right.}{64\sqrt{2}f}$$

```
[Out] (-3*d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(32*Sqrt[2]
*f) + (3*d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(32*Sq
rt[2]*f) + (3*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*T
an[e + f*x]]])/(64*Sqrt[2]*f) - (3*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*
x] + Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(64*Sqrt[2]*f) + (3*d*Cos[e + f*x]^2*(d
*Tan[e + f*x])^(3/2))/(16*f) - (d*Cos[e + f*x]^4*(d*Tan[e + f*x])^(3/2))/(4
*f)
```

Rubi [A] time = 0.179872, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2607, 288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{3d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{32\sqrt{2}f}+\frac{3d^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{32\sqrt{2}f}+\frac{3d^{5/2}\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\right.}{64\sqrt{2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (-3*d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(32*Sqrt[2]
*f) + (3*d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(32*Sq
rt[2]*f) + (3*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*T
an[e + f*x]]])/(64*Sqrt[2]*f) - (3*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*
x] + Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(64*Sqrt[2]*f) + (3*d*Cos[e + f*x]^2*(d
*Tan[e + f*x])^(3/2))/(16*f) - (d*Cos[e + f*x]^4*(d*Tan[e + f*x])^(3/2))/(4
*f)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{\text{Subst}\left(\int \frac{(dx)^{5/2}}{\left(1+x^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{d\cos^4(e+fx)(d\tan(e+fx))^{3/2}}{4f} + \frac{\left(3d^2\right)\text{Subst}\left(\int \frac{\sqrt{dx}}{\left(1+x^2\right)^2}\,dx, x, \tan(e+fx)\right)}{8f}$$

$$= \frac{3d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{16f} - \frac{d\cos^4(e+fx)(d\tan(e+fx))^{3/2}}{4f} + \frac{\left(3d^2\right)\text{Su}}{}$$

$$= \frac{3d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{16f} - \frac{d\cos^4(e+fx)(d\tan(e+fx))^{3/2}}{4f} + \frac{(3d)\,\text{Sub}}{}$$

$$= \frac{3d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{16f} - \frac{d\cos^4(e+fx)(d\tan(e+fx))^{3/2}}{4f} - \frac{(3d)\,\text{Sub}}{}$$

$$= \frac{3d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{16f} - \frac{d\cos^4(e+fx)(d\tan(e+fx))^{3/2}}{4f} + \frac{\left(3d^{5/2}\right)\text{S}}{}$$

$$= \frac{3d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{64\sqrt{2}f} - \frac{3d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan\right.}{}$$

$$= -\frac{3d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{32\sqrt{2}f} + \frac{3d^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{32\sqrt{2}f} + \frac{3d^{5/2}\log\left(\right.}{}$$

Mathematica [A] time = 0.188587, size = 125, normalized size = 0.49

$$-\frac{d^2\sqrt{d\tan(e+fx)}\left(-2\sin(2(e+fx))+2\sin(4(e+fx))+3\sqrt{\sin(2(e+fx))}\csc(e+fx)\sin^{-1}(\cos(e+fx)-\sin(e+f\right.}{64f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] -(d^2*(3*ArcSin[Cos[e + f*x] - Sin[e + f*x]]*Csc[e + f*x]*Sqrt[Sin[2*(e + f
*x)]] + 3*Csc[e + f*x]*Log[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e + f*
x)]]]*Sqrt[Sin[2*(e + f*x)]] - 2*Sin[2*(e + f*x)] + 2*Sin[4*(e + f*x)])*Sqr
```

```
t[d*Tan[e + f*x]])/(64*f)
```

Maple [C] time = 0.145, size = 550, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(d*tan(f*x+e))^(5/2),x)
```

```
[Out] 1/64/f*2^(1/2)*(cos(f*x+e)-1)*(3*I*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((1-co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e
))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,
1/2*2^(1/2))-3*I*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticP
i(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-8*cos
(f*x+e)^4*2^(1/2)+8*cos(f*x+e)^3*2^(1/2)+3*((cos(f*x+e)-1)/sin(f*x+e))^(1/2
)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/s
in(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/
2+1/2*I,1/2*2^(1/2))+3*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))^(1/2)*Ell
ipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))
+6*cos(f*x+e)^2*2^(1/2)-6*cos(f*x+e)*2^(1/2))*cos(f*x+e)^2*(cos(f*x+e)+1)^2
*(d*sin(f*x+e)/cos(f*x+e))^(5/2)/sin(f*x+e)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(d*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33293, size = 366, normalized size = 1.45

$$\frac{1}{128}d^5\left(\frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{d^4f}+\frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{d^4f}-\frac{3\sqrt{2}|d|^{\frac{3}{2}}\log\left(d\tan\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 1/128*d^5*(6*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d))
+ 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(d^4*f) + 6*sqrt(2)*abs(d)^(3/2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(
d)))/(d^4*f) - 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*t
an(f*x + e))*sqrt(abs(d)) + abs(d))/(d^4*f) + 3*sqrt(2)*abs(d)^(3/2)*log(d*
tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(d^4*f)
+ 8*(3*sqrt(d*tan(f*x + e))*d^3*tan(f*x + e)^3 - sqrt(d*tan(f*x + e))*d^3*t
an(f*x + e))/((d^2*tan(f*x + e)^2 + d^2)^2*d^2*f))
```

3.253 $\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$

Optimal. Leaf size=109

$$\frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}+\frac{4\sec(e+fx)\sqrt{d\tan(e+fx)}}{7df}+\frac{4\sqrt{\sin(2e+2fx)}\sec(e+fx)F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{7f\sqrt{d\tan(e+fx)}}$$

```
[Out] (4*EllipticF[e - Pi/4 + f*x, 2]*Sec[e + f*x]*Sqrt[Sin[2*e + 2*f*x]])/(7*f*S
qrt[d*Tan[e + f*x]]) + (4*Sec[e + f*x]*Sqrt[d*Tan[e + f*x]])/(7*d*f) + (2*S
ec[e + f*x]^3*Sqrt[d*Tan[e + f*x]])/(7*d*f)
```

Rubi [A] time = 0.136933, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2613, 2614, 2573, 2641}

$$\frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}+\frac{4\sec(e+fx)\sqrt{d\tan(e+fx)}}{7df}+\frac{4\sqrt{\sin(2e+2fx)}\sec(e+fx)F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{7f\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (4*EllipticF[e - Pi/4 + f*x, 2]*Sec[e + f*x]*Sqrt[Sin[2*e + 2*f*x]])/(7*f*S
qrt[d*Tan[e + f*x]]) + (4*Sec[e + f*x]*Sqrt[d*Tan[e + f*x]])/(7*d*f) + (2*S
ec[e + f*x]^3*Sqrt[d*Tan[e + f*x]])/(7*d*f)
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
```

```
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx &= \frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}+\frac{6}{7}\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx\\
&= \frac{4\sec(e+fx)\sqrt{d\tan(e+fx)}}{7df}+\frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}+\frac{4}{7}\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx\\
&= \frac{4\sec(e+fx)\sqrt{d\tan(e+fx)}}{7df}+\frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}+\frac{\left(4\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}\sqrt{}}}{7\sqrt{\cos(e+fx)}\sqrt{d\tan(e}}\\
&= \frac{4\sec(e+fx)\sqrt{d\tan(e+fx)}}{7df}+\frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}+\frac{\left(4\sec(e+fx)\sqrt{\sin(2e+2fx}\right.}{7\sqrt{d\tan(e+f}}\\
&= \frac{4F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{7f\sqrt{d\tan(e+fx)}}+\frac{4\sec(e+fx)\sqrt{d\tan(e+fx)}}{7df}+\frac{2\sec^3(e+fx}{}
\end{aligned}$$

Mathematica [C] time = 0.541142, size = 79, normalized size = 0.72

$$\frac{2\sin(e+fx)\left(4\sqrt{\sec^2(e+fx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2(e+fx)\right)+(\cos(2(e+fx))+2)\sec^4(e+fx)\right)}{7f\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^5/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*((2 + Cos[2*(e + f*x)])*Sec[e + f*x]^4 + 4*Hypergeometric2F1[1/4, 1/2, 5
/4, -Tan[e + f*x]^2]*Sqrt[Sec[e + f*x]^2])*Sin[e + f*x])/(7*f*Sqrt[d*Tan[e
+ f*x]])
```

Maple [A] time = 0.196, size = 222, normalized size = 2.

$$-\frac{\sqrt{2}\left(\cos\left(fx+e\right)-1\right)\left(\cos\left(fx+e\right)+1\right)^{2}}{7f\left(\sin\left(fx+e\right)\right)^{3}\left(\cos\left(fx+e\right)\right)^{4}}\left(4\sin\left(fx+e\right)\sqrt{\frac{\cos\left(fx+e\right)-1}{\sin\left(fx+e\right)}}\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{\cos(f}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5/(d*tan(f*x+e))^(1/2),x)
```

```
[Out] -1/7/f*2^(1/2)*(cos(f*x+e)-1)*(4*sin(f*x+e)*((cos(f*x+e)-1)/sin(f*x+e))^(1/
2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/
sin(f*x+e))^(1/2)*cos(f*x+e)^3*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))^(1/2),1/2*2^(1/2))-2*cos(f*x+e)^3*2^(1/2)+2*cos(f*x+e)^2*2^(1/2)-cos(f
*x+e)*2^(1/2)+2^(1/2))*(cos(f*x+e)+1)^2/sin(f*x+e)^3/cos(f*x+e)^4/(d*sin(f*
x+e)/cos(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^{5}}{\sqrt{d\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^5/sqrt(d*tan(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\tan\left(fx+e\right)}\sec\left(fx+e\right)^{5}}{d\tan\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(f*x + e))*sec(f*x + e)^5/(d*tan(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^5\left(e+fx\right)}{\sqrt{d\tan\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5/(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**5/sqrt(d*tan(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^5}{\sqrt{d\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^5/sqrt(d*tan(f*x + e)), x)
```

3.254 $\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$

Optimal. Leaf size=79

$$\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}+\frac{2\sqrt{\sin(2e+2fx)}\sec(e+fx)F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{3f\sqrt{d\tan(e+fx)}}$$

```
[Out] (2*EllipticF[e - Pi/4 + f*x, 2]*Sec[e + f*x]*Sqrt[Sin[2*e + 2*f*x]])/(3*f*S
qrt[d*Tan[e + f*x]]) + (2*Sec[e + f*x]*Sqrt[d*Tan[e + f*x]])/(3*d*f)
```

Rubi [A] time = 0.0959896, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2613, 2614, 2573, 2641}

$$\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}+\frac{2\sqrt{\sin(2e+2fx)}\sec(e+fx)F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{3f\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*EllipticF[e - Pi/4 + f*x, 2]*Sec[e + f*x]*Sqrt[Sin[2*e + 2*f*x]])/(3*f*S
qrt[d*Tan[e + f*x]]) + (2*Sec[e + f*x]*Sqrt[d*Tan[e + f*x]])/(3*d*f)
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx &= \frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}+\frac{2}{3}\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx\\ &= \frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}+\frac{\left(2\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}}\,dx}{3\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}\\ &= \frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}+\frac{\left(2\sec(e+fx)\sqrt{\sin(2e+2fx)}\right)\int \frac{1}{\sqrt{\sin(2e+2fx)}}\,dx}{3\sqrt{d\tan(e+fx)}}\\ &= \frac{2F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{3f\sqrt{d\tan(e+fx)}}+\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}\end{aligned}$$

Mathematica [C] time = 0.264323, size = 68, normalized size = 0.86

$$\frac{2\sin(e+fx)\left(2\sqrt{\sec^2(e+fx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2(e+fx)\right)+\sec^2(e+fx)\right)}{3f\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*(Sec[e + f*x]^2 + 2*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[e + f*x]^2]*Sq
rt[Sec[e + f*x]^2])*Sin[e + f*x])/(3*f*Sqrt[d*Tan[e + f*x]])
```

Maple [B] time = 0.171, size = 194, normalized size = 2.5

$$-\frac{\sqrt{2}\left(\cos\left(fx+e\right)-1\right)\left(\cos\left(fx+e\right)+1\right)^{2}}{3f\left(\sin\left(fx+e\right)\right)^{3}\left(\cos\left(fx+e\right)\right)^{2}}\left(2\,\mathrm{EllipticF}\left(\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}},1/2\sqrt{2}\right)\sin\left(fx+e\right)\cos\left(fx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3/(d*tan(f*x+e))^(1/2),x)
```

```
[Out] -1/3/f*2^(1/2)*(cos(f*x+e)-1)*(2*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f
*x+e))^(1/2),1/2*2^(1/2))*sin(f*x+e)*cos(f*x+e)*((cos(f*x+e)-1)/sin(f*x+e))
^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+
e))/sin(f*x+e))^(1/2)-cos(f*x+e)*2^(1/2)+2^(1/2))*(cos(f*x+e)+1)^2/sin(f*x+
e)^3/cos(f*x+e)^2/(d*sin(f*x+e)/cos(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^{3}}{\sqrt{d\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^3/sqrt(d*tan(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{d\tan\left(fx+e\right)}\sec\left(fx+e\right)^{3}}{d\tan\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(f*x + e))*sec(f*x + e)^3/(d*tan(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3\left(e+fx\right)}{\sqrt{d\tan\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3/(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**3/sqrt(d*tan(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^3}{\sqrt{d\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^3/sqrt(d*tan(f*x + e)), x)
```

3.255 $\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{\sin(2e+2fx)}\sec(e+fx)F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{f\sqrt{d\tan(e+fx)}}$$

```
[Out] (EllipticF[e - Pi/4 + f*x, 2]*Sec[e + f*x]*Sqrt[Sin[2*e + 2*f*x]])/(f*Sqrt[
d*Tan[e + f*x]])
```

Rubi [A] time = 0.0587222, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2614, 2573, 2641}

$$\frac{\sqrt{\sin(2e+2fx)}\sec(e+fx)F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{f\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (EllipticF[e - Pi/4 + f*x, 2]*Sec[e + f*x]*Sqrt[Sin[2*e + 2*f*x]])/(f*Sqrt[
d*Tan[e + f*x]])
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx &= \frac{\sqrt{\sin(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}}\,dx}{\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}\\ &= \frac{\left(\sec(e+fx)\sqrt{\sin(2e+2fx)}\right)\int \frac{1}{\sqrt{\sin(2e+2fx)}}\,dx}{\sqrt{d\tan(e+fx)}}\\ &= \frac{F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{f\sqrt{d\tan(e+fx)}}\end{aligned}$$

Mathematica [C] time = 0.124286, size = 77, normalized size = 1.64

$$-\frac{2\sqrt[4]{-1}\sqrt{\tan(e+fx)}\sec^3(e+fx)F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(e+fx)}\right)\middle|-1\right)}{f\left(\tan^2(e+fx)+1\right)^{3/2}\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (-2*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[e + f*x]]], -1]*Sec[
e + f*x]^3*Sqrt[Tan[e + f*x]])/(f*Sqrt[d*Tan[e + f*x]]*(1 + Tan[e + f*x]^2)
^(3/2))
```

Maple [B] time = 0.122, size = 165, normalized size = 3.5

$$-\frac{\sqrt{2}\left(\cos\left(fx+e\right)-1\right)\left(\cos\left(fx+e\right)+1\right)^2}{f\left(\sin\left(fx+e\right)\right)^2\cos\left(fx+e\right)}\mathrm{EllipticF}\left(\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{\cos\left(fx+e\right)-1}{\sin\left(fx+e\right)}}\sqrt{\frac{1-}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(d*tan(f*x+e))^(1/2),x)
```

```
[Out] -1/f*2^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(
1/2))*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/sin(f*x+
e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(cos(f*x+e)-1)/sin(f
*x+e)^2/cos(f*x+e)*(cos(f*x+e)+1)^2/(d*sin(f*x+e)/cos(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(fx+e)}{\sqrt{d\tan(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)/sqrt(d*tan(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{d\tan(fx+e)}\sec(fx+e)}{d\tan(fx+e)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(f*x + e))*sec(f*x + e)/(d*tan(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/sqrt(d*tan(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(fx+e)}{\sqrt{d\tan(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)/sqrt(d*tan(f*x + e)), x)
```

3.256 $\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}} dx$

Optimal. Leaf size=76

$$\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}+\frac{\sqrt{\sin(2e+2fx)}\sec(e+fx)F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{2f\sqrt{d\tan(e+fx)}}$$

```
[Out] (EllipticF[e - Pi/4 + f*x, 2]*Sec[e + f*x]*Sqrt[Sin[2*e + 2*f*x]])/(2*f*Sqr
t[d*Tan[e + f*x]]) + (Cos[e + f*x]*Sqrt[d*Tan[e + f*x]])/(d*f)
```

Rubi [A] time = 0.0932574, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2612, 2614, 2573, 2641}

$$\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}+\frac{\sqrt{\sin(2e+2fx)}\sec(e+fx)F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{2f\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (EllipticF[e - Pi/4 + f*x, 2]*Sec[e + f*x]*Sqrt[Sin[2*e + 2*f*x]])/(2*f*Sqr
t[d*Tan[e + f*x]]) + (Cos[e + f*x]*Sqrt[d*Tan[e + f*x]])/(d*f)
```

Rule 2612

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*m
), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e + f
*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1] &
& EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx &= \frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}+\frac{1}{2}\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx\\ &= \frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}+\frac{\sqrt{\sin(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}}\,dx}{2\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}\\ &= \frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}+\frac{\left(\sec(e+fx)\sqrt{\sin(2e+2fx)}\right)\int \frac{1}{\sqrt{\sin(2e+2fx)}}\,dx}{2\sqrt{d\tan(e+fx)}}\\ &= \frac{F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{2f\sqrt{d\tan(e+fx)}}+\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}\end{aligned}$$

Mathematica [C] time = 0.537385, size = 126, normalized size = 1.66

$$\frac{\cos(2(e+fx))\sqrt{\tan(e+fx)}\sec(e+fx)\left(-\sqrt{\tan(e+fx)}\sqrt{\sec^2(e+fx)}+\sqrt[4]{-1}\sec^2(e+fx)F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(e+\right.\right.}{f\left(\tan^2(e+fx)-1\right)\sqrt{\sec^2(e+fx)}\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (Cos[2*(e + f*x)]*Sec[e + f*x]*((-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*S
qrt[Tan[e + f*x]]], -1]*Sec[e + f*x]^2 - Sqrt[Sec[e + f*x]^2]*Sqrt[Tan[e +
f*x]])*Sqrt[Tan[e + f*x]])/(f*Sqrt[Sec[e + f*x]^2]*Sqrt[d*Tan[e + f*x]]*(-1
 + Tan[e + f*x]^2))
```

Maple [B] time = 0.145, size = 196, normalized size = 2.6

$$-\frac{\sqrt{2}\left(\cos\left(fx+e\right)-1\right)\left(\cos\left(fx+e\right)+1\right)^{2}}{2f\left(\sin\left(fx+e\right)\right)^{3}\cos\left(fx+e\right)}\left(\sin\left(fx+e\right)\sqrt{\frac{\cos\left(fx+e\right)-1}{\sin\left(fx+e\right)}}\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{\cos\left(fx\right.}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)/(d*tan(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*2^(1/2)*(cos(f*x+e)-1)*(sin(f*x+e)*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)
*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))/si
n(f*x+e))^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*
2^(1/2))-cos(f*x+e)^2*2^(1/2)+cos(f*x+e)*2^(1/2))*(cos(f*x+e)+1)^2/sin(f*x+
e)^3/cos(f*x+e)/(d*sin(f*x+e)/cos(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)}{\sqrt{d\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)/sqrt(d*tan(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\tan\left(fx+e\right)}\cos\left(fx+e\right)}{d\tan\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(f*x + e))*cos(f*x + e)/(d*tan(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(e+fx\right)}{\sqrt{d\tan\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(cos(e + f*x)/sqrt(d*tan(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)}{\sqrt{d\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)/sqrt(d*tan(f*x + e)), x)
```

3.257 $\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$

Optimal. Leaf size=109

$$\frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df}+\frac{5\cos(e+fx)\sqrt{d\tan(e+fx)}}{6df}+\frac{5\sqrt{\sin(2e+2fx)}\sec(e+fx)F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{12f\sqrt{d\tan(e+fx)}}$$

```
[Out] (5*EllipticF[e - Pi/4 + f*x, 2]*Sec[e + f*x]*Sqrt[Sin[2*e + 2*f*x]])/(12*f*
Sqrt[d*Tan[e + f*x]]) + (5*Cos[e + f*x]*Sqrt[d*Tan[e + f*x]])/(6*d*f) + (Co
s[e + f*x]^3*Sqrt[d*Tan[e + f*x]])/(3*d*f)
```

Rubi [A] time = 0.127342, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2612, 2614, 2573, 2641}

$$\frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df}+\frac{5\cos(e+fx)\sqrt{d\tan(e+fx)}}{6df}+\frac{5\sqrt{\sin(2e+2fx)}\sec(e+fx)F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{12f\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3/Sqrt[d*Tan[e + f*x]],x]
```


```
[Out] (5*EllipticF[e - Pi/4 + f*x, 2]*Sec[e + f*x]*Sqrt[Sin[2*e + 2*f*x]])/(12*f*
Sqrt[d*Tan[e + f*x]]) + (5*Cos[e + f*x]*Sqrt[d*Tan[e + f*x]])/(6*d*f) + (Co
s[e + f*x]^3*Sqrt[d*Tan[e + f*x]])/(3*d*f)
```


Rule 2612

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*m
), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e + f
*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1] &
& EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df} + \frac{5}{6}\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$$

$$= \frac{5\cos(e+fx)\sqrt{d\tan(e+fx)}}{6df} + \frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df} + \frac{5}{12}\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$$

$$= \frac{5\cos(e+fx)\sqrt{d\tan(e+fx)}}{6df} + \frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df} + \frac{\left(5\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}\sqrt{s}}}{12\sqrt{\cos(e+fx)}\sqrt{d\tan(e}}$$

$$= \frac{5\cos(e+fx)\sqrt{d\tan(e+fx)}}{6df} + \frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df} + \frac{\left(5\sec(e+fx)\sqrt{\sin(2e+2fx)}\right)}{12\sqrt{d\tan(e+f}}$$

$$= \frac{5F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{12f\sqrt{d\tan(e+fx)}} + \frac{5\cos(e+fx)\sqrt{d\tan(e+fx)}}{6df} + \frac{\cos^3(e+fx)}{3}$$

Mathematica [C] time = 0.970626, size = 94, normalized size = 0.86

$$\frac{11\sin(e+fx)+\sin(3(e+fx))-10\sqrt[4]{-1}\cos(e+fx)\sqrt{\tan(e+fx)}\sqrt{\sec^2(e+fx)}F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(e+fx)}\right)\middle|-1\right)}{12f\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (11*Sin[e + f*x] + Sin[3*(e + f*x)] - 10*(-1)^(1/4)*Cos[e + f*x]*EllipticF[
I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[e + f*x]]], -1]*Sqrt[Sec[e + f*x]^2]*Sqrt[Tan
[e + f*x]])/(12*f*Sqrt[d*Tan[e + f*x]])
```

Maple [A] time = 0.174, size = 224, normalized size = 2.1

$$-\frac{\sqrt{2}\left(\cos\left(fx+e\right)-1\right)\left(\cos\left(fx+e\right)+1\right)^{2}}{12f\left(\sin\left(fx+e\right)\right)^{3}\cos\left(fx+e\right)}\left(5\sin\left(fx+e\right)\sqrt{\frac{\cos\left(fx+e\right)-1}{\sin\left(fx+e\right)}}\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{\cos\left(f\right.}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3/(d*tan(f*x+e))^(1/2),x)
```

```
[Out] -1/12/f*2^(1/2)*(cos(f*x+e)-1)*(5*sin(f*x+e)*((cos(f*x+e)-1)/sin(f*x+e))^(1
/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((cos(f*x+e)-1+sin(f*x+e))
/sin(f*x+e))^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1
/2*2^(1/2))-2*cos(f*x+e)^4*2^(1/2)+2*cos(f*x+e)^3*2^(1/2)-5*cos(f*x+e)^2*2^
(1/2)+5*cos(f*x+e)*2^(1/2))*(cos(f*x+e)+1)^2/sin(f*x+e)^3/cos(f*x+e)/(d*sin
(f*x+e)/cos(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^{3}}{\sqrt{d\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^3/sqrt(d*tan(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{d\tan\left(fx+e\right)}\cos\left(fx+e\right)^{3}}{d\tan\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(f*x + e))*cos(f*x + e)^3/(d*tan(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3/(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^{3}}{\sqrt{d\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^3/sqrt(d*tan(f*x + e)), x)
```

3.258 $\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=65

$$\frac{2(d\tan(a+bx))^{7/2}}{7bd^5}+\frac{4(d\tan(a+bx))^{3/2}}{3bd^3}-\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

```
[Out] -2/(b*d*Sqrt[d*Tan[a + b*x]]) + (4*(d*Tan[a + b*x])^(3/2))/(3*b*d^3) + (2*(
d*Tan[a + b*x])^(7/2))/(7*b*d^5)
```

Rubi [A] time = 0.0598842, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2607, 270}

$$\frac{2(d\tan(a+bx))^{7/2}}{7bd^5}+\frac{4(d\tan(a+bx))^{3/2}}{3bd^3}-\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^6/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -2/(b*d*Sqrt[d*Tan[a + b*x]]) + (4*(d*Tan[a + b*x])^(3/2))/(3*b*d^3) + (2*(
d*Tan[a + b*x])^(7/2))/(7*b*d^5)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{(dx)^{3/2}}\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(dx)^{3/2}} + \frac{2\sqrt{dx}}{d^2} + \frac{(dx)^{5/2}}{d^4}\right) dx, x, \tan(a+bx)\right)}{b}$$
$$= -\frac{2}{bd\sqrt{d\tan(a+bx)}} + \frac{4(d\tan(a+bx))^{3/2}}{3bd^3} + \frac{2(d\tan(a+bx))^{7/2}}{7bd^5}$$

Mathematica [A] time = 0.191811, size = 45, normalized size = 0.69

$$\frac{\tan^2(a+bx)\left(6\sec^2(a+bx)+22\right)-42}{21bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^6/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-42 + (22 + 6*Sec[a + b*x]^2)*Tan[a + b*x]^2)/(21*b*d*Sqrt[d*Tan[a + b*x]]
)
```

Maple [A] time = 0.163, size = 60, normalized size = 0.9

$$-\frac{\left(64\,(\cos(bx+a))^4-16\,(\cos(bx+a))^2-6\right)\sin(bx+a)}{21\,b\,(\cos(bx+a))^5}\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^6/(d*tan(b*x+a))^(3/2),x)
```

```
[Out] -2/21/b*(32*cos(b*x+a)^4-8*cos(b*x+a)^2-3)*sin(b*x+a)/cos(b*x+a)^5/(d*sin(b
*x+a)/cos(b*x+a))^(3/2)
```

Maxima [A] time = 0.963428, size = 73, normalized size = 1.12

$$-\frac{2\left(\frac{21}{\sqrt{d\tan(bx+a)}}-\frac{3\,(d\tan(bx+a))^{\frac{7}{2}}+14\,(d\tan(bx+a))^{\frac{3}{2}}d^2}{d^4}\right)}{21\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/21*(21/sqrt(d*tan(b*x + a)) - (3*(d*tan(b*x + a))^(7/2) + 14*(d*tan(b*x
+ a))^(3/2)*d^2)/d^4)/(b*d)
```

Fricas [A] time = 1.87551, size = 162, normalized size = 2.49

$$-\frac{2\left(32\cos\left(bx+a\right)^{4}-8\cos\left(bx+a\right)^{2}-3\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{21\,bd^{2}\cos\left(bx+a\right)^{3}\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/21*(32*cos(b*x + a)^4 - 8*cos(b*x + a)^2 - 3)*sqrt(d*sin(b*x + a)/cos(b*
x + a))/(b*d^2*cos(b*x + a)^3*sin(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^{6}\left(a+bx\right)}{\left(d\tan\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**6/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(sec(a + b*x)**6/(d*tan(a + b*x))**(3/2), x)
```

Giac [A] time = 1.30494, size = 92, normalized size = 1.42

$$\frac{2\left(3\sqrt{d\tan\left(bx+a\right)}d^{3}\tan\left(bx+a\right)^{3}+14\sqrt{d\tan\left(bx+a\right)}d^{3}\tan\left(bx+a\right)-\frac{21\,d^{4}}{\sqrt{d\tan(bx+a)}}\right)}{21\,bd^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2/21*(3*sqrt(d*tan(b*x + a))*d^3*tan(b*x + a)^3 + 14*sqrt(d*tan(b*x + a))*d
^3*tan(b*x + a) - 21*d^4/sqrt(d*tan(b*x + a)))/(b*d^5)
```

3.259 $\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=43

$$\frac{2(d\tan(a+bx))^{3/2}}{3bd^3}-\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

```
[Out] -2/(b*d*Sqrt[d*Tan[a + b*x]]) + (2*(d*Tan[a + b*x])^(3/2))/(3*b*d^3)
```

Rubi [A] time = 0.0522945, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2607, 14}

$$\frac{2(d\tan(a+bx))^{3/2}}{3bd^3}-\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^4/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -2/(b*d*Sqrt[d*Tan[a + b*x]]) + (2*(d*Tan[a + b*x])^(3/2))/(3*b*d^3)
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{(dx)^{3/2}}\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(dx)^{3/2}} + \frac{\sqrt{dx}}{d^2}\right)dx, x, \tan(a+bx)\right)}{b}$$
$$= -\frac{2}{bd\sqrt{d\tan(a+bx)}} + \frac{2(d\tan(a+bx))^{3/2}}{3bd^3}$$

Mathematica [A] time = 0.0958876, size = 32, normalized size = 0.74

$$\frac{2\left(\tan^2(a+bx)-3\right)}{3bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^4/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*(-3 + Tan[a + b*x]^2))/(3*b*d*Sqrt[d*Tan[a + b*x]])
```

Maple [A] time = 0.138, size = 50, normalized size = 1.2

$$-\frac{\left(8\,(\cos(bx+a))^2-2\right)\sin(bx+a)}{3\,b\,(\cos(bx+a))^3}\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4/(d*tan(b*x+a))^(3/2),x)
```

```
[Out] -2/3/b*(4*cos(b*x+a)^2-1)*sin(b*x+a)/cos(b*x+a)^3/(d*sin(b*x+a)/cos(b*x+a))
^(3/2)
```

Maxima [A] time = 0.945544, size = 49, normalized size = 1.14

$$-\frac{2\left(\frac{3}{\sqrt{d\tan(bx+a)}}-\frac{(d\tan(bx+a))^{\frac{3}{2}}}{d^2}\right)}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/3*(3/sqrt(d*tan(b*x + a)) - (d*tan(b*x + a))^(3/2)/d^2)/(b*d)
```

Fricas [A] time = 1.70958, size = 131, normalized size = 3.05

$$-\frac{2 \left(4 \cos \left(bx+a\right)^{2}-1\right) \sqrt{\frac{d \sin(bx+a)}{\cos(bx+a)}}}{3 \, bd^{2} \cos \left(bx+a\right) \sin \left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(4*cos(b*x + a)^2 - 1)*sqrt(d*sin(b*x + a)/cos(b*x + a))/(b*d^2*cos(b*
x + a)*sin(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{4}\left(a+bx\right)}{\left(d \tan \left(a+bx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(sec(a + b*x)**4/(d*tan(a + b*x))**(3/2), x)
```

Giac [A] time = 1.34156, size = 57, normalized size = 1.33

$$\frac{2 \left(\sqrt{d \tan \left(bx+a\right)} d \tan \left(bx+a\right) - \frac{3 \, d^{2}}{\sqrt{d \tan(bx+a)}}\right)}{3 \, bd^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*(sqrt(d*tan(b*x + a))*d*tan(b*x + a) - 3*d^2/sqrt(d*tan(b*x + a)))/(b*d
^3)
```

3.260 $\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=20

$$-\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

```
[Out] -2/(b*d*Sqrt[d*Tan[a + b*x]])
```

Rubi [A] time = 0.0434227, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2607, 32}

$$-\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -2/(b*d*Sqrt[d*Tan[a + b*x]])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(dx)^{3/2}}\,dx, x, \tan(a+bx)\right)}{b}$$
$$= -\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

Mathematica [A] time = 0.0584714, size = 20, normalized size = 1.

$$-\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -2/(b*d*Sqrt[d*Tan[a + b*x]])
```

Maple [A] time = 0.02, size = 19, normalized size = 1.

$$-2\,\frac{1}{bd\sqrt{d\tan(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2/(d*tan(b*x+a))^(3/2),x)
```

```
[Out] -2/b/d/(d*tan(b*x+a))^(1/2)
```

Maxima [A] time = 0.944843, size = 24, normalized size = 1.2

$$-\frac{2}{\sqrt{d\tan(bx+a)}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/(sqrt(d*tan(b*x + a))*b*d)
```

Fricas [B] time = 1.66253, size = 97, normalized size = 4.85

$$-\frac{2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)}{bd^2\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)/(b*d^2*sin(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(sec(a + b*x)**2/(d*tan(a + b*x))**(3/2), x)
```

Giac [A] time = 1.34494, size = 24, normalized size = 1.2

$$-\frac{2}{\sqrt{d\tan(bx+a)}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -2/(sqrt(d*tan(b*x + a))*b*d)
```

3.261 $\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=212

$$\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}bd^{3/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{\sqrt{2}bd^{3/2}}-\frac{\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{2\sqrt{2}bd^{3/2}}+\frac{\log\left(\sqrt{d}\tan\right.}{}$$

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(Sqrt[2]*b*d^(3/2)) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(Sqrt[2]*b*d^(3/2)) - Log[
Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b*x]]]/(2*Sqrt[2]*b
*d^(3/2)) + Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqrt[2]*Sqrt[d*Tan[a + b*x
]]]/(2*Sqrt[2]*b*d^(3/2)) - 2/(b*d*Sqrt[d*Tan[a + b*x]])
```

Rubi [A] time = 0.141506, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3474, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}bd^{3/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{\sqrt{2}bd^{3/2}}-\frac{\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{2\sqrt{2}bd^{3/2}}+\frac{\log\left(\sqrt{d}\tan\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[a + b*x])^(-3/2),x]
```



```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(Sqrt[2]*b*d^(3/2)) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(Sqrt[2]*b*d^(3/2)) - Log[
Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b*x]]]/(2*Sqrt[2]*b
*d^(3/2)) + Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqrt[2]*Sqrt[d*Tan[a + b*x
]]]/(2*Sqrt[2]*b*d^(3/2)) - 2/(b*d*Sqrt[d*Tan[a + b*x]])
```


Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2}{bd\sqrt{d\tan(a+bx)}} - \frac{\int \sqrt{d\tan(a+bx)}\,dx}{d^2}$$

$$= -\frac{2}{bd\sqrt{d\tan(a+bx)}} - \frac{\mathrm{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\tan(a+bx)\right)}{bd}$$

$$= -\frac{2}{bd\sqrt{d\tan(a+bx)}} - \frac{2\,\mathrm{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{bd}$$

$$= -\frac{2}{bd\sqrt{d\tan(a+bx)}} + \frac{\mathrm{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{bd} - \frac{\mathrm{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan}\right.}{bd}$$

$$= -\frac{2}{bd\sqrt{d\tan(a+bx)}} - \frac{\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{2\sqrt{2}bd^{3/2}} - \frac{\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\right.}{2\sqrt{2}}$$

$$= -\frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{2\sqrt{2}bd^{3/2}} + \frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan}\right.}{2\sqrt{2}bd^{3/2}}$$

$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}bd^{3/2}} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}bd^{3/2}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d}\right.}{2\sqrt{2}bd^{3/2}}$$

Mathematica [C] time = 0.0336544, size = 38, normalized size = 0.18

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\tan^2(a+bx)\right)}{bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[a + b*x])^(-3/2),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[a + b*x]^2])/(b*d*Sqrt[d*Tan[a + b
*x]])
```

Maple [A] time = 0.011, size = 184, normalized size = 0.9

$$-2\frac{1}{bd\sqrt{d\tan(bx+a)}}-\frac{\sqrt{2}}{4\,bd}\ln\left(\left(d\tan(bx+a)-\sqrt[4]{d^2}\sqrt{d\tan(bx+a)}\sqrt{2}+\sqrt{d^2}\right)\left(d\tan(bx+a)+\sqrt[4]{d^2}\sqrt{d\tan(bx+a)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*tan(b*x+a))^(3/2),x)
```

```
[Out] -2/b/d/(d*tan(b*x+a))^(1/2)-1/4/b/d/(d^2)^(1/4)*2^(1/2)*ln((d*tan(b*x+a)-(d
^2)^(1/4)*(d*tan(b*x+a))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(b*x+a)+(d^2)^(1/
4)*(d*tan(b*x+a))^(1/2)*2^(1/2)+(d^2)^(1/2)))-1/2/b/d/(d^2)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(d^2)^(1/4)*(d*tan(b*x+a))^(1/2)+1)+1/2/b/d/(d^2)^(1/4)*2^(1/
2)*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(b*x+a))^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.76502, size = 1693, normalized size = 7.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(8*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)*sin(b*x + a) + 4*(sqr
t(2)*b*d^2*cos(b*x + a)^2 - sqrt(2)*b*d^2)*(1/(b^4*d^6))^(1/4)*arctan(-sqrt
(2)*b*d*sqrt(d*sin(b*x + a)/cos(b*x + a))*(1/(b^4*d^6))^(1/4) + sqrt(2)*b*d
*sqrt((sqrt(2)*b^3*d^5*sqrt(d*sin(b*x + a)/cos(b*x + a))*(1/(b^4*d^6))^(3/4
```

```
)*cos(b*x + a) + b^2*d^4*sqrt(1/(b^4*d^6))*cos(b*x + a) + d*sin(b*x + a))/c
os(b*x + a))*(1/(b^4*d^6))^(1/4) - 1) + 4*(sqrt(2)*b*d^2*cos(b*x + a)^2 - s
qrt(2)*b*d^2)*(1/(b^4*d^6))^(1/4)*arctan(-sqrt(2)*b*d*sqrt(d*sin(b*x + a)/c
os(b*x + a))*(1/(b^4*d^6))^(1/4) + sqrt(2)*b*d*sqrt(-(sqrt(2)*b^3*d^5*sqrt(
d*sin(b*x + a)/cos(b*x + a))*(1/(b^4*d^6))^(3/4)*cos(b*x + a) - b^2*d^4*sqr
t(1/(b^4*d^6))*cos(b*x + a) - d*sin(b*x + a))/cos(b*x + a))*(1/(b^4*d^6))^(
1/4) + 1) + (sqrt(2)*b*d^2*cos(b*x + a)^2 - sqrt(2)*b*d^2)*(1/(b^4*d^6))^(1
/4)*log((sqrt(2)*b^3*d^5*sqrt(d*sin(b*x + a)/cos(b*x + a))*(1/(b^4*d^6))^(3
/4)*cos(b*x + a) + b^2*d^4*sqrt(1/(b^4*d^6))*cos(b*x + a) + d*sin(b*x + a))
/cos(b*x + a)) - (sqrt(2)*b*d^2*cos(b*x + a)^2 - sqrt(2)*b*d^2)*(1/(b^4*d^6
))^(1/4)*log(-(sqrt(2)*b^3*d^5*sqrt(d*sin(b*x + a)/cos(b*x + a))*(1/(b^4*d^
6))^(3/4)*cos(b*x + a) - b^2*d^4*sqrt(1/(b^4*d^6))*cos(b*x + a) - d*sin(b*x
 + a))/cos(b*x + a)))/(b*d^2*cos(b*x + a)^2 - b*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d\tan{(a+bx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral((d*tan(a + b*x))**(-3/2), x)
```

Giac [A] time = 1.2477, size = 275, normalized size = 1.3

$$-\frac{1}{4}d\left(\frac{2\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^4}+\frac{2\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^4}-\frac{\sqrt{2}|d|^{\frac{3}{2}}\log\left(d\tan(bx\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*d*(2*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2
*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d^4) + 2*sqrt(2)*abs(d)^(3/2)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d))
```

```
)/(b*d^4) - sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*
x + a))*sqrt(abs(d)) + abs(d))/(b*d^4) + sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x
 + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^4) + 8/(sq
rt(d*tan(b*x + a))*b*d^2))
```

3.262 $\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=249

$$\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}}-\frac{5\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}bd^{3/2}}-\frac{5\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}bd^{3/2}}+\frac{5\log}{}$$

```
[Out] (5*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*b*d^(3/2)
) - (5*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*b*d^(
3/2)) - (5*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b*x]
]])/(8*Sqrt[2]*b*d^(3/2)) + (5*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqrt[2]
*Sqrt[d*Tan[a + b*x]]])/(8*Sqrt[2]*b*d^(3/2)) - 5/(2*b*d*Sqrt[d*Tan[a + b*x
]]) + Cos[a + b*x]^2/(2*b*d*Sqrt[d*Tan[a + b*x]])
```

Rubi [A] time = 0.183214, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2607, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}}-\frac{5\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}bd^{3/2}}-\frac{5\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}bd^{3/2}}+\frac{5\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```


```
[Out] (5*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*b*d^(3/2)
) - (5*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*b*d^(
3/2)) - (5*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b*x]
]])/(8*Sqrt[2]*b*d^(3/2)) + (5*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqrt[2]
*Sqrt[d*Tan[a + b*x]]])/(8*Sqrt[2]*b*d^(3/2)) - 5/(2*b*d*Sqrt[d*Tan[a + b*x
]]) + Cos[a + b*x]^2/(2*b*d*Sqrt[d*Tan[a + b*x]])
```


Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(dx)^{3/2}(1+x^2)^2}\,dx, x, \tan(a+bx)\right)}{b}$$

$$= \frac{\cos^2(a+bx)}{2bd\sqrt{d\tan(a+bx)}} + \frac{5\,\text{Subst}\left(\int \frac{1}{(dx)^{3/2}(1+x^2)}\,dx, x, \tan(a+bx)\right)}{4b}$$

$$= -\frac{5}{2bd\sqrt{d\tan(a+bx)}} + \frac{\cos^2(a+bx)}{2bd\sqrt{d\tan(a+bx)}} - \frac{5\,\text{Subst}\left(\int \frac{\sqrt{dx}}{1+x^2}\,dx, x, \tan(a+bx)\right)}{4bd^2}$$

$$= -\frac{5}{2bd\sqrt{d\tan(a+bx)}} + \frac{\cos^2(a+bx)}{2bd\sqrt{d\tan(a+bx)}} - \frac{5\,\text{Subst}\left(\int \frac{x^2}{1+\frac{x^4}{d^2}}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{2bd^3}$$

$$= -\frac{5}{2bd\sqrt{d\tan(a+bx)}} + \frac{\cos^2(a+bx)}{2bd\sqrt{d\tan(a+bx)}} + \frac{5\,\text{Subst}\left(\int \frac{d-x^2}{1+\frac{x^4}{d^2}}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{4bd^3} - \frac{5\,\text{Sub}}{}$$

$$= -\frac{5}{2bd\sqrt{d\tan(a+bx)}} + \frac{\cos^2(a+bx)}{2bd\sqrt{d\tan(a+bx)}} - \frac{5\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{3/2}}$$

$$= -\frac{5\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{3/2}} + \frac{5\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\,\text{ta}}\right.}{8\sqrt{2}bd^{3/2}}$$

$$= \frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}} - \frac{5\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}} - \frac{5\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\right.}{8\sqrt{2}bd^{3/2}}$$

Mathematica [A] time = 0.282446, size = 115, normalized size = 0.46

$$\frac{\csc(a+bx)\sqrt{d\tan(a+bx)}\left(-17\cos(a+bx)+\cos(3(a+bx))+5\sqrt{\sin(2(a+bx))}\sin^{-1}(\cos(a+bx)-\sin(a+bx))+5\sqrt{s}\right.}{8bd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (Csc[a + b*x]*(-17*Cos[a + b*x] + Cos[3*(a + b*x)] + 5*ArcSin[Cos[a + b*x]
- Sin[a + b*x]]*Sqrt[Sin[2*(a + b*x)]] + 5*Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]*Sqrt[Sin[2*(a + b*x)]])*Sqrt[d*Tan[a + b*x]])/(8*
b*d^2)
```

Maple [C] time = 0.144, size = 982, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/(d*tan(b*x+a))^(3/2),x)
```

```
[Out] 1/8/b*2^(1/2)*(5*I*cos(b*x+a)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*
((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/si
n(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+
a))^(1/2)-5*I*cos(b*x+a)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos
(b*x+a)-1)/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*
EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1
/2))+5*cos(b*x+a)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)
-1)/sin(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1
/2),1/2+1/2*I,1/2*2^(1/2))*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)+5*
cos(b*x+a)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1)/sin
(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(
cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))+5*I*((cos
(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*E
llipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/
2))*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-5*I*((cos(b*x+a)-1+sin(b*
x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-s
in(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x
+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))+2*cos(b*x+a)^3*2^(1/2)+5*((cos(b*x+a)-1)/
sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a
)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/si
n(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))+5*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*
((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/si
n(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/
2-1/2*I,1/2*2^(1/2))-10*cos(b*x+a)*2^(1/2))*sin(b*x+a)/cos(b*x+a)^2/(d*sin(
b*x+a)/cos(b*x+a))^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(cos(a + b*x)**2/(d*tan(a + b*x))**(3/2), x)
```

Giac [A] time = 1.48193, size = 344, normalized size = 1.38

$$-\frac{1}{16}d^3\left(\frac{10\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^6}+\frac{10\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^6}-\frac{5\sqrt{2}|d|^{\frac{3}{2}}\log\left(d\tan\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -1/16*d^3*(10*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d))
 + 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d^6) + 10*sqrt(2)*abs(d)^(3/2)*
```

```
arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(ab
s(d)))/(b*d^6) - 5*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*sqrt(d
*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^6) + 5*sqrt(2)*abs(d)^(3/2)*log(
d*tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^6
) + 8*(5*d^2*tan(b*x + a)^2 + 4*d^2)/((sqrt(d*tan(b*x + a))*d^2*tan(b*x + a
)^2 + sqrt(d*tan(b*x + a))*d^2)*b*d^4))
```

3.263 $\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=138

$$\frac{24\cos(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3}+\frac{12\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3}-\frac{24\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{5bd^2\sqrt{\sin(2a+2bx)}}-$$

```
[Out] (-2*Sec[a + b*x]^3)/(b*d*Sqrt[d*Tan[a + b*x]]) - (24*Cos[a + b*x]*EllipticE
[a - Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(5*b*d^2*Sqrt[Sin[2*a + 2*b*x]])
+ (24*Cos[a + b*x]*(d*Tan[a + b*x])^(3/2))/(5*b*d^3) + (12*Sec[a + b*x]*(d*
Tan[a + b*x])^(3/2))/(5*b*d^3)
```

Rubi [A] time = 0.174187, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2608, 2613, 2615, 2572, 2639}

$$\frac{24\cos(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3}+\frac{12\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3}-\frac{24\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{5bd^2\sqrt{\sin(2a+2bx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^5/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Sec[a + b*x]^3)/(b*d*Sqrt[d*Tan[a + b*x]]) - (24*Cos[a + b*x]*EllipticE
[a - Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(5*b*d^2*Sqrt[Sin[2*a + 2*b*x]])
+ (24*Cos[a + b*x]*(d*Tan[a + b*x])^(3/2))/(5*b*d^3) + (12*Sec[a + b*x]*(d*
Tan[a + b*x])^(3/2))/(5*b*d^3)
```

Rule 2608

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(n + 1)), x] - Dist[(a^2*(m - 2))/(b^2*(n + 1)), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
```

```
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx &= -\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} + \frac{6\int \sec^3(a+bx)\sqrt{d\tan(a+bx)}\,dx}{d^2} \\
&= -\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} + \frac{12\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} + \frac{12\int \sec(a+bx)\sqrt{d\tan(a+bx)}\,dx}{5d^2} \\
&= -\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} + \frac{24\cos(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} + \frac{12\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} - \\
&= -\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} + \frac{24\cos(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} + \frac{12\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} - \\
&= -\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} + \frac{24\cos(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} + \frac{12\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} - \\
&= -\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{24\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{5bd^2\sqrt{\sin(2a+2bx)}} + \frac{24\cos(a+bx)(d\,\mathrm{ta}}{5bd^3}
\end{aligned}$$

Mathematica [C] time = 0.868208, size = 104, normalized size = 0.75

$$\frac{2\csc(a+bx)\sqrt{d\tan(a+bx)}\left(\sqrt{\sec^2(a+bx)}\left(12\sin^2(a+bx)+\tan^2(a+bx)-5\right)-8\tan^2(a+bx)\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)\right)}{5bd^2\sqrt{\sec^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^5/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*Csc[a + b*x]*Sqrt[d*Tan[a + b*x]]*(-8*Hypergeometric2F1[3/4, 3/2, 7/4, -
Tan[a + b*x]^2]*Tan[a + b*x]^2 + Sqrt[Sec[a + b*x]^2]*(-5 + 12*Sin[a + b*x]
^2 + Tan[a + b*x]^2)))/(5*b*d^2*Sqrt[Sec[a + b*x]^2])
```

Maple [B] time = 0.179, size = 529, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^5/(d*tan(b*x+a))^(3/2),x)
```

```
[Out] -1/5/b*2^(1/2)*(12*cos(b*x+a)^3*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2
)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1)/sin(b*x+a))^
(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-2
4*cos(b*x+a)^3*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+
sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*EllipticE((
(1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+12*cos(b*x+a)^2*((
1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b
*x+a))^(1/2)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin
(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-24*cos(b*x+a)^2*((1-cos(b*x+a)+sin(
b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((co
s(b*x+a)-1)/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+
a))^(1/2),1/2*2^(1/2))+12*cos(b*x+a)^3*2^(1/2)-6*cos(b*x+a)^2*2^(1/2)-2^(1/
2))*sin(b*x+a)/cos(b*x+a)^4/(d*sin(b*x+a)/cos(b*x+a))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)^5}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^5/(d*tan(b*x + a))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\tan\left(bx+a\right)}\sec\left(bx+a\right)^{5}}{d^{2}\tan\left(bx+a\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*sec(b*x + a)^5/(d^2*tan(b*x + a)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^{5}\left(a+bx\right)}{\left(d\tan\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**5/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(sec(a + b*x)**5/(d*tan(a + b*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(bx+a\right)^{5}}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^5/(d*tan(b*x + a))^(3/2), x)
```

3.264 $\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=104

$$\frac{4\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd^3}-\frac{4\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}-\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

```
[Out] (-2*Sec[a + b*x])/(b*d*Sqrt[d*Tan[a + b*x]]) - (4*Cos[a + b*x]*EllipticE[a
- Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(b*d^2*Sqrt[Sin[2*a + 2*b*x]]) + (4*
Cos[a + b*x]*(d*Tan[a + b*x])^(3/2))/(b*d^3)
```

Rubi [A] time = 0.133749, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2608, 2613, 2615, 2572, 2639}

$$\frac{4\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd^3}-\frac{4\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}-\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Sec[a + b*x])/(b*d*Sqrt[d*Tan[a + b*x]]) - (4*Cos[a + b*x]*EllipticE[a
- Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(b*d^2*Sqrt[Sin[2*a + 2*b*x]]) + (4*
Cos[a + b*x]*(d*Tan[a + b*x])^(3/2))/(b*d^3)
```

Rule 2608

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(n + 1)), x] - Dist[(a^2*(m - 2))/(b^2*(n + 1)), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
```

```
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}+\frac{2\int \sec(a+bx)\sqrt{d\tan(a+bx)}\,dx}{d^2}$$

$$= -\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}+\frac{4\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd^3}-\frac{4\int \cos(a+bx)\sqrt{d\tan(a+bx)}\,dx}{d^2}$$

$$= -\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}+\frac{4\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd^3}-\frac{\left(4\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \sqrt{c}}{d^2\sqrt{\sin(a+b}}$$

$$= -\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}+\frac{4\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd^3}-\frac{\left(4\cos(a+bx)\sqrt{d\tan(a+bx)}\right)\int \sqrt{si}}{d^2\sqrt{\sin(2a+2bx)}}$$

$$= -\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}-\frac{4\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}+\frac{4\cos(a+bx)(d\tan(a}{bd^3}$$

Mathematica [C] time = 0.447907, size = 93, normalized size = 0.89

$$-\frac{2\csc(a+bx)\sqrt{d\tan(a+bx)}\left(4\tan^2(a+bx)\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)+3\cos(2(a+bx))\sqrt{\sec^2(a+bx)}\right)}{3bd^2\sqrt{\sec^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Csc[a + b*x]*Sqrt[d*Tan[a + b*x]]*(3*Cos[2*(a + b*x)]*Sqrt[Sec[a + b*x]
^2] + 4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Tan[a + b*x]^2))/
(3*b*d^2*Sqrt[Sec[a + b*x]^2])
```

Maple [B] time = 0.177, size = 491, normalized size = 4.7

$$\frac{\sqrt{2}\sin(bx+a)}{b(\cos(bx+a))^2}\left(4\cos(bx+a)\,\mathrm{EllipticE}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},1/2\sqrt{2}\right)\sqrt{\frac{\cos(bx+a)-1}{\sin(bx+a)}}\sqrt{\frac{1-\cos(bx+a)+\ldots}{\sin\ldots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3/(d*tan(b*x+a))^(3/2),x)
```

```
[Out] 1/b*2^(1/2)*(4*cos(b*x+a)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^
(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x
+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-2*cos(b
*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*(
(cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/
2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+4*EllipticE(((1-cos(b*x+a)+
sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2
)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/s
in(b*x+a))^(1/2)-2*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1
/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/si
n(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-2*cos(b*x+a)*2
^(1/2)+2^(1/2))*sin(b*x+a)/cos(b*x+a)^2/(d*sin(b*x+a)/cos(b*x+a))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)^3}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\tan\left(bx+a\right)}\sec\left(bx+a\right)^{3}}{d^{2}\tan\left(bx+a\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*sec(b*x + a)^3/(d^2*tan(b*x + a)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^{3}\left(a+bx\right)}{\left(d\tan\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(sec(a + b*x)**3/(d*tan(a + b*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(bx+a\right)^{3}}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

3.265 $\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{2\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

```
[Out] (-2*Cos[a + b*x])/(b*d*Sqrt[d*Tan[a + b*x]]) - (2*Cos[a + b*x]*EllipticE[a
- Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(b*d^2*Sqrt[Sin[2*a + 2*b*x]])
```

Rubi [A] time = 0.0919042, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2608, 2615, 2572, 2639}

$$-\frac{2\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x])/(b*d*Sqrt[d*Tan[a + b*x]]) - (2*Cos[a + b*x]*EllipticE[a
- Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(b*d^2*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 2608

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(n + 1)), x] - Dist[(a^2*(m - 2))/(b^2*(n + 1)), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx &= -\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}-\frac{2\int \cos(a+bx)\sqrt{d\tan(a+bx)}\,dx}{d^2}\\ &= -\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}-\frac{\left(2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{d^2\sqrt{\sin(a+bx)}}\\ &= -\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}-\frac{\left(2\cos(a+bx)\sqrt{d\tan(a+bx)}\right)\int \sqrt{\sin(2a+2bx)}\,dx}{d^2\sqrt{\sin(2a+2bx)}}\\ &= -\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}-\frac{2\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}\end{aligned}$$

Mathematica [C] time = 0.377695, size = 69, normalized size = 0.88

$$-\frac{2\sin(a+bx)\left(2\tan^2(a+bx)\sqrt{\sec^2(a+bx)}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)+3\right)}{3b(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Sin[a + b*x]*(3 + 2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*S
qrt[Sec[a + b*x]^2]*Tan[a + b*x]^2))/(3*b*(d*Tan[a + b*x])^(3/2))
```

Maple [B] time = 0.133, size = 488, normalized size = 6.3

$$\frac{\sqrt{2}\sin(bx+a)}{b(\cos(bx+a))^2}\left(2\cos(bx+a)\,\mathrm{EllipticE}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},1/2\sqrt{2}\right)\sqrt{\frac{\cos(bx+a)-1}{\sin(bx+a)}}\sqrt{\frac{1-\cos(bx+}{\sin(}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)/(d*tan(b*x+a))^(3/2),x)
```

```
[Out] 1/b*2^(1/2)*(2*cos(b*x+a)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^
(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x
+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-cos(b*x
+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((c
os(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)
*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+2*EllipticE(((1-cos(b*x+a)+si
n(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*
((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin
(b*x+a))^(1/2)-EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2
^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*
x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-cos(b*x+a)*2^(1/2)
)*sin(b*x+a)/cos(b*x+a)^2/(d*sin(b*x+a)/cos(b*x+a))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\tan(bx+a)}\sec(bx+a)}{d^2\tan(bx+a)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*sec(b*x + a)/(d^2*tan(b*x + a)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(sec(a + b*x)/(d*tan(a + b*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

3.266 $\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{3\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

```
[Out] (-2*Cos[a + b*x])/(b*d*Sqrt[d*Tan[a + b*x]]) - (3*Cos[a + b*x]*EllipticE[a
- Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(b*d^2*Sqrt[Sin[2*a + 2*b*x]])
```

Rubi [A] time = 0.0982776, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2609, 2615, 2572, 2639}

$$-\frac{3\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```


```
[Out] (-2*Cos[a + b*x])/(b*d*Sqrt[d*Tan[a + b*x]]) - (3*Cos[a + b*x]*EllipticE[a
- Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(b*d^2*Sqrt[Sin[2*a + 2*b*x]])
```


Rule 2609

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
n + 1)), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx &= -\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{3\int \cos(a+bx)\sqrt{d\tan(a+bx)}\,dx}{d^2} \\
&= -\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{\left(3\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{d^2\sqrt{\sin(a+bx)}} \\
&= -\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{\left(3\cos(a+bx)\sqrt{d\tan(a+bx)}\right)\int \sqrt{\sin(2a+2bx)}\,dx}{d^2\sqrt{\sin(2a+2bx)}} \\
&= -\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{3\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}
\end{aligned}
$$

Mathematica [C] time = 0.395016, size = 66, normalized size = 0.85

$$
-\frac{2\sin(a+bx)\left(\tan^2(a+bx)\sqrt{\sec^2(a+bx)}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)+1\right)}{b(d\tan(a+bx))^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Sin[a + b*x]*(1 + Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sqr
t[Sec[a + b*x]^2]*Tan[a + b*x]^2))/(b*(d*Tan[a + b*x])^(3/2))
```

Maple [B] time = 0.143, size = 501, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(d*tan(b*x+a))^(3/2),x)
```

```
[Out] 1/2/b*2^(1/2)*(6*cos(b*x+a)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a)
)^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b
*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-3*cos
(b*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))
*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(
1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+6*EllipticE(((1-cos(b*x+a
)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1
/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))
/sin(b*x+a))^(1/2)-3*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)
,1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/
sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+cos(b*x+a)^2
*2^(1/2)-3*cos(b*x+a)*2^(1/2))*sin(b*x+a)/cos(b*x+a)^2/(d*sin(b*x+a)/cos(b*
x+a))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (bx + a)}{(d \tan (bx + a))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d \tan (bx + a)} \cos (bx + a)}{d^2 \tan (bx + a)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*cos(b*x + a)/(d^2*tan(b*x + a)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(cos(a + b*x)/(d*tan(a + b*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

3.267 $\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=112

$$-\frac{7\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd^3}-\frac{7\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{2bd^2\sqrt{\sin(2a+2bx)}}-\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

```
[Out] (-2*Cos[a + b*x]^3)/(b*d*Sqrt[d*Tan[a + b*x]]) - (7*Cos[a + b*x]*EllipticE[
a - Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(2*b*d^2*Sqrt[Sin[2*a + 2*b*x]]) -
 (7*Cos[a + b*x]^3*(d*Tan[a + b*x])^(3/2))/(3*b*d^3)
```

Rubi [A] time = 0.149311, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2609, 2612, 2615, 2572, 2639}

$$-\frac{7\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd^3}-\frac{7\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{2bd^2\sqrt{\sin(2a+2bx)}}-\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x]^3)/(b*d*Sqrt[d*Tan[a + b*x]]) - (7*Cos[a + b*x]*EllipticE[
a - Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(2*b*d^2*Sqrt[Sin[2*a + 2*b*x]]) -
 (7*Cos[a + b*x]^3*(d*Tan[a + b*x])^(3/2))/(3*b*d^3)
```

Rule 2609

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
n + 1)), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2612

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*m
), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e + f
*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1] &
```

```
& EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx &= -\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{7\int \cos^3(a+bx)\sqrt{d\tan(a+bx)}\,dx}{d^2} \\
&= -\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{7\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd^3} - \frac{7\int \cos(a+bx)\sqrt{d\tan(a+bx)}\,dx}{2d^2} \\
&= -\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{7\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd^3} - \frac{\left(7\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \sqrt{\ldots}}{2d^2\sqrt{\sin(a+\ldots}} \\
&= -\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{7\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd^3} - \frac{\left(7\cos(a+bx)\sqrt{d\tan(a+bx)}\right)\int \sqrt{\mathrm{s}\ldots}}{2d^2\sqrt{\sin(2a+2bx)}} \\
&= -\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{7\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{2bd^2\sqrt{\sin(2a+2bx)}} - \frac{7\cos^3(a+bx)(d\tan(\ldots}{3bd^3}
\end{aligned}$$

Mathematica [C] time = 0.552562, size = 77, normalized size = 0.69

$$\frac{\sin(a+bx)\left(-14\tan^2(a+bx)\sqrt{\sec^2(a+bx)}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)+\cos(2(a+bx))-13\right)}{6b(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (Sin[a + b*x]*(-13 + Cos[2*(a + b*x)] - 14*Hypergeometric2F1[3/4, 3/2, 7/4,
 -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2]*Tan[a + b*x]^2))/(6*b*(d*Tan[a + b*x
])^(3/2))
```

Maple [B] time = 0.142, size = 515, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/(d*tan(b*x+a))^(3/2),x)
```

```
[Out] 1/12/b*2^(1/2)*(2*cos(b*x+a)^4*2^(1/2)+42*cos(b*x+a)*EllipticE(((1-cos(b*x+
a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(
1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a)
)/sin(b*x+a))^(1/2)-21*cos(b*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(
b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)
+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)
+42*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((c
os(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)
*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-21*EllipticF(((1-cos(b*x+a)+s
in(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)
*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/si
n(b*x+a))^(1/2)+7*cos(b*x+a)^2*2^(1/2)-21*cos(b*x+a)*2^(1/2))*sin(b*x+a)/co
s(b*x+a)^2/(d*sin(b*x+a)/cos(b*x+a))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^3}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\tan\left(bx+a\right)}\cos\left(bx+a\right)^{3}}{d^{2}\tan\left(bx+a\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*cos(b*x + a)^3/(d^2*tan(b*x + a)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(bx+a\right)^{3}}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

3.268 $\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=142

$$-\frac{11\cos^5(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3}-\frac{77\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{30bd^3}-\frac{77\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+b}}{20bd^2\sqrt{\sin(2a+2bx)}}$$

```
[Out] (-2*Cos[a + b*x]^5)/(b*d*Sqrt[d*Tan[a + b*x]]) - (77*Cos[a + b*x]*EllipticE
[a - Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(20*b*d^2*Sqrt[Sin[2*a + 2*b*x]])
 - (77*Cos[a + b*x]^3*(d*Tan[a + b*x])^(3/2))/(30*b*d^3) - (11*Cos[a + b*x]
^5*(d*Tan[a + b*x])^(3/2))/(5*b*d^3)
```

Rubi [A] time = 0.187095, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2609, 2612, 2615, 2572, 2639}

$$-\frac{11\cos^5(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3}-\frac{77\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{30bd^3}-\frac{77\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+b}}{20bd^2\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^5/(d*Tan[a + b*x])^(3/2),x]
```



```
[Out] (-2*Cos[a + b*x]^5)/(b*d*Sqrt[d*Tan[a + b*x]]) - (77*Cos[a + b*x]*EllipticE
[a - Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(20*b*d^2*Sqrt[Sin[2*a + 2*b*x]])
 - (77*Cos[a + b*x]^3*(d*Tan[a + b*x])^(3/2))/(30*b*d^3) - (11*Cos[a + b*x]
^5*(d*Tan[a + b*x])^(3/2))/(5*b*d^3)
```


Rule 2609

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
n + 1)), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2612

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*m
```

```
), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e + f
*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1] &
& EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{11\int \cos^5(a+bx)\sqrt{d\tan(a+bx)}\,dx}{d^2}$$

$$= -\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{11\cos^5(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} - \frac{77\int \cos^3(a+bx)\sqrt{d\tan(a+bx)}\,dx}{10d^2}$$

$$= -\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{77\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{30bd^3} - \frac{11\cos^5(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} -$$

$$= -\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{77\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{30bd^3} - \frac{11\cos^5(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} -$$

$$= -\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{77\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{30bd^3} - \frac{11\cos^5(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} -$$

$$= -\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{77\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{20bd^2\sqrt{\sin(2a+2bx)}} - \frac{77\cos^3(a+bx)(d\tan}{30bd^3}$$

Mathematica [C] time = 0.838343, size = 89, normalized size = 0.63

$$\frac{\sin(a+bx)\left(-308\tan^2(a+bx)\sqrt{\sec^2(a+bx)}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)+34\cos(2(a+bx))+3\cos(4(a+bx))-277\right)}{120b(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^5/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (Sin[a + b*x]*(-277 + 34*Cos[2*(a + b*x)] + 3*Cos[4*(a + b*x)] - 308*Hyperg
eometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2]*Tan[a + b*
x]^2))/(120*b*(d*Tan[a + b*x])^(3/2))
```

Maple [B] time = 0.165, size = 536, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^5/(d*tan(b*x+a))^(3/2),x)
```

```
[Out] 1/120/b*2^(1/2)*(12*cos(b*x+a)^6*2^(1/2)+22*cos(b*x+a)^4*2^(1/2)+462*cos(b*
x+a)*EllipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*(
(cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/
2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-231*cos(b*x+a)*EllipticF((
-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/s
in(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)
-1-sin(b*x+a))/sin(b*x+a))^(1/2)+462*EllipticE((-(cos(b*x+a)-1-sin(b*x+a))/
sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+
a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^
(1/2)-231*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/
2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a)
)^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)+77*cos(b*x+a)^2*2^(1/
2)-231*cos(b*x+a)*2^(1/2))*sin(b*x+a)/cos(b*x+a)^2/(d*sin(b*x+a)/cos(b*x+a)
)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^5}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^5/(d*tan(b*x + a))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\tan(bx+a)}\cos(bx+a)^5}{d^2\tan(bx+a)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*cos(b*x + a)^5/(d^2*tan(b*x + a)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**5/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^5}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^5/(d*tan(b*x + a))^(3/2), x)
```

3.269 $\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=82

$$-\frac{\sqrt{\sin(2a+2bx)}\sec(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{3bd^2\sqrt{d\tan(a+bx)}}-\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$

```
[Out] (-2*Sec[a + b*x])/(3*b*d*(d*Tan[a + b*x])^(3/2)) - (EllipticF[a - Pi/4 + b*
x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(3*b*d^2*Sqrt[d*Tan[a + b*x]])
```

Rubi [A] time = 0.0873308, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2609, 2614, 2573, 2641}

$$-\frac{\sqrt{\sin(2a+2bx)}\sec(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{3bd^2\sqrt{d\tan(a+bx)}}-\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*Sec[a + b*x])/(3*b*d*(d*Tan[a + b*x])^(3/2)) - (EllipticF[a - Pi/4 + b*
x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(3*b*d^2*Sqrt[d*Tan[a + b*x]])
```

Rule 2609

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
n + 1)), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2614

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx &= -\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}} - \frac{\int \frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx}{3d^2}\\
&= -\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}} - \frac{\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}\,dx}{3d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}\\
&= -\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}} - \frac{\left(\sec(a+bx)\sqrt{\sin(2a+2bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{3d^2\sqrt{d\tan(a+bx)}}\\
&= -\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}} - \frac{F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{3bd^2\sqrt{d\tan(a+bx)}}\end{aligned}$$

Mathematica [C] time = 0.694892, size = 113, normalized size = 1.38

$$\frac{2\cos(2(a+bx))\csc(a+bx)\sqrt{\sec^2(a+bx)}\left(\sqrt{\sec^2(a+bx)}-\sqrt[4]{-1}\tan^{\frac{3}{2}}(a+bx)F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right)\middle|-1\right)\right)}{3bd^2\left(\tan^2(a+bx)-1\right)\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (2*Cos[2*(a + b*x)]*Csc[a + b*x]*Sqrt[Sec[a + b*x]^2]*(Sqrt[Sec[a + b*x]^2]
 - (-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1]*Tan[a
 + b*x]^(3/2)))/(3*b*d^2*Sqrt[d*Tan[a + b*x]]*(-1 + Tan[a + b*x]^2))
```

Maple [B] time = 0.135, size = 302, normalized size = 3.7

$$-\frac{\sqrt{2}\left(\cos\left(bx+a\right)-1\right)^{2}\left(\cos\left(bx+a\right)+1\right)^{2}}{3\,b\left(\sin\left(bx+a\right)\right)^{3}\left(\cos\left(bx+a\right)\right)^{3}}\left(\sin\left(bx+a\right)\cos\left(bx+a\right)\mathrm{EllipticF}\left(\sqrt{\frac{1-\cos\left(bx+a\right)+\sin\left(bx+a\right)}{\sin\left(bx+a\right)}},\frac{\sqrt{2}}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)/(d*tan(b*x+a))^(5/2),x)
```

```
[Out] -1/3/b*2^(1/2)*(cos(b*x+a)-1)^2*(sin(b*x+a)*cos(b*x+a)*EllipticF(((1-cos(b*
x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))
^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+
a))/sin(b*x+a))^(1/2)+sin(b*x+a)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(
b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))
^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+
cos(b*x+a)*2^(1/2))*(cos(b*x+a)+1)^2/sin(b*x+a)^3/cos(b*x+a)^3/(d*sin(b*x+a
)/cos(b*x+a))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(bx+a\right)}{\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)/(d*tan(b*x + a))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{d\tan\left(bx+a\right)}\sec\left(bx+a\right)}{d^{3}\tan\left(bx+a\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*sec(b*x + a)/(d^3*tan(b*x + a)^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Integral(sec(a + b*x)/(d*tan(a + b*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)/(d*tan(b*x + a))^(5/2), x)
```

3.270 $\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx$

Optimal. Leaf size=110

$$-\frac{4\cos(a+bx)}{5bd^3\sqrt{d\tan(a+bx)}}-\frac{4\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{5bd^4\sqrt{\sin(2a+2bx)}}-\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}$$

```
[Out] (-2*Sec[a + b*x])/(5*b*d*(d*Tan[a + b*x])^(5/2)) - (4*Cos[a + b*x])/(5*b*d^
3*Sqrt[d*Tan[a + b*x]]) - (4*Cos[a + b*x]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt
[d*Tan[a + b*x]])/(5*b*d^4*Sqrt[Sin[2*a + 2*b*x]])
```

Rubi [A] time = 0.137395, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2608, 2615, 2572, 2639}

$$-\frac{4\cos(a+bx)}{5bd^3\sqrt{d\tan(a+bx)}}-\frac{4\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{5bd^4\sqrt{\sin(2a+2bx)}}-\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^3/(d*Tan[a + b*x])^(7/2),x]
```

```
[Out] (-2*Sec[a + b*x])/(5*b*d*(d*Tan[a + b*x])^(5/2)) - (4*Cos[a + b*x])/(5*b*d^
3*Sqrt[d*Tan[a + b*x]]) - (4*Cos[a + b*x]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt
[d*Tan[a + b*x]])/(5*b*d^4*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 2608

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(n + 1)), x] - Dist[(a^2*(m - 2))/(b^2*(n + 1)), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2615

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/Sqrt[Sin[e + f*x]], Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx = -\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}+\frac{2\int\frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx}{5d^2}$$
$$= -\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}-\frac{4\cos(a+bx)}{5bd^3\sqrt{d\tan(a+bx)}}-\frac{4\int\cos(a+bx)\sqrt{d\tan(a+bx)}\,dx}{5d^4}$$
$$= -\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}-\frac{4\cos(a+bx)}{5bd^3\sqrt{d\tan(a+bx)}}-\frac{\left(4\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int\sqrt{\cos(a+}}{5d^4\sqrt{\sin(a+bx)}}$$
$$= -\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}-\frac{4\cos(a+bx)}{5bd^3\sqrt{d\tan(a+bx)}}-\frac{\left(4\cos(a+bx)\sqrt{d\tan(a+bx)}\right)\int\sqrt{\sin(2a+}}{5d^4\sqrt{\sin(2a+2bx)}}$$
$$= -\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}-\frac{4\cos(a+bx)}{5bd^3\sqrt{d\tan(a+bx)}}-\frac{4\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+b}}{5bd^4\sqrt{\sin(2a+2bx)}}$$

Mathematica [C] time = 1.5268, size = 103, normalized size = 0.94

$$-\frac{2\sin(a+bx)\sqrt{d\tan(a+bx)}\left(4\sec^2(a+bx)\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)+3\left(\csc^4(a+bx)+\csc^2(a+bx)-2\right)\sqrt{\sec^2(a}}{15bd^4\sqrt{\sec^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^3/(d*Tan[a + b*x])^(7/2),x]
```

```
[Out] (-2*(4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]^2 + 3
*(-2 + Csc[a + b*x]^2 + Csc[a + b*x]^4)*Sqrt[Sec[a + b*x]^2])*Sin[a + b*x]*
Sqrt[d*Tan[a + b*x]])/(15*b*d^4*Sqrt[Sec[a + b*x]^2])
```

Maple [B] time = 0.177, size = 970, normalized size = 8.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3/(d*tan(b*x+a))^(7/2),x)
```

```
[Out] -1/5/b*2^(1/2)*(4*cos(b*x+a)^3*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)
*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1)/sin(b*x+a))^(
1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-2*
cos(b*x+a)^3*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+si
n(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*EllipticF(((1
-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+4*cos(b*x+a)^2*((1-c
os(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+
a))^(1/2)*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*
x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-2*cos(b*x+a)^2*((1-cos(b*x+a)+sin(b*x+
a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*
x+a)-1)/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^
(1/2),1/2*2^(1/2))-4*cos(b*x+a)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*
x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+s
in(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+2
*cos(b*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1
/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a
))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-2*cos(b*x+a)^3*2^(1/2
)-4*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((c
os(b*x+a)-1)/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)
*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+2*EllipticF(((1-cos(b*x+a)+si
n(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1)/sin(b*x+a))^(1/2)*
((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin
(b*x+a))^(1/2)+cos(b*x+a)^2*2^(1/2)+2*cos(b*x+a)*2^(1/2))*sin(b*x+a)/cos(b*
x+a)^4/(d*sin(b*x+a)/cos(b*x+a))^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)^3}{(d\tan(bx+a))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/(d*tan(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^3/(d*tan(b*x + a))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{d\tan\left(bx+a\right)}\sec\left(bx+a\right)^{3}}{d^{4}\tan\left(bx+a\right)^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/(d*tan(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*sec(b*x + a)^3/(d^4*tan(b*x + a)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3/(d*tan(b*x+a))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(bx+a\right)^{3}}{\left(d\tan\left(bx+a\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/(d*tan(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^3/(d*tan(b*x + a))^(7/2), x)
```

3.271 $\int \sec^{\frac{10}{3}}(e+fx)\sin^2(e+fx)\,dx$

Optimal. Leaf size=53

$$\frac{3\sin(e+fx)\sec^{\frac{7}{3}}(e+fx)\,{}_2F_1\left(-\frac{7}{6},-\frac{1}{2};-\frac{1}{6};\cos^2(e+fx)\right)}{7f\sqrt{\sin^2(e+fx)}}$$

```
[Out] (3*Hypergeometric2F1[-7/6, -1/2, -1/6, Cos[e + f*x]^2]*Sec[e + f*x]^(7/3)*S
in[e + f*x])/(7*f*Sqrt[Sin[e + f*x]^2])
```

Rubi [A] time = 0.058961, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2632, 2576}

$$\frac{3\sin(e+fx)\sec^{\frac{7}{3}}(e+fx)\,{}_2F_1\left(-\frac{7}{6},-\frac{1}{2};-\frac{1}{6};\cos^2(e+fx)\right)}{7f\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^(10/3)*Sin[e + f*x]^2,x]
```

```
[Out] (3*Hypergeometric2F1[-7/6, -1/2, -1/6, Cos[e + f*x]^2]*Sec[e + f*x]^(7/3)*S
in[e + f*x])/(7*f*Sqrt[Sin[e + f*x]^2])
```

Rule 2632

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(n + 1
)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))/b^2, Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^2(e+fx)\,dx = \left(\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\right)\int \frac{\sin^2(e+fx)}{\cos^{\frac{10}{3}}(e+fx)}\,dx$$
$$= \frac{3\,{}_2F_1\left(-\frac{7}{6},-\frac{1}{2};-\frac{1}{6};\cos^2(e+fx)\right)\sec^{\frac{7}{3}}(e+fx)\sin(e+fx)}{7f\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 0.207093, size = 77, normalized size = 1.45

$$\frac{3\sqrt[3]{\sec(e+fx)}\left(2\sin(e+fx)\sqrt[6]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)-3\sin(e+fx)+\tan(e+fx)\sec(e+fx)\right)}{7f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^(10/3)*Sin[e + f*x]^2,x]
```

```
[Out] (3*Sec[e + f*x]^(1/3)*(-3*Sin[e + f*x] + 2*(Cos[e + f*x]^2)^(1/6)*Hypergeom
etric2F1[1/6, 1/2, 3/2, Sin[e + f*x]^2]*Sin[e + f*x] + Sec[e + f*x]*Tan[e +
 f*x]))/(7*f)
```

Maple [F] time = 0.067, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^{\frac{4}{3}}\left(\tan\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^(4/3)*tan(f*x+e)^2,x)
```

```
[Out] int(sec(f*x+e)^(4/3)*tan(f*x+e)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)^{\frac{4}{3}}\tan\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(4/3)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^(4/3)*tan(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\sec\left(fx+e\right)^{\frac{4}{3}}\tan\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(4/3)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(sec(f*x + e)^(4/3)*tan(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**(4/3)*tan(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)^{\frac{4}{3}}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(4/3)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^(4/3)*tan(f*x + e)^2, x)
```

3.272 $\int \sec^{\frac{8}{3}}(e+fx)\sin^2(e+fx)\,dx$

Optimal. Leaf size=53

$$\frac{3\sin(e+fx)\sec^{\frac{5}{3}}(e+fx)\,{}_2F_1\left(-\frac{5}{6},-\frac{1}{2};\frac{1}{6};\cos^2(e+fx)\right)}{5f\sqrt{\sin^2(e+fx)}}$$

```
[Out] (3*Hypergeometric2F1[-5/6, -1/2, 1/6, Cos[e + f*x]^2]*Sec[e + f*x]^(5/3)*Si
n[e + f*x])/(5*f*Sqrt[Sin[e + f*x]^2])
```

Rubi [A] time = 0.0588771, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2632, 2576}

$$\frac{3\sin(e+fx)\sec^{\frac{5}{3}}(e+fx)\,{}_2F_1\left(-\frac{5}{6},-\frac{1}{2};\frac{1}{6};\cos^2(e+fx)\right)}{5f\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^(8/3)*Sin[e + f*x]^2,x]
```

```
[Out] (3*Hypergeometric2F1[-5/6, -1/2, 1/6, Cos[e + f*x]^2]*Sec[e + f*x]^(5/3)*Si
n[e + f*x])/(5*f*Sqrt[Sin[e + f*x]^2])
```

Rule 2632

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(n + 1
)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))/b^2, Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int \sec^{\frac{8}{3}}(e+fx)\sin^2(e+fx)\,dx = \left(\cos^{\frac{2}{3}}(e+fx)\sec^{\frac{2}{3}}(e+fx)\right)\int \frac{\sin^2(e+fx)}{\cos^{\frac{8}{3}}(e+fx)}\,dx$$
$$= \frac{3\,{}_2F_1\left(-\frac{5}{6},-\frac{1}{2};\frac{1}{6};\cos^2(e+fx)\right)\sec^{\frac{5}{3}}(e+fx)\sin(e+fx)}{5f\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 0.0826136, size = 56, normalized size = 1.06

$$-\frac{3\sin(e+fx)\sec^{\frac{5}{3}}(e+fx)\left(\cos^2(e+fx)^{5/6}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\sin^2(e+fx)\right)-1\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^(8/3)*Sin[e + f*x]^2,x]
```

```
[Out] (-3*(-1 + (Cos[e + f*x]^2)^(5/6)*Hypergeometric2F1[1/2, 5/6, 3/2, Sin[e + f
*x]^2])*Sec[e + f*x]^(5/3)*Sin[e + f*x])/(5*f)
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^{\frac{2}{3}}\left(\tan\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^(2/3)*tan(f*x+e)^2,x)
```

```
[Out] int(sec(f*x+e)^(2/3)*tan(f*x+e)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)^{\frac{2}{3}}\tan\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(2/3)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^(2/3)*tan(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sec\left(fx+e\right)^{\frac{2}{3}}\tan\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(2/3)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(sec(f*x + e)^(2/3)*tan(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \tan^{2}\left(e+fx\right)\sec^{\frac{2}{3}}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**(2/3)*tan(f*x+e)**2,x)
```

```
[Out] Integral(tan(e + f*x)**2*sec(e + f*x)**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)^{\frac{2}{3}}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(2/3)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^(2/3)*tan(f*x + e)^2, x)
```

3.273 $\int \sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx$

Optimal. Leaf size=53

$$\frac{3\sin(e+fx)\sec^{\frac{4}{3}}(e+fx)\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{2};\frac{1}{3};\cos^2(e+fx)\right)}{4f\sqrt{\sin^2(e+fx)}}$$

```
[Out] (3*Hypergeometric2F1[-2/3, -1/2, 1/3, Cos[e + f*x]^2]*Sec[e + f*x]^(4/3)*Si
n[e + f*x])/(4*f*Sqrt[Sin[e + f*x]^2])
```

Rubi [A] time = 0.0581687, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2632, 2576}

$$\frac{3\sin(e+fx)\sec^{\frac{4}{3}}(e+fx)\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{2};\frac{1}{3};\cos^2(e+fx)\right)}{4f\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^(7/3)*Sin[e + f*x]^2,x]
```

```
[Out] (3*Hypergeometric2F1[-2/3, -1/2, 1/3, Cos[e + f*x]^2]*Sec[e + f*x]^(4/3)*Si
n[e + f*x])/(4*f*Sqrt[Sin[e + f*x]^2])
```

Rule 2632

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(n + 1
)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))/b^2, Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int \sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx = \left(\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\right)\int \frac{\sin^2(e+fx)}{\cos^{\frac{7}{3}}(e+fx)}\,dx$$
$$= \frac{3\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{2};\frac{1}{3};\cos^2(e+fx)\right)\sec^{\frac{4}{3}}(e+fx)\sin(e+fx)}{4f\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 0.0850869, size = 56, normalized size = 1.06

$$-\frac{3\sin(e+fx)\sec^{\frac{4}{3}}(e+fx)\left(\cos^2(e+fx)^{2/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};\sin^2(e+fx)\right)-1\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^(7/3)*Sin[e + f*x]^2,x]
```

```
[Out] (-3*(-1 + (Cos[e + f*x]^2)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2, Sin[e + f
*x]^2])*Sec[e + f*x]^(4/3)*Sin[e + f*x])/(4*f)
```

Maple [F] time = 0.061, size = 0, normalized size = 0.

$$\int \sqrt[3]{\sec\left(fx+e\right)}\left(\tan\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^(1/3)*tan(f*x+e)^2,x)
```

```
[Out] int(sec(f*x+e)^(1/3)*tan(f*x+e)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)^{\frac{1}{3}}\tan\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(1/3)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^(1/3)*tan(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sec\left(fx+e\right)^{\frac{1}{3}}\tan\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(1/3)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(sec(f*x + e)^(1/3)*tan(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \tan^{2}\left(e+fx\right)\sqrt[3]{\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**(1/3)*tan(f*x+e)**2,x)
```

```
[Out] Integral(tan(e + f*x)**2*sec(e + f*x)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)^{\frac{1}{3}}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(1/3)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^(1/3)*tan(f*x + e)^2, x)
```

3.274 $\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx$

Optimal. Leaf size=53

$$\frac{3\sin(e+fx)\sec^{\frac{2}{3}}(e+fx)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{3};\frac{2}{3};\cos^2(e+fx)\right)}{2f\sqrt{\sin^2(e+fx)}}$$

```
[Out] (3*Hypergeometric2F1[-1/2, -1/3, 2/3, Cos[e + f*x]^2]*Sec[e + f*x]^(2/3)*Si
n[e + f*x])/(2*f*Sqrt[Sin[e + f*x]^2])
```

Rubi [A] time = 0.0579557, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2632, 2576}

$$\frac{3\sin(e+fx)\sec^{\frac{2}{3}}(e+fx)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{3};\frac{2}{3};\cos^2(e+fx)\right)}{2f\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^(5/3)*Sin[e + f*x]^2,x]
```

```
[Out] (3*Hypergeometric2F1[-1/2, -1/3, 2/3, Cos[e + f*x]^2]*Sec[e + f*x]^(2/3)*Si
n[e + f*x])/(2*f*Sqrt[Sin[e + f*x]^2])
```

Rule 2632

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(n + 1
)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))/b^2, Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx = \left(\cos^{\frac{2}{3}}(e+fx)\sec^{\frac{2}{3}}(e+fx)\right)\int \frac{\sin^2(e+fx)}{\cos^{\frac{5}{3}}(e+fx)}\,dx$$
$$= \frac{3\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{3};\frac{2}{3};\cos^2(e+fx)\right)\sec^{\frac{2}{3}}(e+fx)\sin(e+fx)}{2f\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 0.0804503, size = 56, normalized size = 1.06

$$-\frac{3\sin(e+fx)\sec^{\frac{2}{3}}(e+fx)\left(\sqrt[3]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)-1\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^(5/3)*Sin[e + f*x]^2,x]
```

```
[Out] (-3*(-1 + (Cos[e + f*x]^2)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, Sin[e + f
*x]^2])*Sec[e + f*x]^(2/3)*Sin[e + f*x])/(2*f)
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int \left(\tan\left(fx+e\right)\right)^2 \frac{1}{\sqrt[3]{\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/sec(f*x+e)^(1/3),x)
```

```
[Out] int(tan(f*x+e)^2/sec(f*x+e)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^2}{\sec\left(fx+e\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/sec(f*x+e)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^2/sec(f*x + e)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\tan\left(fx+e\right)^{2}}{\sec\left(fx+e\right)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/sec(f*x+e)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(tan(f*x + e)^2/sec(f*x + e)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^{2}\left(e+fx\right)}{\sqrt[3]{\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/sec(f*x+e)**(1/3),x)
```

```
[Out] Integral(tan(e + f*x)**2/sec(e + f*x)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^{2}}{\sec\left(fx+e\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/sec(f*x+e)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/sec(f*x + e)^(1/3), x)
```

3.275 $\int \sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\,dx$

Optimal. Leaf size=51

$$\frac{3\sin(e+fx)\sqrt[3]{\sec(e+fx)}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{6};\frac{5}{6};\cos^2(e+fx)\right)}{f\sqrt{\sin^2(e+fx)}}$$

```
[Out] (3*Hypergeometric2F1[-1/2, -1/6, 5/6, Cos[e + f*x]^2]*Sec[e + f*x]^(1/3)*Si
n[e + f*x])/(f*Sqrt[Sin[e + f*x]^2])
```

Rubi [A] time = 0.0581419, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2632, 2576}

$$\frac{3\sin(e+fx)\sqrt[3]{\sec(e+fx)}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{6};\frac{5}{6};\cos^2(e+fx)\right)}{f\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^(4/3)*Sin[e + f*x]^2,x]
```

```
[Out] (3*Hypergeometric2F1[-1/2, -1/6, 5/6, Cos[e + f*x]^2]*Sec[e + f*x]^(1/3)*Si
n[e + f*x])/(f*Sqrt[Sin[e + f*x]^2])
```

Rule 2632

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(n + 1
)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))/b^2, Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int \sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\,dx = \left(\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\right)\int \frac{\sin^2(e+fx)}{\cos^{\frac{4}{3}}(e+fx)}\,dx$$
$$= \frac{3\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{6};\frac{5}{6};\cos^2(e+fx)\right)\sqrt[3]{\sec(e+fx)}\sin(e+fx)}{f\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 0.0714524, size = 54, normalized size = 1.06

$$-\frac{3\sin(e+fx)\sqrt[3]{\sec(e+fx)}\left(\sqrt[6]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)-1\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^(4/3)*Sin[e + f*x]^2,x]
```

```
[Out] (-3*(-1 + (Cos[e + f*x]^2)^(1/6)*Hypergeometric2F1[1/6, 1/2, 3/2, Sin[e + f
*x]^2])*Sec[e + f*x]^(1/3)*Sin[e + f*x])/f
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int \left(\tan\left(fx+e\right)\right)^2\left(\sec\left(fx+e\right)\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/sec(f*x+e)^(2/3),x)
```

```
[Out] int(tan(f*x+e)^2/sec(f*x+e)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^2}{\sec\left(fx+e\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/sec(f*x+e)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^2/sec(f*x + e)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\tan\left(fx+e\right)^2}{\sec\left(fx+e\right)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/sec(f*x+e)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(tan(f*x + e)^2/sec(f*x + e)^(2/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2\left(e+fx\right)}{\sec^{\frac{2}{3}}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/sec(f*x+e)**(2/3),x)
```

```
[Out] Integral(tan(e + f*x)**2/sec(e + f*x)**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^2}{\sec\left(fx+e\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/sec(f*x+e)^(2/3),x, algorithm="giac")

[Out] integrate(tan(f*x + e)^2/sec(f*x + e)^(2/3), x)
```

3.276 $\int \sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx$

Optimal. Leaf size=53

$$\frac{3\sin(e+fx)\sec^{\frac{13}{3}}(e+fx)\,{}_2F_1\left(-\frac{13}{6},-\frac{3}{2};-\frac{7}{6};\cos^2(e+fx)\right)}{13f\sqrt{\sin^2(e+fx)}}$$

```
[Out] (3*Hypergeometric2F1[-13/6, -3/2, -7/6, Cos[e + f*x]^2]*Sec[e + f*x]^(13/3)
*Sin[e + f*x])/(13*f*Sqrt[Sin[e + f*x]^2])
```

Rubi [A] time = 0.0594236, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2632, 2576}

$$\frac{3\sin(e+fx)\sec^{\frac{13}{3}}(e+fx)\,{}_2F_1\left(-\frac{13}{6},-\frac{3}{2};-\frac{7}{6};\cos^2(e+fx)\right)}{13f\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^(16/3)*Sin[e + f*x]^4,x]
```


```
[Out] (3*Hypergeometric2F1[-13/6, -3/2, -7/6, Cos[e + f*x]^2]*Sec[e + f*x]^(13/3)
*Sin[e + f*x])/(13*f*Sqrt[Sin[e + f*x]^2])
```


Rule 2632

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(n + 1
)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))/b^2, Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int \sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx = \left(\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\right)\int \frac{\sin^4(e+fx)}{\cos^{\frac{16}{3}}(e+fx)}\,dx$$
$$= \frac{3\,{}_2F_1\left(-\frac{13}{6},-\frac{3}{2};-\frac{7}{6};\cos^2(e+fx)\right)\sec^{\frac{13}{3}}(e+fx)\sin(e+fx)}{13f\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 1.10049, size = 89, normalized size = 1.68

$$\frac{3\sqrt[3]{\sec(e+fx)}\left(-18\sin(e+fx)\sqrt[6]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)+27\sin(e+fx)+\tan(e+fx)\sec(e+fx)\left(7\right.\right.}{91f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^(16/3)*Sin[e + f*x]^4,x]
```

```
[Out] (3*Sec[e + f*x]^(1/3)*(27*Sin[e + f*x] - 18*(Cos[e + f*x]^2)^(1/6)*Hypergeo
metric2F1[1/6, 1/2, 3/2, Sin[e + f*x]^2]*Sin[e + f*x] + Sec[e + f*x]*(-16 +
 7*Sec[e + f*x]^2)*Tan[e + f*x]))/(91*f)
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^{\frac{4}{3}}\left(\tan\left(fx+e\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^(4/3)*tan(f*x+e)^4,x)
```

```
[Out] int(sec(f*x+e)^(4/3)*tan(f*x+e)^4,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(4/3)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sec\left(fx+e\right)^{\frac{4}{3}}\tan\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(4/3)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral(sec(f*x + e)^(4/3)*tan(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**(4/3)*tan(f*x+e)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)^{\frac{4}{3}}\tan\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(4/3)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^(4/3)*tan(f*x + e)^4, x)
```

3.277 $\int \sec^{\frac{14}{3}}(e+fx)\sin^4(e+fx)\,dx$

Optimal. Leaf size=53

$$\frac{3\sin(e+fx)\sec^{\frac{11}{3}}(e+fx)\,{}_2F_1\left(-\frac{11}{6},-\frac{3}{2};-\frac{5}{6};\cos^2(e+fx)\right)}{11f\sqrt{\sin^2(e+fx)}}$$

```
[Out] (3*Hypergeometric2F1[-11/6, -3/2, -5/6, Cos[e + f*x]^2]*Sec[e + f*x]^(11/3)
*Sin[e + f*x])/(11*f*Sqrt[Sin[e + f*x]^2])
```

Rubi [A] time = 0.05791, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2632, 2576}

$$\frac{3\sin(e+fx)\sec^{\frac{11}{3}}(e+fx)\,{}_2F_1\left(-\frac{11}{6},-\frac{3}{2};-\frac{5}{6};\cos^2(e+fx)\right)}{11f\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^(14/3)*Sin[e + f*x]^4,x]
```

```
[Out] (3*Hypergeometric2F1[-11/6, -3/2, -5/6, Cos[e + f*x]^2]*Sec[e + f*x]^(11/3)
*Sin[e + f*x])/(11*f*Sqrt[Sin[e + f*x]^2])
```

Rule 2632

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(n + 1
)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))/b^2, Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int \sec^{\frac{14}{3}}(e+fx)\sin^4(e+fx)\,dx = \left(\cos^{\frac{2}{3}}(e+fx)\sec^{\frac{2}{3}}(e+fx)\right)\int \frac{\sin^4(e+fx)}{\cos^{\frac{14}{3}}(e+fx)}\,dx$$
$$= \frac{3\,{}_2F_1\left(-\frac{11}{6},-\frac{3}{2};-\frac{5}{6};\cos^2(e+fx)\right)\sec^{\frac{11}{3}}(e+fx)\sin(e+fx)}{11f\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 0.859298, size = 78, normalized size = 1.47

$$\frac{3\sin(e+fx)\left(\frac{9\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\sin^2(e+fx)\right)}{\sqrt[6]{\cos^2(e+fx)}}-(7\cos(2(e+fx))+2)\sec^4(e+fx)\right)}{55f\sqrt[3]{\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^(14/3)*Sin[e + f*x]^4,x]
```

```
[Out] (3*((9*Hypergeometric2F1[1/2, 5/6, 3/2, Sin[e + f*x]^2])/(Cos[e + f*x]^2)^(
1/6) - (2 + 7*Cos[2*(e + f*x)])*Sec[e + f*x]^4)*Sin[e + f*x])/(55*f*Sec[e +
 f*x]^(1/3))
```

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^{\frac{2}{3}}\left(\tan\left(fx+e\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^(2/3)*tan(f*x+e)^4,x)
```

```
[Out] int(sec(f*x+e)^(2/3)*tan(f*x+e)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)^{\frac{2}{3}}\tan\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(2/3)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^(2/3)*tan(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sec\left(fx+e\right)^{\frac{2}{3}}\tan\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(2/3)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral(sec(f*x + e)^(2/3)*tan(f*x + e)^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \tan^{4}\left(e+fx\right)\sec^{\frac{2}{3}}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**(2/3)*tan(f*x+e)**4,x)
```

```
[Out] Integral(tan(e + f*x)**4*sec(e + f*x)**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)^{\frac{2}{3}}\tan\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(2/3)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^(2/3)*tan(f*x + e)^4, x)
```

3.278 $\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx$

Optimal. Leaf size=53

$$\frac{3\sin(e+fx)\sec^{\frac{10}{3}}(e+fx)\,{}_2F_1\left(-\frac{5}{3},-\frac{3}{2};-\frac{2}{3};\cos^2(e+fx)\right)}{10f\sqrt{\sin^2(e+fx)}}$$

```
[Out] (3*Hypergeometric2F1[-5/3, -3/2, -2/3, Cos[e + f*x]^2]*Sec[e + f*x]^(10/3)*
Sin[e + f*x])/(10*f*Sqrt[Sin[e + f*x]^2])
```

Rubi [A] time = 0.0572237, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2632, 2576}

$$\frac{3\sin(e+fx)\sec^{\frac{10}{3}}(e+fx)\,{}_2F_1\left(-\frac{5}{3},-\frac{3}{2};-\frac{2}{3};\cos^2(e+fx)\right)}{10f\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^(13/3)*Sin[e + f*x]^4,x]
```

```
[Out] (3*Hypergeometric2F1[-5/3, -3/2, -2/3, Cos[e + f*x]^2]*Sec[e + f*x]^(10/3)*
Sin[e + f*x])/(10*f*Sqrt[Sin[e + f*x]^2])
```

Rule 2632

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(n + 1
)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))/b^2, Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx = \left(\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\right)\int \frac{\sin^4(e+fx)}{\cos^{\frac{13}{3}}(e+fx)}\,dx$$
$$= \frac{3\,{}_2F_1\left(-\frac{5}{3},-\frac{3}{2};-\frac{2}{3};\cos^2(e+fx)\right)\sec^{\frac{10}{3}}(e+fx)\sin(e+fx)}{10f\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 0.744251, size = 77, normalized size = 1.45

$$\frac{3\sin(e+fx)\left(\frac{9\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};\sin^2(e+fx)\right)}{\sqrt[3]{\cos^2(e+fx)}}+\left(4\sec^2(e+fx)-13\right)\sec^2(e+fx)\right)}{40f\sec^{\frac{2}{3}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^(13/3)*Sin[e + f*x]^4,x]
```

```
[Out] (3*((9*Hypergeometric2F1[1/2, 2/3, 3/2, Sin[e + f*x]^2])/(Cos[e + f*x]^2)^(
1/3) + Sec[e + f*x]^2*(-13 + 4*Sec[e + f*x]^2))*Sin[e + f*x])/(40*f*Sec[e +
 f*x]^(2/3))
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int \sqrt[3]{\sec\left(fx+e\right)}\left(\tan\left(fx+e\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^(1/3)*tan(f*x+e)^4,x)
```

```
[Out] int(sec(f*x+e)^(1/3)*tan(f*x+e)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)^{\frac{1}{3}}\tan\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(1/3)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^(1/3)*tan(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sec\left(fx+e\right)^{\frac{1}{3}}\tan\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(1/3)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral(sec(f*x + e)^(1/3)*tan(f*x + e)^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \tan^{4}\left(e+fx\right)\sqrt[3]{\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**(1/3)*tan(f*x+e)**4,x)
```

```
[Out] Integral(tan(e + f*x)**4*sec(e + f*x)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)^{\frac{1}{3}}\tan\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(1/3)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^(1/3)*tan(f*x + e)^4, x)
```

3.279 $\int \sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx$

Optimal. Leaf size=53

$$\frac{3\sin(e+fx)\sec^{\frac{8}{3}}(e+fx)\,{}_2F_1\left(-\frac{3}{2},-\frac{4}{3};-\frac{1}{3};\cos^2(e+fx)\right)}{8f\sqrt{\sin^2(e+fx)}}$$

```
[Out] (3*Hypergeometric2F1[-3/2, -4/3, -1/3, Cos[e + f*x]^2]*Sec[e + f*x]^(8/3)*S
in[e + f*x])/(8*f*Sqrt[Sin[e + f*x]^2])
```

Rubi [A] time = 0.0602202, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2632, 2576}

$$\frac{3\sin(e+fx)\sec^{\frac{8}{3}}(e+fx)\,{}_2F_1\left(-\frac{3}{2},-\frac{4}{3};-\frac{1}{3};\cos^2(e+fx)\right)}{8f\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^(11/3)*Sin[e + f*x]^4,x]
```


```
[Out] (3*Hypergeometric2F1[-3/2, -4/3, -1/3, Cos[e + f*x]^2]*Sec[e + f*x]^(8/3)*S
in[e + f*x])/(8*f*Sqrt[Sin[e + f*x]^2])
```


Rule 2632

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(n + 1
)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))/b^2, Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int \sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx = \left(\cos^{\frac{2}{3}}(e+fx)\sec^{\frac{2}{3}}(e+fx)\right)\int \frac{\sin^4(e+fx)}{\cos^{\frac{11}{3}}(e+fx)}\,dx$$
$$= \frac{3\,{}_2F_1\left(-\frac{3}{2},-\frac{4}{3};-\frac{1}{3};\cos^2(e+fx)\right)\sec^{\frac{8}{3}}(e+fx)\sin(e+fx)}{8f\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 0.194314, size = 78, normalized size = 1.47

$$\frac{3\sec^{\frac{2}{3}}(e+fx)\left(9\sin(e+fx)\sqrt[3]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)-11\sin(e+fx)+2\tan(e+fx)\sec(e+fx)\right)}{16f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^(11/3)*Sin[e + f*x]^4,x]
```

```
[Out] (3*Sec[e + f*x]^(2/3)*(-11*Sin[e + f*x] + 9*(Cos[e + f*x]^2)^(1/3)*Hypergeo
metric2F1[1/3, 1/2, 3/2, Sin[e + f*x]^2]*Sin[e + f*x] + 2*Sec[e + f*x]*Tan[
e + f*x]))/(16*f)
```

Maple [F] time = 0.076, size = 0, normalized size = 0.

$$\int \left(\tan\left(fx+e\right)\right)^4 \frac{1}{\sqrt[3]{\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/sec(f*x+e)^(1/3),x)
```

```
[Out] int(tan(f*x+e)^4/sec(f*x+e)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^4}{\sec\left(fx+e\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/sec(f*x+e)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^4/sec(f*x + e)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\tan\left(fx+e\right)^{4}}{\sec\left(fx+e\right)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/sec(f*x+e)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(tan(f*x + e)^4/sec(f*x + e)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^{4}\left(e+fx\right)}{\sqrt[3]{\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/sec(f*x+e)**(1/3),x)
```

```
[Out] Integral(tan(e + f*x)**4/sec(e + f*x)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^{4}}{\sec\left(fx+e\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/sec(f*x+e)^(1/3),x, algorithm="giac")

[Out] integrate(tan(f*x + e)^4/sec(f*x + e)^(1/3), x)
```

3.280 $\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx$

Optimal. Leaf size=53

$$\frac{3\sin(e+fx)\sec^{\frac{7}{3}}(e+fx)\,{}_2F_1\left(-\frac{3}{2},-\frac{7}{6};-\frac{1}{6};\cos^2(e+fx)\right)}{7f\sqrt{\sin^2(e+fx)}}$$

```
[Out] (3*Hypergeometric2F1[-3/2, -7/6, -1/6, Cos[e + f*x]^2]*Sec[e + f*x]^(7/3)*S
in[e + f*x])/(7*f*Sqrt[Sin[e + f*x]^2])
```

Rubi [A] time = 0.0585169, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2632, 2576}

$$\frac{3\sin(e+fx)\sec^{\frac{7}{3}}(e+fx)\,{}_2F_1\left(-\frac{3}{2},-\frac{7}{6};-\frac{1}{6};\cos^2(e+fx)\right)}{7f\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^(10/3)*Sin[e + f*x]^4,x]
```

```
[Out] (3*Hypergeometric2F1[-3/2, -7/6, -1/6, Cos[e + f*x]^2]*Sec[e + f*x]^(7/3)*S
in[e + f*x])/(7*f*Sqrt[Sin[e + f*x]^2])
```

Rule 2632

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(n + 1
)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))/b^2, Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx = \left(\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\right)\int \frac{\sin^4(e+fx)}{\cos^{\frac{10}{3}}(e+fx)}\,dx$$
$$= \frac{3\,{}_2F_1\left(-\frac{3}{2},-\frac{7}{6};-\frac{1}{6};\cos^2(e+fx)\right)\sec^{\frac{7}{3}}(e+fx)\sin(e+fx)}{7f\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 0.254815, size = 77, normalized size = 1.45

$$\frac{3\sqrt[3]{\sec(e+fx)}\left(9\sin(e+fx)\sqrt[6]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)-10\sin(e+fx)+\tan(e+fx)\sec(e+fx)\right)}{7f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^(10/3)*Sin[e + f*x]^4,x]
```

```
[Out] (3*Sec[e + f*x]^(1/3)*(-10*Sin[e + f*x] + 9*(Cos[e + f*x]^2)^(1/6)*Hypergeo
metric2F1[1/6, 1/2, 3/2, Sin[e + f*x]^2]*Sin[e + f*x] + Sec[e + f*x]*Tan[e
+ f*x]))/(7*f)
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int \left(\tan\left(fx+e\right)\right)^4\left(\sec\left(fx+e\right)\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/sec(f*x+e)^(2/3),x)
```

```
[Out] int(tan(f*x+e)^4/sec(f*x+e)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^4}{\sec\left(fx+e\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/sec(f*x+e)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^4/sec(f*x + e)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\tan\left(fx+e\right)^4}{\sec\left(fx+e\right)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/sec(f*x+e)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(tan(f*x + e)^4/sec(f*x + e)^(2/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^4\left(e+fx\right)}{\sec^{\frac{2}{3}}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/sec(f*x+e)**(2/3),x)
```

```
[Out] Integral(tan(e + f*x)**4/sec(e + f*x)**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^4}{\sec\left(fx+e\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/sec(f*x+e)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^4/sec(f*x + e)^(2/3), x)
```

3.281 $\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx$

Optimal. Leaf size=57

$$\frac{\cos^2(e+fx)^{13/6}\tan^3(e+fx)(d\sec(e+fx))^{4/3}\,{}_2F_1\left(\frac{3}{2},\frac{13}{6};\frac{5}{2};\sin^2(e+fx)\right)}{3f}$$

```
[Out] ((Cos[e + f*x]^2)^(13/6)*Hypergeometric2F1[3/2, 13/6, 5/2, Sin[e + f*x]^2]*
(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.043448, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2617}

$$\frac{\cos^2(e+fx)^{13/6}\tan^3(e+fx)(d\sec(e+fx))^{4/3}\,{}_2F_1\left(\frac{3}{2},\frac{13}{6};\frac{5}{2};\sin^2(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^2,x]
```

```
[Out] ((Cos[e + f*x]^2)^(13/6)*Hypergeometric2F1[3/2, 13/6, 5/2, Sin[e + f*x]^2]*
(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^3)/(3*f)
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx=\frac{\cos^2(e+fx)^{13/6}\,{}_2F_1\left(\frac{3}{2},\frac{13}{6};\frac{5}{2};\sin^2(e+fx)\right)(d\sec(e+fx))^{4/3}\tan^3(e+fx)}{3f}$$

Mathematica [A] time = 0.280316, size = 80, normalized size = 1.4

$$\frac{3d\sqrt[3]{d\sec(e+fx)}\left(2\sin(e+fx)\sqrt[6]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)-3\sin(e+fx)+\tan(e+fx)\sec(e+fx)\right)}{7f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^2,x]
```

```
[Out] (3*d*(d*Sec[e + f*x])^(1/3)*(-3*Sin[e + f*x] + 2*(Cos[e + f*x]^2)^(1/6)*Hyp
ergeometric2F1[1/6, 1/2, 3/2, Sin[e + f*x]^2]*Sin[e + f*x] + Sec[e + f*x]*T
an[e + f*x]))/(7*f)
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{4}{3}}\left(\tan\left(fx+e\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(4/3)*tan(f*x+e)^2,x)
```

```
[Out] int((d*sec(f*x+e))^(4/3)*tan(f*x+e)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{4}{3}}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(4/3)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(4/3)*tan(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}d\sec\left(fx+e\right)\tan\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(4/3)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^(1/3)*d*sec(f*x + e)*tan(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(4/3)*tan(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\sec\left(fx+e\right)\right)^{\frac{4}{3}}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(4/3)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(4/3)*tan(f*x + e)^2, x)
```

3.282 $\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx$

Optimal. Leaf size=57

$$\frac{\cos^2(e+fx)^{11/6}\tan^3(e+fx)(d\sec(e+fx))^{2/3}\,{}_2F_1\left(\frac{3}{2},\frac{11}{6};\frac{5}{2};\sin^2(e+fx)\right)}{3f}$$

```
[Out] ((Cos[e + f*x]^2)^(11/6)*Hypergeometric2F1[3/2, 11/6, 5/2, Sin[e + f*x]^2]*
(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.0421866, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2617}

$$\frac{\cos^2(e+fx)^{11/6}\tan^3(e+fx)(d\sec(e+fx))^{2/3}\,{}_2F_1\left(\frac{3}{2},\frac{11}{6};\frac{5}{2};\sin^2(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^2,x]
```

```
[Out] ((Cos[e + f*x]^2)^(11/6)*Hypergeometric2F1[3/2, 11/6, 5/2, Sin[e + f*x]^2]*
(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^3)/(3*f)
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx = \frac{\cos^2(e+fx)^{11/6}\,{}_2F_1\left(\frac{3}{2},\frac{11}{6};\frac{5}{2};\sin^2(e+fx)\right)(d\sec(e+fx))^{2/3}\tan^3(e+fx)}{3f}$$

Mathematica [A] time = 0.303094, size = 80, normalized size = 1.4

$$\frac{3(d\sec(e+fx))^{2/3}\left(2\sqrt[6]{\cos^2(e+fx)}\tan(e+fx)-\sin(2(e+fx))\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\sin^2(e+fx)\right)\right)}{10f\sqrt[6]{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^2,x]
```

```
[Out] (3*(d*Sec[e + f*x])^(2/3)*(-(Hypergeometric2F1[1/2, 5/6, 3/2, Sin[e + f*x]^
2]*Sin[2*(e + f*x)]) + 2*(Cos[e + f*x]^2)^(1/6)*Tan[e + f*x]))/(10*f*(Cos[e
 + f*x]^2)^(1/6))
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}}\left(\tan\left(fx+e\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(2/3)*tan(f*x+e)^2,x)
```

```
[Out] int((d*sec(f*x+e))^(2/3)*tan(f*x+e)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(2/3)*tan(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}}\tan\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^(2/3)*tan(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\sec\left(e+fx\right)\right)^{\frac{2}{3}}\tan^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(2/3)*tan(f*x+e)**2,x)
```

```
[Out] Integral((d*sec(e + f*x))**(2/3)*tan(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(2/3)*tan(f*x + e)^2, x)
```

3.283 $\int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx$

Optimal. Leaf size=57

$$\frac{\cos^2(e+fx)^{5/3}\tan^3(e+fx)\sqrt[3]{d\sec(e+fx)}\,{}_2F_1\left(\frac{3}{2},\frac{5}{3};\frac{5}{2};\sin^2(e+fx)\right)}{3f}$$

```
[Out] ((Cos[e + f*x]^2)^(5/3)*Hypergeometric2F1[3/2, 5/3, 5/2, Sin[e + f*x]^2]*(d
*Sec[e + f*x])^(1/3)*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.0364821, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2617}

$$\frac{\cos^2(e+fx)^{5/3}\tan^3(e+fx)\sqrt[3]{d\sec(e+fx)}\,{}_2F_1\left(\frac{3}{2},\frac{5}{3};\frac{5}{2};\sin^2(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(1/3)*Tan[e + f*x]^2,x]
```

```
[Out] ((Cos[e + f*x]^2)^(5/3)*Hypergeometric2F1[3/2, 5/3, 5/2, Sin[e + f*x]^2]*(d
*Sec[e + f*x])^(1/3)*Tan[e + f*x]^3)/(3*f)
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx=\frac{\cos^2(e+fx)^{5/3}\,{}_2F_1\left(\frac{3}{2},\frac{5}{3};\frac{5}{2};\sin^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan^3(e+fx)}{3f}$$

Mathematica [A] time = 0.280346, size = 80, normalized size = 1.4

$$\frac{3\sqrt[3]{d\sec(e+fx)}\left(2\sqrt[3]{\cos^2(e+fx)}\tan(e+fx)-\sin(2(e+fx))\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};\sin^2(e+fx)\right)\right)}{8f\sqrt[3]{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(1/3)*Tan[e + f*x]^2,x]
```

```
[Out] (3*(d*Sec[e + f*x])^(1/3)*(-(Hypergeometric2F1[1/2, 2/3, 3/2, Sin[e + f*x]^
2]*Sin[2*(e + f*x)]) + 2*(Cos[e + f*x]^2)^(1/3)*Tan[e + f*x]))/(8*f*(Cos[e
+ f*x]^2)^(1/3))
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int\sqrt[3]{d\sec\left(fx+e\right)}\left(\tan\left(fx+e\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/3)*tan(f*x+e)^2,x)
```

```
[Out] int((d*sec(f*x+e))^(1/3)*tan(f*x+e)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(1/3)*tan(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\tan\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^(1/3)*tan(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt[3]{d\sec\left(e+fx\right)}\tan^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/3)*tan(f*x+e)**2,x)
```

```
[Out] Integral((d*sec(e + f*x))**(1/3)*tan(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(1/3)*tan(f*x + e)^2, x)
```

3.284 $\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$

Optimal. Leaf size=57

$$\frac{\cos^2(e+fx)^{4/3}\tan^3(e+fx)\,{}_2F_1\left(\frac{4}{3},\frac{3}{2};\frac{5}{2};\sin^2(e+fx)\right)}{3f\sqrt[3]{d\sec(e+fx)}}$$

```
[Out] ((Cos[e + f*x]^2)^(4/3)*Hypergeometric2F1[4/3, 3/2, 5/2, Sin[e + f*x]^2]*Ta
n[e + f*x]^3)/(3*f*(d*Sec[e + f*x])^(1/3))
```

Rubi [A] time = 0.0425088, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2617}

$$\frac{\cos^2(e+fx)^{4/3}\tan^3(e+fx)\,{}_2F_1\left(\frac{4}{3},\frac{3}{2};\frac{5}{2};\sin^2(e+fx)\right)}{3f\sqrt[3]{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(d*Sec[e + f*x])^(1/3),x]
```

```
[Out] ((Cos[e + f*x]^2)^(4/3)*Hypergeometric2F1[4/3, 3/2, 5/2, Sin[e + f*x]^2]*Ta
n[e + f*x]^3)/(3*f*(d*Sec[e + f*x])^(1/3))
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \frac{\cos^2(e+fx)^{4/3}\,{}_2F_1\left(\frac{4}{3},\frac{3}{2};\frac{5}{2};\sin^2(e+fx)\right)\tan^3(e+fx)}{3f\sqrt[3]{d\sec(e+fx)}}$$

Mathematica [A] time = 0.213616, size = 80, normalized size = 1.4

$$\frac{3\left(2\cos^2(e+fx)^{2/3}\tan(e+fx)-\sin(2(e+fx))\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)\right)}{4f\cos^2(e+fx)^{2/3}\sqrt[3]{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(d*Sec[e + f*x])^(1/3),x]
```

```
[Out] (3*(-(Hypergeometric2F1[1/3, 1/2, 3/2, Sin[e + f*x]^2]*Sin[2*(e + f*x)]) +
2*(Cos[e + f*x]^2)^(2/3)*Tan[e + f*x]))/(4*f*(Cos[e + f*x]^2)^(2/3)*(d*Sec[
e + f*x])^(1/3))
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \left(\tan\left(fx+e\right)\right)^2\frac{1}{\sqrt[3]{d\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(d*sec(f*x+e))^(1/3),x)
```

```
[Out] int(tan(f*x+e)^2/(d*sec(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^2}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(d*sec(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^2/(d*sec(f*x + e))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}}\tan\left(fx+e\right)^{2}}{d\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(d*sec(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^(2/3)*tan(f*x + e)^2/(d*sec(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tan^{2}\left(e+fx\right)}{\sqrt[3]{d\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(d*sec(f*x+e))**(1/3),x)
```

```
[Out] Integral(tan(e + f*x)**2/(d*sec(e + f*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tan\left(fx+e\right)^{2}}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(d*sec(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/(d*sec(f*x + e))^(1/3), x)
```

3.285 $\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx$

Optimal. Leaf size=57

$$\frac{\cos^2(e+fx)^{7/6}\tan^3(e+fx)\,{}_2F_1\left(\frac{7}{6},\frac{3}{2};\frac{5}{2};\sin^2(e+fx)\right)}{3f(d\sec(e+fx))^{2/3}}$$

```
[Out] ((Cos[e + f*x]^2)^(7/6)*Hypergeometric2F1[7/6, 3/2, 5/2, Sin[e + f*x]^2]*Ta
n[e + f*x]^3)/(3*f*(d*Sec[e + f*x])^(2/3))
```

Rubi [A] time = 0.0444188, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2617}

$$\frac{\cos^2(e+fx)^{7/6}\tan^3(e+fx)\,{}_2F_1\left(\frac{7}{6},\frac{3}{2};\frac{5}{2};\sin^2(e+fx)\right)}{3f(d\sec(e+fx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(d*Sec[e + f*x])^(2/3),x]
```

```
[Out] ((Cos[e + f*x]^2)^(7/6)*Hypergeometric2F1[7/6, 3/2, 5/2, Sin[e + f*x]^2]*Ta
n[e + f*x]^3)/(3*f*(d*Sec[e + f*x])^(2/3))
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \frac{\cos^2(e+fx)^{7/6}\,{}_2F_1\left(\frac{7}{6},\frac{3}{2};\frac{5}{2};\sin^2(e+fx)\right)\tan^3(e+fx)}{3f(d\sec(e+fx))^{2/3}}$$

Mathematica [A] time = 0.194593, size = 79, normalized size = 1.39

$$\frac{3\cos^2(e+fx)^{5/6}\tan(e+fx)-\frac{3}{2}\sin(2(e+fx))\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)}{f\cos^2(e+fx)^{5/6}(d\sec(e+fx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(d*Sec[e + f*x])^(2/3),x]
```

```
[Out] ((-3*Hypergeometric2F1[1/6, 1/2, 3/2, Sin[e + f*x]^2]*Sin[2*(e + f*x)])/2 +
 3*(Cos[e + f*x]^2)^(5/6)*Tan[e + f*x])/(f*(Cos[e + f*x]^2)^(5/6)*(d*Sec[e
+ f*x])^(2/3))
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \left(\tan\left(fx+e\right)\right)^{2}\left(d\sec\left(fx+e\right)\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(d*sec(f*x+e))^(2/3),x)
```

```
[Out] int(tan(f*x+e)^2/(d*sec(f*x+e))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^{2}}{\left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(d*sec(f*x+e))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^2/(d*sec(f*x + e))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\tan\left(fx+e\right)^{2}}{d\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(d*sec(f*x+e))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^(1/3)*tan(f*x + e)^2/(d*sec(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tan^{2}\left(e+fx\right)}{\left(d\sec\left(e+fx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(d*sec(f*x+e))**(2/3),x)
```

```
[Out] Integral(tan(e + f*x)**2/(d*sec(e + f*x))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tan\left(fx+e\right)^{2}}{\left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(d*sec(f*x+e))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/(d*sec(f*x + e))^(2/3), x)
```

3.286 $\int (d\sec(e+fx))^{4/3}\tan^4(e+fx)\,dx$

Optimal. Leaf size=57

$$\frac{\cos^2(e+fx)^{19/6}\tan^5(e+fx)(d\sec(e+fx))^{4/3}\,{}_2F_1\left(\frac{5}{2},\frac{19}{6};\frac{7}{2};\sin^2(e+fx)\right)}{5f}$$

```
[Out] ((Cos[e + f*x]^2)^(19/6)*Hypergeometric2F1[5/2, 19/6, 7/2, Sin[e + f*x]^2]*
(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^5)/(5*f)
```

Rubi [A] time = 0.04203, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2617}

$$\frac{\cos^2(e+fx)^{19/6}\tan^5(e+fx)(d\sec(e+fx))^{4/3}\,{}_2F_1\left(\frac{5}{2},\frac{19}{6};\frac{7}{2};\sin^2(e+fx)\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^4,x]
```

```
[Out] ((Cos[e + f*x]^2)^(19/6)*Hypergeometric2F1[5/2, 19/6, 7/2, Sin[e + f*x]^2]*
(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^5)/(5*f)
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (d\sec(e+fx))^{4/3}\tan^4(e+fx)\,dx=\frac{\cos^2(e+fx)^{19/6}\,{}_2F_1\left(\frac{5}{2},\frac{19}{6};\frac{7}{2};\sin^2(e+fx)\right)(d\sec(e+fx))^{4/3}\tan^5(e+fx)}{5f}$$

Mathematica [A] time = 1.0984, size = 92, normalized size = 1.61

$$\frac{3d\sqrt[3]{d\sec(e+fx)}\left(-18\sin(e+fx)\sqrt[6]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)+27\sin(e+fx)+\tan(e+fx)\sec(e+fx)\right.}{91f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^4,x]
```

```
[Out] (3*d*(d*Sec[e + f*x])^(1/3)*(27*Sin[e + f*x] - 18*(Cos[e + f*x]^2)^(1/6)*Hy
pergeometric2F1[1/6, 1/2, 3/2, Sin[e + f*x]^2]*Sin[e + f*x] + Sec[e + f*x]*
(-16 + 7*Sec[e + f*x]^2)*Tan[e + f*x]))/(91*f)
```

Maple [F] time = 0.078, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{4}{3}}\left(\tan\left(fx+e\right)\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(4/3)*tan(f*x+e)^4,x)
```

```
[Out] int((d*sec(f*x+e))^(4/3)*tan(f*x+e)^4,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(4/3)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}d\sec\left(fx+e\right)\tan\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(4/3)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^(1/3)*d*sec(f*x + e)*tan(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(4/3)*tan(f*x+e)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{4}{3}}\tan\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(4/3)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(4/3)*tan(f*x + e)^4, x)
```

3.287 $\int (d\sec(e+fx))^{2/3}\tan^4(e+fx)\,dx$

Optimal. Leaf size=57

$$\frac{\cos^2(e+fx)^{17/6}\tan^5(e+fx)(d\sec(e+fx))^{2/3}\,{}_2F_1\left(\frac{5}{2},\frac{17}{6};\frac{7}{2};\sin^2(e+fx)\right)}{5f}$$

```
[Out] ((Cos[e + f*x]^2)^(17/6)*Hypergeometric2F1[5/2, 17/6, 7/2, Sin[e + f*x]^2]*
(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^5)/(5*f)
```

Rubi [A] time = 0.0434981, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2617}

$$\frac{\cos^2(e+fx)^{17/6}\tan^5(e+fx)(d\sec(e+fx))^{2/3}\,{}_2F_1\left(\frac{5}{2},\frac{17}{6};\frac{7}{2};\sin^2(e+fx)\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^4,x]
```

```
[Out] ((Cos[e + f*x]^2)^(17/6)*Hypergeometric2F1[5/2, 17/6, 7/2, Sin[e + f*x]^2]*
(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^5)/(5*f)
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (d\sec(e+fx))^{2/3}\tan^4(e+fx)\,dx=\frac{\cos^2(e+fx)^{17/6}\,{}_2F_1\left(\frac{5}{2},\frac{17}{6};\frac{7}{2};\sin^2(e+fx)\right)(d\sec(e+fx))^{2/3}\tan^5(e+fx)}{5f}$$

Mathematica [A] time = 0.154955, size = 69, normalized size = 1.21

$$\frac{3\tan(e+fx)(d\sec(e+fx))^{2/3}\left(9\cos^2(e+fx)^{5/6}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\sin^2(e+fx)\right)+5\sec^2(e+fx)-14\right)}{55f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^4,x]
```

```
[Out] (3*(d*Sec[e + f*x])^(2/3)*(-14 + 9*(Cos[e + f*x]^2)^(5/6)*Hypergeometric2F1
[1/2, 5/6, 3/2, Sin[e + f*x]^2] + 5*Sec[e + f*x]^2)*Tan[e + f*x])/(55*f)
```

Maple [F] time = 0.081, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}}\left(\tan\left(fx+e\right)\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(2/3)*tan(f*x+e)^4,x)
```

```
[Out] int((d*sec(f*x+e))^(2/3)*tan(f*x+e)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}}\tan\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(2/3)*tan(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}}\tan\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^(2/3)*tan(f*x + e)^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec \left(e+fx\right)\right)^{\frac{2}{3}}\tan^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(2/3)*tan(f*x+e)**4,x)
```

```
[Out] Integral((d*sec(e + f*x))**(2/3)*tan(e + f*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec \left(fx+e\right)\right)^{\frac{2}{3}}\tan \left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(2/3)*tan(f*x + e)^4, x)
```

3.288 $\int \sqrt[3]{d\sec(e+fx)}\tan^4(e+fx)\,dx$

Optimal. Leaf size=57

$$\frac{\cos^2(e+fx)^{8/3}\tan^5(e+fx)\sqrt[3]{d\sec(e+fx)}\,{}_2F_1\left(\frac{5}{2},\frac{8}{3};\frac{7}{2};\sin^2(e+fx)\right)}{5f}$$

```
[Out] ((Cos[e + f*x]^2)^(8/3)*Hypergeometric2F1[5/2, 8/3, 7/2, Sin[e + f*x]^2]*(d
*Sec[e + f*x])^(1/3)*Tan[e + f*x]^5)/(5*f)
```

Rubi [A] time = 0.0374419, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2617}

$$\frac{\cos^2(e+fx)^{8/3}\tan^5(e+fx)\sqrt[3]{d\sec(e+fx)}\,{}_2F_1\left(\frac{5}{2},\frac{8}{3};\frac{7}{2};\sin^2(e+fx)\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(1/3)*Tan[e + f*x]^4,x]
```

```
[Out] ((Cos[e + f*x]^2)^(8/3)*Hypergeometric2F1[5/2, 8/3, 7/2, Sin[e + f*x]^2]*(d
*Sec[e + f*x])^(1/3)*Tan[e + f*x]^5)/(5*f)
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

Rubi steps

$$\int \sqrt[3]{d\sec(e+fx)}\tan^4(e+fx)\,dx=\frac{\cos^2(e+fx)^{8/3}\,{}_2F_1\left(\frac{5}{2},\frac{8}{3};\frac{7}{2};\sin^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan^5(e+fx)}{5f}$$

Mathematica [A] time = 0.150734, size = 69, normalized size = 1.21

$$\frac{3\tan(e+fx)\sqrt[3]{d\sec(e+fx)}\left(9\cos^2(e+fx)^{2/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};\sin^2(e+fx)\right)+4\sec^2(e+fx)-13\right)}{40f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(1/3)*Tan[e + f*x]^4,x]
```

```
[Out] (3*(d*Sec[e + f*x])^(1/3)*(-13 + 9*(Cos[e + f*x]^2)^(2/3)*Hypergeometric2F1
[1/2, 2/3, 3/2, Sin[e + f*x]^2] + 4*Sec[e + f*x]^2)*Tan[e + f*x])/(40*f)
```

Maple [F] time = 0.077, size = 0, normalized size = 0.

$$\int \sqrt[3]{d\sec\left(fx+e\right)}\left(\tan\left(fx+e\right)\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/3)*tan(f*x+e)^4,x)
```

```
[Out] int((d*sec(f*x+e))^(1/3)*tan(f*x+e)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\tan\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(1/3)*tan(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\tan\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^(1/3)*tan(f*x + e)^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{d\sec\left(e+fx\right)}\tan^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/3)*tan(f*x+e)**4,x)
```

```
[Out] Integral((d*sec(e + f*x))**(1/3)*tan(e + f*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\tan\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(1/3)*tan(f*x + e)^4, x)
```

3.289 $\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$

Optimal. Leaf size=57

$$\frac{\cos^2(e+fx)^{7/3}\tan^5(e+fx)\,{}_2F_1\left(\frac{7}{3},\frac{5}{2};\frac{7}{2};\sin^2(e+fx)\right)}{5f\sqrt[3]{d\sec(e+fx)}}$$

```
[Out] ((Cos[e + f*x]^2)^(7/3)*Hypergeometric2F1[7/3, 5/2, 7/2, Sin[e + f*x]^2]*Ta
n[e + f*x]^5)/(5*f*(d*Sec[e + f*x])^(1/3))
```

Rubi [A] time = 0.0386355, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2617}

$$\frac{\cos^2(e+fx)^{7/3}\tan^5(e+fx)\,{}_2F_1\left(\frac{7}{3},\frac{5}{2};\frac{7}{2};\sin^2(e+fx)\right)}{5f\sqrt[3]{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(d*Sec[e + f*x])^(1/3),x]
```

```
[Out] ((Cos[e + f*x]^2)^(7/3)*Hypergeometric2F1[7/3, 5/2, 7/2, Sin[e + f*x]^2]*Ta
n[e + f*x]^5)/(5*f*(d*Sec[e + f*x])^(1/3))
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \frac{\cos^2(e+fx)^{7/3}\,{}_2F_1\left(\frac{7}{3},\frac{5}{2};\frac{7}{2};\sin^2(e+fx)\right)\tan^5(e+fx)}{5f\sqrt[3]{d\sec(e+fx)}}$$

Mathematica [A] time = 0.148885, size = 69, normalized size = 1.21

$$\frac{3\tan(e+fx)\left(9\sqrt[3]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)+2\sec^2(e+fx)-11\right)}{16f\sqrt[3]{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(d*Sec[e + f*x])^(1/3),x]
```

```
[Out] (3*(-11 + 9*(Cos[e + f*x]^2)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, Sin[e +
 f*x]^2] + 2*Sec[e + f*x]^2)*Tan[e + f*x])/(16*f*(d*Sec[e + f*x])^(1/3))
```

Maple [F] time = 0.076, size = 0, normalized size = 0.

$$\int\left(\tan\left(fx+e\right)\right)^4\frac{1}{\sqrt[3]{d\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(d*sec(f*x+e))^(1/3),x)
```

```
[Out] int(tan(f*x+e)^4/(d*sec(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tan\left(fx+e\right)^4}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(d*sec(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^4/(d*sec(f*x + e))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}}\tan\left(fx+e\right)^{4}}{d\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(d*sec(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^(2/3)*tan(f*x + e)^4/(d*sec(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tan^{4}\left(e+fx\right)}{\sqrt[3]{d\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(d*sec(f*x+e))**(1/3),x)
```

```
[Out] Integral(tan(e + f*x)**4/(d*sec(e + f*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tan\left(fx+e\right)^{4}}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(d*sec(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^4/(d*sec(f*x + e))^(1/3), x)
```

3.290 $\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx$

Optimal. Leaf size=57

$$\frac{\cos^2(e+fx)^{13/6}\tan^5(e+fx)\,{}_2F_1\left(\frac{13}{6},\frac{5}{2};\frac{7}{2};\sin^2(e+fx)\right)}{5f(d\sec(e+fx))^{2/3}}$$

```
[Out] ((Cos[e + f*x]^2)^(13/6)*Hypergeometric2F1[13/6, 5/2, 7/2, Sin[e + f*x]^2]*
Tan[e + f*x]^5)/(5*f*(d*Sec[e + f*x])^(2/3))
```

Rubi [A] time = 0.0446449, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2617}

$$\frac{\cos^2(e+fx)^{13/6}\tan^5(e+fx)\,{}_2F_1\left(\frac{13}{6},\frac{5}{2};\frac{7}{2};\sin^2(e+fx)\right)}{5f(d\sec(e+fx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(d*Sec[e + f*x])^(2/3),x]
```


```
[Out] ((Cos[e + f*x]^2)^(13/6)*Hypergeometric2F1[13/6, 5/2, 7/2, Sin[e + f*x]^2]*
Tan[e + f*x]^5)/(5*f*(d*Sec[e + f*x])^(2/3))
```


Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \frac{\cos^2(e+fx)^{13/6}\,{}_2F_1\left(\frac{13}{6},\frac{5}{2};\frac{7}{2};\sin^2(e+fx)\right)\tan^5(e+fx)}{5f(d\sec(e+fx))^{2/3}}$$

Mathematica [A] time = 0.14309, size = 67, normalized size = 1.18

$$\frac{3\tan(e+fx)\left(9\sqrt[6]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)+\sec^2(e+fx)-10\right)}{7f(d\sec(e+fx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(d*Sec[e + f*x])^(2/3),x]
```

```
[Out] (3*(-10 + 9*(Cos[e + f*x]^2)^(1/6)*Hypergeometric2F1[1/6, 1/2, 3/2, Sin[e +
 f*x]^2] + Sec[e + f*x]^2)*Tan[e + f*x])/(7*f*(d*Sec[e + f*x])^(2/3))
```

Maple [F] time = 0.076, size = 0, normalized size = 0.

$$\int \left(\tan\left(fx+e\right)\right)^4\left(d\sec\left(fx+e\right)\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(d*sec(f*x+e))^(2/3),x)
```

```
[Out] int(tan(f*x+e)^4/(d*sec(f*x+e))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^4}{\left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(d*sec(f*x+e))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^4/(d*sec(f*x + e))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\tan\left(fx+e\right)^{4}}{d\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(d*sec(f*x+e))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^(1/3)*tan(f*x + e)^4/(d*sec(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^{4}\left(e+fx\right)}{\left(d\sec\left(e+fx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(d*sec(f*x+e))**(2/3),x)
```

```
[Out] Integral(tan(e + f*x)**4/(d*sec(e + f*x))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^{4}}{\left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(d*sec(f*x+e))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^4/(d*sec(f*x + e))^(2/3), x)
```

3.291 $\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx$

Optimal. Leaf size=178

$$\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}-\frac{\sqrt{b}d^3\sqrt{b\tan(e+fx)}\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{4f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{\sqrt{b}d^3\sqrt{b\tan(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{4f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

```
[Out] -(Sqrt[b]*d^3*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(4
*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]]) + (Sqrt[b]*d^3*ArcTanh[Sqrt[b
*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(4*f*Sqrt[d*Sec[e + f*x]]*Sqr
t[b*Sin[e + f*x]]) + (d^2*Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2))/(2*b
*f)
```

Rubi [A] time = 0.162123, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2613, 2616, 2564, 329, 298, 203, 206}

$$\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}-\frac{\sqrt{b}d^3\sqrt{b\tan(e+fx)}\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{4f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{\sqrt{b}d^3\sqrt{b\tan(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{4f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] -(Sqrt[b]*d^3*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(4
*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]]) + (Sqrt[b]*d^3*ArcTanh[Sqrt[b
*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(4*f*Sqrt[d*Sec[e + f*x]]*Sqr
t[b*Sin[e + f*x]]) + (d^2*Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2))/(2*b
*f)
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \frac{d^2\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{2bf} + \frac{1}{4}d^2\int\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx$$

$$= \frac{d^2\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{2bf} + \frac{\left(d^3\sqrt{b\tan(e+fx)}\right)\int\sec(e+fx)\sqrt{b\sin}}{4\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$= \frac{d^2\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{2bf} + \frac{\left(d^3\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int\frac{\sqrt{x}}{1-\frac{x^2}{b^2}}\,dx,x\right.}{4bf\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+}}$$

$$= \frac{d^2\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{2bf} + \frac{\left(d^3\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int\frac{x^2}{1-\frac{x^4}{b^2}}\,dx,x\right.}{2bf\sqrt{d\sec(e+fx)}\sqrt{b\sin(e}}$$

$$= \frac{d^2\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{2bf} + \frac{\left(bd^3\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int\frac{1}{b-x^2}\,dx,x\right.}{4f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e}}$$

$$= -\frac{\sqrt{b}d^3\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{4f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} + \frac{\sqrt{b}d^3\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan}}{4f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+}}$$

Mathematica [A] time = 1.90293, size = 174, normalized size = 0.98

$$\frac{b(d\sec(e+fx))^{5/2}\left(4\sec^{\frac{5}{2}}(e+fx)-4\sqrt{\sec(e+fx)}+2\sqrt[4]{\tan^2(e+fx)}\tan^{-1}\left(\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)+\sqrt[4]{\tan^2(e+fx)}\left(\log\left(\frac{\sqrt{\text{se}}}{\sqrt[4]{\tan}}\right.\right.\right.}{8f\sec^{\frac{5}{2}}(e+fx)\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (b*(d*Sec[e + f*x])^(5/2)*(-4*Sqrt[Sec[e + f*x]] + 4*Sec[e + f*x]^(5/2) + 2
*ArcTan[Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^(1/4)]*(Tan[e + f*x]^2)^(1/4) +
 (-Log[1 - Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^(1/4)] + Log[1 + Sqrt[Sec[e
+ f*x]]/(Tan[e + f*x]^2)^(1/4)])*(Tan[e + f*x]^2)^(1/4)))/(8*f*Sec[e + f*x]
^(5/2)*Sqrt[b*Tan[e + f*x]])
```

Maple [C] time = 0.316, size = 600, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x)
```

```
[Out] 1/8/f*2^(1/2)*(I*cos(f*x+e)^2*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2
)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f*
x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1
/2*I,1/2*2^(1/2))-I*cos(f*x+e)^2*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(
1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin
(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/
2+1/2*I,1/2*2^(1/2))-cos(f*x+e)^2*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^
(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/si
n(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1
/2-1/2*I,1/2*2^(1/2))-cos(f*x+e)^2*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))
^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/s
in(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),
1/2+1/2*I,1/2*2^(1/2))+2*cos(f*x+e)*2^(1/2)-2*2^(1/2))*cos(f*x+e)*(d/cos(f*
x+e))^(5/2)*(b*sin(f*x+e)/cos(f*x+e))^(1/2)*sin(f*x+e)/(cos(f*x+e)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}\sqrt{b\tan\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)*sqrt(b*tan(f*x + e)), x)
```

Fricas [B] time = 2.90462, size = 2044, normalized size = 11.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(2*sqrt(-b*d)*d^2*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (c
os(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt
```

```
(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(f*x
+ e)^2 - b*d - (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e) - sqrt(
-b*d)*d^2*cos(f*x + e)*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2 - 8*
(7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(
f*x + e))*sqrt(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))
 + 72*b*d + 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x + e)^4 -
 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) - 16*d^2*sqrt
(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x
 + e)), -1/32*(2*sqrt(b*d)*d^2*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^
2 + (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4
)*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos
(f*x + e)^2 - b*d + (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e) -
sqrt(b*d)*d^2*cos(f*x + e)*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2
- 8*(7*cos(f*x + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*
cos(f*x + e))*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x +
e)) + 72*b*d - 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x + e)^
4 - 8*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) - 16*d^2*s
qrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*sin(f*x + e))/(f*cos(
f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/2)*(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}\sqrt{b\tan\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)*sqrt(b*tan(f*x + e)), x)
```

3.292 $\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx$

Optimal. Leaf size=93

$$\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

```
[Out] -((d^2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[d*Sec
[e + f*x]]*Sqrt[Sin[e + f*x]])) + (d^2*(b*Tan[e + f*x])^(3/2))/(b*f*Sqrt[d*
Sec[e + f*x]])
```

Rubi [A] time = 0.11169, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2613, 2616, 2640, 2639}

$$\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] -((d^2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[d*Sec
[e + f*x]]*Sqrt[Sin[e + f*x]])) + (d^2*(b*Tan[e + f*x])^(3/2))/(b*f*Sqrt[d*
Sec[e + f*x]])
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
```

```
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx &= \frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{1}{2}d^2\int\frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx\\ &= \frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{\left(d^2\sqrt{b\tan(e+fx)}\right)\int\sqrt{b\sin(e+fx)}\,dx}{2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}\\ &= \frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{\left(d^2\sqrt{b\tan(e+fx)}\right)\int\sqrt{\sin(e+fx)}\,dx}{2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}\\ &= -\frac{d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}\end{aligned}$$

Mathematica [C] time = 1.00158, size = 71, normalized size = 0.76

$$\frac{d\sin(e+fx)\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}\left(1-\frac{{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\sec^2(e+fx)\right)}{\left(-\tan^2(e+fx)\right)^{3/4}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (d*Sqrt[d*Sec[e + f*x]]*Sin[e + f*x]*Sqrt[b*Tan[e + f*x]]*(1 - Hypergeometr
ic2F1[-1/4, 1/4, 3/4, Sec[e + f*x]^2]/(-Tan[e + f*x]^2)^(3/4)))/f
```

Maple [C] time = 0.277, size = 572, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*2^(1/2)*(b*sin(f*x+e)/cos(f*x+e))^(1/2)*(d/cos(f*x+e))^(3/2)*cos(f*x+
e)*(2*cos(f*x+e)^2*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f
*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*((I*cos(f*x+e)-I+sin(f*x
+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)-cos(
f*x+e)^2*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/
sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF
(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*cos(f*x+e)*(
-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)-I+sin(f*x+e))/
sin(f*x+e))^(1/2),1/2*2^(1/2))*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/
2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)-cos(f*x+e)*(-I*(cos(f*x+
e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I
*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)-I+sin(
f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-cos(f*x+e)*2^(1/2)+2^(1/2))/sin(f*x+
e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{b\tan\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)*sqrt(b*tan(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{d\sec\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}d\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e))*d*sec(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(3/2)*(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{b\tan\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)*sqrt(b*tan(f*x + e)), x)
```

3.293 $\int \sqrt{d \sec(e+fx)}\sqrt{b \tan(e+fx)}\,dx$

Optimal. Leaf size=132

$$\frac{\sqrt{b}d\sqrt{b\tan(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{\sqrt{b}d\sqrt{b\tan(e+fx)}\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

```
[Out] -((Sqrt[b]*d*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(f*
Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])) + (Sqrt[b]*d*ArcTanh[Sqrt[b*Sin
[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Si
n[e + f*x]])
```

Rubi [A] time = 0.0979791, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2616, 2564, 329, 298, 203, 206}

$$\frac{\sqrt{b}d\sqrt{b\tan(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{\sqrt{b}d\sqrt{b\tan(e+fx)}\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] -((Sqrt[b]*d*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(f*
Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])) + (Sqrt[b]*d*ArcTanh[Sqrt[b*Sin
[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Si
n[e + f*x]])
```

Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx = \frac{\left(d\sqrt{b\tan(e+fx)}\right)\int \sec(e+fx)\sqrt{b\sin(e+fx)}\,dx}{\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$= \frac{\left(d\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{b^2}}\,dx, x, b\sin(e+fx)\right)}{bf\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$= \frac{\left(2d\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sin(e+fx)}\right)}{bf\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$= \frac{\left(bd\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sin(e+fx)}\right)}{f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} - \frac{\left(bd\sqrt{b\tan(e+fx)}\right)\text{S}}{f\sqrt{d\sec(}}$$

$$= -\frac{\sqrt{b}d\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} + \frac{\sqrt{b}d\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+}}{f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

Mathematica [A] time = 0.788373, size = 136, normalized size = 1.03

$$\frac{b\sqrt[4]{\tan^2(e+fx)}\sqrt{d\sec(e+fx)}\left(2\tan^{-1}\left(\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)-\log\left(1-\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)+\log\left(\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}+1\right)\right)}{2f\sqrt{\sec(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (b*(2*ArcTan[Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^(1/4)] - Log[1 - Sqrt[Sec[
e + f*x]]/(Tan[e + f*x]^2)^(1/4)] + Log[1 + Sqrt[Sec[e + f*x]]/(Tan[e + f*x
]^2)^(1/4)])*Sqrt[d*Sec[e + f*x]]*(Tan[e + f*x]^2)^(1/4))/(2*f*Sqrt[Sec[e +
 f*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [C] time = 0.232, size = 306, normalized size = 2.3

$$\frac{\sin\left(fx+e\right)\sqrt{2}\cos\left(fx+e\right)}{2f\left(\cos\left(fx+e\right)-1\right)}\sqrt{\frac{d}{\cos\left(fx+e\right)}}\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{i\cos\left(fx+e\right)-i+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{-\frac{i\cos\left(fx+e\right)-i-\text{si}}{\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*(d/cos(f*x+e))^(1/2)*(b*sin(f*x+e)/cos(f*x+e))^(1/2)*sin(f*x+e)*((I*c
os(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin
(f*x+e))^(1/2)*2^(1/2)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*cos(f*x+e)*(I*E
llipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1
/2))-I*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,
1/2*2^(1/2))-EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-
1/2*I,1/2*2^(1/2))-EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2
),1/2+1/2*I,1/2*2^(1/2)))/(cos(f*x+e)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \sec\left(fx+e\right)}\sqrt{b \tan\left(fx+e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e)), x)
```

Fricas [B] time = 2.80112, size = 1701, normalized size = 12.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(2*sqrt(-b*d)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (cos(f*
x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(-b*d
)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(f*x + e)^
2 - b*d - (b*d*cos(f*x + e) + b*d)*sin(f*x + e))) - sqrt(-b*d)*log((b*d*cos
(f*x + e)^4 - 72*b*d*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3 - (cos(f*x + e)^3
 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(-b*d)*sqrt(b*sin(f*x
 + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)) + 72*b*d + 28*(b*d*cos(f*x + e)^2
- 2*b*d)*sin(f*x + e))/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f*x + e)
```

```
^2 - 2)*sin(f*x + e) + 8)))/f, -1/8*(2*sqrt(b*d)*arctan(1/4*(cos(f*x + e)^3
 - 5*cos(f*x + e)^2 + (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) -
2*cos(f*x + e) + 4)*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(
f*x + e))/(b*d*cos(f*x + e)^2 - b*d + (b*d*cos(f*x + e) + b*d)*sin(f*x + e)
)) - sqrt(b*d)*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2 - 8*(7*cos(f
*x + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e)
)*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)) + 72*b*d
 - 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x + e)^4 - 8*cos(f*
x + e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\tan\left(e+fx\right)}\sqrt{d\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/2)*(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(b*tan(e + f*x))*sqrt(d*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\sec\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e)), x)
```

3.294 $\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx$

Optimal. Leaf size=55

$$\frac{2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

```
[Out] (2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[d*Sec[e +
 f*x]]*Sqrt[Sin[e + f*x]])
```

Rubi [A] time = 0.061064, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2616, 2640, 2639}

$$\frac{2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[d*Sec[e +
 f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx = \frac{\sqrt{b\tan(e+fx)}\int\sqrt{b\sin(e+fx)}\,dx}{\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$
$$= \frac{\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}\,dx}{\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}$$
$$= \frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}$$

Mathematica [C] time = 0.513585, size = 62, normalized size = 1.13

$$-\frac{2b\sqrt[4]{-\tan^2(e+fx)}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\sec^2(e+fx)\right)}{f\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (-2*b*Hypergeometric2F1[-1/4, 1/4, 3/4, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(
1/4))/(f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [C] time = 0.229, size = 551, normalized size = 10.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/f*2^(1/2)*(2*cos(f*x+e)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*EllipticE((
(I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*((I*cos(f*x+e)-I
+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1
```

```
/2)-cos(f*x+e)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*
x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*Ell
ipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*((I*co
s(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(
f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*
2^(1/2))*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)-((I*cos(f*x+e)-I+sin(f*x+e))/
sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF
(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-I*(cos(f*x+e
)-1)/sin(f*x+e))^(1/2)+cos(f*x+e)*2^(1/2)-2^(1/2))*(b*sin(f*x+e)/cos(f*x+e)
)^(1/2)/(d/cos(f*x+e))^(1/2)/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\tan\left(fx+e\right)}}{\sqrt{d\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))/sqrt(d*sec(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{d\sec\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}}{d\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e))/(d*sec(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\tan\left(e+fx\right)}}{\sqrt{d\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(1/2)/(d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(b*tan(e + f*x))/sqrt(d*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\tan\left(fx+e\right)}}{\sqrt{d\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))/sqrt(d*sec(f*x + e)), x)
```

3.295 $\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=34

$$\frac{2(b\tan(e+fx))^{3/2}}{3bf(d\sec(e+fx))^{3/2}}$$

```
[Out] (2*(b*Tan[e + f*x])^(3/2))/(3*b*f*(d*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.0510625, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2605}

$$\frac{2(b\tan(e+fx))^{3/2}}{3bf(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*(b*Tan[e + f*x])^(3/2))/(3*b*f*(d*Sec[e + f*x])^(3/2))
```

Rule 2605

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f
*m), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx = \frac{2(b\tan(e+fx))^{3/2}}{3bf(d\sec(e+fx))^{3/2}}$$

Mathematica [A] time = 0.119699, size = 34, normalized size = 1.

$$\frac{2(b\tan(e+fx))^{3/2}}{3bf(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*(b*Tan[e + f*x])^(3/2))/(3*b*f*(d*Sec[e + f*x])^(3/2))
```

Maple [A] time = 0.18, size = 50, normalized size = 1.5

$$\frac{2\sin\left(fx+e\right)}{3f\cos\left(fx+e\right)}\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\left(\frac{d}{\cos\left(fx+e\right)}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(3/2),x)
```

```
[Out] 2/3/f*sin(f*x+e)*(b*sin(f*x+e)/cos(f*x+e))^(1/2)/cos(f*x+e)/(d/cos(f*x+e))^
(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(3/2), x)
```

Fricas [A] time = 1.7156, size = 127, normalized size = 3.74

$$\frac{2\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{d}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\sin\left(fx+e\right)}{3\,d^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)*sin
(f*x + e)/(d^2*f)
```

Sympy [A] time = 42.9155, size = 53, normalized size = 1.56

$$\begin{cases} \frac{2\sqrt{b}\tan^{\frac{3}{2}}\left(e+fx\right)}{3d^{\frac{3}{2}}f\sec^{\frac{3}{2}}\left(e+fx\right)} & \text{for } f \neq 0 \\ \frac{x\sqrt{b\tan\left(e\right)}}{\left(d\sec\left(e\right)\right)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(1/2)/(d*sec(f*x+e))**(3/2),x)
```

```
[Out] Piecewise((2*sqrt(b)*tan(e + f*x)**(3/2)/(3*d**(3/2)*f*sec(e + f*x)**(3/2))
, Ne(f, 0)), (x*sqrt(b*tan(e))/(d*sec(e))**(3/2), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(3/2), x)
```

3.296 $\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=95

$$\frac{4E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}$$

```
[Out] (4*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(5*d^2*f*Sqrt[d*S
ec[e + f*x]]*Sqrt[Sin[e + f*x]]) + (2*(b*Tan[e + f*x])^(3/2))/(5*b*f*(d*Sec
[e + f*x])^(5/2))
```

Rubi [A] time = 0.113922, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2612, 2616, 2640, 2639}

$$\frac{4E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (4*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(5*d^2*f*Sqrt[d*S
ec[e + f*x]]*Sqrt[Sin[e + f*x]]) + (2*(b*Tan[e + f*x])^(3/2))/(5*b*f*(d*Sec
[e + f*x])^(5/2))
```

Rule 2612

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*m
), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e + f
*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1] &
& EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
```

```
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx &= \frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}+\frac{2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx}{5d^2}\\
&= \frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}+\frac{\left(2\sqrt{b\tan(e+fx)}\right)\int \sqrt{b\sin(e+fx)}\,dx}{5d^2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}\\
&= \frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}+\frac{\left(2\sqrt{b\tan(e+fx)}\right)\int \sqrt{\sin(e+fx)}\,dx}{5d^2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}\\
&= \frac{4E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.670819, size = 79, normalized size = 0.83

$$-\frac{b\left(4\sqrt[4]{-\tan^2(e+fx)}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\sec^2(e+fx)\right)+\cos(2(e+fx))-1\right)}{5d^2f\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] -(b*(-1 + Cos[2*(e + f*x)] + 4*Hypergeometric2F1[-1/4, 1/4, 3/4, Sec[e + f*
x]^2]*(-Tan[e + f*x]^2)^(1/4)))/(5*d^2*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e
+ f*x]])
```

Maple [C] time = 0.242, size = 573, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(5/2),x)
```

```
[Out] 1/5/f*2^(1/2)*(2*cos(f*x+e)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*
x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+
e))^(1/2)*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1
/2))-4*cos(f*x+e)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*
x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*((I*cos(f*x+e)-I+sin(f*x+
e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)+2*((I
*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/s
in(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1
/2*2^(1/2))*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)-4*((I*cos(f*x+e)-I+sin(f*x
+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*Elli
pticE(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-I*(cos(
f*x+e)-1)/sin(f*x+e))^(1/2)-cos(f*x+e)^3*2^(1/2)-cos(f*x+e)*2^(1/2)+2*2^(1/
2))*(b*sin(f*x+e)/cos(f*x+e))^(1/2)/(d/cos(f*x+e))^(5/2)/cos(f*x+e)^2/sin(f
*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\sec\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}}{d^3\sec\left(fx+e\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e))/(d^3*sec(f*x + e)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(1/2)/(d*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(5/2), x)
```

3.297 $\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=72

$$\frac{8(b\tan(e+fx))^{3/2}}{21bd^2f(d\sec(e+fx))^{3/2}}+\frac{2(b\tan(e+fx))^{3/2}}{7bf(d\sec(e+fx))^{7/2}}$$

```
[Out] (2*(b*Tan[e + f*x])^(3/2))/(7*b*f*(d*Sec[e + f*x])^(7/2)) + (8*(b*Tan[e + f
*x])^(3/2))/(21*b*d^2*f*(d*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.103374, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2612, 2605}

$$\frac{8(b\tan(e+fx))^{3/2}}{21bd^2f(d\sec(e+fx))^{3/2}}+\frac{2(b\tan(e+fx))^{3/2}}{7bf(d\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(7/2),x]
```

```
[Out] (2*(b*Tan[e + f*x])^(3/2))/(7*b*f*(d*Sec[e + f*x])^(7/2)) + (8*(b*Tan[e + f
*x])^(3/2))/(21*b*d^2*f*(d*Sec[e + f*x])^(3/2))
```

Rule 2612

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*m
), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e + f
*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1] &
& EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2605

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f
*m), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx = \frac{2(b\tan(e+fx))^{3/2}}{7bf(d\sec(e+fx))^{7/2}} + \frac{4\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx}{7d^2}$$
$$= \frac{2(b\tan(e+fx))^{3/2}}{7bf(d\sec(e+fx))^{7/2}} + \frac{8(b\tan(e+fx))^{3/2}}{21bd^2f(d\sec(e+fx))^{3/2}}$$

Mathematica [A] time = 0.169996, size = 53, normalized size = 0.74

$$\frac{(19\sin(e+fx)+3\sin(3(e+fx)))\sqrt{b\tan(e+fx)}}{42d^3f\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(7/2),x]
```

```
[Out] ((19*Sin[e + f*x] + 3*Sin[3*(e + f*x)])*Sqrt[b*Tan[e + f*x]])/(42*d^3*f*Sqr
t[d*Sec[e + f*x]])
```

Maple [A] time = 0.191, size = 62, normalized size = 0.9

$$\frac{\left(6\left(\cos\left(fx+e\right)\right)^2+8\right)\sin\left(fx+e\right)}{21f\left(\cos\left(fx+e\right)\right)^3}\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\left(\frac{d}{\cos\left(fx+e\right)}\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(7/2),x)
```

```
[Out] 2/21/f*(3*cos(f*x+e)^2+4)*(b*sin(f*x+e)/cos(f*x+e))^(1/2)*sin(f*x+e)/(d/cos
(f*x+e))^(7/2)/cos(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(d\sec\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(7/2), x)
```

Fricas [A] time = 1.7139, size = 159, normalized size = 2.21

$$\frac{2\left(3\,\cos\left(fx+e\right)^3+4\,\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{d}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)}{21\,d^4 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*cos(f*x + e)^3 + 4*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*
sqrt(d/cos(f*x + e))*sin(f*x + e)/(d^4*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(1/2)/(d*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(d\sec\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(7/2), x)
```

3.298 $\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=132

$$\frac{4(b\tan(e+fx))^{3/2}}{15bd^2f(d\sec(e+fx))^{5/2}}+\frac{8E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{15d^4f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}$$

```
[Out] (8*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(15*d^4*f*Sqrt[d*
Sec[e + f*x]]*Sqrt[Sin[e + f*x]]) + (2*(b*Tan[e + f*x])^(3/2))/(9*b*f*(d*Se
c[e + f*x])^(9/2)) + (4*(b*Tan[e + f*x])^(3/2))/(15*b*d^2*f*(d*Sec[e + f*x]
)^(5/2))
```

Rubi [A] time = 0.167954, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2612, 2616, 2640, 2639}

$$\frac{4(b\tan(e+fx))^{3/2}}{15bd^2f(d\sec(e+fx))^{5/2}}+\frac{8E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{15d^4f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(9/2),x]
```

```
[Out] (8*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(15*d^4*f*Sqrt[d*
Sec[e + f*x]]*Sqrt[Sin[e + f*x]]) + (2*(b*Tan[e + f*x])^(3/2))/(9*b*f*(d*Se
c[e + f*x])^(9/2)) + (4*(b*Tan[e + f*x])^(3/2))/(15*b*d^2*f*(d*Sec[e + f*x]
)^(5/2))
```

Rule 2612

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*m
), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e + f
*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1] &
& EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
```

```
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx &= \frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}+\frac{2\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx}{3d^2}\\
&= \frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}+\frac{4(b\tan(e+fx))^{3/2}}{15bd^2f(d\sec(e+fx))^{5/2}}+\frac{4\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx}{15d^4}\\
&= \frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}+\frac{4(b\tan(e+fx))^{3/2}}{15bd^2f(d\sec(e+fx))^{5/2}}+\frac{\left(4\sqrt{b\tan(e+fx)}\right)\int \sqrt{b\sin(e+fx)}\,dx}{15d^4\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}\\
&= \frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}+\frac{4(b\tan(e+fx))^{3/2}}{15bd^2f(d\sec(e+fx))^{5/2}}+\frac{\left(4\sqrt{b\tan(e+fx)}\right)\int \sqrt{\sin(e+fx)}\,dx}{15d^4\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}\\
&= \frac{8E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{15d^4f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}+\frac{4(b\tan(e+fx))^{3/2}}{15bd^2f(d\sec(e+fx))^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.931741, size = 92, normalized size = 0.7

$$\frac{b\sin^2(e+fx)(5\cos(2(e+fx))+17)-24b\sqrt[4]{-\tan^2(e+fx)}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\sec^2(e+fx)\right)}{45d^4f\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(9/2),x]
```

```
[Out] (b*(17 + 5*Cos[2*(e + f*x)])*Sin[e + f*x]^2 - 24*b*Hypergeometric2F1[-1/4,
1/4, 3/4, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(1/4))/(45*d^4*f*Sqrt[d*Sec[e +
 f*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [C] time = 0.216, size = 585, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(9/2),x)
```

```
[Out] -1/45/f*2^(1/2)*(5*cos(f*x+e)^5*2^(1/2)-12*cos(f*x+e)*(-I*(cos(f*x+e)-1)/si
n(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x
+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/
sin(f*x+e))^(1/2),1/2*2^(1/2))+24*cos(f*x+e)*(-I*(cos(f*x+e)-1)/sin(f*x+e))
^(1/2)*EllipticE(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2)
)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+
e))/sin(f*x+e))^(1/2)+cos(f*x+e)^3*2^(1/2)-12*((I*cos(f*x+e)-I+sin(f*x+e))/
sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF
(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-I*(cos(f*x+e
)-1)/sin(f*x+e))^(1/2)+24*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-
(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)-I+si
n(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/
2)+6*cos(f*x+e)*2^(1/2)-12*2^(1/2))*(b*sin(f*x+e)/cos(f*x+e))^(1/2)/(d/cos(
f*x+e))^(9/2)/cos(f*x+e)^4/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(d\sec\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\sec\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}}{d^5\sec\left(fx+e\right)^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e))/(d^5*sec(f*x + e)^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(1/2)/(d*sec(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{b\tan\left(fx+e\right)}}{\left(d\sec\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(9/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(9/2), x)
```

3.299 $\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=131

$$-\frac{b^2d^2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\sec(e+fx)}}{6f\sqrt{b\tan(e+fx)}}-\frac{bd^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{6f}+\frac{b\sqrt{b\tan(e+fx)}(d\sec(e}{3f}$$

```
[Out] -(b^2*d^2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e
+ f*x]])/(6*f*Sqrt[b*Tan[e + f*x]]) - (b*d^2*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Ta
n[e + f*x]])/(6*f) + (b*(d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]])/(3*f)
```

Rubi [A] time = 0.17649, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2611, 2613, 2616, 2642, 2641}

$$-\frac{b^2d^2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\sec(e+fx)}}{6f\sqrt{b\tan(e+fx)}}-\frac{bd^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{6f}+\frac{b\sqrt{b\tan(e+fx)}(d\sec(e}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2),x]
```


```
[Out] -(b^2*d^2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e
+ f*x]])/(6*f*Sqrt[b*Tan[e + f*x]]) - (b*d^2*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Ta
n[e + f*x]])/(6*f) + (b*(d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]])/(3*f)
```


Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
```

```
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \frac{b(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{3f} - \frac{1}{6}b^2\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx$$

$$= -\frac{bd^2\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{6f} + \frac{b(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{3f} - \cdots$$

$$= -\frac{bd^2\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{6f} + \frac{b(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{3f} - \cdots$$

$$= -\frac{bd^2\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{6f} + \frac{b(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{3f} - \cdots$$

$$= -\frac{b^2d^2F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{6f\sqrt{b\tan(e+fx)}} - \frac{bd^2\sqrt{d\sec(e+f\cdots}}{6\cdots}$$

Mathematica [C] time = 0.769971, size = 95, normalized size = 0.73

$$\frac{bd^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}\left({}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};\sec^2(e+fx)\right)+\sqrt[4]{-\tan^2(e+fx)}\left(2\sec^2(e+fx)-1\right)\right)}{6f\sqrt[4]{-\tan^2(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (b*d^2*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]]*(Hypergeometric2F1[1/4, 3/
4, 5/4, Sec[e + f*x]^2] + (-1 + 2*Sec[e + f*x]^2)*(-Tan[e + f*x]^2)^(1/4)))
/(6*f*(-Tan[e + f*x]^2)^(1/4))
```

Maple [C] time = 0.249, size = 239, normalized size = 1.8

$$\frac{\cos\left(fx+e\right)\sqrt{2}}{12\,f\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\left(\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}\left(\frac{d}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}\left(i\left(\cos\left(fx+e\right)\right)^3\sin\left(fx+e\right)\sqrt{\frac{-i\left(\cos\left(fx+e\right)\right.}{\sin\left(fx+e\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x)
```

```
[Out] 1/12/f*2^(1/2)*(b*sin(f*x+e)/cos(f*x+e))^(3/2)*(d/cos(f*x+e))^(5/2)*cos(f*x
+e)*(I*cos(f*x+e)^3*sin(f*x+e)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos
(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f
*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2
^(1/2))-cos(f*x+e)^3*2^(1/2)+cos(f*x+e)^2*2^(1/2)+2*cos(f*x+e)*2^(1/2)-2*2^
(1/2))/(cos(f*x+e)-1)/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

[Out] integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\sec\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}bd^2\sec\left(fx+e\right)^2\tan\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e))*b*d^2*sec(f*x + e)^2*tan(f*x + e), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sec(f*x+e))**(5/2)*(b*tan(f*x+e))**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="giac")

[Out] integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2), x)

3.300 $\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=169

$$-\frac{b^{3/2}d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{4f\sqrt{b\tan(e+fx)}}-\frac{b^{3/2}d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{4f\sqrt{b\tan(e+fx)}}+\frac{b\sqrt{}}{}$$

```
[Out] -(b^(3/2)*d*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[d*Sec[e + f*x]]*Sqrt[
b*Sin[e + f*x]])/(4*f*Sqrt[b*Tan[e + f*x]]) - (b^(3/2)*d*ArcTanh[Sqrt[b*Sin
[e + f*x]]/Sqrt[b]]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])/(4*f*Sqrt[b*
Tan[e + f*x]]) + (b*(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]])/(2*f)
```

Rubi [A] time = 0.173737, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2611, 2616, 2564, 329, 212, 206, 203}

$$-\frac{b^{3/2}d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{4f\sqrt{b\tan(e+fx)}}-\frac{b^{3/2}d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{4f\sqrt{b\tan(e+fx)}}+\frac{b\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] -(b^(3/2)*d*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[d*Sec[e + f*x]]*Sqrt[
b*Sin[e + f*x]])/(4*f*Sqrt[b*Tan[e + f*x]]) - (b^(3/2)*d*ArcTanh[Sqrt[b*Sin
[e + f*x]]/Sqrt[b]]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])/(4*f*Sqrt[b*
Tan[e + f*x]]) + (b*(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]])/(2*f)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
```

```
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \frac{b(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2f} - \frac{1}{4}b^2\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx$$

$$= \frac{b(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2f} - \frac{\left(b^2 d\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\int \frac{\text{se}}{\sqrt{b}}}{4\sqrt{b\tan(e+fx)}}$$

$$= \frac{b(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2f} - \frac{\left(bd\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\text{Subst}}{4f\sqrt{b\tan(e}}$$

$$= \frac{b(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2f} - \frac{\left(bd\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\text{Subst}}{2f\sqrt{b\tan(e+}}$$

$$= \frac{b(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2f} - \frac{\left(b^2 d\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\text{Subs}}{4f\sqrt{b\tan(e+}}$$

$$= -\frac{b^{3/2}d\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{4f\sqrt{b\tan(e+fx)}} - \frac{b^{3/2}d\tanh^{-1}\left(\frac{\sqrt{b\text{si}}}{}\right.}{}$$

Mathematica [A] time = 6.29854, size = 129, normalized size = 0.76

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}\left(2\sqrt[4]{\tan^2(e+fx)}\sec^{\frac{3}{2}}(e+fx)+\tan^{-1}\left(\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)-\tanh^{-1}\left(\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)\right)}{4f\sqrt[4]{\tan^2(e+fx)}\sec^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (b*(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]*(ArcTan[Sqrt[Sec[e + f*x]]/(
Tan[e + f*x]^2)^(1/4)] - ArcTanh[Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^(1/4)]
 + 2*Sec[e + f*x]^(3/2)*(Tan[e + f*x]^2)^(1/4)))/(4*f*Sec[e + f*x]^(3/2)*(T
an[e + f*x]^2)^(1/4))
```

Maple [C] time = 0.183, size = 759, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x)
```

```
[Out] 1/8/f*2^(1/2)*(I*cos(f*x+e)^2*sin(f*x+e)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f
*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e
)-1)/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^
(1/2),1/2+1/2*I,1/2*2^(1/2))+I*cos(f*x+e)^2*sin(f*x+e)*((I*cos(f*x+e)-I+sin
(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*
(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e)
)/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-2*I*cos(f*x+e)^2*sin(f*x+e)*((I*
cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/si
n(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+
e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+cos(f*x+e)^2*sin(f*x+e)*((I
*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/s
in(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*
x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-cos(f*x+e)^2*si
n(f*x+e)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-s
in(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*EllipticP
i(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))+2*c
os(f*x+e)*2^(1/2)-2*2^(1/2))*cos(f*x+e)*(d/cos(f*x+e))^(3/2)*(b*sin(f*x+e)/
cos(f*x+e))^(3/2)/(cos(f*x+e)-1)/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d \sec \left(fx+e\right)\right)^{\frac{3}{2}} \left(b \tan \left(fx+e\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2), x)
```

Fricas [B] time = 2.99851, size = 2007, normalized size = 11.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(2*sqrt(-b*d)*b*d*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (co
s(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(
-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(f*x +
 e)^2 - b*d - (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e) + sqrt(-
b*d)*b*d*cos(f*x + e)*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2 - 8*(
7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f
*x + e))*sqrt(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))
+ 72*b*d + 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x + e)^4 -
8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) + 16*b*d*sqrt(
b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/(f*cos(f*x + e)), -1/32*
(2*sqrt(b*d)*b*d*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 + (cos(f*x +
 e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(b*d)*sq
rt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(f*x + e)^2 -
b*d + (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e) - sqrt(b*d)*b*d*
cos(f*x + e)*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2 + 8*(7*cos(f*x
 + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*
sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)) + 72*b*d -
 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x + e)^4 - 8*cos(f*x
+ e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) - 16*b*d*sqrt(b*sin(f*x
+ e)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/(f*cos(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(3/2)*(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x, algorithm="giac")

[Out] integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2), x)
```

3.301 $\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=88

$$\frac{b\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{f}-\frac{b^2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\sec(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

```
[Out] -((b^2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f
*x]])/(f*Sqrt[b*Tan[e + f*x]])) + (b*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*
x]])/f
```

Rubi [A] time = 0.115462, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2611, 2616, 2642, 2641}

$$\frac{b\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{f}-\frac{b^2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\sec(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] -((b^2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f
*x]])/(f*Sqrt[b*Tan[e + f*x]])) + (b*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*
x]])/f
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
```

```
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}\,dx = \frac{b\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{f} - \frac{1}{2}b^2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx$$

$$= \frac{b\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{f} - \frac{\left(b^2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\int \frac{1}{\sqrt{b\sin(e}}}{2\sqrt{b\tan(e+fx)}}$$

$$= \frac{b\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{f} - \frac{\left(b^2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+f}}}{2\sqrt{b\tan(e+fx)}}$$

$$= -\frac{b^2F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}} + \frac{b\sqrt{d\sec(e+fx)}\sqrt{b\,t}}{f}$$

Mathematica [C] time = 0.73783, size = 105, normalized size = 1.19

$$\frac{b\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}\left(\sec^{\frac{3}{2}}(e+fx)-\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}{\sqrt{2}}\right)}{f\sec^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (b*Sqrt[d*Sec[e + f*x]]*(Sec[e + f*x]^(3/2) - (Hypergeometric2F1[1/4, 1/2,
5/4, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Sqrt[1 + Sec[e + f*x]])/Sqrt[2
```

```
])*Sqrt[b*Tan[e + f*x]])/(f*Sec[e + f*x]^(3/2))
```

Maple [C] time = 0.226, size = 211, normalized size = 2.4

$$\frac{\cos\left(fx+e\right)\sqrt{2}}{2f\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\left(\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}\sqrt{\frac{d}{\cos\left(fx+e\right)}}\left(i\cos\left(fx+e\right)\sin\left(fx+e\right)\sqrt{\frac{i\cos\left(fx+e\right)-i+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x)
```

```
[Out] 1/2/f*2^(1/2)*(b*sin(f*x+e)/cos(f*x+e))^(3/2)*(d/cos(f*x+e))^(1/2)*cos(f*x+
e)*(I*cos(f*x+e)*sin(f*x+e)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*
(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)-I+
sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(
1/2)+cos(f*x+e)*2^(1/2)-2^(1/2))/(cos(f*x+e)-1)/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{d\sec\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}b\tan\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e))*b*tan(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/2)*(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \sec(fx+e)} (b \tan(fx+e))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(3/2), x)
```

3.302 $\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx$

Optimal. Leaf size=167

$$\frac{b^{3/2}d(b\tan(e+fx))^{3/2}\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{f(b\sin(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}+\frac{b^{3/2}d(b\tan(e+fx))^{3/2}\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{f(b\sin(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}-\frac{2d\csc(e+fx)(b\tan(e+fx))^{3/2}}{f(d\sec(e+fx))^{3/2}}$$

```
[Out] (-2*d*Csc[e + f*x]*(b*Tan[e + f*x])^(3/2))/(f*(d*Sec[e + f*x])^(3/2)) + (b^
(3/2)*d*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*(b*Tan[e + f*x])^(3/2))/(f*(d*
Sec[e + f*x])^(3/2)*(b*Sin[e + f*x])^(3/2)) + (b^(3/2)*d*ArcTanh[Sqrt[b*Sin
[e + f*x]]/Sqrt[b]]*(b*Tan[e + f*x])^(3/2))/(f*(d*Sec[e + f*x])^(3/2)*(b*Si
n[e + f*x])^(3/2))
```

Rubi [A] time = 0.125362, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2616, 2564, 321, 329, 212, 206, 203}

$$\frac{b^{3/2}d(b\tan(e+fx))^{3/2}\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{f(b\sin(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}+\frac{b^{3/2}d(b\tan(e+fx))^{3/2}\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{f(b\sin(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}-\frac{2d\csc(e+fx)(b\tan(e+fx))^{3/2}}{f(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x])^(3/2)/Sqrt[d*Sec[e + f*x]],x]
```


```
[Out] (-2*d*Csc[e + f*x]*(b*Tan[e + f*x])^(3/2))/(f*(d*Sec[e + f*x])^(3/2)) + (b^
(3/2)*d*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*(b*Tan[e + f*x])^(3/2))/(f*(d*
Sec[e + f*x])^(3/2)*(b*Sin[e + f*x])^(3/2)) + (b^(3/2)*d*ArcTanh[Sqrt[b*Sin
[e + f*x]]/Sqrt[b]]*(b*Tan[e + f*x])^(3/2))/(f*(d*Sec[e + f*x])^(3/2)*(b*Si
n[e + f*x])^(3/2))
```


Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx = \frac{\left(d(b\tan(e+fx))^{3/2}\right)\int \sec(e+fx)(b\sin(e+fx))^{3/2}\,dx}{(d\sec(e+fx))^{3/2}(b\sin(e+fx))^{3/2}}$$

$$= \frac{\left(d(b\tan(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{x^{3/2}}{1-\frac{x^2}{b^2}}\,dx, x, b\sin(e+fx)\right)}{bf(d\sec(e+fx))^{3/2}(b\sin(e+fx))^{3/2}}$$

$$= -\frac{2d\csc(e+fx)(b\tan(e+fx))^{3/2}}{f(d\sec(e+fx))^{3/2}} + \frac{\left(bd(b\tan(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{b^2}\right)}\,dx, x, b\sin(e+$$

$$= -\frac{2d\csc(e+fx)(b\tan(e+fx))^{3/2}}{f(d\sec(e+fx))^{3/2}} + \frac{\left(2bd(b\tan(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sin(e+f}$$

$$= -\frac{2d\csc(e+fx)(b\tan(e+fx))^{3/2}}{f(d\sec(e+fx))^{3/2}} + \frac{\left(b^2d(b\tan(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sin(e+f}$$

$$= -\frac{2d\csc(e+fx)(b\tan(e+fx))^{3/2}}{f(d\sec(e+fx))^{3/2}} + \frac{b^{3/2}d\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)(b\tan(e+fx))^{3/2}}{f(d\sec(e+fx))^{3/2}(b\sin(e+fx))^{3/2}} + \frac{b^{3/2}d\tanh^{-}}{f(d\sec}$$

Mathematica [C] time = 4.1733, size = 64, normalized size = 0.38

$$\frac{2(b\tan(e+fx))^{5/2}\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};\sec^2(e+fx)\right)}{bf\left(-\tan^2(e+fx)\right)^{5/4}\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x])^(3/2)/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (2*Hypergeometric2F1[-1/4, -1/4, 3/4, Sec[e + f*x]^2]*(b*Tan[e + f*x])^(5/2
))/(b*f*Sqrt[d*Sec[e + f*x]]*(-Tan[e + f*x]^2)^(5/4))
```

Maple [C] time = 0.227, size = 719, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*2^(1/2)*(2*I*sin(f*x+e)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2
)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)-
I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-I*(cos(f*x+e)-1)/sin(f*x+e))
^(1/2)-I*sin(f*x+e)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos
(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x
+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-I*(cos(f*x+e)-1)/sin(f*x+e)
)^(1/2)-I*sin(f*x+e)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*co
s(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*
x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-I*(cos(f*x+e)-1)/sin(f*x+e
))^(1/2)+sin(f*x+e)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos
(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x
+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-I*(cos(f*x+e)-1)/sin(f*x+e)
)^(1/2)-sin(f*x+e)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(
f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+
e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-I*(cos(f*x+e)-1)/sin(f*x+e))
^(1/2)-2*cos(f*x+e)*2^(1/2)+2*2^(1/2))*(b*sin(f*x+e)/cos(f*x+e))^(3/2)*cos(
f*x+e)/(cos(f*x+e)-1)/(d/cos(f*x+e))^(1/2)/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{d\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^(3/2)/sqrt(d*sec(f*x + e)), x)
```

Fricas [B] time = 5.35585, size = 1886, normalized size = 11.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(2*b*d*sqrt(-b/d)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (co
s(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(
b*sin(f*x + e)/cos(f*x + e))*sqrt(-b/d)*sqrt(d/cos(f*x + e))/(b*cos(f*x + e
)^2 - (b*cos(f*x + e) + b)*sin(f*x + e) - b)) - b*d*sqrt(-b/d)*log((b*cos(f
*x + e)^4 - 72*b*cos(f*x + e)^2 + 8*(7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8
*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x +
 e))*sqrt(-b/d)*sqrt(d/cos(f*x + e)) + 28*(b*cos(f*x + e)^2 - 2*b)*sin(f*x
+ e) + 72*b)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*si
n(f*x + e) + 8)) + 16*b*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x +
e))*cos(f*x + e))/(d*f), 1/8*(2*b*d*sqrt(b/d)*arctan(1/4*(cos(f*x + e)^3 -
5*cos(f*x + e)^2 + (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*c
os(f*x + e) + 4)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(b/d)*sqrt(d/cos(f*x
 + e))/(b*cos(f*x + e)^2 + (b*cos(f*x + e) + b)*sin(f*x + e) - b)) + b*d*sq
rt(b/d)*log((b*cos(f*x + e)^4 - 72*b*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3 +
 (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*si
n(f*x + e)/cos(f*x + e))*sqrt(b/d)*sqrt(d/cos(f*x + e)) - 28*(b*cos(f*x + e
)^2 - 2*b)*sin(f*x + e) + 72*b)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 + 4*(cos
(f*x + e)^2 - 2)*sin(f*x + e) + 8)) - 16*b*sqrt(b*sin(f*x + e)/cos(f*x + e)
)*sqrt(d/cos(f*x + e))*cos(f*x + e))/(d*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(3/2)/(d*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{d\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^(3/2)/sqrt(d*sec(f*x + e)), x)
```

3.303 $\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=96

$$\frac{2b^2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\sec(e+fx)}}{3d^2f\sqrt{b\tan(e+fx)}}-\frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}$$

```
[Out] (2*b^2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f
*x]])/(3*d^2*f*Sqrt[b*Tan[e + f*x]]) - (2*b*Sqrt[b*Tan[e + f*x]])/(3*f*(d*S
ec[e + f*x])^(3/2))
```

Rubi [A] time = 0.12114, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {2610, 2616, 2642, 2641}

$$\frac{2b^2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\sec(e+fx)}}{3d^2f\sqrt{b\tan(e+fx)}}-\frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*b^2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f
*x]])/(3*d^2*f*Sqrt[b*Tan[e + f*x]]) - (2*b*Sqrt[b*Tan[e + f*x]])/(3*f*(d*S
ec[e + f*x])^(3/2))
```

Rule 2610

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m)
, x] - Dist[(b^2*(n - 1))/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1]
 || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
```

```
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx &= -\frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}+\frac{b^2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx}{3d^2}\\
&= -\frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}+\frac{\left(b^2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\int \frac{1}{\sqrt{b\sin(e+fx)}}\,dx}{3d^2\sqrt{b\tan(e+fx)}}\\
&= -\frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}+\frac{\left(b^2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx}{3d^2\sqrt{b\tan(e+fx)}}\\
&= \frac{2b^2F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{3d^2f\sqrt{b\tan(e+fx)}}-\frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.595159, size = 98, normalized size = 1.02

$$-\frac{2b\sqrt{b\tan(e+fx)}\left(\sqrt{\sec(e+fx)+1}-\sqrt{2}\sec^{\frac{3}{2}}(e+fx)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)}{3f\sqrt{\sec(e+fx)+1}(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*b*(-(Sqrt[2]*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[(e + f*x)/2]^2]*Sec[
e + f*x]^(3/2)) + Sqrt[1 + Sec[e + f*x]])*Sqrt[b*Tan[e + f*x]])/(3*f*(d*Sec
[e + f*x])^(3/2)*Sqrt[1 + Sec[e + f*x]])
```

Maple [C] time = 0.202, size = 207, normalized size = 2.2

$$-\frac{\sqrt{2}}{3 f \left(\cos \left(f x+e\right)-1\right) \sin \left(f x+e\right)}\left(i \sin \left(f x+e\right) \sqrt{\frac{-i \left(\cos \left(f x+e\right)-1\right)}{\sin \left(f x+e\right)}}\, \sqrt{\frac{i \cos \left(f x+e\right)-i+\sin \left(f x+e\right)}{\sin \left(f x+e\right)}}\, \sqrt{-\frac{i \cos \left(f\right.}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/3/f*2^(1/2)*(I*sin(f*x+e)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f
*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x
+e))^(1/2)*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(
1/2))+cos(f*x+e)^2*2^(1/2)-cos(f*x+e)*2^(1/2))*(b*sin(f*x+e)/cos(f*x+e))^(3
/2)/(cos(f*x+e)-1)/(d/cos(f*x+e))^(3/2)/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \tan \left(f x+e\right)\right)^{\frac{3}{2}}}{\left(d \sec \left(f x+e\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{d \sec \left(f x+e\right)} \sqrt{b \tan \left(f x+e\right)} b \tan \left(f x+e\right)}{d^{2} \sec \left(f x+e\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e))*b*tan(f*x + e)/(d^2*sec(
f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(3/2)/(d*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(3/2), x)
```

3.304 $\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=34

$$\frac{2(b\tan(e+fx))^{5/2}}{5bf(d\sec(e+fx))^{5/2}}$$

```
[Out] (2*(b*Tan[e + f*x])^(5/2))/(5*b*f*(d*Sec[e + f*x])^(5/2))
```

Rubi [A] time = 0.0570212, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2605}

$$\frac{2(b\tan(e+fx))^{5/2}}{5bf(d\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*(b*Tan[e + f*x])^(5/2))/(5*b*f*(d*Sec[e + f*x])^(5/2))
```

Rule 2605

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f
*m), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}}\,dx = \frac{2(b\tan(e+fx))^{5/2}}{5bf(d\sec(e+fx))^{5/2}}$$

Mathematica [B] time = 1.33815, size = 141, normalized size = 4.15

$$-\frac{b\sec^{\frac{3}{2}}(e+fx)\sqrt{b\tan(e+fx)}\left(-\sqrt{\sec(e+fx)+1}\sec^2\left(\frac{1}{2}(e+fx)\right)+\sqrt{\frac{1}{\cos(e+fx)+1}}\cos(3(e+fx))\sec^{\frac{3}{2}}(e+fx)+\sqrt{\frac{}{\cos}}\right.}{10f\sqrt{\frac{1}{\cos(e+fx)+1}}(d\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] -(b*Sec[e + f*x]^(3/2)*(Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[Sec[e + f*x]] +
Sqrt[(1 + Cos[e + f*x])^(-1)]*Cos[3*(e + f*x)]*Sec[e + f*x]^(3/2) - Sec[(e
+ f*x)/2]^2*Sqrt[1 + Sec[e + f*x]])*Sqrt[b*Tan[e + f*x]])/(10*f*Sqrt[(1 + C
os[e + f*x])^(-1)]*(d*Sec[e + f*x])^(5/2))
```

Maple [A] time = 0.151, size = 50, normalized size = 1.5

$$\frac{2\sin\left(fx+e\right)}{5f\cos\left(fx+e\right)}\left(\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}\left(\frac{d}{\cos\left(fx+e\right)}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(5/2),x)
```

```
[Out] 2/5/f*(b*sin(f*x+e)/cos(f*x+e))^(3/2)*sin(f*x+e)/cos(f*x+e)/(d/cos(f*x+e))^
(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(5/2), x)
```

Fricas [B] time = 2.01878, size = 142, normalized size = 4.18

$$-\frac{2\left(b\cos\left(fx+e\right)^{3}-b\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{d}{\cos\left(fx+e\right)}}}{5\,d^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/5*(b*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*
sqrt(d/cos(f*x + e))/(d^3*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(3/2)/(d*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(5/2), x)
```

3.305 $\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=131

$$\frac{4b^2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\sec(e+fx)}}{21d^4f\sqrt{b\tan(e+fx)}}+\frac{2b\sqrt{b\tan(e+fx)}}{21d^2f(d\sec(e+fx))^{3/2}}-\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}}$$

```
[Out] (4*b^2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f
*x]])/(21*d^4*f*Sqrt[b*Tan[e + f*x]]) - (2*b*Sqrt[b*Tan[e + f*x]])/(7*f*(d*
Sec[e + f*x])^(7/2)) + (2*b*Sqrt[b*Tan[e + f*x]])/(21*d^2*f*(d*Sec[e + f*x]
)^(3/2))
```

Rubi [A] time = 0.179561, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2610, 2612, 2616, 2642, 2641}

$$\frac{4b^2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\sec(e+fx)}}{21d^4f\sqrt{b\tan(e+fx)}}+\frac{2b\sqrt{b\tan(e+fx)}}{21d^2f(d\sec(e+fx))^{3/2}}-\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(7/2),x]
```


```
[Out] (4*b^2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f
*x]])/(21*d^4*f*Sqrt[b*Tan[e + f*x]]) - (2*b*Sqrt[b*Tan[e + f*x]])/(7*f*(d*
Sec[e + f*x])^(7/2)) + (2*b*Sqrt[b*Tan[e + f*x]])/(21*d^2*f*(d*Sec[e + f*x]
)^(3/2))
```


Rule 2610

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m)
, x] - Dist[(b^2*(n - 1))/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1]
 || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

Rule 2612

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*m
```

```
), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e + f
*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1] &
& EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx = -\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}} + \frac{b^2\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx}{7d^2}$$

$$= -\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}} + \frac{2b\sqrt{b\tan(e+fx)}}{21d^2f(d\sec(e+fx))^{3/2}} + \frac{\left(2b^2\right)\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx}{21d^4}$$

$$= -\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}} + \frac{2b\sqrt{b\tan(e+fx)}}{21d^2f(d\sec(e+fx))^{3/2}} + \frac{\left(2b^2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\int \frac{}{\sqrt{b\sin}}}{21d^4\sqrt{b\tan(e+fx)}}$$

$$= -\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}} + \frac{2b\sqrt{b\tan(e+fx)}}{21d^2f(d\sec(e+fx))^{3/2}} + \frac{\left(2b^2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+}}}{21d^4\sqrt{b\tan(e+fx)}}$$

$$= \frac{4b^2F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{21d^4f\sqrt{b\tan(e+fx)}} - \frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}} + \frac{2b\sqrt{b\tan(e}}{21d^2f(d\sec(e}$$

Mathematica [C] time = 1.26467, size = 105, normalized size = 0.8

$$-\frac{b\sqrt{b\tan(e+fx)}\left(4\sec^2(e+fx)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};\sec^2(e+fx)\right)+(3\cos(2(e+fx))+1)\sqrt[4]{-\tan^2(e+fx)}\right)}{21d^2f\sqrt[4]{-\tan^2(e+fx)}(d\sec(e+fx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(7/2),x]
```

```
[Out] -(b*Sqrt[b*Tan[e + f*x]]*(4*Hypergeometric2F1[1/4, 3/4, 5/4, Sec[e + f*x]^2
]*Sec[e + f*x]^2 + (1 + 3*Cos[2*(e + f*x)])*(-Tan[e + f*x]^2)^(1/4)))/(21*d
^2*f*(d*Sec[e + f*x])^(3/2)*(-Tan[e + f*x]^2)^(1/4))
```

Maple [C] time = 0.233, size = 241, normalized size = 1.8

$$-\frac{\sqrt{2}}{21\,f\left(\cos\left(fx+e\right)-1\right)\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)}\left(2\,i\sin\left(fx+e\right)\sqrt{\frac{i\cos\left(fx+e\right)-i+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{-\frac{i\cos\left(fx+\right.}{\mathrm{s}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(7/2),x)
```

```
[Out] -1/21/f*2^(1/2)*(2*I*sin(f*x+e)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1
/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e
)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-I*(cos(f*x+e)-1)/sin(f*x+e
))^(1/2)+3*cos(f*x+e)^4*2^(1/2)-3*cos(f*x+e)^3*2^(1/2)-cos(f*x+e)^2*2^(1/2)
+cos(f*x+e)*2^(1/2))*(b*sin(f*x+e)/cos(f*x+e))^(3/2)/(cos(f*x+e)-1)/cos(f*x
+e)^2/sin(f*x+e)/(d/cos(f*x+e))^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(d\sec\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\sec\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}b\tan\left(fx+e\right)}{d^4\sec\left(fx+e\right)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e))*b*tan(f*x + e)/(d^4*sec(
f*x + e)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(3/2)/(d*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(d\sec\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(7/2), x)
```

3.306 $\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=103

$$\frac{8b\sqrt{b\tan(e+fx)}}{45d^4f\sqrt{d\sec(e+fx)}}+\frac{2b\sqrt{b\tan(e+fx)}}{45d^2f(d\sec(e+fx))^{5/2}}-\frac{2b\sqrt{b\tan(e+fx)}}{9f(d\sec(e+fx))^{9/2}}$$

```
[Out] (-2*b*Sqrt[b*Tan[e + f*x]])/(9*f*(d*Sec[e + f*x])^(9/2)) + (2*b*Sqrt[b*Tan[
e + f*x]])/(45*d^2*f*(d*Sec[e + f*x])^(5/2)) + (8*b*Sqrt[b*Tan[e + f*x]])/(
45*d^4*f*Sqrt[d*Sec[e + f*x]])
```

Rubi [A] time = 0.162904, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2610, 2612, 2605}

$$\frac{8b\sqrt{b\tan(e+fx)}}{45d^4f\sqrt{d\sec(e+fx)}}+\frac{2b\sqrt{b\tan(e+fx)}}{45d^2f(d\sec(e+fx))^{5/2}}-\frac{2b\sqrt{b\tan(e+fx)}}{9f(d\sec(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(9/2),x]
```


```
[Out] (-2*b*Sqrt[b*Tan[e + f*x]])/(9*f*(d*Sec[e + f*x])^(9/2)) + (2*b*Sqrt[b*Tan[
e + f*x]])/(45*d^2*f*(d*Sec[e + f*x])^(5/2)) + (8*b*Sqrt[b*Tan[e + f*x]])/(
45*d^4*f*Sqrt[d*Sec[e + f*x]])
```


Rule 2610

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m)
, x] - Dist[(b^2*(n - 1))/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1]
 || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

Rule 2612

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*m
), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e + f
*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1] &
```

```
& EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2605

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f
*m), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx &= -\frac{2b\sqrt{b\tan(e+fx)}}{9f(d\sec(e+fx))^{9/2}}+\frac{b^2\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx}{9d^2}\\ &= -\frac{2b\sqrt{b\tan(e+fx)}}{9f(d\sec(e+fx))^{9/2}}+\frac{2b\sqrt{b\tan(e+fx)}}{45d^2f(d\sec(e+fx))^{5/2}}+\frac{\left(4b^2\right)\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx}{45d^4}\\ &= -\frac{2b\sqrt{b\tan(e+fx)}}{9f(d\sec(e+fx))^{9/2}}+\frac{2b\sqrt{b\tan(e+fx)}}{45d^2f(d\sec(e+fx))^{5/2}}+\frac{8b\sqrt{b\tan(e+fx)}}{45d^4f\sqrt{d\sec(e+fx)}}\end{aligned}$$

Mathematica [A] time = 3.3127, size = 158, normalized size = 1.53

$$-\frac{b\sqrt{\sec(e+fx)}\sqrt{b\tan(e+fx)}\left(-21\sqrt{\sec(e+fx)+1}\sec^2\left(\frac{1}{2}(e+fx)\right)+\sqrt{\frac{1}{\cos(e+fx)+1}}(21\cos(3(e+fx))+5\cos(5(e+\right.}{360d^3f\sqrt{\frac{1}{\cos(e+fx)+1}}(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(9/2),x]
```

```
[Out] -(b*Sqrt[Sec[e + f*x]]*(16*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[Sec[e + f*x]]
 + Sqrt[(1 + Cos[e + f*x])^(-1)]*(21*Cos[3*(e + f*x)] + 5*Cos[5*(e + f*x)])
*Sec[e + f*x]^(3/2) - 21*Sec[(e + f*x)/2]^2*Sqrt[1 + Sec[e + f*x]])*Sqrt[b*
Tan[e + f*x]])/(360*d^3*f*Sqrt[(1 + Cos[e + f*x])^(-1)]*(d*Sec[e + f*x])^(3
/2))
```

Maple [A] time = 0.162, size = 62, normalized size = 0.6

$$\frac{\left(10 \left(\cos \left(fx+e\right)\right)^{2}+8\right) \sin \left(fx+e\right)}{45 f \left(\cos \left(fx+e\right)\right)^{3}} \left(\frac{b \sin \left(fx+e\right)}{\cos \left(fx+e\right)}\right)^{\frac{3}{2}} \left(\frac{d}{\cos \left(fx+e\right)}\right)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(9/2),x)
```

```
[Out] 2/45/f*(5*cos(f*x+e)^2+4)*(b*sin(f*x+e)/cos(f*x+e))^(3/2)*sin(f*x+e)/cos(f*
x+e)^3/(d/cos(f*x+e))^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \tan \left(fx+e\right)\right)^{\frac{3}{2}}}{\left(d \sec \left(fx+e\right)\right)^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(9/2), x)
```

Fricas [A] time = 1.81583, size = 174, normalized size = 1.69

$$-\frac{2 \left(5 b \cos \left(fx+e\right)^{5}-b \cos \left(fx+e\right)^{3}-4 b \cos \left(fx+e\right)\right) \sqrt{\frac{b \sin \left(fx+e\right)}{\cos \left(fx+e\right)}} \sqrt{\frac{d}{\cos \left(fx+e\right)}}}{45 d^{5} f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] -2/45*(5*b*cos(f*x + e)^5 - b*cos(f*x + e)^3 - 4*b*cos(f*x + e))*sqrt(b*sin
(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(d^5*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(3/2)/(d*sec(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(d\sec\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(9/2), x)
```

3.307 $\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=208

$$\frac{3b^{5/2}d^3\sqrt{b\tan(e+fx)}\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{32f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{3b^{5/2}d^3\sqrt{b\tan(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{32f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{3bd^2(b\tan(e+fx))^{3/2}\sqrt{d\sec}}{16f}$$

```
[Out] (3*b^(5/2)*d^3*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(
32*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]]) - (3*b^(5/2)*d^3*ArcTanh[Sq
rt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(32*f*Sqrt[d*Sec[e + f*x]
]*Sqrt[b*Sin[e + f*x]]) - (3*b*d^2*Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3
/2))/(16*f) + (b*(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2))/(4*f)
```

Rubi [A] time = 0.228034, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {2611, 2613, 2616, 2564, 329, 298, 203, 206}

$$\frac{3b^{5/2}d^3\sqrt{b\tan(e+fx)}\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{32f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{3b^{5/2}d^3\sqrt{b\tan(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{32f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{3bd^2(b\tan(e+fx))^{3/2}\sqrt{d\sec}}{16f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(5/2),x]
```


```
[Out] (3*b^(5/2)*d^3*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(
32*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]]) - (3*b^(5/2)*d^3*ArcTanh[Sq
rt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(32*f*Sqrt[d*Sec[e + f*x]
]*Sqrt[b*Sin[e + f*x]]) - (3*b*d^2*Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3
/2))/(16*f) + (b*(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2))/(4*f)
```


Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx = \frac{b(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}{4f} - \frac{1}{8}\left(3b^2\right)\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+$$

$$= -\frac{3bd^2\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{16f} + \frac{b(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}{4f}$$

$$= -\frac{3bd^2\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{16f} + \frac{b(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}{4f}$$

$$= -\frac{3bd^2\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{16f} + \frac{b(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}{4f}$$

$$= -\frac{3bd^2\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{16f} + \frac{b(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}{4f}$$

$$= -\frac{3bd^2\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{16f} + \frac{b(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}{4f}$$

$$= \frac{3b^{5/2}d^3\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{32f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} - \frac{3b^{5/2}d^3\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b}}{32f\sqrt{d\sec(e+fx)}\sqrt{b\sin(}}$$

Mathematica [A] time = 3.14155, size = 189, normalized size = 0.91

$$\frac{b^3(d\sec(e+fx))^{5/2}\left(16\sec^{\frac{9}{2}}(e+fx) - 28\sec^{\frac{5}{2}}(e+fx) + 12\sqrt{\sec(e+fx)} - 6\sqrt[4]{\tan^2(e+fx)}\tan^{-1}\left(\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right) + 3\sqrt[4]{\mathrm{ta}}\right.}{64f\sec^{\frac{5}{2}}(e+fx)\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (b^3*(d*Sec[e + f*x])^(5/2)*(12*Sqrt[Sec[e + f*x]] - 28*Sec[e + f*x]^(5/2)
+ 16*Sec[e + f*x]^(9/2) - 6*ArcTan[Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^(1/4
)]*(Tan[e + f*x]^2)^(1/4) + 3*(Log[1 - Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^
(1/4)] - Log[1 + Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^(1/4)])*(Tan[e + f*x]^
```

```
2)^(1/4)))/(64*f*Sec[e + f*x]^(5/2)*Sqrt[b*Tan[e + f*x]])
```

Maple [C] time = 0.229, size = 628, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(5/2),x)
```

```
[Out] 1/64/f*2^(1/2)*(3*I*cos(f*x+e)^4*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(
1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin
(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/
2+1/2*I,1/2*2^(1/2))-3*I*cos(f*x+e)^4*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+
e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1
)/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/
2),1/2-1/2*I,1/2*2^(1/2))+3*cos(f*x+e)^4*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f
*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e
)-1)/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^
(1/2),1/2+1/2*I,1/2*2^(1/2))+3*cos(f*x+e)^4*((I*cos(f*x+e)-I+sin(f*x+e))/si
n(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*
x+e)-1)/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e
))^(1/2),1/2-1/2*I,1/2*2^(1/2))-6*cos(f*x+e)^3*2^(1/2)+6*cos(f*x+e)^2*2^(1/
2)+8*cos(f*x+e)*2^(1/2)-8*2^(1/2))*cos(f*x+e)*(d/cos(f*x+e))^(5/2)*(b*sin(f
*x+e)/cos(f*x+e))^(5/2)/(cos(f*x+e)-1)/sin(f*x+e)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 3.56756, size = 2182, normalized size = 10.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/256*(6*sqrt(-b*d)*b^2*d^2*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2
- (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*
sqrt(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(
f*x + e)^2 - b*d - (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e)^3 +
 3*sqrt(-b*d)*b^2*d^2*cos(f*x + e)^3*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f
*x + e)^2 + 8*(7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x
 + e) - 8*cos(f*x + e))*sqrt(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d
/cos(f*x + e)) + 72*b*d + 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(co
s(f*x + e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8))
 - 16*(3*b^2*d^2*cos(f*x + e)^2 - 4*b^2*d^2)*sqrt(b*sin(f*x + e)/cos(f*x +
e))*sqrt(d/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3), 1/256*(6*sqrt(b*
d)*b^2*d^2*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 + (cos(f*x + e)^2
+ 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(b*d)*sqrt(b*s
in(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(f*x + e)^2 - b*d +
(b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e)^3 + 3*sqrt(b*d)*b^2*d^
2*cos(f*x + e)^3*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2 + 8*(7*cos
(f*x + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x +
e))*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)) + 72*b
*d - 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x + e)^4 - 8*cos(
f*x + e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) - 16*(3*b^2*d^2*cos(
f*x + e)^2 - 4*b^2*d^2)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x +
e))*sin(f*x + e))/(f*cos(f*x + e)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/2)*(b*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\tan\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(5/2), x)
```

3.308 $\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=131

$$\frac{b^2d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{bd^2(b\tan(e+fx))^{3/2}}{2f\sqrt{d\sec(e+fx)}}+\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}{3f}$$

```
[Out] (b^2*d^2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(2*f*Sqrt[d
*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]) - (b*d^2*(b*Tan[e + f*x])^(3/2))/(2*f*Sq
rt[d*Sec[e + f*x]]) + (b*(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2))/(3*
f)
```

Rubi [A] time = 0.173201, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2611, 2613, 2616, 2640, 2639}

$$\frac{b^2d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{bd^2(b\tan(e+fx))^{3/2}}{2f\sqrt{d\sec(e+fx)}}+\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(5/2),x]
```


```
[Out] (b^2*d^2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(2*f*Sqrt[d
*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]) - (b*d^2*(b*Tan[e + f*x])^(3/2))/(2*f*Sq
rt[d*Sec[e + f*x]]) + (b*(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2))/(3*
f)
```


Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
```

```
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx = \frac{b(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}{3f} - \frac{1}{2}b^2\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+f}$$

$$= -\frac{bd^2(b\tan(e+fx))^{3/2}}{2f\sqrt{d\sec(e+fx)}} + \frac{b(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}{3f} + \frac{1}{4}\left(b^2d^2\right)\int$$

$$= -\frac{bd^2(b\tan(e+fx))^{3/2}}{2f\sqrt{d\sec(e+fx)}} + \frac{b(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}{3f} + \frac{\left(b^2d^2\sqrt{b\,\mathrm{ta}}\right.}{4\sqrt{d\,\mathrm{s}}}$$

$$= -\frac{bd^2(b\tan(e+fx))^{3/2}}{2f\sqrt{d\sec(e+fx)}} + \frac{b(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}{3f} + \frac{\left(b^2d^2\sqrt{b\,\mathrm{ta}}\right.}{4\sqrt{d\,\mathrm{s}}}$$

$$= \frac{b^2d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}} - \frac{bd^2(b\tan(e+fx))^{3/2}}{2f\sqrt{d\sec(e+fx)}} + \frac{b(d\sec}{}$$

Mathematica [C] time = 2.33937, size = 93, normalized size = 0.71

$$\frac{b^3d^2\left(-3\sqrt[4]{-\tan^2(e+fx)}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\sec^2(e+fx)\right)+2\sec^4(e+fx)-5\sec^2(e+fx)+3\right)}{6f\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (b^3*d^2*(3 - 5*Sec[e + f*x]^2 + 2*Sec[e + f*x]^4 - 3*Hypergeometric2F1[-1/
4, 1/4, 3/4, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(1/4)))/(6*f*Sqrt[d*Sec[e +
f*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [C] time = 0.179, size = 593, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(5/2),x)
```

```
[Out] 1/12/f*2^(1/2)*(3*cos(f*x+e)^4*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/
2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f
*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2
^(1/2))-6*cos(f*x+e)^4*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*
cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(
1/2)*EllipticE(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+
3*cos(f*x+e)^3*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+
e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*Ell
ipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-6*cos(f*
x+e)^3*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin
(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*EllipticE((
(I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+3*cos(f*x+e)^3*2
^(1/2)-5*cos(f*x+e)^2*2^(1/2)+2*2^(1/2))*cos(f*x+e)*(d/cos(f*x+e))^(3/2)*(b
*sin(f*x+e)/cos(f*x+e))^(5/2)/sin(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\tan\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\sec\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}b^2d\sec\left(fx+e\right)\tan\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e))*b^2*d*sec(f*x + e)*tan(f
*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(3/2)*(b*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\tan\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(5/2), x)
```

3.309 $\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=169

$$\frac{3b^{5/2}d\sqrt{b\tan(e+fx)}\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{4f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{3b^{5/2}d\sqrt{b\tan(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{4f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{b(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2f}$$

```
[Out] (3*b^(5/2)*d*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(4*
f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]]) - (3*b^(5/2)*d*ArcTanh[Sqrt[b*
Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(4*f*Sqrt[d*Sec[e + f*x]]*Sqrt
[b*Sin[e + f*x]]) + (b*Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2))/(2*f)
```

Rubi [A] time = 0.152934, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2611, 2616, 2564, 329, 298, 203, 206}

$$\frac{3b^{5/2}d\sqrt{b\tan(e+fx)}\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{4f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{3b^{5/2}d\sqrt{b\tan(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{4f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{b(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(5/2),x]
```


```
[Out] (3*b^(5/2)*d*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(4*
f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]]) - (3*b^(5/2)*d*ArcTanh[Sqrt[b*
Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(4*f*Sqrt[d*Sec[e + f*x]]*Sqrt
[b*Sin[e + f*x]]) + (b*Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2))/(2*f)
```


Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
```

```
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\,dx = \frac{b\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{2f} - \frac{1}{4}\left(3b^2\right)\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,d$$

$$= \frac{b\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{2f} - \frac{\left(3b^2d\sqrt{b\tan(e+fx)}\right)\int \sec(e+fx)\sqrt{b\sin}}{4\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$= \frac{b\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{2f} - \frac{\left(3bd\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{b^2}}\,dx,x,\right.}{4f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+f}}$$

$$= \frac{b\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{2f} - \frac{\left(3bd\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{b^2}}\,dx,x,\right.}{2f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+}}$$

$$= \frac{b\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{2f} - \frac{\left(3b^3d\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{1}{b-x^2}\,dx,x,\right.}{4f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+}}$$

$$= \frac{3b^{5/2}d\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{4f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} - \frac{3b^{5/2}d\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan}}{4f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+}}$$

Mathematica [A] time = 1.67164, size = 182, normalized size = 1.08

$$\frac{\csc^3(e+fx)(b\tan(e+fx))^{5/2}\sqrt{d\sec(e+fx)}\left(4\sec^{\frac{5}{2}}(e+fx) - 4\sqrt{\sec(e+fx)} - 6\sqrt[4]{\tan^2(e+fx)}\tan^{-1}\left(\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right) + 3\right.}{8f\sec^{\frac{7}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (Csc[e + f*x]^3*Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(5/2)*(-4*Sqrt[Sec[e
+ f*x]] + 4*Sec[e + f*x]^(5/2) - 6*ArcTan[Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^
2)^(1/4)]*(Tan[e + f*x]^2)^(1/4) + 3*(Log[1 - Sqrt[Sec[e + f*x]]/(Tan[e + f
*x]^2)^(1/4)] - Log[1 + Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^(1/4)])*(Tan[e
+ f*x]^2)^(1/4)))/(8*f*Sec[e + f*x]^(7/2))
```

Maple [C] time = 0.196, size = 602, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(5/2),x)
```

```
[Out] 1/8/f*2^(1/2)*(b*sin(f*x+e)/cos(f*x+e))^(5/2)*(d/cos(f*x+e))^(1/2)*cos(f*x+
e)*(3*I*cos(f*x+e)^2*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*co
s(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/
2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*
2^(1/2))-3*I*cos(f*x+e)^2*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-
(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f*x+e)
)^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I
,1/2*2^(1/2))+3*cos(f*x+e)^2*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)
*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f*x
+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/
2*I,1/2*2^(1/2))+3*cos(f*x+e)^2*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1
/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(
f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2
-1/2*I,1/2*2^(1/2))+2*cos(f*x+e)*2^(1/2)-2*2^(1/2))/(cos(f*x+e)-1)/sin(f*x+
e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(5/2), x)
```

Fricas [B] time = 2.9858, size = 2047, normalized size = 12.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/32*(6*sqrt(-b*d)*b^2*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (co
s(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(
-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(f*x +
 e)^2 - b*d - (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e) + 3*sqrt
(-b*d)*b^2*cos(f*x + e)*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2 + 8
*(7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos
(f*x + e))*sqrt(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)
) + 72*b*d + 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x + e)^4
- 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) + 16*b^2*sqr
t(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*sin(f*x + e))/(f*cos(f*
x + e)), 1/32*(6*sqrt(b*d)*b^2*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^
2 + (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4
)*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos
(f*x + e)^2 - b*d + (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e) +
3*sqrt(b*d)*b^2*cos(f*x + e)*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^
2 + 8*(7*cos(f*x + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) -
8*cos(f*x + e))*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x
+ e)) + 72*b*d - 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x + e
)^4 - 8*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) + 16*b^2
*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*sin(f*x + e))/(f*co
s(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/2)*(b*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\sec(fx+e)}\,(b\tan(fx+e))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(5/2), x)
```

3.310 $\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx$

Optimal. Leaf size=88

$$\frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}}-\frac{3b^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

```
[Out] (-3*b^2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[d*Se
c[e + f*x]]*Sqrt[Sin[e + f*x]]) + (b*(b*Tan[e + f*x])^(3/2))/(f*Sqrt[d*Sec[
e + f*x]])
```

Rubi [A] time = 0.112627, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2611, 2616, 2640, 2639}

$$\frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}}-\frac{3b^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x])^(5/2)/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (-3*b^2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[d*Se
c[e + f*x]]*Sqrt[Sin[e + f*x]]) + (b*(b*Tan[e + f*x])^(3/2))/(f*Sqrt[d*Sec[
e + f*x]])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
```

```
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx &= \frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}} - \frac{1}{2}\left(3b^2\right)\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx \\
&= \frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}} - \frac{\left(3b^2\sqrt{b\tan(e+fx)}\right)\int \sqrt{b\sin(e+fx)}\,dx}{2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} \\
&= \frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}} - \frac{\left(3b^2\sqrt{b\tan(e+fx)}\right)\int \sqrt{\sin(e+fx)}\,dx}{2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}} \\
&= -\frac{3b^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}} + \frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}}
\end{aligned}$$

Mathematica [C] time = 0.710106, size = 74, normalized size = 0.84

$$\frac{b^3\left(3\sqrt[4]{-\tan^2(e+fx)}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\sec^2(e+fx)\right)+\tan^2(e+fx)\right)}{f\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Tan[e + f*x])^(5/2)/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (b^3*(Tan[e + f*x]^2 + 3*Hypergeometric2F1[-1/4, 1/4, 3/4, Sec[e + f*x]^2]*
(-Tan[e + f*x]^2)^(1/4)))/(f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [C] time = 0.223, size = 585, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*2^(1/2)*(6*cos(f*x+e)^2*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*Elliptic
E(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*((I*cos(f*x+e
)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))
^(1/2)-3*cos(f*x+e)^2*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I
+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1
/2)*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+6
*cos(f*x+e)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)-I
+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*((I*cos(f*x+e)-I+sin(f*x+e))/si
n(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)-3*cos(f*x+e
)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x
+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((I*co
s(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*cos(f*x+e)^2*2^(1/2
)-3*cos(f*x+e)*2^(1/2)+2^(1/2))*(b*sin(f*x+e)/cos(f*x+e))^(5/2)*cos(f*x+e)/
(d/cos(f*x+e))^(1/2)/sin(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)\right)^{\frac{5}{2}}}{\sqrt{d\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^(5/2)/sqrt(d*sec(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{d\sec\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}b^{2}\tan\left(fx+e\right)^{2}}{d\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e))*b^2*tan(f*x + e)^2/(d*se
c(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(5/2)/(d*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)\right)^{\frac{5}{2}}}{\sqrt{d\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^(5/2)/sqrt(d*sec(f*x + e)), x)
```

3.311 $\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=168

$$-\frac{b^{5/2}\sqrt{b\tan(e+fx)}\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{df\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{b^{5/2}\sqrt{b\tan(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{df\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}$$

```
[Out] -((b^(5/2)*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(d*f*
Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])) + (b^(5/2)*ArcTanh[Sqrt[b*Sin[e
 + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(d*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Si
n[e + f*x]]) - (2*b*(b*Tan[e + f*x])^(3/2))/(3*f*(d*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.165321, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2610, 2616, 2564, 329, 298, 203, 206}

$$-\frac{b^{5/2}\sqrt{b\tan(e+fx)}\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{df\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{b^{5/2}\sqrt{b\tan(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{df\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((b^(5/2)*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(d*f*
Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])) + (b^(5/2)*ArcTanh[Sqrt[b*Sin[e
 + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(d*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Si
n[e + f*x]]) - (2*b*(b*Tan[e + f*x])^(3/2))/(3*f*(d*Sec[e + f*x])^(3/2))
```

Rule 2610

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m)
, x] - Dist[(b^2*(n - 1))/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1]
 || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
```

```
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx = -\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}} + \frac{b^2\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx}{d^2}$$

$$= -\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}} + \frac{\left(b^2\sqrt{b\tan(e+fx)}\right)\int \sec(e+fx)\sqrt{b\sin(e+fx)}\,dx}{d\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$= -\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}} + \frac{\left(b\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{b^2}}\,dx, x, b\sin(e+fx)\right)}{df\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$= -\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}} + \frac{\left(2b\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sin(e+fx)}\right)}{df\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$= -\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}} + \frac{\left(b^3\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sin(e+fx)}\right)}{df\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} - \frac{\left(b^3\sqrt{b\tan\right.}{}$$

$$= -\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{df\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} + \frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{df\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} - \frac{2b(b\,\text{ta}}{3f(d\,\text{se}}$$

Mathematica [A] time = 1.05884, size = 181, normalized size = 1.08

$$\frac{\csc^3(e+fx)(b\tan(e+fx))^{5/2}\sqrt{d\sec(e+fx)}\left(-4\sin^2(e+fx)\sqrt{\sec(e+fx)}+6\sqrt[4]{\tan^2(e+fx)}\tan^{-1}\left(\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)+3\sqrt[4]{\right.}}{6d^2f\sec^{\frac{7}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (Csc[e + f*x]^3*Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(5/2)*(-4*Sqrt[Sec[e
+ f*x]]*Sin[e + f*x]^2 + 6*ArcTan[Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^(1/4)
]*(Tan[e + f*x]^2)^(1/4) + 3*(-Log[1 - Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^
(1/4)] + Log[1 + Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^(1/4)])*(Tan[e + f*x]^
2)^(1/4)))/(6*d^2*f*Sec[e + f*x]^(7/2))
```

Maple [C] time = 0.211, size = 570, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/6/f*2^(1/2)*(3*I*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+s
in(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2
)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2
^(1/2))-3*I*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e
))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))+
3*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x
+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((I*c
os(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*(-I*(cos
(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)
*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-
I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))+2*cos(f*x+e)*2^(1/2)
-2*2^(1/2))*(b*sin(f*x+e)/cos(f*x+e))^(5/2)*cos(f*x+e)/(cos(f*x+e)-1)/(d/co
s(f*x+e))^(3/2)/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(3/2), x)
```

Fricas [B] time = 5.52909, size = 1952, normalized size = 11.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(16*b^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f
*x + e)*sin(f*x + e) + 6*b^2*d*sqrt(-b/d)*arctan(1/4*(cos(f*x + e)^3 - 5*co
s(f*x + e)^2 - (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f
*x + e) + 4)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-b/d)*sqrt(d/cos(f*x +
e))/(b*cos(f*x + e)^2 - (b*cos(f*x + e) + b)*sin(f*x + e) - b)) - 3*b^2*d*s
qrt(-b/d)*log((b*cos(f*x + e)^4 - 72*b*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3
 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*
sin(f*x + e)/cos(f*x + e))*sqrt(-b/d)*sqrt(d/cos(f*x + e)) + 28*(b*cos(f*x
+ e)^2 - 2*b)*sin(f*x + e) + 72*b)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 - 4*(
cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)))/(d^2*f), -1/24*(16*b^2*sqrt(b*sin(f
*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 6*b^
2*d*sqrt(b/d)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 + (cos(f*x + e)
^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(b*sin(f*x
+ e)/cos(f*x + e))*sqrt(b/d)*sqrt(d/cos(f*x + e))/(b*cos(f*x + e)^2 + (b*co
s(f*x + e) + b)*sin(f*x + e) - b)) - 3*b^2*d*sqrt(b/d)*log((b*cos(f*x + e)^
4 - 72*b*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x
 + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqr
t(b/d)*sqrt(d/cos(f*x + e)) - 28*(b*cos(f*x + e)^2 - 2*b)*sin(f*x + e) + 72
*b)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x + e
) + 8)))/(d^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(5/2)/(d*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(3/2), x)
```

3.312 $\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=96

$$\frac{6b^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}$$

```
[Out] (6*b^2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(5*d^2*f*Sqrt
[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]) - (2*b*(b*Tan[e + f*x])^(3/2))/(5*f*(d
*Sec[e + f*x])^(5/2))
```

Rubi [A] time = 0.120262, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2610, 2616, 2640, 2639}

$$\frac{6b^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (6*b^2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(5*d^2*f*Sqrt
[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]) - (2*b*(b*Tan[e + f*x])^(3/2))/(5*f*(d
*Sec[e + f*x])^(5/2))
```

Rule 2610

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m)
, x] - Dist[(b^2*(n - 1))/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1]
 || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
```

```
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx &= -\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}+\frac{\left(3b^2\right)\int\frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx}{5d^2}\\
&= -\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}+\frac{\left(3b^2\sqrt{b\tan(e+fx)}\right)\int\sqrt{b\sin(e+fx)}\,dx}{5d^2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}\\
&= -\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}+\frac{\left(3b^2\sqrt{b\tan(e+fx)}\right)\int\sqrt{\sin(e+fx)}\,dx}{5d^2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}\\
&= \frac{6b^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.738884, size = 81, normalized size = 0.84

$$\frac{b^3\left(-6\sqrt[4]{-\tan^2(e+fx)}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\sec^2(e+fx)\right)+\cos(2(e+fx))-1\right)}{5d^2f\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (b^3*(-1 + Cos[2*(e + f*x)] - 6*Hypergeometric2F1[-1/4, 1/4, 3/4, Sec[e + f
*x]^2]*(-Tan[e + f*x]^2)^(1/4)))/(5*d^2*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e
 + f*x]])
```

Maple [C] time = 0.213, size = 564, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(5/2),x)
```

```
[Out] 1/5/f*2^(1/2)*(-6*cos(f*x+e)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*EllipticE
(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*((I*cos(f*x+e)
-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^
(1/2)+3*cos(f*x+e)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+si
n(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)
*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+cos(
f*x+e)^3*2^(1/2)-6*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(
f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)-I+sin(f*x+e
))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)+3*((
I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/
sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),
1/2*2^(1/2))*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)-4*cos(f*x+e)*2^(1/2)+3*2^
(1/2))*(b*sin(f*x+e)/cos(f*x+e))^(5/2)/(d/cos(f*x+e))^(5/2)/sin(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \tan\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(d \sec\left(fx+e\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d \sec\left(fx+e\right)}\sqrt{b \tan\left(fx+e\right)}b^2 \tan\left(fx+e\right)^2}{d^3 \sec\left(fx+e\right)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e))*b^2*tan(f*x + e)^2/(d^3*
sec(f*x + e)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(5/2)/(d*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(5/2), x)
```

3.313 $\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=34

$$\frac{2(b\tan(e+fx))^{7/2}}{7bf(d\sec(e+fx))^{7/2}}$$

```
[Out] (2*(b*Tan[e + f*x])^(7/2))/(7*b*f*(d*Sec[e + f*x])^(7/2))
```

Rubi [A] time = 0.0549945, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2605}

$$\frac{2(b\tan(e+fx))^{7/2}}{7bf(d\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(7/2),x]
```

```
[Out] (2*(b*Tan[e + f*x])^(7/2))/(7*b*f*(d*Sec[e + f*x])^(7/2))
```

Rule 2605

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f
*m), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx = \frac{2(b\tan(e+fx))^{7/2}}{7bf(d\sec(e+fx))^{7/2}}$$

Mathematica [A] time = 0.157927, size = 45, normalized size = 1.32

$$\frac{2b^2\sin^3(e+fx)\sqrt{b\tan(e+fx)}}{7d^3f\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(7/2),x]
```

```
[Out] (2*b^2*Sin[e + f*x]^3*Sqrt[b*Tan[e + f*x]])/(7*d^3*f*Sqrt[d*Sec[e + f*x]])
```

Maple [A] time = 0.149, size = 50, normalized size = 1.5

$$\frac{2\sin\left(fx+e\right)}{7f\cos\left(fx+e\right)}\left(\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}\left(\frac{d}{\cos\left(fx+e\right)}\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(7/2),x)
```

```
[Out] 2/7/f*sin(f*x+e)*(b*sin(f*x+e)/cos(f*x+e))^(5/2)/cos(f*x+e)/(d/cos(f*x+e))^
(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\tan\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(d\sec\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(7/2), x)
```

Fricas [B] time = 1.71274, size = 165, normalized size = 4.85

$$-\frac{2\left(b^{2}\cos\left(fx+e\right)^{3}-b^{2}\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{d}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)}{7\,d^{4}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] -2/7*(b^2*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x +
e))*sqrt(d/cos(f*x + e))*sin(f*x + e)/(d^4*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(5/2)/(d*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(d\sec\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(7/2), x)
```

3.314 $\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=131

$$\frac{4b^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{15d^4f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2b(b\tan(e+fx))^{3/2}}{15d^2f(d\sec(e+fx))^{5/2}}-\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}$$

```
[Out] (4*b^2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(15*d^4*f*Sqr
t[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]) - (2*b*(b*Tan[e + f*x])^(3/2))/(9*f*(
d*Sec[e + f*x])^(9/2)) + (2*b*(b*Tan[e + f*x])^(3/2))/(15*d^2*f*(d*Sec[e +
f*x])^(5/2))
```

Rubi [A] time = 0.176889, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2610, 2612, 2616, 2640, 2639}

$$\frac{4b^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{15d^4f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2b(b\tan(e+fx))^{3/2}}{15d^2f(d\sec(e+fx))^{5/2}}-\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(9/2),x]
```


```
[Out] (4*b^2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(15*d^4*f*Sqr
t[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]) - (2*b*(b*Tan[e + f*x])^(3/2))/(9*f*(
d*Sec[e + f*x])^(9/2)) + (2*b*(b*Tan[e + f*x])^(3/2))/(15*d^2*f*(d*Sec[e +
f*x])^(5/2))
```


Rule 2610

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m)
, x] - Dist[(b^2*(n - 1))/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1]
 || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

Rule 2612

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*m
```

```
), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e + f
*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1] &
& EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx &= -\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}+\frac{b^2\int\frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx}{3d^2}\\
&= -\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}+\frac{2b(b\tan(e+fx))^{3/2}}{15d^2f(d\sec(e+fx))^{5/2}}+\frac{\left(2b^2\right)\int\frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx}{15d^4}\\
&= -\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}+\frac{2b(b\tan(e+fx))^{3/2}}{15d^2f(d\sec(e+fx))^{5/2}}+\frac{\left(2b^2\sqrt{b\tan(e+fx)}\right)\int\sqrt{b\sin(e+fx)}\,dx}{15d^4\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}\\
&= -\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}+\frac{2b(b\tan(e+fx))^{3/2}}{15d^2f(d\sec(e+fx))^{5/2}}+\frac{\left(2b^2\sqrt{b\tan(e+fx)}\right)\int\sqrt{\sin(e+fx)}\,dx}{15d^4\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}\\
&= \frac{4b^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{15d^4f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}+\frac{2b(b\tan(e+fx))^{3/2}}{15d^2f(d\sec(e+fx))^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.950966, size = 99, normalized size = 0.76

$$-\frac{b^2 \sin(2(e+fx))\sqrt{b\tan(e+fx)}\left(12\sqrt[4]{-\tan^2(e+fx)}\csc^2(e+fx)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\sec^2(e+fx)\right)+5\cos(2(e+fx))-1\right)}{90d^4f\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(9/2),x]
```

```
[Out] -(b^2*Sin[2*(e + f*x)]*Sqrt[b*Tan[e + f*x]]*(-1 + 5*Cos[2*(e + f*x)] + 12*C
sc[e + f*x]^2*Hypergeometric2F1[-1/4, 1/4, 3/4, Sec[e + f*x]^2]*(-Tan[e + f
*x]^2)^(1/4)))/(90*d^4*f*Sqrt[d*Sec[e + f*x]])
```

Maple [C] time = 0.186, size = 586, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(9/2),x)
```

```
[Out] 1/45/f*2^(1/2)*(5*cos(f*x+e)^5*2^(1/2)-12*cos(f*x+e)*(-I*(cos(f*x+e)-1)/sin
(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2
*2^(1/2))*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-
sin(f*x+e))/sin(f*x+e))^(1/2)+6*cos(f*x+e)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(
1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f
*x+e))/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))
^(1/2),1/2*2^(1/2))-8*cos(f*x+e)^3*2^(1/2)-12*((I*cos(f*x+e)-I+sin(f*x+e))/
sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticE
(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-I*(cos(f*x+e
)-1)/sin(f*x+e))^(1/2)+6*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(
I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)-I+sin
(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2
)-3*cos(f*x+e)*2^(1/2)+6*2^(1/2))*(b*sin(f*x+e)/cos(f*x+e))^(5/2)/(d/cos(f*
x+e))^(9/2)/cos(f*x+e)^2/sin(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(d\sec\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{d\sec\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}b^{2}\tan\left(fx+e\right)^{2}}{d^{5}\sec\left(fx+e\right)^{5}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e))*b^2*tan(f*x + e)^2/(d^5*
sec(f*x + e)^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(5/2)/(d*sec(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(d\sec\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(9/2), x)
```

3.315 $\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal. Leaf size=178

$$\frac{d^2\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2bf}+\frac{3d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{4\sqrt{b}f\sqrt{b\tan(e+fx)}}+\frac{3d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(}{4\sqrt{b}f\sqrt{b\,t}}$$

```
[Out] (3*d^3*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin
[e + f*x]])/(4*Sqrt[b]*f*Sqrt[b*Tan[e + f*x]]) + (3*d^3*ArcTanh[Sqrt[b*Sin[
e + f*x]]/Sqrt[b]]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])/(4*Sqrt[b]*f*
Sqrt[b*Tan[e + f*x]]) + (d^2*(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]])/(
2*b*f)
```

Rubi [A] time = 0.16788, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2613, 2616, 2564, 329, 212, 206, 203}

$$\frac{d^2\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2bf}+\frac{3d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{4\sqrt{b}f\sqrt{b\tan(e+fx)}}+\frac{3d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(}{4\sqrt{b}f\sqrt{b\,t}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(7/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (3*d^3*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin
[e + f*x]])/(4*Sqrt[b]*f*Sqrt[b*Tan[e + f*x]]) + (3*d^3*ArcTanh[Sqrt[b*Sin[
e + f*x]]/Sqrt[b]]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])/(4*Sqrt[b]*f*
Sqrt[b*Tan[e + f*x]]) + (d^2*(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]])/(
2*b*f)
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{d^2(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2bf} + \frac{1}{4}\left(3d^2\right)\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx$$

$$= \frac{d^2(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2bf} + \frac{\left(3d^3\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{b\sin(e+fx)}}\,dx}{4\sqrt{b\tan(e+fx)}}$$

$$= \frac{d^2(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2bf} + \frac{\left(3d^3\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{b^2}\right)}\,d\right.}{4bf\sqrt{b\tan(e+fx)}}$$

$$= \frac{d^2(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2bf} + \frac{\left(3d^3\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{b^2}}\,dx, x,\right.}{2bf\sqrt{b\tan(e+fx)}}$$

$$= \frac{d^2(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2bf} + \frac{\left(3d^3\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x,\right.}{4f\sqrt{b\tan(e+fx)}}$$

$$= \frac{3d^3\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{4\sqrt{b}f\sqrt{b\tan(e+fx)}} + \frac{3d^3\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+f}}{4\sqrt{b}f\sqrt{b\tan(e+fx)}}$$

Mathematica [A] time = 6.35564, size = 136, normalized size = 0.76

$$\frac{d^3\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}\left(2\sqrt[4]{\tan^2(e+fx)}\sec^{\frac{3}{2}}(e+fx) - 3\tan^{-1}\left(\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right) + 3\tanh^{-1}\left(\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)\right)}{4bf\sqrt[4]{\tan^2(e+fx)}\sqrt{\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(7/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (d^3*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]]*(-3*ArcTan[Sqrt[Sec[e + f*x]
]/(Tan[e + f*x]^2)^(1/4)] + 3*ArcTanh[Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^(
1/4)] + 2*Sec[e + f*x]^(3/2)*(Tan[e + f*x]^2)^(1/4)))/(4*b*f*Sqrt[Sec[e + f
*x]]*(Tan[e + f*x]^2)^(1/4))
```

Maple [C] time = 0.215, size = 758, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x)
```

```
[Out] 1/8/f*2^(1/2)*(6*I*cos(f*x+e)^2*sin(f*x+e)*((I*cos(f*x+e)-I+sin(f*x+e))/sin
(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x
+e)-1)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))
^(1/2),1/2*2^(1/2))-3*I*cos(f*x+e)^2*sin(f*x+e)*((I*cos(f*x+e)-I+sin(f*x+e)
)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(co
s(f*x+e)-1)/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f
*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*I*cos(f*x+e)^2*sin(f*x+e)*((I*cos(f*x
+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e
))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+s
in(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*cos(f*x+e)^2*sin(f*x+
e)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x
+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*EllipticPi(((I*
cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*cos(f*x
+e)^2*sin(f*x+e)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*
x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*E
llipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1
/2))+2*cos(f*x+e)*2^(1/2)-2*2^(1/2))*(d/cos(f*x+e))^(7/2)*sin(f*x+e)*cos(f*
x+e)/(cos(f*x+e)-1)/(b*sin(f*x+e)/cos(f*x+e))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 3.32371, size = 1980, normalized size = 11.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(6*b*d^3*sqrt(-d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 -
(cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sq
rt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-d/b)*sqrt(d/cos(f*x + e))/(d*cos(f*x
+ e)^2 - (d*cos(f*x + e) + d)*sin(f*x + e) - d))*cos(f*x + e) - 3*b*d^3*sqr
t(-d/b)*cos(f*x + e)*log((d*cos(f*x + e)^4 - 72*d*cos(f*x + e)^2 + 8*(7*cos
(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x +
e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-d/b)*sqrt(d/cos(f*x + e)) + 28*
(d*cos(f*x + e)^2 - 2*d)*sin(f*x + e) + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x +
 e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) - 16*d^3*sqrt(b*sin(f*x +
 e)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/(b*f*cos(f*x + e)), 1/32*(6*b*d^3*s
qrt(d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 + (cos(f*x + e)^2 +
6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(b*sin(f*x + e)/
cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e))/(d*cos(f*x + e)^2 + (d*cos(f*x
 + e) + d)*sin(f*x + e) - d))*cos(f*x + e) + 3*b*d^3*sqrt(d/b)*cos(f*x + e)
*log((d*cos(f*x + e)^4 - 72*d*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3 + (cos(f
*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x +
 e)/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e)) - 28*(d*cos(f*x + e)^2 - 2
*d)*sin(f*x + e) + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 + 4*(cos(f*x +
e)^2 - 2)*sin(f*x + e) + 8)) + 16*d^3*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqr
t(d/cos(f*x + e)))/(b*f*cos(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(7/2)/(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{\frac{7}{2}}}{\sqrt{b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(7/2)/sqrt(b*tan(f*x + e)), x)
```

3.316 $\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal. Leaf size=92

$$\frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf}+\frac{d^2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\sec(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

```
[Out] (d^2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x
]])/(f*Sqrt[b*Tan[e + f*x]]) + (d^2*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x
]])/(b*f)
```

Rubi [A] time = 0.112805, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2613, 2616, 2642, 2641}

$$\frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf}+\frac{d^2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\sec(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (d^2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x
]])/(f*Sqrt[b*Tan[e + f*x]]) + (d^2*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x
]])/(b*f)
```

Rule 2613

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(m + n - 1)), x] + Dist[(a^2*(m - 2))/(m + n - 1), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
```

```
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx &= \frac{d^2\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{bf}+\frac{1}{2}d^2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx\\
&= \frac{d^2\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{bf}+\frac{\left(d^2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\int \frac{1}{\sqrt{b\sin(e+fx)}}\,dx}{2\sqrt{b\tan(e+fx)}}\\
&= \frac{d^2\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{bf}+\frac{\left(d^2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx}{2\sqrt{b\tan(e+fx)}}\\
&= \frac{d^2F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{bf}
\end{aligned}$$

Mathematica [C] time = 2.34878, size = 83, normalized size = 0.9

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\sin(e+fx)\cos(e+fx)\sec^2(e+fx)^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\tan^2(e+fx)\right)+\tan(e+fx)\right)}{f\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (d^2*Sqrt[d*Sec[e + f*x]]*(Cos[e + f*x]*Hypergeometric2F1[1/4, 3/4, 5/4, -T
an[e + f*x]^2]*(Sec[e + f*x]^2)^(3/4)*Sin[e + f*x] + Tan[e + f*x]))/(f*Sqrt
```

```
[b*Tan[e + f*x]])
```

Maple [C] time = 0.21, size = 208, normalized size = 2.3

$$-\frac{\sqrt{2}\sin\left(fx+e\right)\cos\left(fx+e\right)}{2f\left(\cos\left(fx+e\right)-1\right)}\left(i\cos\left(fx+e\right)\sin\left(fx+e\right)\sqrt{\frac{i\cos\left(fx+e\right)-i+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{-\frac{i\cos\left(fx+e\right)-i-\sin\left(\right.}{\sin\left(fx+e\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*2^(1/2)*(I*cos(f*x+e)*sin(f*x+e)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*
x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((I*c
os(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-I*(cos(f*x+e)-1)/s
in(f*x+e))^(1/2)-cos(f*x+e)*2^(1/2)+2^(1/2))*(d/cos(f*x+e))^(5/2)*sin(f*x+e
)*cos(f*x+e)/(cos(f*x+e)-1)/(b*sin(f*x+e)/cos(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}}{\sqrt{b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)/sqrt(b*tan(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{d\sec\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}d^{2}\sec\left(fx+e\right)^{2}}{b\tan\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e))*d^2*sec(f*x + e)^2/(b*ta
n(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/2)/(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}}{\sqrt{b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)/sqrt(b*tan(f*x + e)), x)
```

3.317 $\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal. Leaf size=131

$$\frac{d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f\sqrt{b\tan(e+fx)}}+\frac{d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f\sqrt{b\tan(e+fx)}}$$

```
[Out] (d*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e +
 f*x]])/(Sqrt[b]*f*Sqrt[b*Tan[e + f*x]]) + (d*ArcTanh[Sqrt[b*Sin[e + f*x]]/
Sqrt[b]]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])/(Sqrt[b]*f*Sqrt[b*Tan[e
 + f*x]])
```

Rubi [A] time = 0.106436, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2616, 2564, 329, 212, 206, 203}

$$\frac{d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f\sqrt{b\tan(e+fx)}}+\frac{d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(3/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (d*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e +
 f*x]])/(Sqrt[b]*f*Sqrt[b*Tan[e + f*x]]) + (d*ArcTanh[Sqrt[b*Sin[e + f*x]]/
Sqrt[b]]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])/(Sqrt[b]*f*Sqrt[b*Tan[e
 + f*x]])
```

Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
```

```
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{\left(d\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{b\sin(e+fx)}}\,dx}{\sqrt{b\tan(e+fx)}}$$

$$= \frac{\left(d\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{b^2}\right)}\,dx, x, b\sin(e+fx)\right)}{bf\sqrt{b\tan(e+fx)}}$$

$$= \frac{\left(2d\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sin(e+fx)}\right)}{bf\sqrt{b\tan(e+fx)}}$$

$$= \frac{\left(d\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sin(e+fx)}\right)}{f\sqrt{b\tan(e+fx)}} + \frac{\left(d\sqrt{d\sec(e+fx)}\sqrt{b\right.}{}$$

$$= \frac{d\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{\sqrt{b}f\sqrt{b\tan(e+fx)}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{}}{\sqrt{b}f\sqrt{b\tan(e+fx)}}$$

Mathematica [A] time = 4.25268, size = 105, normalized size = 0.8

$$-\frac{\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}\left(\tan^{-1}\left(\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)-\tanh^{-1}\left(\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)\right)}{bf\sqrt[4]{\tan^2(e+fx)}\sec^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] -(((ArcTan[Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^(1/4)] - ArcTanh[Sqrt[Sec[e
+ f*x]]/(Tan[e + f*x]^2)^(1/4)])*(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]
])/(b*f*Sec[e + f*x]^(3/2)*(Tan[e + f*x]^2)^(1/4)))
```

Maple [C] time = 0.214, size = 344, normalized size = 2.6

$$\frac{\cos(fx+e)\sqrt{2}(\sin(fx+e))^2}{2f(\cos(fx+e)-1)}\left(2i\text{EllipticF}\left(\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}}, \frac{\sqrt{2}}{2}\right) - i\text{EllipticPi}\left(\sqrt{\frac{i\cos(fx+e)-}{\sin(}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*2^(1/2)*(2*I*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)
,1/2*2^(1/2))-I*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1
/2-1/2*I,1/2*2^(1/2))-I*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))
^(1/2),1/2+1/2*I,1/2*2^(1/2))+EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f
*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))
/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2)))*cos(f*x+e)*(-I*(cos(f*x+e)-1)/si
n(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x
+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(d/cos(f*x+e))^(3/2)*sin(f*x+e)^2/(cos(
f*x+e)-1)/(b*sin(f*x+e)/cos(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)/sqrt(b*tan(f*x + e)), x)
```

Fricas [B] time = 3.01246, size = 1662, normalized size = 12.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(2*d*sqrt(-d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (cos(
f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(b*
sin(f*x + e)/cos(f*x + e))*sqrt(-d/b)*sqrt(d/cos(f*x + e))/(d*cos(f*x + e)^
2 - (d*cos(f*x + e) + d)*sin(f*x + e) - d)) - d*sqrt(-d/b)*log((d*cos(f*x +
 e)^4 - 72*d*cos(f*x + e)^2 + 8*(7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos
```

```
(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))
*sqrt(-d/b)*sqrt(d/cos(f*x + e)) + 28*(d*cos(f*x + e)^2 - 2*d)*sin(f*x + e)
 + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*
x + e) + 8)))/f, 1/8*(2*d*sqrt(d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x
+ e)^2 + (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e
) + 4)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e))/(d*
cos(f*x + e)^2 + (d*cos(f*x + e) + d)*sin(f*x + e) - d)) + d*sqrt(d/b)*log(
(d*cos(f*x + e)^4 - 72*d*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3 + (cos(f*x +
e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x + e)/c
os(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e)) - 28*(d*cos(f*x + e)^2 - 2*d)*s
in(f*x + e) + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 + 4*(cos(f*x + e)^2
- 2)*sin(f*x + e) + 8)))/f]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(3/2)/(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)/sqrt(b*tan(f*x + e)), x)
```

3.318 $\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal. Leaf size=55

$$\frac{2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\sec(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

```
[Out] (2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]
)/(f*Sqrt[b*Tan[e + f*x]])
```

Rubi [A] time = 0.061367, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2616, 2642, 2641}

$$\frac{2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\sec(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sec[e + f*x]]/Sqrt[b*Tan[e + f*x]],x]
```


```
[Out] (2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]
)/(f*Sqrt[b*Tan[e + f*x]])
```


Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx &= \frac{\left(\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\int \frac{1}{\sqrt{b\sin(e+fx)}}\,dx}{\sqrt{b\tan(e+fx)}}\\
&= \frac{\left(\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx}{\sqrt{b\tan(e+fx)}}\\
&= \frac{2F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}}
\end{aligned}
$$

Mathematica [C] time = 0.444065, size = 89, normalized size = 1.62

$$
\frac{2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}{bf\sqrt{\sec(e+fx)}\sqrt{\cos^2\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (2*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[(e + f*x)/2]^2]*Sqrt[d*Sec[e + f*x
]]*Sqrt[b*Tan[e + f*x]])/(b*f*Sqrt[Sec[e + f*x]]*Sqrt[Cos[(e + f*x)/2]^2*Se
c[e + f*x]])
```

Maple [C] time = 0.207, size = 175, normalized size = 3.2

$$
\frac{-i\left(\sin\left(fx+e\right)\right)^2\sqrt{2}}{f\left(\cos\left(fx+e\right)-1\right)}\sqrt{\frac{i\cos\left(fx+e\right)-i+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{-\frac{i\cos\left(fx+e\right)-i-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\mathrm{EllipticF}\left(\sqrt{\frac{i\cos\left(fx+e\right)-}{\sin\left(\right.}}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x)
```

```
[Out] -I/f*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*
(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+
e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(d/cos(f*x+e))^(1/
2)*sin(f*x+e)^2*2^(1/2)/(cos(f*x+e)-1)/(b*sin(f*x+e)/cos(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d \sec (fx+e)}}{\sqrt{b \tan (fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))/sqrt(b*tan(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d \sec (fx+e)}\sqrt{b \tan (fx+e)}}{b \tan (fx+e)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e))/(b*tan(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d \sec (e+fx)}}{\sqrt{b \tan (e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/2)/(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*sec(e + f*x))/sqrt(b*tan(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sec\left(fx+e\right)}}{\sqrt{b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))/sqrt(b*tan(f*x + e)), x)
```

3.319 $\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx$

Optimal. Leaf size=32

$$\frac{2\sqrt{b\tan(e+fx)}}{bf\sqrt{d\sec(e+fx)}}$$

```
[Out] (2*Sqrt[b*Tan[e + f*x]])/(b*f*Sqrt[d*Sec[e + f*x]])
```

Rubi [A] time = 0.045485, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2605}

$$\frac{2\sqrt{b\tan(e+fx)}}{bf\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]]),x]
```

```
[Out] (2*Sqrt[b*Tan[e + f*x]])/(b*f*Sqrt[d*Sec[e + f*x]])
```

Rule 2605

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f
*m), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \frac{2\sqrt{b\tan(e+fx)}}{bf\sqrt{d\sec(e+fx)}}$$

Mathematica [A] time = 0.38786, size = 32, normalized size = 1.

$$\frac{2\sqrt{b\tan(e+fx)}}{bf\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]]),x]
```

```
[Out] (2*Sqrt[b*Tan[e + f*x]])/(b*f*Sqrt[d*Sec[e + f*x]])
```

Maple [A] time = 0.165, size = 50, normalized size = 1.6

$$2\frac{\sin\left(fx+e\right)}{f\cos\left(fx+e\right)}\frac{1}{\sqrt{\frac{d}{\cos\left(fx+e\right)}}}\frac{1}{\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x)
```

```
[Out] 2/f*sin(f*x+e)/(d/cos(f*x+e))^(1/2)/(b*sin(f*x+e)/cos(f*x+e))^(1/2)/cos(f*x
+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d\sec\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e))), x)
```

Fricas [A] time = 1.67059, size = 107, normalized size = 3.34

$$\frac{2\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{d}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)}{bdf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)/(b*d*
f)
```

Sympy [A] time = 22.735, size = 51, normalized size = 1.59

$$\begin{cases} \frac{2\sqrt{\tan\left(e+fx\right)}}{\sqrt{b}\sqrt{d}f\sqrt{\sec\left(e+fx\right)}} & \text{for}\ f\neq 0 \\ \frac{x}{\sqrt{b\tan\left(e\right)}\sqrt{d\sec\left(e\right)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(1/2)/(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(tan(e + f*x))/(sqrt(b)*sqrt(d)*f*sqrt(sec(e + f*x))), Ne(
f, 0)), (x/(sqrt(b*tan(e))*sqrt(d*sec(e))), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d\sec\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e))), x)
```

3.320 $\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx$

Optimal. Leaf size=95

$$\frac{4\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\sec(e+fx)}}{3d^2f\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}$$

```
[Out] (4*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]
)/(3*d^2*f*Sqrt[b*Tan[e + f*x]]) + (2*Sqrt[b*Tan[e + f*x]])/(3*b*f*(d*Sec[e
 + f*x])^(3/2))
```

Rubi [A] time = 0.115107, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2612, 2616, 2642, 2641}

$$\frac{4\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\sec(e+fx)}}{3d^2f\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]),x]
```

```
[Out] (4*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]
)/(3*d^2*f*Sqrt[b*Tan[e + f*x]]) + (2*Sqrt[b*Tan[e + f*x]])/(3*b*f*(d*Sec[e
 + f*x])^(3/2))
```

Rule 2612

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*m
), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e + f
*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1] &
& EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
```

```
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx &= \frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}+\frac{2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx}{3d^2}\\
&= \frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}+\frac{\left(2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\int \frac{1}{\sqrt{b\sin(e+fx)}}\,dx}{3d^2\sqrt{b\tan(e+fx)}}\\
&= \frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}+\frac{\left(2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx}{3d^2\sqrt{b\tan(e+fx)}}\\
&= \frac{4F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{3d^2f\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}
\end{aligned}$$

Mathematica [C] time = 1.00144, size = 91, normalized size = 0.96

$$\frac{2\sqrt{b\tan(e+fx)}\left(\sqrt[4]{-\tan^2(e+fx)}-2\sec^2(e+fx)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};\sec^2(e+fx)\right)\right)}{3bf\sqrt[4]{-\tan^2(e+fx)}(d\sec(e+fx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]),x]
```

```
[Out] (2*Sqrt[b*Tan[e + f*x]]*(-2*Hypergeometric2F1[1/4, 3/4, 5/4, Sec[e + f*x]^2
]*Sec[e + f*x]^2 + (-Tan[e + f*x]^2)^(1/4)))/(3*b*f*(d*Sec[e + f*x])^(3/2)*
```

```
(-Tan[e + f*x]^2)^(1/4))
```

Maple [C] time = 0.211, size = 213, normalized size = 2.2

$$-\frac{\sqrt{2}\sin\left(fx+e\right)}{3f\left(\cos\left(fx+e\right)-1\right)\left(\cos\left(fx+e\right)\right)^{2}}\left(2i\sin\left(fx+e\right)\sqrt{\frac{i\cos\left(fx+e\right)-i+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{-\frac{i\cos\left(fx+e\right)-i-\sin\left(fx\right.}{\sin\left(fx+e\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x)
```

```
[Out] -1/3/f*2^(1/2)*(2*I*sin(f*x+e)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/
2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)
-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-I*(cos(f*x+e)-1)/sin(f*x+e)
)^(1/2)-cos(f*x+e)^2*2^(1/2)+cos(f*x+e)*2^(1/2))*sin(f*x+e)/(cos(f*x+e)-1)/
(d/cos(f*x+e))^(3/2)/(b*sin(f*x+e)/cos(f*x+e))^(1/2)/cos(f*x+e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*sqrt(b*tan(f*x + e))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\sec\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}}{bd^{2}\sec\left(fx+e\right)^{2}\tan\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e))/(b*d^2*sec(f*x + e)^2*ta
n(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(3/2)/(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*sqrt(b*tan(f*x + e))), x)
```

3.321 $\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx$

Optimal. Leaf size=72

$$\frac{8\sqrt{b\tan(e+fx)}}{5bd^2f\sqrt{d\sec(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}}$$

```
[Out] (2*Sqrt[b*Tan[e + f*x]])/(5*b*f*(d*Sec[e + f*x])^(5/2)) + (8*Sqrt[b*Tan[e +
 f*x]])/(5*b*d^2*f*Sqrt[d*Sec[e + f*x]])
```

Rubi [A] time = 0.0973784, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2612, 2605}

$$\frac{8\sqrt{b\tan(e+fx)}}{5bd^2f\sqrt{d\sec(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]),x]
```

```
[Out] (2*Sqrt[b*Tan[e + f*x]])/(5*b*f*(d*Sec[e + f*x])^(5/2)) + (8*Sqrt[b*Tan[e +
 f*x]])/(5*b*d^2*f*Sqrt[d*Sec[e + f*x]])
```

Rule 2612

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*m
), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e + f
*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1] &
& EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2605

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f
*m), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}} + \frac{4\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx}{5d^2}$$
$$= \frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}} + \frac{8\sqrt{b\tan(e+fx)}}{5bd^2f\sqrt{d\sec(e+fx)}}$$

Mathematica [A] time = 1.15039, size = 112, normalized size = 1.56

$$\frac{\sqrt{\frac{1}{\cos(e+fx)+1}}\cos(2(e+fx))\tan(e+fx) + 9\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)}\sqrt{\sec(e+fx)+1}}{5d^2f\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]),x]
```

```
[Out] (9*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2] + Sqrt[(1 + C
os[e + f*x])^(-1)]*Cos[2*(e + f*x)]*Tan[e + f*x])/(5*d^2*f*Sqrt[(1 + Cos[e
+ f*x])^(-1)]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [A] time = 0.17, size = 60, normalized size = 0.8

$$\frac{2\sin(fx+e)\left((\cos(fx+e))^2+4\right)}{5f(\cos(fx+e))^3}\left(\frac{d}{\cos(fx+e)}\right)^{-\frac{5}{2}}\frac{1}{\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x)
```

```
[Out] 2/5/f*sin(f*x+e)*(cos(f*x+e)^2+4)/cos(f*x+e)^3/(d/cos(f*x+e))^(5/2)/(b*sin(
f*x+e)/cos(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}\sqrt{b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*sec(f*x + e))^(5/2)*sqrt(b*tan(f*x + e))), x)
```

Fricas [A] time = 1.696, size = 140, normalized size = 1.94

$$\frac{2\left(\cos\left(fx+e\right)^{3}+4\,\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{d}{\cos\left(fx+e\right)}}}{5\,bd^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2/5*(cos(f*x + e)^3 + 4*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqr
t(d/cos(f*x + e))/(b*d^3*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(5/2)/(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}\sqrt{b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(5/2)*sqrt(b*tan(f*x + e))), x)
```

3.322 $\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=171

$$-\frac{d^3\sqrt{b\tan(e+fx)}\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^3\sqrt{b\tan(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

```
[Out] (-2*d^2*Sqrt[d*Sec[e + f*x]])/(b*f*Sqrt[b*Tan[e + f*x]]) - (d^3*ArcTan[Sqrt
[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(b^(3/2)*f*Sqrt[d*Sec[e + f
*x]]*Sqrt[b*Sin[e + f*x]]) + (d^3*ArcTanh[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqr
t[b*Tan[e + f*x]])/(b^(3/2)*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])
```

Rubi [A] time = 0.166765, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2608, 2616, 2564, 329, 298, 203, 206}

$$-\frac{d^3\sqrt{b\tan(e+fx)}\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^3\sqrt{b\tan(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*d^2*Sqrt[d*Sec[e + f*x]])/(b*f*Sqrt[b*Tan[e + f*x]]) - (d^3*ArcTan[Sqrt
[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(b^(3/2)*f*Sqrt[d*Sec[e + f
*x]]*Sqrt[b*Sin[e + f*x]]) + (d^3*ArcTanh[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqr
t[b*Tan[e + f*x]])/(b^(3/2)*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])
```

Rule 2608

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(n + 1)), x] - Dist[(a^2*(m - 2))/(b^2*(n + 1)), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}} + \frac{d^2\int\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx}{b^2}$$

$$= -\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}} + \frac{\left(d^3\sqrt{b\tan(e+fx)}\right)\int\sec(e+fx)\sqrt{b\sin(e+fx)}\,dx}{b^2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$= -\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}} + \frac{\left(d^3\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int\frac{\sqrt{x}}{1-\frac{x^2}{b^2}}\,dx, x, b\sin(e+fx)\right)}{b^3f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$= -\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}} + \frac{\left(2d^3\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int\frac{x^2}{1-\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sin(e+fx)}\right)}{b^3f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$= -\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}} + \frac{\left(d^3\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int\frac{1}{b-x^2}\,dx, x, \sqrt{b\sin(e+fx)}\right)}{bf\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} - \frac{\left(d^3\sqrt{b\tan\right.}{}$$

$$= -\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}} - \frac{d^3\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{b^{3/2}f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} + \frac{d^3\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan}}{b^{3/2}f\sqrt{d\sec(e+fx)}\sqrt{b\sin(}}$$

Mathematica [A] time = 1.03718, size = 211, normalized size = 1.23

$$-\frac{d^3\sin(e+fx)\left(-4\csc^2(e+fx)+16\csc^2(2(e+fx))-2\sqrt[4]{\tan^2(e+fx)}\sec^{\frac{3}{2}}(e+fx)\tan^{-1}\left(\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)+\sqrt[4]{\tan^2(e+}\right.}{2f(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] -(d^3*Sin[e + f*x]*(-4*Csc[e + f*x]^2 + 16*Csc[2*(e + f*x)]^2 - 2*ArcTan[Sq
rt[Sec[e + f*x]]/(Tan[e + f*x]^2)^(1/4)]*Sec[e + f*x]^(3/2)*(Tan[e + f*x]^2
)^(1/4) + Log[1 - Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^(1/4)]*Sec[e + f*x]^(
3/2)*(Tan[e + f*x]^2)^(1/4) - Log[1 + Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^(
1/4)]*Sec[e + f*x]^(3/2)*(Tan[e + f*x]^2)^(1/4)))/(2*f*Sqrt[d*Sec[e + f*x]]
*(b*Tan[e + f*x])^(3/2))
```

Maple [C] time = 0.21, size = 1061, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x)
```

```
[Out] -1/2/f*2^(1/2)*(I*cos(f*x+e)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f
*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x
+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/
2*I,1/2*2^(1/2))-I*cos(f*x+e)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(
f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*
x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1
/2*I,1/2*2^(1/2))-cos(f*x+e)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f
*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x
+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/
2*I,1/2*2^(1/2))-cos(f*x+e)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*
x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+
e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2
*I,1/2*2^(1/2))+I*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin
(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*
EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(
1/2))-I*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/s
in(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi
(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-(-I*
(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(
1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x
+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-(-I*(cos(f*x+e)-
1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*co
s(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*
x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))+2*2^(1/2))*(d/cos(f*x+e))^(5
/2)*sin(f*x+e)*cos(f*x+e)/(b*sin(f*x+e)/cos(f*x+e))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \sec\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b \tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e))^(3/2), x)
```

Fricas [B] time = 3.26436, size = 2014, normalized size = 11.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(2*b*d^2*sqrt(-d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (
cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqr
t(b*sin(f*x + e)/cos(f*x + e))*sqrt(-d/b)*sqrt(d/cos(f*x + e))/(d*cos(f*x +
 e)^2 - (d*cos(f*x + e) + d)*sin(f*x + e) - d))*sin(f*x + e) - b*d^2*sqrt(-
d/b)*log((d*cos(f*x + e)^4 - 72*d*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3 - (c
os(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f
*x + e)/cos(f*x + e))*sqrt(-d/b)*sqrt(d/cos(f*x + e)) + 28*(d*cos(f*x + e)^
2 - 2*d)*sin(f*x + e) + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f
*x + e)^2 - 2)*sin(f*x + e) + 8))*sin(f*x + e) + 16*d^2*sqrt(b*sin(f*x + e)
/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e))/(b^2*f*sin(f*x + e)), -1/
8*(2*b*d^2*sqrt(d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 + (cos(f
*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(b*s
in(f*x + e)/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e))/(d*cos(f*x + e)^2
+ (d*cos(f*x + e) + d)*sin(f*x + e) - d))*sin(f*x + e) - b*d^2*sqrt(d/b)*lo
g((d*cos(f*x + e)^4 - 72*d*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3 + (cos(f*x
+ e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x + e)
/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e)) - 28*(d*cos(f*x + e)^2 - 2*d)
*sin(f*x + e) + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 + 4*(cos(f*x + e)^
2 - 2)*sin(f*x + e) + 8))*sin(f*x + e) + 16*d^2*sqrt(b*sin(f*x + e)/cos(f*x
 + e))*sqrt(d/cos(f*x + e))*cos(f*x + e))/(b^2*f*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\sec(fx+e))^{\frac{5}{2}}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e))^(3/2), x)
```

3.323 $\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{2d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

```
[Out] (-2*d^2)/(b*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]]) - (2*d^2*EllipticE
[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(b^2*f*Sqrt[d*Sec[e + f*x]]*S
qrt[Sin[e + f*x]])
```

Rubi [A] time = 0.122558, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2608, 2616, 2640, 2639}

$$-\frac{2d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(3/2)/(b*Tan[e + f*x])^(3/2),x]
```


```
[Out] (-2*d^2)/(b*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]]) - (2*d^2*EllipticE
[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(b^2*f*Sqrt[d*Sec[e + f*x]]*S
qrt[Sin[e + f*x]])
```


Rule 2608

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(n + 1)), x] - Dist[(a^2*(m - 2))/(b^2*(n + 1)), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
```

```
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx &= -\frac{2d^2}{bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{d^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx}{b^2}\\
&= -\frac{2d^2}{bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{\left(d^2\sqrt{b\tan(e+fx)}\right)\int \sqrt{b\sin(e+fx)}\,dx}{b^2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}\\
&= -\frac{2d^2}{bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{\left(d^2\sqrt{b\tan(e+fx)}\right)\int \sqrt{\sin(e+fx)}\,dx}{b^2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}\\
&= -\frac{2d^2}{bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{2d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}
\end{aligned}
$$

Mathematica [C] time = 0.608908, size = 70, normalized size = 0.72

$$
\frac{2d^2\left(\sqrt[4]{-\tan^2(e+fx)}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\sec^2(e+fx)\right)-1\right)}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*d^2*(-1 + Hypergeometric2F1[-1/4, 1/4, 3/4, Sec[e + f*x]^2]*(-Tan[e + f*
x]^2)^(1/4)))/(b*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [C] time = 0.185, size = 538, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x)
```

```
[Out] 1/f*2^(1/2)*(2*cos(f*x+e)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*EllipticE(((
I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*((I*cos(f*x+e)-I+
sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/
2)-cos(f*x+e)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x
+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*Elli
pticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*((I*cos
(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f
*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2
^(1/2))*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)-((I*cos(f*x+e)-I+sin(f*x+e))/s
in(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(
((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-I*(cos(f*x+e)
-1)/sin(f*x+e))^(1/2)-2^(1/2))*(d/cos(f*x+e))^(3/2)*sin(f*x+e)/(b*sin(f*x+e
)/cos(f*x+e))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\sec\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}d\sec\left(fx+e\right)}{b^2\tan\left(fx+e\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e))*d*sec(f*x + e)/(b^2*tan(
f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(3/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \sec\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b \tan\left(fx+e\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e))^(3/2), x)
```

3.324 $\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=32

$$-\frac{2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

```
[Out] (-2*Sqrt[d*Sec[e + f*x]])/(b*f*Sqrt[b*Tan[e + f*x]])
```

Rubi [A] time = 0.0503519, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2605}

$$-\frac{2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sec[e + f*x]]/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[d*Sec[e + f*x]])/(b*f*Sqrt[b*Tan[e + f*x]])
```

Rule 2605

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f
*m), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

Mathematica [A] time = 0.123912, size = 32, normalized size = 1.

$$-\frac{2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[d*Sec[e + f*x]])/(b*f*Sqrt[b*Tan[e + f*x]])
```

Maple [A] time = 0.154, size = 50, normalized size = 1.6

$$-2\,{\frac {\sin \left(fx+e \right) }{f\cos \left(fx+e \right) }\sqrt {{\frac {d}{\cos \left(fx+e \right) }}} \left({\frac {b\sin \left(fx+e \right) }{\cos \left(fx+e \right) }} \right) ^{-3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x)
```

```
[Out] -2/f*sin(f*x+e)*(d/cos(f*x+e))^(1/2)/(b*sin(f*x+e)/cos(f*x+e))^(3/2)/cos(f*
x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d \sec\left(f x + e\right)}}{\left(b \tan\left(f x + e\right)\right)^{\frac{3}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e))^(3/2), x)
```

Fricas [A] time = 1.65273, size = 126, normalized size = 3.94

$$-\frac{2 \, \sqrt{\frac{b \sin\left(f x + e\right)}{\cos\left(f x + e\right)}} \sqrt{\frac{d}{\cos\left(f x + e\right)}} \cos\left(f x + e\right)}{b^{2} f \sin\left(f x + e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)/(b^2
*f*sin(f*x + e))
```

Sympy [A] time = 24.0339, size = 53, normalized size = 1.66

$$\begin{cases} -\frac{2\sqrt{d}\sqrt{\sec\left(e+fx\right)}}{b^{\frac{3}{2}}f\sqrt{\tan\left(e+fx\right)}} & \text{for } f \neq 0 \\ \frac{x\sqrt{d\sec\left(e\right)}}{\left(b\tan\left(e\right)\right)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Piecewise((-2*sqrt(d)*sqrt(sec(e + f*x))/(b**(3/2)*f*sqrt(tan(e + f*x))), N
e(f, 0)), (x*sqrt(d*sec(e))/(b*tan(e))**(3/2), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sec\left(fx+e\right)}}{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e))^(3/2), x)
```

3.325 $\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{4E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

```
[Out] -2/(b*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]]) - (4*EllipticE[(e - Pi/2
 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(b^2*f*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e
+ f*x]])
```

Rubi [A] time = 0.116357, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2609, 2616, 2640, 2639}

$$-\frac{4E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -2/(b*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]]) - (4*EllipticE[(e - Pi/2
 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(b^2*f*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e
+ f*x]])
```

Rule 2609

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
n + 1)), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
```

```
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}\,dx &= -\frac{2}{bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{2\int\frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx}{b^2}\\ &= -\frac{2}{bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{\left(2\sqrt{b\tan(e+fx)}\right)\int\sqrt{b\sin(e+fx)}\,dx}{b^2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}\\ &= -\frac{2}{bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{\left(2\sqrt{b\tan(e+fx)}\right)\int\sqrt{\sin(e+fx)}\,dx}{b^2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}\\ &= -\frac{2}{bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{4E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}\end{aligned}$$

Mathematica [C] time = 0.562015, size = 67, normalized size = 0.74

$$\frac{4\sqrt[4]{-\tan^2(e+fx)}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\sec^2(e+fx)\right)-2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] (-2 + 4*Hypergeometric2F1[-1/4, 1/4, 3/4, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)
^(1/4))/(b*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [C] time = 0.206, size = 556, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x)
```

```
[Out] 1/f*2^(1/2)*(4*cos(f*x+e)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*EllipticE(((
I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*((I*cos(f*x+e)-I+
sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/
2)-2*cos(f*x+e)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f
*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*El
lipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+4*((I*c
os(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin
(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2
*2^(1/2))*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)-2*((I*cos(f*x+e)-I+sin(f*x+e
))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-I*(cos(f*
x+e)-1)/sin(f*x+e))^(1/2)+cos(f*x+e)*2^(1/2)-2*2^(1/2))*sin(f*x+e)/(d/cos(f
*x+e))^(1/2)/(b*sin(f*x+e)/cos(f*x+e))^(3/2)/cos(f*x+e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d\sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\sec\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}}{b^{2}d\sec\left(fx+e\right)\tan\left(fx+e\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e))/(b^2*d*sec(f*x + e)*tan(
f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(1/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d \sec\left(fx+e\right)}\left(b \tan\left(fx+e\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(3/2)), x)
```

3.326 $\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2}{3bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}-\frac{8\sqrt{d\sec(e+fx)}}{3bd^2f\sqrt{b\tan(e+fx)}}$$

```
[Out] 2/(3*b*f*(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]) - (8*Sqrt[d*Sec[e + f
*x]])/(3*b*d^2*f*Sqrt[b*Tan[e + f*x]])
```

Rubi [A] time = 0.108494, antiderivative size = 67, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2609, 2605}

$$-\frac{8(b\tan(e+fx))^{3/2}}{3b^3f(d\sec(e+fx))^{3/2}}-\frac{2}{bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -2/(b*f*(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]) - (8*(b*Tan[e + f*x])^
(3/2))/(3*b^3*f*(d*Sec[e + f*x])^(3/2))
```

Rule 2609

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
n + 1)), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2605

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f
*m), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = -\frac{2}{bf(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}} - \frac{4\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx}{b^2}$$
$$= -\frac{2}{bf(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}} - \frac{8(b\tan(e+fx))^{3/2}}{3b^3f(d\sec(e+fx))^{3/2}}$$

Mathematica [A] time = 0.176048, size = 52, normalized size = 0.72

$$\frac{(\cos(2(e+fx))-7)\sec^2(e+fx)}{3bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] ((-7 + Cos[2*(e + f*x)])*Sec[e + f*x]^2)/(3*b*f*(d*Sec[e + f*x])^(3/2)*Sqrt
[b*Tan[e + f*x]])
```

Maple [A] time = 0.147, size = 60, normalized size = 0.8

$$\frac{2\sin(fx+e)\left(-4+(\cos(fx+e))^2\right)}{3f(\cos(fx+e))^3}\left(\frac{d}{\cos(fx+e)}\right)^{-\frac{3}{2}}\left(\frac{b\sin(fx+e)}{\cos(fx+e)}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x)
```

```
[Out] 2/3/f*sin(f*x+e)*(-4+cos(f*x+e)^2)/(d/cos(f*x+e))^(3/2)/(b*sin(f*x+e)/cos(f
*x+e))^(3/2)/cos(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d\sec(fx+e))^{\frac{3}{2}}(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2)), x)
```

Fricas [A] time = 1.67619, size = 161, normalized size = 2.24

$$\frac{2\left(\cos\left(fx+e\right)^{3}-4\,\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{d}{\cos\left(fx+e\right)}}}{3\,b^{2}d^{2}f\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 2/3*(cos(f*x + e)^3 - 4*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqr
t(d/cos(f*x + e))/(b^2*d^2*f*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(3/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2)), x)
```

3.327 $\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=130

$$-\frac{24E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5b^2d^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{12(b\tan(e+fx))^{3/2}}{5b^3f(d\sec(e+fx))^{5/2}}-\frac{2}{bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}$$

```
[Out] -2/(b*f*(d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]) - (24*EllipticE[(e - P
i/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(5*b^2*d^2*f*Sqrt[d*Sec[e + f*x]]*Sq
rt[Sin[e + f*x]]) - (12*(b*Tan[e + f*x])^(3/2))/(5*b^3*f*(d*Sec[e + f*x])^(
5/2))
```

Rubi [A] time = 0.186373, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2609, 2612, 2616, 2640, 2639}

$$-\frac{24E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5b^2d^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{12(b\tan(e+fx))^{3/2}}{5b^3f(d\sec(e+fx))^{5/2}}-\frac{2}{bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -2/(b*f*(d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]) - (24*EllipticE[(e - P
i/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(5*b^2*d^2*f*Sqrt[d*Sec[e + f*x]]*Sq
rt[Sin[e + f*x]]) - (12*(b*Tan[e + f*x])^(3/2))/(5*b^3*f*(d*Sec[e + f*x])^(
5/2))
```

Rule 2609

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
n + 1)), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2612

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*m
```

```
), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e + f
*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1] &
& EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = -\frac{2}{bf(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} - \frac{6\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx}{b^2}$$

$$= -\frac{2}{bf(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} - \frac{12(b\tan(e+fx))^{3/2}}{5b^3f(d\sec(e+fx))^{5/2}} - \frac{12\int \frac{\sqrt{b\tan(\cdots}}{\sqrt{d\sec(\cdots}}}{5b^2d^{\cdots}}$$

$$= -\frac{2}{bf(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} - \frac{12(b\tan(e+fx))^{3/2}}{5b^3f(d\sec(e+fx))^{5/2}} - \frac{\left(12\sqrt{b\tan(\cdots}\right.}{5b^2d^2\sqrt{d\cdots}}$$

$$= -\frac{2}{bf(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} - \frac{12(b\tan(e+fx))^{3/2}}{5b^3f(d\sec(e+fx))^{5/2}} - \frac{\left(12\sqrt{b\tan(\cdots}\right.}{5b^2d^2\sqrt{d\cdots}}$$

$$= -\frac{2}{bf(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} - \frac{24E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+\cdots}}{5b^2d^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx\cdots}}$$

Mathematica [C] time = 0.656595, size = 81, normalized size = 0.62

$$\frac{24\sqrt[4]{-\tan^2(e+fx)}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\sec^2(e+fx)\right)+\cos(2(e+fx))-11}{5bd^2f\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] (-11 + Cos[2*(e + f*x)] + 24*Hypergeometric2F1[-1/4, 1/4, 3/4, Sec[e + f*x]
^2]*(-Tan[e + f*x]^2)^(1/4))/(5*b*d^2*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e +
 f*x]])
```

Maple [C] time = 0.217, size = 570, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x)
```

```
[Out] 1/5/f*2^(1/2)*(24*cos(f*x+e)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*EllipticE
(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*((I*cos(f*x+e)
-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^
(1/2)-12*cos(f*x+e)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+s
in(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2
)*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+cos
(f*x+e)^3*2^(1/2)+24*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*co
s(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)-I+sin(f*x
+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)-12
*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e
))/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/
2),1/2*2^(1/2))*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)+6*cos(f*x+e)*2^(1/2)-1
2*2^(1/2))*sin(f*x+e)/(d/cos(f*x+e))^(5/2)/(b*sin(f*x+e)/cos(f*x+e))^(3/2)/
cos(f*x+e)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\sec\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}}{b^2d^3\sec\left(fx+e\right)^3\tan\left(fx+e\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e))/(b^2*d^3*sec(f*x + e)^3*
tan(f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(5/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")

[Out] integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2)), x)
```

3.328 $\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=172

$$\frac{d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{b^{5/2}f\sqrt{b\tan(e+fx)}}+\frac{d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{b^{5/2}f\sqrt{b\tan(e+fx)}}-\frac{2d^2(d\sec}{3bf(b\tan}$$

```
[Out] (-2*d^2*(d*Sec[e + f*x])^(3/2))/(3*b*f*(b*Tan[e + f*x])^(3/2)) + (d^3*ArcTa
n[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])/
(b^(5/2)*f*Sqrt[b*Tan[e + f*x]]) + (d^3*ArcTanh[Sqrt[b*Sin[e + f*x]]/Sqrt[b
]]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])/(b^(5/2)*f*Sqrt[b*Tan[e + f*x
]])
```

Rubi [A] time = 0.180373, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2608, 2616, 2564, 329, 212, 206, 203}

$$\frac{d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{b^{5/2}f\sqrt{b\tan(e+fx)}}+\frac{d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{b^{5/2}f\sqrt{b\tan(e+fx)}}-\frac{2d^2(d\sec}{3bf(b\tan}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(7/2)/(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (-2*d^2*(d*Sec[e + f*x])^(3/2))/(3*b*f*(b*Tan[e + f*x])^(3/2)) + (d^3*ArcTa
n[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])/
(b^(5/2)*f*Sqrt[b*Tan[e + f*x]]) + (d^3*ArcTanh[Sqrt[b*Sin[e + f*x]]/Sqrt[b
]]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])/(b^(5/2)*f*Sqrt[b*Tan[e + f*x
]])
```

Rule 2608

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(n + 1)), x] - Dist[(a^2*(m - 2))/(b^2*(n + 1)), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx = -\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}} + \frac{d^2\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx}{b^2}$$

$$= -\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}} + \frac{\left(d^3\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{b\sin(e+fx)}}\,dx}{b^2\sqrt{b\tan(e+fx)}}$$

$$= -\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}} + \frac{\left(d^3\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{b^2}\right)}\,dx, x, b\sin(e+\right.}{b^3 f\sqrt{b\tan(e+fx)}}$$

$$= -\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}} + \frac{\left(2d^3\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\operatorname{Subst}\left(\int \frac{1}{1-\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sin(e+\right.}{b^3 f\sqrt{b\tan(e+fx)}}$$

$$= -\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}} + \frac{\left(d^3\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\operatorname{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sin(e+f\right.}{b^2 f\sqrt{b\tan(e+fx)}}$$

$$= -\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}} + \frac{d^3\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{b^{5/2}f\sqrt{b\tan(e+fx)}} + \frac{d^3\tanh^{-1}\left(\right.}{}$$

Mathematica [A] time = 1.20722, size = 144, normalized size = 0.84

$$-\frac{d^4\sqrt{b\tan(e+fx)}\left(2\sqrt[4]{\tan^2(e+fx)}\csc^2(e+fx)+3\sqrt{\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)-3\sqrt{\sec(e+fx)}\tanh^{-1}\left(\frac{\sqrt{\sec(e+}}{\sqrt[4]{\tan^2(e}}\right.\right.}{3b^3f\sqrt[4]{\tan^2(e+fx)}\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(7/2)/(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] -(d^4*Sqrt[b*Tan[e + f*x]]*(3*ArcTan[Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^(1
/4)]*Sqrt[Sec[e + f*x]] - 3*ArcTanh[Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^(1/
4)]*Sqrt[Sec[e + f*x]] + 2*Csc[e + f*x]^2*(Tan[e + f*x]^2)^(1/4)))/(3*b^3*f
*Sqrt[d*Sec[e + f*x]]*(Tan[e + f*x]^2)^(1/4))
```

Maple [C] time = 0.247, size = 1367, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(5/2),x)
```

```
[Out] 1/6/f*2^(1/2)*(3*I*cos(f*x+e)*sin(f*x+e)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/
2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x
+e))/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^
(1/2),1/2-1/2*I,1/2*2^(1/2))-6*I*cos(f*x+e)*sin(f*x+e)*((I*cos(f*x+e)-I+sin
(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*
EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-I*(
cos(f*x+e)-1)/sin(f*x+e))^(1/2)+3*I*cos(f*x+e)*sin(f*x+e)*(-I*(cos(f*x+e)-1
)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos
(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x
+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*cos(f*x+e)*sin(f*x+e)*(-I*(
cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1
/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+
e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))+3*cos(f*x+e)*sin(
f*x+e)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/si
n(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(
((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*I*s
in(f*x+e)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))
/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*Elliptic
Pi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-6*
I*sin(f*x+e)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+
e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*Ellip
ticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+3*I*sin(f*
x+e)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(
f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((
I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*sin(f
*x+e)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(
f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e
))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)+3*sin(
f*x+e)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin
(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+
e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)-2*2^(
1/2))*(d/cos(f*x+e))^(7/2)*sin(f*x+e)*cos(f*x+e)/(b*sin(f*x+e)/cos(f*x+e))^
(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{\frac{7}{2}}}{\left(b\tan\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(7/2)/(b*tan(f*x + e))^(5/2), x)
```

Fricas [B] time = 4.12252, size = 2111, normalized size = 12.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(16*d^3*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*
x + e) - 6*(b*d^3*cos(f*x + e)^2 - b*d^3)*sqrt(-d/b)*arctan(1/4*(cos(f*x +
e)^3 - 5*cos(f*x + e)^2 - (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e
) - 2*cos(f*x + e) + 4)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-d/b)*sqrt(d
/cos(f*x + e))/(d*cos(f*x + e)^2 - (d*cos(f*x + e) + d)*sin(f*x + e) - d))
+ 3*(b*d^3*cos(f*x + e)^2 - b*d^3)*sqrt(-d/b)*log((d*cos(f*x + e)^4 - 72*d*
cos(f*x + e)^2 + 8*(7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*si
n(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-d/b)*s
qrt(d/cos(f*x + e)) + 28*(d*cos(f*x + e)^2 - 2*d)*sin(f*x + e) + 72*d)/(cos
(f*x + e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)))
/(b^3*f*cos(f*x + e)^2 - b^3*f), 1/24*(16*d^3*sqrt(b*sin(f*x + e)/cos(f*x +
 e))*sqrt(d/cos(f*x + e))*cos(f*x + e) + 6*(b*d^3*cos(f*x + e)^2 - b*d^3)*s
qrt(d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 + (cos(f*x + e)^2 +
6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(b*sin(f*x + e)/
cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e))/(d*cos(f*x + e)^2 + (d*cos(f*x
 + e) + d)*sin(f*x + e) - d)) + 3*(b*d^3*cos(f*x + e)^2 - b*d^3)*sqrt(d/b)*
log((d*cos(f*x + e)^4 - 72*d*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3 + (cos(f*
x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x +
e)/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e)) - 28*(d*cos(f*x + e)^2 - 2*
d)*sin(f*x + e) + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 + 4*(cos(f*x + e
)^2 - 2)*sin(f*x + e) + 8)))/(b^3*f*cos(f*x + e)^2 - b^3*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(7/2)/(b*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{\frac{7}{2}}}{\left(b\tan\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(7/2)/(b*tan(f*x + e))^(5/2), x)
```

3.329 $\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=101

$$\frac{2d^2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\sec(e+fx)}}{3b^2f\sqrt{b\tan(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

```
[Out] (-2*d^2*Sqrt[d*Sec[e + f*x]])/(3*b*f*(b*Tan[e + f*x])^(3/2)) + (2*d^2*Ellip
ticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]])/(3*b^2
*f*Sqrt[b*Tan[e + f*x]])
```

Rubi [A] time = 0.126031, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2608, 2616, 2642, 2641}

$$\frac{2d^2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\sec(e+fx)}}{3b^2f\sqrt{b\tan(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/2)/(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (-2*d^2*Sqrt[d*Sec[e + f*x]])/(3*b*f*(b*Tan[e + f*x])^(3/2)) + (2*d^2*Ellip
ticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]])/(3*b^2
*f*Sqrt[b*Tan[e + f*x]])
```

Rule 2608

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a^2*(a*Sec[e + f*x])^(m - 2)*(b*Tan[e + f*x])^(n +
1))/(b*f*(n + 1)), x] - Dist[(a^2*(m - 2))/(b^2*(n + 1)), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
```

```
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx &= -\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}+\frac{d^2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx}{3b^2}\\
&= -\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}+\frac{\left(d^2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\int \frac{1}{\sqrt{b\sin(e+fx)}}\,dx}{3b^2\sqrt{b\tan(e+fx)}}\\
&= -\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}+\frac{\left(d^2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx}{3b^2\sqrt{b\tan(e+fx)}}\\
&= -\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}+\frac{2d^2F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{3b^2f\sqrt{b\tan(e+fx)}}
\end{aligned}
$$

Mathematica [C] time = 0.443671, size = 116, normalized size = 1.15

$$
\frac{2d^3\sqrt{b\tan(e+fx)}\left(\sqrt{2}\sqrt{\sec(e+fx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-\cot(e+fx)\csc(e+fx)\sqrt{\sec(e+fx)+1}\right)}{3b^3f\sqrt{\sec(e+fx)+1}\sqrt{d\sec(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)/(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (2*d^3*(Sqrt[2]*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[(e + f*x)/2]^2]*Sqrt[
Sec[e + f*x]] - Cot[e + f*x]*Csc[e + f*x]*Sqrt[1 + Sec[e + f*x]])*Sqrt[b*Ta
```

```
n[e + f*x]])/(3*b^3*f*Sqrt[d*Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]])
```

Maple [C] time = 0.205, size = 315, normalized size = 3.1

$$\frac{\sqrt{2}\sin\left(fx+e\right)}{3f}\left(i\cos\left(fx+e\right)\sin\left(fx+e\right)\sqrt{\frac{i\cos\left(fx+e\right)-i+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{-\frac{i\cos\left(fx+e\right)-i-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\text{Ellipt}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x)
```

```
[Out] 1/3/f*2^(1/2)*(I*cos(f*x+e)*sin(f*x+e)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x
+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((I*co
s(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-I*(cos(f*x+e)-1)/si
n(f*x+e))^(1/2)+I*sin(f*x+e)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f
*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x
+e))^(1/2)*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(
1/2))-cos(f*x+e)*2^(1/2))*(d/cos(f*x+e))^(5/2)*sin(f*x+e)/(b*sin(f*x+e)/cos
(f*x+e))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b\tan\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\sec\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}d^{2}\sec\left(fx+e\right)^{2}}{b^{3}\tan\left(fx+e\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e))*d^2*sec(f*x + e)^2/(b^3*
tan(f*x + e)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/2)/(b*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b\tan\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e))^(5/2), x)
```

3.330 $\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=34

$$-\frac{2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

```
[Out] (-2*(d*Sec[e + f*x])^(3/2))/(3*b*f*(b*Tan[e + f*x])^(3/2))
```

Rubi [A] time = 0.0568758, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2605}

$$-\frac{2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(3/2)/(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (-2*(d*Sec[e + f*x])^(3/2))/(3*b*f*(b*Tan[e + f*x])^(3/2))
```

Rule 2605

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f
*m), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}\,dx = -\frac{2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

Mathematica [A] time = 0.156271, size = 34, normalized size = 1.

$$-\frac{2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)/(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (-2*(d*Sec[e + f*x])^(3/2))/(3*b*f*(b*Tan[e + f*x])^(3/2))
```

Maple [A] time = 0.133, size = 50, normalized size = 1.5

$$-\frac{2\sin\left(fx+e\right)}{3f\cos\left(fx+e\right)}\left(\frac{d}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}\left(\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x)
```

```
[Out] -2/3/f*(d/cos(f*x+e))^(3/2)*sin(f*x+e)/(b*sin(f*x+e)/cos(f*x+e))^(5/2)/cos(
f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\tan\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e))^(5/2), x)
```

Fricas [B] time = 1.62757, size = 143, normalized size = 4.21

$$\frac{2\,d\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{d}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)}{3\left(b^3f\cos\left(fx+e\right)^2-b^3f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*d*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)/(
b^3*f*cos(f*x + e)^2 - b^3*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(3/2)/(b*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \sec \left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b \tan \left(fx+e\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e))^(5/2), x)
```

3.331 $\int \frac{\sqrt{d \sec(e+fx)}}{(b \tan(e+fx))^{5/2}} \, dx$

Optimal. Leaf size=95

$$-\frac{4\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\sec(e+fx)}}{3b^2f\sqrt{b\tan(e+fx)}}-\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

```
[Out] (-2*Sqrt[d*Sec[e + f*x]])/(3*b*f*(b*Tan[e + f*x])^(3/2)) - (4*EllipticF[(e
- Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]])/(3*b^2*f*Sqrt[
b*Tan[e + f*x]])
```

Rubi [A] time = 0.114301, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2609, 2616, 2642, 2641}

$$-\frac{4\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\sec(e+fx)}}{3b^2f\sqrt{b\tan(e+fx)}}-\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sec[e + f*x]]/(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (-2*Sqrt[d*Sec[e + f*x]])/(3*b*f*(b*Tan[e + f*x])^(3/2)) - (4*EllipticF[(e
- Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]])/(3*b^2*f*Sqrt[
b*Tan[e + f*x]])
```

Rule 2609

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
n + 1)), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
```

```
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx &= -\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}} - \frac{2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx}{3b^2} \\
&= -\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}} - \frac{\left(2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\int \frac{1}{\sqrt{b\sin(e+fx)}}\,dx}{3b^2\sqrt{b\tan(e+fx)}} \\
&= -\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}} - \frac{\left(2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx}{3b^2\sqrt{b\tan(e+fx)}} \\
&= -\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}} - \frac{4F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{3b^2f\sqrt{b\tan(e+fx)}}
\end{aligned}$$

Mathematica [C] time = 0.705384, size = 70, normalized size = 0.74

$$-\frac{2\sqrt{d\sec(e+fx)}\left(2\left(-\tan^2(e+fx)\right)^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};\sec^2(e+fx)\right)+1\right)}{3bf(b\tan(e+fx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]/(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (-2*Sqrt[d*Sec[e + f*x]]*(1 + 2*Hypergeometric2F1[1/4, 3/4, 5/4, Sec[e + f*
x]^2]*(-Tan[e + f*x]^2)^(3/4)))/(3*b*f*(b*Tan[e + f*x])^(3/2))
```

Maple [C] time = 0.214, size = 322, normalized size = 3.4

$$-\frac{\sqrt{2}\,\sin\left(fx+e\right)}{3\,f\left(\cos\left(fx+e\right)\right)^{2}}\left(2\,i\cos\left(fx+e\right)\sin\left(fx+e\right)\sqrt{\frac{i\cos\left(fx+e\right)-i+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{-\frac{i\cos\left(fx+e\right)-i-\sin\left(fx+e}{\sin\left(fx+e\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x)
```

```
[Out] -1/3/f*2^(1/2)*(2*I*cos(f*x+e)*sin(f*x+e)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(
f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((I
*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-I*(cos(f*x+e)-1)
/sin(f*x+e))^(1/2)+2*I*sin(f*x+e)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^
(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x
+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-I*(cos(f*x+e)-1)/sin(f*x
+e))^(1/2)+cos(f*x+e)*2^(1/2))*(d/cos(f*x+e))^(1/2)*sin(f*x+e)/cos(f*x+e)^2
/(b*sin(f*x+e)/cos(f*x+e))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sec\left(fx+e\right)}}{\left(b\tan\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\sec\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}}{b^{3}\tan\left(fx+e\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e))/(b^3*tan(f*x + e)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/2)/(b*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d \sec \left(fx+e\right)}}{\left(b \tan \left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e))^(5/2), x)
```

3.332 $\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=69

$$-\frac{8\sqrt{b\tan(e+fx)}}{3b^3f\sqrt{d\sec(e+fx)}}-\frac{2}{3bf(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}$$

```
[Out] -2/(3*b*f*Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2)) - (8*Sqrt[b*Tan[e +
f*x]])/(3*b^3*f*Sqrt[d*Sec[e + f*x]])
```

Rubi [A] time = 0.10089, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2609, 2605}

$$-\frac{8\sqrt{b\tan(e+fx)}}{3b^3f\sqrt{d\sec(e+fx)}}-\frac{2}{3bf(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(5/2)),x]
```

```
[Out] -2/(3*b*f*Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2)) - (8*Sqrt[b*Tan[e +
f*x]])/(3*b^3*f*Sqrt[d*Sec[e + f*x]])
```

Rule 2609

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
n + 1)), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2605

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f
*m), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}}\,dx = -\frac{2}{3bf\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}} - \frac{4\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx}{3b^2}$$
$$= -\frac{2}{3bf\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}} - \frac{8\sqrt{b\tan(e+fx)}}{3b^3f\sqrt{d\sec(e+fx)}}$$

Mathematica [A] time = 0.851484, size = 110, normalized size = 1.59

$$-\frac{2\left(3\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)+1}\sqrt{\sec(e+fx)}+\sqrt{\frac{1}{\cos(e+fx)+1}}\csc(e+fx)\sec(e+fx)\right)}{3b^2f\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(5/2)),x]
```

```
[Out] (-2*(Sqrt[(1 + Cos[e + f*x])^(-1)]*Csc[e + f*x]*Sec[e + f*x] + 3*Sqrt[Sec[e
 + f*x]]*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]))/(3*b^2*f*Sqrt[(1 + Cos[e
 + f*x])^(-1)]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [A] time = 0.177, size = 62, normalized size = 0.9

$$\frac{2\sin(fx+e)\left(3(\cos(fx+e))^2-4\right)}{3f(\cos(fx+e))^3}\frac{1}{\sqrt{\frac{d}{\cos(fx+e)}}}\left(\frac{b\sin(fx+e)}{\cos(fx+e)}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x)
```

```
[Out] 2/3/f*sin(f*x+e)*(3*cos(f*x+e)^2-4)/cos(f*x+e)^3/(d/cos(f*x+e))^(1/2)/(b*si
n(f*x+e)/cos(f*x+e))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d\sec(fx+e)}\,(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(5/2)), x)
```

Fricas [A] time = 1.74557, size = 178, normalized size = 2.58

$$-\frac{2\left(3\,\cos\left(fx+e\right)^{3}-4\,\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{d}{\cos\left(fx+e\right)}}}{3\left(b^{3}df\cos\left(fx+e\right)^{2}-b^{3}df\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] -2/3*(3*cos(f*x + e)^3 - 4*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*
sqrt(d/cos(f*x + e))/(b^3*d*f*cos(f*x + e)^2 - b^3*d*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(1/2)/(b*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d\sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(5/2)), x)
```

3.333 $\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=132

$$-\frac{8\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\sec(e+fx)}}{3b^2d^2f\sqrt{b\tan(e+fx)}}-\frac{4\sqrt{b\tan(e+fx)}}{3b^3f(d\sec(e+fx))^{3/2}}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

```
[Out] -2/(3*b*f*(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2)) - (8*EllipticF[(e
- Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]])/(3*b^2*d^2*f*S
qrt[b*Tan[e + f*x]]) - (4*Sqrt[b*Tan[e + f*x]])/(3*b^3*f*(d*Sec[e + f*x])^(
3/2))
```

Rubi [A] time = 0.184395, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2609, 2612, 2616, 2642, 2641}

$$-\frac{8\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\sec(e+fx)}}{3b^2d^2f\sqrt{b\tan(e+fx)}}-\frac{4\sqrt{b\tan(e+fx)}}{3b^3f(d\sec(e+fx))^{3/2}}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(5/2)),x]
```


```
[Out] -2/(3*b*f*(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2)) - (8*EllipticF[(e
- Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]])/(3*b^2*d^2*f*S
qrt[b*Tan[e + f*x]]) - (4*Sqrt[b*Tan[e + f*x]])/(3*b^3*f*(d*Sec[e + f*x])^(
3/2))
```


Rule 2609

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
n + 1)), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2612

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*m
```

```
), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e + f
*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1] &
& EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2616

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^(m + n)*(b*Tan[e + f*x])^n)/((a*Sec[e + f*x])^n*(b
*Sin[e + f*x])^n), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx = -\frac{2}{3bf(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}} - \frac{2\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx}{b^2}$$

$$= -\frac{2}{3bf(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}} - \frac{4\sqrt{b\tan(e+fx)}}{3b^3f(d\sec(e+fx))^{3/2}} - \frac{4\int \frac{\sqrt{d\,\mathrm{se}}}{\sqrt{b\,\mathrm{ta}}}}{3b}$$

$$= -\frac{2}{3bf(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}} - \frac{4\sqrt{b\tan(e+fx)}}{3b^3f(d\sec(e+fx))^{3/2}} - \frac{\left(4\sqrt{d\,\mathrm{se}}\right.}{}$$

$$= -\frac{2}{3bf(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}} - \frac{4\sqrt{b\tan(e+fx)}}{3b^3f(d\sec(e+fx))^{3/2}} - \frac{\left(4\sqrt{d\,\mathrm{se}}\right.}{}$$

$$= -\frac{2}{3bf(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}} - \frac{8F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{d\sec(e+}}{3b^2d^2f\sqrt{b\tan(e+}}$$

Mathematica [C] time = 2.37566, size = 112, normalized size = 0.85

$$\frac{\left(-\tan^2(e+fx)\right)^{3/4}\csc^2(e+fx)\sqrt{b\tan(e+fx)}\left(\sqrt[4]{-\tan^2(e+fx)}\left(\cos(2(e+fx))+2\csc^2(e+fx)-1\right)-8\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4}\right.\right.}{3b^3f(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(5/2)),x]
```

```
[Out] (Csc[e + f*x]^2*Sqrt[b*Tan[e + f*x]]*(-Tan[e + f*x]^2)^(3/4)*(-8*Hypergeome
tric2F1[1/4, 3/4, 5/4, Sec[e + f*x]^2] + (-1 + Cos[2*(e + f*x)] + 2*Csc[e +
 f*x]^2)*(-Tan[e + f*x]^2)^(1/4)))/(3*b^3*f*(d*Sec[e + f*x])^(3/2))
```

Maple [C] time = 0.209, size = 335, normalized size = 2.5

$$\frac{\sqrt{2}\sin\left(fx+e\right)}{3f\left(\cos\left(fx+e\right)\right)^4}\left(-4i\cos\left(fx+e\right)\sin\left(fx+e\right)\sqrt{\frac{i\cos\left(fx+e\right)-i+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{-\frac{i\cos\left(fx+e\right)-i-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x)
```

```
[Out] 1/3/f*2^(1/2)*(-4*I*cos(f*x+e)*sin(f*x+e)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(
f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((I
*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-I*(cos(f*x+e)-1)
/sin(f*x+e))^(1/2)-4*I*sin(f*x+e)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*
cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/si
n(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/
2*2^(1/2))+cos(f*x+e)^3*2^(1/2)-2*cos(f*x+e)*2^(1/2))*sin(f*x+e)/(d/cos(f*x
+e))^(3/2)/(b*sin(f*x+e)/cos(f*x+e))^(5/2)/cos(f*x+e)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\tan\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\sec\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}}{b^3d^2\sec\left(fx+e\right)^2\tan\left(fx+e\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e))/(b^3*d^2*sec(f*x + e)^2*
tan(f*x + e)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(3/2)/(b*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\tan\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")

[Out] integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(5/2)), x)
```

3.334 $\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=106

$$-\frac{64\sqrt{b\tan(e+fx)}}{15b^3d^2f\sqrt{d\sec(e+fx)}}-\frac{16\sqrt{b\tan(e+fx)}}{15b^3f(d\sec(e+fx))^{5/2}}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}$$

```
[Out] -2/(3*b*f*(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2)) - (16*Sqrt[b*Tan[e
 + f*x]])/(15*b^3*f*(d*Sec[e + f*x])^(5/2)) - (64*Sqrt[b*Tan[e + f*x]])/(15
*b^3*d^2*f*Sqrt[d*Sec[e + f*x]])
```

Rubi [A] time = 0.165777, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2609, 2612, 2605}

$$-\frac{64\sqrt{b\tan(e+fx)}}{15b^3d^2f\sqrt{d\sec(e+fx)}}-\frac{16\sqrt{b\tan(e+fx)}}{15b^3f(d\sec(e+fx))^{5/2}}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(5/2)),x]
```

```
[Out] -2/(3*b*f*(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2)) - (16*Sqrt[b*Tan[e
 + f*x]])/(15*b^3*f*(d*Sec[e + f*x])^(5/2)) - (64*Sqrt[b*Tan[e + f*x]])/(15
*b^3*d^2*f*Sqrt[d*Sec[e + f*x]])
```

Rule 2609

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
n + 1)), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2612

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*m
), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e + f
*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1] &
```

```
& EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2605

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f
*m), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx = -\frac{2}{3bf(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}} - \frac{8\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx}{3b^2}$$

$$= -\frac{2}{3bf(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}} - \frac{16\sqrt{b\tan(e+fx)}}{15b^3f(d\sec(e+fx))^{5/2}} - \frac{32\int$$

$$= -\frac{2}{3bf(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}} - \frac{16\sqrt{b\tan(e+fx)}}{15b^3f(d\sec(e+fx))^{5/2}} - \frac{64}{15b^3d$$

Mathematica [A] time = 3.05235, size = 159, normalized size = 1.5

$$\frac{-6\sqrt{\frac{1}{\cos(e+fx)+1}}(2\cos(2(e+fx))-1)\tan(e+fx) - 228\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)+1}\sqrt{\sec(e+fx)} + \sqrt{\frac{1}{\cos(e+fx)+1}}}{60b^2d^2f\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(5/2)),x]
```

```
[Out] (Sqrt[(1 + Cos[e + f*x])^(-1)]*(-43 + 3*Cos[2*(e + f*x)])*Csc[e + f*x]*Sec[
e + f*x] - 228*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2] -
 6*Sqrt[(1 + Cos[e + f*x])^(-1)]*(-1 + 2*Cos[2*(e + f*x)])*Tan[e + f*x])/(6
0*b^2*d^2*f*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e
 + f*x]])
```

Maple [A] time = 0.156, size = 72, normalized size = 0.7

$$\frac{2\sin\left(fx+e\right)\left(3\left(\cos\left(fx+e\right)\right)^{4}+24\left(\cos\left(fx+e\right)\right)^{2}-32\right)}{15f\left(\cos\left(fx+e\right)\right)^{5}}\left(\frac{d}{\cos\left(fx+e\right)}\right)^{-\frac{5}{2}}\left(\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x)
```

```
[Out] 2/15/f*sin(f*x+e)*(3*cos(f*x+e)^4+24*cos(f*x+e)^2-32)/(d/cos(f*x+e))^(5/2)/
(b*sin(f*x+e)/cos(f*x+e))^(5/2)/cos(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\tan\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(5/2)), x)
```

Fricas [A] time = 1.93458, size = 213, normalized size = 2.01

$$-\frac{2\left(3\cos\left(fx+e\right)^{5}+24\cos\left(fx+e\right)^{3}-32\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{d}{\cos\left(fx+e\right)}}}{15\left(b^{3}d^{3}f\cos\left(fx+e\right)^{2}-b^{3}d^{3}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] -2/15*(3*cos(f*x + e)^5 + 24*cos(f*x + e)^3 - 32*cos(f*x + e))*sqrt(b*sin(f
*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b^3*d^3*f*cos(f*x + e)^2 - b^3*
```

```
d^3*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(5/2)/(b*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\tan\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(5/2)), x)
```

3.335 $\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx$

Optimal. Leaf size=64

$$\frac{2\cos^2(e+fx)^{17/12}(b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{17}{12};\frac{7}{4};\sin^2(e+fx)\right)}{3df}$$

```
[Out] (2*(Cos[e + f*x]^2)^(17/12)*Hypergeometric2F1[3/4, 17/12, 7/4, Sin[e + f*x]
^2]*(b*Sec[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2))/(3*d*f)
```

Rubi [A] time = 0.0530586, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2617}

$$\frac{2\cos^2(e+fx)^{17/12}(b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{17}{12};\frac{7}{4};\sin^2(e+fx)\right)}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(4/3)*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(17/12)*Hypergeometric2F1[3/4, 17/12, 7/4, Sin[e + f*x]
^2]*(b*Sec[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2))/(3*d*f)
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx=\frac{2\cos^2(e+fx)^{17/12}\,{}_2F_1\left(\frac{3}{4},\frac{17}{12};\frac{7}{4};\sin^2(e+fx)\right)(b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/}}{3df}$$

Mathematica [A] time = 0.0994543, size = 64, normalized size = 1.

$$\frac{3d\sqrt[4]{-\tan^2(e+fx)}(b\sec(e+fx))^{4/3}\,{}_2F_1\left(\frac{1}{4},\frac{2}{3};\frac{5}{3};\sec^2(e+fx)\right)}{4f\sqrt{d\tan(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Sec[e + f*x])^(4/3)*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (3*d*Hypergeometric2F1[1/4, 2/3, 5/3, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(4/3
)*(-Tan[e + f*x]^2)^(1/4))/(4*f*Sqrt[d*Tan[e + f*x]])
```

Maple [F] time = 0.173, size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{4}{3}}\sqrt{d\tan\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x)
```

```
[Out] int((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{4}{3}}\sqrt{d\tan\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(4/3)*sqrt(d*tan(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sec\left(fx+e\right)\right)^{\frac{1}{3}}\sqrt{d\tan\left(fx+e\right)}b\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^(1/3)*sqrt(d*tan(f*x + e))*b*sec(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(4/3)*(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{4}{3}}\sqrt{d\tan\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(4/3)*sqrt(d*tan(f*x + e)), x)
```

3.336 $\int \sqrt[3]{b \sec(e+fx)}\sqrt{d \tan(e+fx)}\,dx$

Optimal. Leaf size=64

$$\frac{2\cos^2(e+fx)^{11/12}\sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{11}{12};\frac{7}{4};\sin^2(e+fx)\right)}{3df}$$

```
[Out] (2*(Cos[e + f*x]^2)^(11/12)*Hypergeometric2F1[3/4, 11/12, 7/4, Sin[e + f*x]
^2]*(b*Sec[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2))/(3*d*f)
```

Rubi [A] time = 0.04489, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2617}

$$\frac{2\cos^2(e+fx)^{11/12}\sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{11}{12};\frac{7}{4};\sin^2(e+fx)\right)}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(1/3)*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(11/12)*Hypergeometric2F1[3/4, 11/12, 7/4, Sin[e + f*x]
^2]*(b*Sec[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2))/(3*d*f)
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \frac{2\cos^2(e+fx)^{11/12}\,{}_2F_1\left(\frac{3}{4},\frac{11}{12};\frac{7}{4};\sin^2(e+fx)\right)\sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}}{3df}$$

Mathematica [A] time = 0.0784136, size = 62, normalized size = 0.97

$$\frac{3d\sqrt[4]{-\tan^2(e+fx)}\sqrt[3]{b\sec(e+fx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{4};\frac{7}{6};\sec^2(e+fx)\right)}{f\sqrt{d\tan(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Sec[e + f*x])^(1/3)*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (3*d*Hypergeometric2F1[1/6, 1/4, 7/6, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(1/3
)*(-Tan[e + f*x]^2)^(1/4))/(f*Sqrt[d*Tan[e + f*x]])
```

Maple [F] time = 0.262, size = 0, normalized size = 0.

$$\int \sqrt[3]{b\sec\left(fx+e\right)}\sqrt{d\tan\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x)
```

```
[Out] int((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{1}{3}}\sqrt{d\tan\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(1/3)*sqrt(d*tan(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sec\left(fx+e\right)\right)^{\frac{1}{3}}\sqrt{d\tan\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^(1/3)*sqrt(d*tan(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{b \sec \left(e + fx\right)} \sqrt{d \tan \left(e + fx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(1/3)*(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral((b*sec(e + f*x))**(1/3)*sqrt(d*tan(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(fx + e\right)\right)^{\frac{1}{3}} \sqrt{d \tan \left(fx + e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(1/3)*sqrt(d*tan(f*x + e)), x)
```

3.337 $\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=64

$$\frac{2\cos^2(e+fx)^{7/12}(d\tan(e+fx))^{3/2}\,{}_2F_1\left(\frac{7}{12},\frac{3}{4};\frac{7}{4};\sin^2(e+fx)\right)}{3df\sqrt[3]{b\sec(e+fx)}}$$

```
[Out] (2*(Cos[e + f*x]^2)^(7/12)*Hypergeometric2F1[7/12, 3/4, 7/4, Sin[e + f*x]^2
]*(d*Tan[e + f*x])^(3/2))/(3*d*f*(b*Sec[e + f*x])^(1/3))
```

Rubi [A] time = 0.0484381, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2617}

$$\frac{2\cos^2(e+fx)^{7/12}(d\tan(e+fx))^{3/2}\,{}_2F_1\left(\frac{7}{12},\frac{3}{4};\frac{7}{4};\sin^2(e+fx)\right)}{3df\sqrt[3]{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Tan[e + f*x]]/(b*Sec[e + f*x])^(1/3),x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(7/12)*Hypergeometric2F1[7/12, 3/4, 7/4, Sin[e + f*x]^2
]*(d*Tan[e + f*x])^(3/2))/(3*d*f*(b*Sec[e + f*x])^(1/3))
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \frac{2\cos^2(e+fx)^{7/12}\,{}_2F_1\left(\frac{7}{12},\frac{3}{4};\frac{7}{4};\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{3df\sqrt[3]{b\sec(e+fx)}}$$

Mathematica [A] time = 0.0929996, size = 62, normalized size = 0.97

$$-\frac{3d\sqrt[4]{-\tan^2(e+fx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{4};\frac{5}{6};\sec^2(e+fx)\right)}{f\sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d*Tan[e + f*x]]/(b*Sec[e + f*x])^(1/3),x]
```

```
[Out] (-3*d*Hypergeometric2F1[-1/6, 1/4, 5/6, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(
1/4))/(f*(b*Sec[e + f*x])^(1/3)*Sqrt[d*Tan[e + f*x]])
```

Maple [F] time = 0.241, size = 0, normalized size = 0.

$$\int \sqrt{d\tan\left(fx+e\right)}\frac{1}{\sqrt[3]{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/3),x)
```

```
[Out] int((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\tan\left(fx+e\right)}}{\left(b\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*tan(f*x + e))/(b*sec(f*x + e))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\sec\left(fx+e\right)\right)^{\frac{2}{3}}\sqrt{d\tan\left(fx+e\right)}}{b\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))/(b*sec(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{d\tan\left(e+fx\right)}}{\sqrt[3]{b\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(1/2)/(b*sec(f*x+e))**(1/3),x)
```

```
[Out] Integral(sqrt(d*tan(e + f*x))/(b*sec(e + f*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{d\tan\left(fx+e\right)}}{\left(b\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*tan(f*x + e))/(b*sec(f*x + e))^(1/3), x)
```

3.338 $\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx$

Optimal. Leaf size=64

$$\frac{2\sqrt[12]{\cos^2(e+fx)}(d\tan(e+fx))^{3/2}\,{}_2F_1\left(\frac{1}{12},\frac{3}{4};\frac{7}{4};\sin^2(e+fx)\right)}{3df(b\sec(e+fx))^{4/3}}$$

```
[Out] (2*(Cos[e + f*x]^2)^(1/12)*Hypergeometric2F1[1/12, 3/4, 7/4, Sin[e + f*x]^2
]*(d*Tan[e + f*x])^(3/2))/(3*d*f*(b*Sec[e + f*x])^(4/3))
```

Rubi [A] time = 0.0541346, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2617}

$$\frac{2\sqrt[12]{\cos^2(e+fx)}(d\tan(e+fx))^{3/2}\,{}_2F_1\left(\frac{1}{12},\frac{3}{4};\frac{7}{4};\sin^2(e+fx)\right)}{3df(b\sec(e+fx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Tan[e + f*x]]/(b*Sec[e + f*x])^(4/3),x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(1/12)*Hypergeometric2F1[1/12, 3/4, 7/4, Sin[e + f*x]^2
]*(d*Tan[e + f*x])^(3/2))/(3*d*f*(b*Sec[e + f*x])^(4/3))
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx = \frac{2\sqrt[12]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{12},\frac{3}{4};\frac{7}{4};\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{3df(b\sec(e+fx))^{4/3}}$$

Mathematica [A] time = 0.123525, size = 64, normalized size = 1.

$$-\frac{3d\sqrt[4]{-\tan^2(e+fx)}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{4};\frac{1}{3};\sec^2(e+fx)\right)}{4f(b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d*Tan[e + f*x]]/(b*Sec[e + f*x])^(4/3),x]
```

```
[Out] (-3*d*Hypergeometric2F1[-2/3, 1/4, 1/3, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(
1/4))/(4*f*(b*Sec[e + f*x])^(4/3)*Sqrt[d*Tan[e + f*x]])
```

Maple [F] time = 0.207, size = 0, normalized size = 0.

$$\int \sqrt{d\tan\left(fx+e\right)}\left(b\sec\left(fx+e\right)\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(4/3),x)
```

```
[Out] int((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\tan\left(fx+e\right)}}{\left(b\sec\left(fx+e\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*tan(f*x + e))/(b*sec(f*x + e))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\sec\left(fx+e\right)\right)^{\frac{2}{3}}\sqrt{d\tan\left(fx+e\right)}}{b^2\sec\left(fx+e\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))/(b^2*sec(f*x + e)^2),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(1/2)/(b*sec(f*x+e))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{d\tan\left(fx+e\right)}}{\left(b\sec\left(fx+e\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*tan(f*x + e))/(b*sec(f*x + e))^(4/3), x)
```

3.339 $\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=64

$$\frac{2\cos^2(e+fx)^{23/12}(b\sec(e+fx))^{4/3}(d\tan(e+fx))^{5/2}\,{}_2F_1\left(\frac{5}{4},\frac{23}{12};\frac{9}{4};\sin^2(e+fx)\right)}{5df}$$

```
[Out] (2*(Cos[e + f*x]^2)^(23/12)*Hypergeometric2F1[5/4, 23/12, 9/4, Sin[e + f*x]
^2]*(b*Sec[e + f*x])^(4/3)*(d*Tan[e + f*x])^(5/2))/(5*d*f)
```

Rubi [A] time = 0.0580486, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2617}

$$\frac{2\cos^2(e+fx)^{23/12}(b\sec(e+fx))^{4/3}(d\tan(e+fx))^{5/2}\,{}_2F_1\left(\frac{5}{4},\frac{23}{12};\frac{9}{4};\sin^2(e+fx)\right)}{5df}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(23/12)*Hypergeometric2F1[5/4, 23/12, 9/4, Sin[e + f*x]
^2]*(b*Sec[e + f*x])^(4/3)*(d*Tan[e + f*x])^(5/2))/(5*d*f)
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx=\frac{2\cos^2(e+fx)^{23/12}\,{}_2F_1\left(\frac{5}{4},\frac{23}{12};\frac{9}{4};\sin^2(e+fx)\right)(b\sec(e+fx))^{4/3}(d\tan(e+fx)}{5df}$$

Mathematica [A] time = 0.144881, size = 64, normalized size = 1.

$$\frac{3d(b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,{}_2F_1\left(-\frac{1}{4},\frac{2}{3};\frac{5}{3};\sec^2(e+fx)\right)}{4f\sqrt[4]{-\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (3*d*Hypergeometric2F1[-1/4, 2/3, 5/3, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(4/
3)*Sqrt[d*Tan[e + f*x]])/(4*f*(-Tan[e + f*x]^2)^(1/4))
```

Maple [F] time = 0.146, size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{4}{3}}\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x)
```

```
[Out] int((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{4}{3}}\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(4/3)*(d*tan(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)\right)^{\frac{1}{3}}\sqrt{d\tan\left(fx+e\right)}bd\sec\left(fx+e\right)\tan\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^(1/3)*sqrt(d*tan(f*x + e))*b*d*sec(f*x + e)*tan(f
*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(4/3)*(d*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(fx+e\right)\right)^{\frac{4}{3}} \left(d \tan \left(fx+e\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(4/3)*(d*tan(f*x + e))^(3/2), x)
```

3.340 $\int \sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=64

$$\frac{2\cos^2(e+fx)^{17/12}\sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{5/2}\,{}_2F_1\left(\frac{5}{4},\frac{17}{12};\frac{9}{4};\sin^2(e+fx)\right)}{5df}$$

```
[Out] (2*(Cos[e + f*x]^2)^(17/12)*Hypergeometric2F1[5/4, 17/12, 9/4, Sin[e + f*x]
^2]*(b*Sec[e + f*x])^(1/3)*(d*Tan[e + f*x])^(5/2))/(5*d*f)
```

Rubi [A] time = 0.0511223, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2617}

$$\frac{2\cos^2(e+fx)^{17/12}\sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{5/2}\,{}_2F_1\left(\frac{5}{4},\frac{17}{12};\frac{9}{4};\sin^2(e+fx)\right)}{5df}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(17/12)*Hypergeometric2F1[5/4, 17/12, 9/4, Sin[e + f*x]
^2]*(b*Sec[e + f*x])^(1/3)*(d*Tan[e + f*x])^(5/2))/(5*d*f)
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,dx=\frac{2\cos^2(e+fx)^{17/12}\,{}_2F_1\left(\frac{5}{4},\frac{17}{12};\frac{9}{4};\sin^2(e+fx)\right)\sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{5/}}{5df}$$

Mathematica [A] time = 0.132816, size = 62, normalized size = 0.97

$$\frac{3d\sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{6};\frac{7}{6};\sec^2(e+fx)\right)}{f\sqrt[4]{-\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (3*d*Hypergeometric2F1[-1/4, 1/6, 7/6, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(1/
3)*Sqrt[d*Tan[e + f*x]])/(f*(-Tan[e + f*x]^2)^(1/4))
```

Maple [F] time = 0.14, size = 0, normalized size = 0.

$$\int\sqrt[3]{b\sec\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x)
```

```
[Out] int((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(fx+e\right)\right)^{\frac{1}{3}}\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(1/3)*(d*tan(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sec\left(fx+e\right)\right)^{\frac{1}{3}}\sqrt{d\tan\left(fx+e\right)}d\tan\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^(1/3)*sqrt(d*tan(f*x + e))*d*tan(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(1/3)*(d*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(f x+e\right)\right)^{\frac{1}{3}} \left(d \tan \left(f x+e\right)\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(1/3)*(d*tan(f*x + e))^(3/2), x)
```

3.341 $\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=64

$$\frac{2\cos^2(e+fx)^{13/12}(d\tan(e+fx))^{5/2}\,{}_2F_1\left(\frac{13}{12},\frac{5}{4};\frac{9}{4};\sin^2(e+fx)\right)}{5df\sqrt[3]{b\sec(e+fx)}}$$

```
[Out] (2*(Cos[e + f*x]^2)^(13/12)*Hypergeometric2F1[13/12, 5/4, 9/4, Sin[e + f*x]
^2]*(d*Tan[e + f*x])^(5/2))/(5*d*f*(b*Sec[e + f*x])^(1/3))
```

Rubi [A] time = 0.0523521, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2617}

$$\frac{2\cos^2(e+fx)^{13/12}(d\tan(e+fx))^{5/2}\,{}_2F_1\left(\frac{13}{12},\frac{5}{4};\frac{9}{4};\sin^2(e+fx)\right)}{5df\sqrt[3]{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(3/2)/(b*Sec[e + f*x])^(1/3),x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(13/12)*Hypergeometric2F1[13/12, 5/4, 9/4, Sin[e + f*x]
^2]*(d*Tan[e + f*x])^(5/2))/(5*d*f*(b*Sec[e + f*x])^(1/3))
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \frac{2\cos^2(e+fx)^{13/12}\,{}_2F_1\left(\frac{13}{12},\frac{5}{4};\frac{9}{4};\sin^2(e+fx)\right)(d\tan(e+fx))^{5/2}}{5df\sqrt[3]{b\sec(e+fx)}}$$

Mathematica [A] time = 0.0730565, size = 69, normalized size = 1.08

$$\frac{3\left(-\tan^2(e+fx)\right)^{3/4}\cot^3(e+fx)(d\tan(e+fx))^{3/2}\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{6};\frac{5}{6};\sec^2(e+fx)\right)}{f\sqrt[3]{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(3/2)/(b*Sec[e + f*x])^(1/3),x]
```

```
[Out] (3*Cot[e + f*x]^3*Hypergeometric2F1[-1/4, -1/6, 5/6, Sec[e + f*x]^2]*(d*Tan
[e + f*x])^(3/2)*(-Tan[e + f*x]^2)^(3/4))/(f*(b*Sec[e + f*x])^(1/3))
```

Maple [F] time = 0.141, size = 0, normalized size = 0.

$$\int \left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}\frac{1}{\sqrt[3]{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(1/3),x)
```

```
[Out] int((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(f*x + e))^(3/2)/(b*sec(f*x + e))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\sec\left(fx+e\right)\right)^{\frac{2}{3}}\sqrt{d\tan\left(fx+e\right)}d\tan\left(fx+e\right)}{b\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))*d*tan(f*x + e)/(b*sec(
f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(3/2)/(b*sec(f*x+e))**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((d*tan(f*x + e))^(3/2)/(b*sec(f*x + e))^(1/3), x)
```

3.342 $\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx$

Optimal. Leaf size=64

$$\frac{2\cos^2(e+fx)^{7/12}(d\tan(e+fx))^{5/2}\,{}_2F_1\left(\frac{7}{12},\frac{5}{4};\frac{9}{4};\sin^2(e+fx)\right)}{5df(b\sec(e+fx))^{4/3}}$$

```
[Out] (2*(Cos[e + f*x]^2)^(7/12)*Hypergeometric2F1[7/12, 5/4, 9/4, Sin[e + f*x]^2
]*(d*Tan[e + f*x])^(5/2))/(5*d*f*(b*Sec[e + f*x])^(4/3))
```

Rubi [A] time = 0.0597472, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2617}

$$\frac{2\cos^2(e+fx)^{7/12}(d\tan(e+fx))^{5/2}\,{}_2F_1\left(\frac{7}{12},\frac{5}{4};\frac{9}{4};\sin^2(e+fx)\right)}{5df(b\sec(e+fx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(3/2)/(b*Sec[e + f*x])^(4/3),x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(7/12)*Hypergeometric2F1[7/12, 5/4, 9/4, Sin[e + f*x]^2
]*(d*Tan[e + f*x])^(5/2))/(5*d*f*(b*Sec[e + f*x])^(4/3))
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx=\frac{2\cos^2(e+fx)^{7/12}\,{}_2F_1\left(\frac{7}{12},\frac{5}{4};\frac{9}{4};\sin^2(e+fx)\right)(d\tan(e+fx))^{5/2}}{5df(b\sec(e+fx))^{4/3}}$$

Mathematica [A] time = 0.0768728, size = 71, normalized size = 1.11

$$\frac{3\left(-\tan^2(e+fx)\right)^{3/4}\cot^3(e+fx)(d\tan(e+fx))^{3/2}\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{4};\frac{1}{3};\sec^2(e+fx)\right)}{4f(b\sec(e+fx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(3/2)/(b*Sec[e + f*x])^(4/3),x]
```

```
[Out] (3*Cot[e + f*x]^3*Hypergeometric2F1[-2/3, -1/4, 1/3, Sec[e + f*x]^2]*(d*Tan
[e + f*x])^(3/2)*(-Tan[e + f*x]^2)^(3/4))/(4*f*(b*Sec[e + f*x])^(4/3))
```

Maple [F] time = 0.186, size = 0, normalized size = 0.

$$\int \left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\sec\left(fx+e\right)\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(4/3),x)
```

```
[Out] int((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sec\left(fx+e\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(f*x + e))^(3/2)/(b*sec(f*x + e))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\sec\left(fx+e\right)\right)^{\frac{2}{3}}\sqrt{d\tan\left(fx+e\right)}d\tan\left(fx+e\right)}{b^2\sec\left(fx+e\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))*d*tan(f*x + e)/(b^2*se
c(f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(3/2)/(b*sec(f*x+e))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sec\left(fx+e\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((d*tan(f*x + e))^(3/2)/(b*sec(f*x + e))^(4/3), x)
```

3.343 $\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx$

Optimal. Leaf size=64

$$\frac{3\cos^2(e+fx)^{17/12}\sqrt{b\sec(e+fx)}(d\tan(e+fx))^{7/3}\,{}_2F_1\left(\frac{7}{6},\frac{17}{12};\frac{13}{6};\sin^2(e+fx)\right)}{7df}$$

```
[Out] (3*(Cos[e + f*x]^2)^(17/12)*Hypergeometric2F1[7/6, 17/12, 13/6, Sin[e + f*x
]^2]*Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(7/3))/(7*d*f)
```

Rubi [A] time = 0.0503131, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2617}

$$\frac{3\cos^2(e+fx)^{17/12}\sqrt{b\sec(e+fx)}(d\tan(e+fx))^{7/3}\,{}_2F_1\left(\frac{7}{6},\frac{17}{12};\frac{13}{6};\sin^2(e+fx)\right)}{7df}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (3*(Cos[e + f*x]^2)^(17/12)*Hypergeometric2F1[7/6, 17/12, 13/6, Sin[e + f*x
]^2]*Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(7/3))/(7*d*f)
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

Rubi steps

$$\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \frac{3\cos^2(e+fx)^{17/12}\,{}_2F_1\left(\frac{7}{6},\frac{17}{12};\frac{13}{6};\sin^2(e+fx)\right)\sqrt{b\sec(e+fx)}(d\tan(e+fx))^{7/3}}{7df}$$

Mathematica [A] time = 0.141539, size = 62, normalized size = 0.97

$$\frac{2d\sqrt{b\sec(e+fx)}\sqrt[3]{d\tan(e+fx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{4};\frac{5}{4};\sec^2(e+fx)\right)}{f\sqrt[6]{-\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (2*d*Hypergeometric2F1[-1/6, 1/4, 5/4, Sec[e + f*x]^2]*Sqrt[b*Sec[e + f*x]]
*(d*Tan[e + f*x])^(1/3))/(f*(-Tan[e + f*x]^2)^(1/6))
```

Maple [F] time = 0.173, size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x)
```

```
[Out] int((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}d\tan\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(1/3)*d*tan(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(1/2)*(d*tan(f*x+e))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(4/3), x)
```

3.344 $\int \sqrt{b \sec(e+fx)}\sqrt[3]{d \tan(e+fx)}\,dx$

Optimal. Leaf size=64

$$\frac{3\cos^2(e+fx)^{11/12}\sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{11}{12};\frac{5}{3};\sin^2(e+fx)\right)}{4df}$$

```
[Out] (3*(Cos[e + f*x]^2)^(11/12)*Hypergeometric2F1[2/3, 11/12, 5/3, Sin[e + f*x]
^2]*Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(4/3))/(4*d*f)
```

Rubi [A] time = 0.042857, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2617}

$$\frac{3\cos^2(e+fx)^{11/12}\sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{11}{12};\frac{5}{3};\sin^2(e+fx)\right)}{4df}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (3*(Cos[e + f*x]^2)^(11/12)*Hypergeometric2F1[2/3, 11/12, 5/3, Sin[e + f*x]
^2]*Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(4/3))/(4*d*f)
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \sqrt{b\sec(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx=\frac{3\cos^2(e+fx)^{11/12}\,{}_2F_1\left(\frac{2}{3},\frac{11}{12};\frac{5}{3};\sin^2(e+fx)\right)\sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}}{4df}$$

Mathematica [A] time = 0.0845744, size = 62, normalized size = 0.97

$$\frac{2d\sqrt[3]{-\tan^2(e+fx)}\sqrt{b\sec(e+fx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{5}{4};\sec^2(e+fx)\right)}{f(d\tan(e+fx))^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (2*d*Hypergeometric2F1[1/4, 1/3, 5/4, Sec[e + f*x]^2]*Sqrt[b*Sec[e + f*x]]*
(-Tan[e + f*x]^2)^(1/3))/(f*(d*Tan[e + f*x])^(2/3))
```

Maple [F] time = 0.247, size = 0, normalized size = 0.

$$\int\sqrt{b\sec\left(fx+e\right)}\sqrt[3]{d\tan\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x)
```

```
[Out] int((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sec\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sec\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(e+fx\right)}\sqrt[3]{d\tan\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(1/2)*(d*tan(f*x+e))**(1/3),x)
```

```
[Out] Integral(sqrt(b*sec(e + f*x))*(d*tan(e + f*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(1/3), x)
```

3.345 $\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx$

Optimal. Leaf size=64

$$\frac{3\cos^2(e+fx)^{7/12}\sqrt{b\sec(e+fx)}(d\tan(e+fx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{7}{12};\frac{4}{3};\sin^2(e+fx)\right)}{2df}$$

```
[Out] (3*(Cos[e + f*x]^2)^(7/12)*Hypergeometric2F1[1/3, 7/12, 4/3, Sin[e + f*x]^2
]*Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(2/3))/(2*d*f)
```

Rubi [A] time = 0.0466348, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2617}

$$\frac{3\cos^2(e+fx)^{7/12}\sqrt{b\sec(e+fx)}(d\tan(e+fx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{7}{12};\frac{4}{3};\sin^2(e+fx)\right)}{2df}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]/(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (3*(Cos[e + f*x]^2)^(7/12)*Hypergeometric2F1[1/3, 7/12, 4/3, Sin[e + f*x]^2
]*Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(2/3))/(2*d*f)
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{3\cos^2(e+fx)^{7/12}\,{}_2F_1\left(\frac{1}{3},\frac{7}{12};\frac{4}{3};\sin^2(e+fx)\right)\sqrt{b\sec(e+fx)}(d\tan(e+fx))^{2/3}}{2df}$$

Mathematica [A] time = 0.112473, size = 62, normalized size = 0.97

$$\frac{2d\left(-\tan^2(e+fx)\right)^{2/3}\sqrt{b\sec(e+fx)}\,{}_2F_1\left(\frac{1}{4},\frac{2}{3};\frac{5}{4};\sec^2(e+fx)\right)}{f(d\tan(e+fx))^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]/(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (2*d*Hypergeometric2F1[1/4, 2/3, 5/4, Sec[e + f*x]^2]*Sqrt[b*Sec[e + f*x]]*
(-Tan[e + f*x]^2)^(2/3))/(f*(d*Tan[e + f*x])^(4/3))
```

Maple [F] time = 0.221, size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)}\frac{1}{\sqrt[3]{d\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x)
```

```
[Out] int((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))/(d*tan(f*x + e))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{2}{3}}}{d\tan\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(2/3)/(d*tan(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{b\sec\left(e+fx\right)}}{\sqrt[3]{d\tan\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(1/2)/(d*tan(f*x+e))**(1/3),x)
```

```
[Out] Integral(sqrt(b*sec(e + f*x))/(d*tan(e + f*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{b\sec\left(fx+e\right)}}{\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))/(d*tan(f*x + e))^(1/3), x)
```

3.346 $\int \frac{\sqrt{b \sec(e+fx)}}{(d \tan(e+fx))^{4/3}} dx$

Optimal. Leaf size=62

$$-\frac{3 \sqrt[12]{\cos^2(e+fx)}\sqrt{b \sec(e+fx)}\, _2F_1\left(-\frac{1}{6},\frac{1}{12};\frac{5}{6};\sin^2(e+fx)\right)}{df\sqrt[3]{d \tan(e+fx)}}$$

```
[Out] (-3*(Cos[e + f*x]^2)^(1/12)*Hypergeometric2F1[-1/6, 1/12, 5/6, Sin[e + f*x]
^2]*Sqrt[b*Sec[e + f*x]])/(d*f*(d*Tan[e + f*x])^(1/3))
```

Rubi [A] time = 0.0527477, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2617}

$$-\frac{3 \sqrt[12]{\cos^2(e+fx)}\sqrt{b \sec(e+fx)}\, _2F_1\left(-\frac{1}{6},\frac{1}{12};\frac{5}{6};\sin^2(e+fx)\right)}{df\sqrt[3]{d \tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]/(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (-3*(Cos[e + f*x]^2)^(1/12)*Hypergeometric2F1[-1/6, 1/12, 5/6, Sin[e + f*x]
^2]*Sqrt[b*Sec[e + f*x]])/(d*f*(d*Tan[e + f*x])^(1/3))
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sqrt{b \sec(e+fx)}}{(d \tan(e+fx))^{4/3}} dx = -\frac{3 \sqrt[12]{\cos^2(e+fx)}\, _2F_1\left(-\frac{1}{6},\frac{1}{12};\frac{5}{6};\sin^2(e+fx)\right)\sqrt{b \sec(e+fx)}}{df\sqrt[3]{d \tan(e+fx)}}$$

Mathematica [A] time = 0.228715, size = 62, normalized size = 1.

$$\frac{2d\left(-\tan^2(e+fx)\right)^{7/6}\sqrt{b\sec(e+fx)}\,{}_2F_1\left(\frac{1}{4},\frac{7}{6};\frac{5}{4};\sec^2(e+fx)\right)}{f(d\tan(e+fx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]/(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (2*d*Hypergeometric2F1[1/4, 7/6, 5/4, Sec[e + f*x]^2]*Sqrt[b*Sec[e + f*x]]*
(-Tan[e + f*x]^2)^(7/6))/(f*(d*Tan[e + f*x])^(7/3))
```

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int \sqrt{b\sec(fx+e)}\,(d\tan(fx+e))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x)
```

```
[Out] int((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec(fx+e)}}{(d\tan(fx+e))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))/(d*tan(f*x + e))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{2}{3}}}{d^2\tan\left(fx+e\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(2/3)/(d^2*tan(f*x + e)^2),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(1/2)/(d*tan(f*x+e))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{b\sec\left(fx+e\right)}}{\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))/(d*tan(f*x + e))^(4/3), x)
```

3.347 $\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx$

Optimal. Leaf size=64

$$\frac{3\cos^2(e+fx)^{23/12}(b\sec(e+fx))^{3/2}(d\tan(e+fx))^{7/3}\,{}_2F_1\left(\frac{7}{6},\frac{23}{12};\frac{13}{6};\sin^2(e+fx)\right)}{7df}$$

```
[Out] (3*(Cos[e + f*x]^2)^(23/12)*Hypergeometric2F1[7/6, 23/12, 13/6, Sin[e + f*x
]^2]*(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(7/3))/(7*d*f)
```

Rubi [A] time = 0.0579168, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2617}

$$\frac{3\cos^2(e+fx)^{23/12}(b\sec(e+fx))^{3/2}(d\tan(e+fx))^{7/3}\,{}_2F_1\left(\frac{7}{6},\frac{23}{12};\frac{13}{6};\sin^2(e+fx)\right)}{7df}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3),x]
```


```
[Out] (3*(Cos[e + f*x]^2)^(23/12)*Hypergeometric2F1[7/6, 23/12, 13/6, Sin[e + f*x
]^2]*(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(7/3))/(7*d*f)
```


Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx=\frac{3\cos^2(e+fx)^{23/12}\,{}_2F_1\left(\frac{7}{6},\frac{23}{12};\frac{13}{6};\sin^2(e+fx)\right)(b\sec(e+fx))^{3/2}(d\tan(e+fx$$

Mathematica [A] time = 0.145581, size = 64, normalized size = 1.

$$\frac{2d(b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,{}_2F_1\left(-\frac{1}{6},\frac{3}{4};\frac{7}{4};\sec^2(e+fx)\right)}{3f\sqrt[6]{-\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (2*d*Hypergeometric2F1[-1/6, 3/4, 7/4, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(3/
2)*(d*Tan[e + f*x])^(1/3))/(3*f*(-Tan[e + f*x]^2)^(1/6))
```

Maple [F] time = 0.129, size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x)
```

```
[Out] int((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)*(d*tan(f*x + e))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}bd\sec\left(fx+e\right)\tan\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(1/3)*b*d*sec(f*x + e)*tan(f
*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(3/2)*(d*tan(f*x+e))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)*(d*tan(f*x + e))^(4/3), x)
```

3.348 $\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx$

Optimal. Leaf size=64

$$\frac{3\cos^2(e+fx)^{17/12}(b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{17}{12};\frac{5}{3};\sin^2(e+fx)\right)}{4df}$$

```
[Out] (3*(Cos[e + f*x]^2)^(17/12)*Hypergeometric2F1[2/3, 17/12, 5/3, Sin[e + f*x]
^2]*(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3))/(4*d*f)
```

Rubi [A] time = 0.0512265, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2617}

$$\frac{3\cos^2(e+fx)^{17/12}(b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{17}{12};\frac{5}{3};\sin^2(e+fx)\right)}{4df}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (3*(Cos[e + f*x]^2)^(17/12)*Hypergeometric2F1[2/3, 17/12, 5/3, Sin[e + f*x]
^2]*(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3))/(4*d*f)
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx=\frac{3\cos^2(e+fx)^{17/12}\,{}_2F_1\left(\frac{2}{3},\frac{17}{12};\frac{5}{3};\sin^2(e+fx)\right)(b\sec(e+fx))^{3/2}(d\tan(e+fx)}{4df}$$

Mathematica [A] time = 0.0892856, size = 64, normalized size = 1.

$$\frac{2d\sqrt[3]{-\tan^2(e+fx)}(b\sec(e+fx))^{3/2}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{7}{4};\sec^2(e+fx)\right)}{3f(d\tan(e+fx))^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (2*d*Hypergeometric2F1[1/3, 3/4, 7/4, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(3/2
)*(-Tan[e + f*x]^2)^(1/3))/(3*f*(d*Tan[e + f*x])^(2/3))
```

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt[3]{d\tan\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x)
```

```
[Out] int((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)*(d*tan(f*x + e))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}b\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(1/3)*b*sec(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(3/2)*(d*tan(f*x+e))**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}} \left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)*(d*tan(f*x + e))^(1/3), x)
```

3.349 $\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx$

Optimal. Leaf size=64

$$\frac{3\cos^2(e+fx)^{13/12}(b\sec(e+fx))^{3/2}(d\tan(e+fx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{13}{12};\frac{4}{3};\sin^2(e+fx)\right)}{2df}$$

```
[Out] (3*(Cos[e + f*x]^2)^(13/12)*Hypergeometric2F1[1/3, 13/12, 4/3, Sin[e + f*x]
^2]*(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(2/3))/(2*d*f)
```

Rubi [A] time = 0.0551391, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2617}

$$\frac{3\cos^2(e+fx)^{13/12}(b\sec(e+fx))^{3/2}(d\tan(e+fx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{13}{12};\frac{4}{3};\sin^2(e+fx)\right)}{2df}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(3/2)/(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (3*(Cos[e + f*x]^2)^(13/12)*Hypergeometric2F1[1/3, 13/12, 4/3, Sin[e + f*x]
^2]*(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(2/3))/(2*d*f)
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{3\cos^2(e+fx)^{13/12}\,{}_2F_1\left(\frac{1}{3},\frac{13}{12};\frac{4}{3};\sin^2(e+fx)\right)(b\sec(e+fx))^{3/2}(d\tan(e+fx))^{2/3}}{2df}$$

Mathematica [A] time = 0.102991, size = 64, normalized size = 1.

$$\frac{2d\left(-\tan^2(e+fx)\right)^{2/3}(b\sec(e+fx))^{3/2}\,{}_2F_1\left(\frac{2}{3},\frac{3}{4};\frac{7}{4};\sec^2(e+fx)\right)}{3f(d\tan(e+fx))^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)/(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (2*d*Hypergeometric2F1[2/3, 3/4, 7/4, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(3/2
)*(-Tan[e + f*x]^2)^(2/3))/(3*f*(d*Tan[e + f*x])^(4/3))
```

Maple [F] time = 0.115, size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\frac{1}{\sqrt[3]{d\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x)
```

```
[Out] int((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)/(d*tan(f*x + e))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{2}{3}}b\sec\left(fx+e\right)}{d\tan\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(2/3)*b*sec(f*x + e)/(d*tan(
f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(3/2)/(d*tan(f*x+e))**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)/(d*tan(f*x + e))^(1/3), x)
```

3.350 $\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx$

Optimal. Leaf size=62

$$-\frac{3\cos^2(e+fx)^{7/12}(b\sec(e+fx))^{3/2}\,{}_2F_1\left(-\frac{1}{6},\frac{7}{12};\frac{5}{6};\sin^2(e+fx)\right)}{df\sqrt[3]{d\tan(e+fx)}}$$

```
[Out] (-3*(Cos[e + f*x]^2)^(7/12)*Hypergeometric2F1[-1/6, 7/12, 5/6, Sin[e + f*x]
^2]*(b*Sec[e + f*x])^(3/2))/(d*f*(d*Tan[e + f*x])^(1/3))
```

Rubi [A] time = 0.0585234, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2617}

$$-\frac{3\cos^2(e+fx)^{7/12}(b\sec(e+fx))^{3/2}\,{}_2F_1\left(-\frac{1}{6},\frac{7}{12};\frac{5}{6};\sin^2(e+fx)\right)}{df\sqrt[3]{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(3/2)/(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (-3*(Cos[e + f*x]^2)^(7/12)*Hypergeometric2F1[-1/6, 7/12, 5/6, Sin[e + f*x]
^2]*(b*Sec[e + f*x])^(3/2))/(d*f*(d*Tan[e + f*x])^(1/3))
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = -\frac{3\cos^2(e+fx)^{7/12}\,{}_2F_1\left(-\frac{1}{6},\frac{7}{12};\frac{5}{6};\sin^2(e+fx)\right)(b\sec(e+fx))^{3/2}}{df\sqrt[3]{d\tan(e+fx)}}$$

Mathematica [A] time = 0.202587, size = 64, normalized size = 1.03

$$\frac{2d\left(-\tan^2(e+fx)\right)^{7/6}(b\sec(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{7}{6};\frac{7}{4};\sec^2(e+fx)\right)}{3f(d\tan(e+fx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)/(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (2*d*Hypergeometric2F1[3/4, 7/6, 7/4, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(3/2
)*(-Tan[e + f*x]^2)^(7/6))/(3*f*(d*Tan[e + f*x])^(7/3))
```

Maple [F] time = 0.113, size = 0, normalized size = 0.

$$\int\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(d\tan\left(fx+e\right)\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x)
```

```
[Out] int((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)/(d*tan(f*x + e))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{2}{3}}b\sec\left(fx+e\right)}{d^2\tan\left(fx+e\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(2/3)*b*sec(f*x + e)/(d^2*ta
n(f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(3/2)/(d*tan(f*x+e))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)/(d*tan(f*x + e))^(4/3), x)
```

3.351 $\int (b\sec(e+fx))^m \tan^5(e+fx)\,dx$

Optimal. Leaf size=67

$$-\frac{2(b\sec(e+fx))^{m+2}}{b^2f(m+2)}+\frac{(b\sec(e+fx))^{m+4}}{b^4f(m+4)}+\frac{(b\sec(e+fx))^m}{fm}$$

```
[Out] (b*Sec[e + f*x])^m/(f*m) - (2*(b*Sec[e + f*x])^(2 + m))/(b^2*f*(2 + m)) + (
b*Sec[e + f*x])^(4 + m)/(b^4*f*(4 + m))
```

Rubi [A] time = 0.0623153, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2606, 270}

$$-\frac{2(b\sec(e+fx))^{m+2}}{b^2f(m+2)}+\frac{(b\sec(e+fx))^{m+4}}{b^4f(m+4)}+\frac{(b\sec(e+fx))^m}{fm}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^m*Tan[e + f*x]^5,x]
```

```
[Out] (b*Sec[e + f*x])^m/(f*m) - (2*(b*Sec[e + f*x])^(2 + m))/(b^2*f*(2 + m)) + (
b*Sec[e + f*x])^(4 + m)/(b^4*f*(4 + m))
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (b\sec(e+fx))^m \tan^5(e+fx)\,dx = \frac{b\,\mathrm{Subst}\left(\int (bx)^{-1+m}\left(-1+x^2\right)^2\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{b\,\mathrm{Subst}\left(\int \left((bx)^{-1+m} - \frac{2(bx)^{1+m}}{b^2} + \frac{(bx)^{3+m}}{b^4}\right)dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{(b\sec(e+fx))^m}{fm} - \frac{2(b\sec(e+fx))^{2+m}}{b^2f(2+m)} + \frac{(b\sec(e+fx))^{4+m}}{b^4f(4+m)}$$

Mathematica [A] time = 0.351632, size = 47, normalized size = 0.7

$$\frac{\left(\frac{\sec^4(e+fx)}{m+4} - \frac{2\sec^2(e+fx)}{m+2} + \frac{1}{m}\right)(b\sec(e+fx))^m}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^m*Tan[e + f*x]^5,x]
```

```
[Out] ((b*Sec[e + f*x])^m*(m^(-1) - (2*Sec[e + f*x]^2)/(2 + m) + Sec[e + f*x]^4/(
4 + m)))/f
```

Maple [C] time = 0.603, size = 6797, normalized size = 101.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^m*tan(f*x+e)^5,x)
```

```
[Out] result too large to display
```

Maxima [A] time = 1.0213, size = 104, normalized size = 1.55

$$\frac{\frac{b^m\cos(fx+e)^{-m}}{m} - \frac{2\,b^m\cos(fx+e)^{-m}}{(m+2)\cos(fx+e)^2} + \frac{b^m\cos(fx+e)^{-m}}{(m+4)\cos(fx+e)^4}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] (b^m*cos(f*x + e)^(-m)/m - 2*b^m*cos(f*x + e)^(-m)/((m + 2)*cos(f*x + e)^2)
 + b^m*cos(f*x + e)^(-m)/((m + 4)*cos(f*x + e)^4))/f
```

Fricas [A] time = 1.72904, size = 188, normalized size = 2.81

$$\frac{\left(\left(m^2+6\,m+8\right)\cos\left(fx+e\right)^4-2\left(m^2+4\,m\right)\cos\left(fx+e\right)^2+m^2+2\,m\right)\left(\frac{b}{\cos\left(fx+e\right)}\right)^m}{\left(fm^3+6\,fm^2+8\,fm\right)\cos\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] ((m^2 + 6*m + 8)*cos(f*x + e)^4 - 2*(m^2 + 4*m)*cos(f*x + e)^2 + m^2 + 2*m)
*(b/cos(f*x + e))^m/((f*m^3 + 6*f*m^2 + 8*f*m)*cos(f*x + e)^4)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**m*tan(f*x+e)**5,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(fx+e\right)\right)^m\tan\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^m*tan(f*x + e)^5, x)
```

3.352 $\int (b\sec(e+fx))^m \tan^3(e+fx)\,dx$

Optimal. Leaf size=43

$$\frac{(b\sec(e+fx))^{m+2}}{b^2f(m+2)}-\frac{(b\sec(e+fx))^m}{fm}$$

```
[Out] -((b*Sec[e + f*x])^m/(f*m)) + (b*Sec[e + f*x])^(2 + m)/(b^2*f*(2 + m))
```

Rubi [A] time = 0.0488089, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2606, 14}

$$\frac{(b\sec(e+fx))^{m+2}}{b^2f(m+2)}-\frac{(b\sec(e+fx))^m}{fm}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^m*Tan[e + f*x]^3,x]
```

```
[Out] -((b*Sec[e + f*x])^m/(f*m)) + (b*Sec[e + f*x])^(2 + m)/(b^2*f*(2 + m))
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int (b\sec(e+fx))^m\tan^3(e+fx)\,dx = \frac{b\,\text{Subst}\left(\int (bx)^{-1+m}\left(-1+x^2\right)dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{b\,\text{Subst}\left(\int \left(-(bx)^{-1+m}+\frac{(bx)^{1+m}}{b^2}\right)dx, x, \sec(e+fx)\right)}{f}$$
$$= -\frac{(b\sec(e+fx))^m}{fm}+\frac{(b\sec(e+fx))^{2+m}}{b^2f(2+m)}$$

Mathematica [A] time = 0.106865, size = 34, normalized size = 0.79

$$\frac{\left(\frac{\sec^2(e+fx)}{m+2}-\frac{1}{m}\right)(b\sec(e+fx))^m}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^m*Tan[e + f*x]^3,x]
```

```
[Out] ((b*Sec[e + f*x])^m*(-m^(-1) + Sec[e + f*x]^2/(2 + m)))/f
```

Maple [C] time = 0.175, size = 2707, normalized size = 63.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^m*tan(f*x+e)^3,x)
```

```
[Out] -1/(2+m)/f/(exp(2*I*(f*x+e))+1)^2/m*(2^m*b^m*exp(I*(Re(f*x)+Re(e)))^m/((exp
(2*I*(f*x+e))+1)^m)*m*exp(-m*Im(f*x)-m*Im(e))*exp(-1/2*I*Pi*csgn(I*exp(I*(f
*x+e))/(exp(2*I*(f*x+e))+1))^3*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2
*I*(f*x+e))+1))^2*csgn(I*exp(I*(f*x+e)))*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+
e))/(exp(2*I*(f*x+e))+1))^2*csgn(I/(exp(2*I*(f*x+e))+1))*m)*exp(-1/2*I*Pi*c
sgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*exp(I*(f*x+e)))*csgn(I/(e
xp(2*I*(f*x+e))+1))*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))
+1))*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+e)))^2*m)*exp(-1/2*I*Pi*csgn(
I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*
(f*x+e)))*csgn(I*b)*m)*exp(-1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f
```

```
*x+e)))^3*m)*exp(1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+e)))^2*c
sgn(I*b)*m)*exp(4*I*f*x)*exp(4*I*e)+2*2^m*b^m*exp(I*(Re(f*x)+Re(e)))^m/((ex
p(2*I*(f*x+e))+1)^m)*exp(-m*Im(f*x)-m*Im(e))*exp(-1/2*I*Pi*csgn(I*exp(I*(f*
x+e))/(exp(2*I*(f*x+e))+1))^3*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*
I*(f*x+e))+1))^2*csgn(I*exp(I*(f*x+e)))*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e
))/(exp(2*I*(f*x+e))+1))^2*csgn(I/(exp(2*I*(f*x+e))+1))*m)*exp(-1/2*I*Pi*cs
gn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*exp(I*(f*x+e)))*csgn(I/(ex
p(2*I*(f*x+e))+1))*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+
1))*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+e)))^2*m)*exp(-1/2*I*Pi*csgn(I
*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(
f*x+e)))*csgn(I*b)*m)*exp(-1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f*
x+e)))^3*m)*exp(1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+e)))^2*cs
gn(I*b)*m)*exp(4*I*f*x)*exp(4*I*e)-2*2^m*b^m*exp(I*(Re(f*x)+Re(e)))^m/((exp
(2*I*(f*x+e))+1)^m)*m*exp(-m*Im(f*x)-m*Im(e))*exp(-1/2*I*Pi*csgn(I*exp(I*(f
*x+e))/(exp(2*I*(f*x+e))+1))^3*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2
*I*(f*x+e))+1))^2*csgn(I*exp(I*(f*x+e)))*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+
e))/(exp(2*I*(f*x+e))+1))^2*csgn(I/(exp(2*I*(f*x+e))+1))*m)*exp(-1/2*I*Pi*c
sgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*exp(I*(f*x+e)))*csgn(I/(e
xp(2*I*(f*x+e))+1))*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))
+1))*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+e)))^2*m)*exp(-1/2*I*Pi*csgn(
I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*
(f*x+e)))*csgn(I*b)*m)*exp(-1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f
*x+e)))^3*m)*exp(1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+e)))^2*c
sgn(I*b)*m)*exp(2*I*f*x)*exp(2*I*e)+4*2^m*b^m*exp(I*(Re(f*x)+Re(e)))^m/((ex
p(2*I*(f*x+e))+1)^m)*exp(-m*Im(f*x)-m*Im(e))*exp(-1/2*I*Pi*csgn(I*exp(I*(f*
x+e))/(exp(2*I*(f*x+e))+1))^3*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*
I*(f*x+e))+1))^2*csgn(I*exp(I*(f*x+e)))*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e
))/(exp(2*I*(f*x+e))+1))^2*csgn(I/(exp(2*I*(f*x+e))+1))*m)*exp(-1/2*I*Pi*cs
gn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*exp(I*(f*x+e)))*csgn(I/(ex
p(2*I*(f*x+e))+1))*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+
1))*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+e)))^2*m)*exp(-1/2*I*Pi*csgn(I
*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(
f*x+e)))*csgn(I*b)*m)*exp(-1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f*
x+e)))^3*m)*exp(1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+e)))^2*cs
gn(I*b)*m)*exp(2*I*f*x)*exp(2*I*e)+2^m*b^m*exp(I*(Re(f*x)+Re(e)))^m/((exp(2
*I*(f*x+e))+1)^m)*m*exp(-1/2*m*(I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e)
)+1))*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+e)))*csgn(I*b)-I*Pi*csgn(I*b
/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+e)))^2*csgn(I*b)+I*Pi*csgn(I*exp(I*(f*x+e)
)/(exp(2*I*(f*x+e))+1))^3-I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^
2*csgn(I*exp(I*(f*x+e)))-I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2
*csgn(I/(exp(2*I*(f*x+e))+1))+I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+
1))*csgn(I*exp(I*(f*x+e)))*csgn(I/(exp(2*I*(f*x+e))+1))-I*Pi*csgn(I*exp(I*(
f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+e)))
^2+I*Pi*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+e)))^3+2*Im(f*x)+2*Im(e)))
+2*2^m*b^m*exp(I*(Re(f*x)+Re(e)))^m/((exp(2*I*(f*x+e))+1)^m)*exp(-1/2*m*(I*
```

```
Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b/(exp(2*I*(f*x+e))+1
)*exp(I*(f*x+e)))*csgn(I*b)-I*Pi*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+e
)))^2*csgn(I*b)+I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3-I*Pi*csg
n(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2*csgn(I*exp(I*(f*x+e)))-I*Pi*csgn
(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2*csgn(I/(exp(2*I*(f*x+e))+1))+I*Pi
*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*exp(I*(f*x+e)))*csgn(I/
(exp(2*I*(f*x+e))+1))-I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn
(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+e)))^2+I*Pi*csgn(I*b/(exp(2*I*(f*x+e))
+1)*exp(I*(f*x+e)))^3+2*Im(f*x)+2*Im(e))))
```

Maxima [A] time = 1.00428, size = 69, normalized size = 1.6

$$-\frac{\frac{b^m \cos\left(fx+e\right)^{-m}}{m} - \frac{b^m \cos\left(fx+e\right)^{-m}}{(m+2)\cos\left(fx+e\right)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] -(b^m*cos(f*x + e)^(-m)/m - b^m*cos(f*x + e)^(-m)/((m + 2)*cos(f*x + e)^2))
/f
```

Fricas [A] time = 1.64096, size = 112, normalized size = 2.6

$$-\frac{\left((m+2)\cos\left(fx+e\right)^2 - m\right)\left(\frac{b}{\cos\left(fx+e\right)}\right)^m}{\left(fm^2+2\,fm\right)\cos\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] -((m + 2)*cos(f*x + e)^2 - m)*(b/cos(f*x + e))^m/((f*m^2 + 2*f*m)*cos(f*x +
 e)^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**m*tan(f*x+e)**3,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(f x + e\right)\right)^{m} \tan \left(f x + e\right)^{3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^m*tan(f*x + e)^3, x)
```

3.353 $\int (b\sec(e+fx))^m \tan(e+fx)\,dx$

Optimal. Leaf size=17

$$\frac{(b\sec(e+fx))^m}{fm}$$

```
[Out] (b*Sec[e + f*x])^m/(f*m)
```

Rubi [A] time = 0.0210696, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2606, 32}

$$\frac{(b\sec(e+fx))^m}{fm}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^m*Tan[e + f*x],x]
```

```
[Out] (b*Sec[e + f*x])^m/(f*m)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (b\sec(e+fx))^m \tan(e+fx)\,dx &= \frac{b\,\text{Subst}\left(\int (bx)^{-1+m}\,dx, x, \sec(e+fx)\right)}{f}\\ &= \frac{(b\sec(e+fx))^m}{fm}\end{aligned}$$

Mathematica [A] time = 0.0216875, size = 17, normalized size = 1.

$$\frac{(b\sec(e+fx))^m}{fm}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^m*Tan[e + f*x],x]
```

```
[Out] (b*Sec[e + f*x])^m/(f*m)
```

Maple [A] time = 0.01, size = 18, normalized size = 1.1

$$\frac{\left(b\sec\left(fx+e\right)\right)^m}{fm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^m*tan(f*x+e),x)
```

```
[Out] (b*sec(f*x+e))^m/f/m
```

Maxima [A] time = 0.999073, size = 27, normalized size = 1.59

$$\frac{b^m\cos\left(fx+e\right)^{-m}}{fm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] b^m*cos(f*x + e)^(-m)/(f*m)
```

Fricas [A] time = 1.6066, size = 35, normalized size = 2.06

$$\frac{\left(\frac{b}{\cos\left(fx+e\right)}\right)^m}{fm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] (b/cos(f*x + e))^m/(f*m)
```

Sympy [A] time = 0.538158, size = 44, normalized size = 2.59

$$\begin{cases} x\tan(e) & \text{for}\ f=0 \wedge m=0 \\ x\left(b\sec(e)\right)^m \tan(e) & \text{for}\ f=0 \\ \frac{\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} & \text{for}\ m=0 \\ \frac{b^m \sec^m\left(e+fx\right)}{fm} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**m*tan(f*x+e),x)
```

```
[Out] Piecewise((x*tan(e), Eq(f, 0) & Eq(m, 0)), (x*(b*sec(e))**m*tan(e), Eq(f, 0
)), (log(tan(e + f*x)**2 + 1)/(2*f), Eq(m, 0)), (b**m*sec(e + f*x)**m/(f*m)
, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^m \tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^m*tan(f*x + e), x)
```

3.354 $\int \cot(e+fx)(b\sec(e+fx))^m\,dx$

Optimal. Leaf size=40

$$-\frac{(b\sec(e+fx))^m\,{}_2F_1\left(1,\frac{m}{2};\frac{m+2}{2};\sec^2(e+fx)\right)}{fm}$$

```
[Out] -((Hypergeometric2F1[1, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^m)
/(f*m))
```

Rubi [A] time = 0.0416297, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2606, 364}

$$-\frac{(b\sec(e+fx))^m\,{}_2F_1\left(1,\frac{m}{2};\frac{m+2}{2};\sec^2(e+fx)\right)}{fm}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(b*Sec[e + f*x])^m,x]
```

```
[Out] -((Hypergeometric2F1[1, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^m)
/(f*m))
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \cot(e+fx)(b\sec(e+fx))^m\,dx = \frac{b\,\text{Subst}\left(\int \frac{(bx)^{-1+m}}{-1+x^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{{}_2F_1\left(1,\frac{m}{2};\frac{2+m}{2};\sec^2(e+fx)\right)(b\sec(e+fx))^m}{fm}$$

Mathematica [B] time = 0.804233, size = 124, normalized size = 3.1

$$\frac{b\sec^2\left(\frac{1}{2}(e+fx)\right)(b\sec(e+fx))^{m-1}\left((\cos(e+fx)+1)\,{}_2F_1(1,1-m;2-m;\cos(e+fx)) - 2^m\sec^2\left(\frac{1}{2}(e+fx)\right)^{-m}\,{}_2F_1\left(1\right.\right.}{4f(m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(b*Sec[e + f*x])^m,x]
```

```
[Out] (b*Sec[(e + f*x)/2]^2*((1 + Cos[e + f*x])*Hypergeometric2F1[1, 1 - m, 2 - m
, Cos[e + f*x]] - (2^m*Hypergeometric2F1[1 - m, 1 - m, 2 - m, (Cos[e + f*x]
*Sec[(e + f*x)/2]^2)/2])/(Sec[(e + f*x)/2]^2)^m)*(b*Sec[e + f*x])^(-1 + m))
/(4*f*(-1 + m))
```

Maple [F] time = 0.521, size = 0, normalized size = 0.

$$\int \cot\left(fx+e\right)\left(b\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(b*sec(f*x+e))^m,x)
```

```
[Out] int(cot(f*x+e)*(b*sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^m\cot\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(b*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sec\left(fx+e\right)\right)^{m}\cot\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(b*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^m*cot(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(e+fx\right)\right)^{m}\cot\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(b*sec(f*x+e))**m,x)
```

```
[Out] Integral((b*sec(e + f*x))**m*cot(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(fx+e\right)\right)^{m}\cot\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(b*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e), x)
```

3.355 $\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx$

Optimal. Leaf size=39

$$\frac{(b\sec(e+fx))^m\,{}_2F_1\left(2,\frac{m}{2};\frac{m+2}{2};\sec^2(e+fx)\right)}{fm}$$

```
[Out] (Hypergeometric2F1[2, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^m)/(
f*m)
```

Rubi [A] time = 0.0444063, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2606, 364}

$$\frac{(b\sec(e+fx))^m\,{}_2F_1\left(2,\frac{m}{2};\frac{m+2}{2};\sec^2(e+fx)\right)}{fm}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(b*Sec[e + f*x])^m,x]
```

```
[Out] (Hypergeometric2F1[2, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^m)/(
f*m)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx = \frac{b\,\mathrm{Subst}\left(\int \frac{(bx)^{-1+m}}{(-1+x^2)^2}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{{}_2F_1\left(2,\frac{m}{2};\frac{2+m}{2};\sec^2(e+fx)\right)(b\sec(e+fx))^m}{fm}$$

Mathematica [C] time = 17.4608, size = 3020, normalized size = 77.44

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^3*(b*Sec[e + f*x])^m,x]
```

```
[Out] -(Cot[e + f*x]^3*(2^(2 + m)*AppellF1[1 - m, -m, 1, 2 - m, (Cos[e + f*x]*Sec
[(e + f*x)/2]^2)/2, Cos[e + f*x]*Sec[(e + f*x)/2]^2]*Cos[e + f*x]*(Csc[(e +
 f*x)/2]^2)^(1 + m) + (-1 + m)*(AppellF1[1, m, -m, 2, Cot[(e + f*x)/2]^2, -
Cot[(e + f*x)/2]^2]*Cot[(e + f*x)/2]^4*(-(Cos[e + f*x]*Csc[(e + f*x)/2]^2))
^m*(Sec[(e + f*x)/2]^2)^m + (-8*AppellF1[1, m, 1 - m, 2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2] + AppellF1[1, m, -m, 2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2])*(Csc[(e + f*x)/2]^2)^m*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^m))*
(b*Sec[e + f*x])^m*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^m*Tan[(e + f*x)/2]^2)/
(8*f*(-1 + m)*(Csc[(e + f*x)/2]^2)^m*(-(m*(2^(2 + m)*AppellF1[1 - m, -m, 1,
 2 - m, (Cos[e + f*x]*Sec[(e + f*x)/2]^2)/2, Cos[e + f*x]*Sec[(e + f*x)/2]^
2]*Cos[e + f*x]*(Csc[(e + f*x)/2]^2)^(1 + m) + (-1 + m)*(AppellF1[1, m, -m,
 2, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f*x)/2]^4*(-(Cos[e +
f*x]*Csc[(e + f*x)/2]^2))^m*(Sec[(e + f*x)/2]^2)^m + (-8*AppellF1[1, m, 1 -
 m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[1, m, -m, 2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*(Csc[(e + f*x)/2]^2)^m*(Cos[e + f*x]
*Sec[(e + f*x)/2]^2)^m))*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^m*Tan[(e + f*x)/
2])/(8*(-1 + m)*(Csc[(e + f*x)/2]^2)^m) - (Sec[(e + f*x)/2]^2*(2^(2 + m)*Ap
pellF1[1 - m, -m, 1, 2 - m, (Cos[e + f*x]*Sec[(e + f*x)/2]^2)/2, Cos[e + f*
x]*Sec[(e + f*x)/2]^2]*Cos[e + f*x]*(Csc[(e + f*x)/2]^2)^(1 + m) + (-1 + m)
*(AppellF1[1, m, -m, 2, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f
*x)/2]^4*(-(Cos[e + f*x]*Csc[(e + f*x)/2]^2))^m*(Sec[(e + f*x)/2]^2)^m + (-
8*AppellF1[1, m, 1 - m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + Appel
lF1[1, m, -m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*(Csc[(e + f*x)/2
]^2)^m*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^m))*(Cos[(e + f*x)/2]^2*Sec[e + f*
x])^m*Tan[(e + f*x)/2])/(8*(-1 + m)*(Csc[(e + f*x)/2]^2)^m) - ((Cos[(e + f*
x)/2]^2*Sec[e + f*x])^m*Tan[(e + f*x)/2]^2*(-(2^(2 + m)*(1 + m)*AppellF1[1
```

```
- m, -m, 1, 2 - m, (Cos[e + f*x]*Sec[(e + f*x)/2]^2)/2, Cos[e + f*x]*Sec[(e
 + f*x)/2]^2]*Cos[e + f*x]*Cot[(e + f*x)/2]*(Csc[(e + f*x)/2]^2)^(1 + m)) -
 2^(2 + m)*AppellF1[1 - m, -m, 1, 2 - m, (Cos[e + f*x]*Sec[(e + f*x)/2]^2)/
2, Cos[e + f*x]*Sec[(e + f*x)/2]^2]*(Csc[(e + f*x)/2]^2)^(1 + m)*Sin[e + f*
x] + 2^(2 + m)*Cos[e + f*x]*(Csc[(e + f*x)/2]^2)^(1 + m)*(-(((1 - m)*m*Appe
llF1[2 - m, 1 - m, 1, 3 - m, (Cos[e + f*x]*Sec[(e + f*x)/2]^2)/2, Cos[e + f
*x]*Sec[(e + f*x)/2]^2]*(-(Sec[(e + f*x)/2]^2*Sin[e + f*x])/2 + (Cos[e + f*
x]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/2))/(2 - m)) + ((1 - m)*AppellF1[2
- m, -m, 2, 3 - m, (Cos[e + f*x]*Sec[(e + f*x)/2]^2)/2, Cos[e + f*x]*Sec[(e
 + f*x)/2]^2]*(-(Sec[(e + f*x)/2]^2*Sin[e + f*x]) + Cos[e + f*x]*Sec[(e + f
*x)/2]^2*Tan[(e + f*x)/2]))/(2 - m)) + (-1 + m)*(m*AppellF1[1, m, -m, 2, Co
t[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f*x)/2]^3*(-(Cos[e + f*x]*C
sc[(e + f*x)/2]^2))^m*(Sec[(e + f*x)/2]^2)^m - 2*AppellF1[1, m, -m, 2, Cot[
(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f*x)/2]^3*Csc[(e + f*x)/2]^2*
(-(Cos[e + f*x]*Csc[(e + f*x)/2]^2))^m*(Sec[(e + f*x)/2]^2)^m + Cot[(e + f*
x)/2]^4*(-(Cos[e + f*x]*Csc[(e + f*x)/2]^2))^m*(-(m*AppellF1[2, m, 1 - m, 3
, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f*x)/2]*Csc[(e + f*x)/2
]^2)/2 - (m*AppellF1[2, 1 + m, -m, 3, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]
^2]*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^2)/2)*(Sec[(e + f*x)/2]^2)^m - m*(-8*
AppellF1[1, m, 1 - m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF
1[1, m, -m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Cot[(e + f*x)/2]*(
Csc[(e + f*x)/2]^2)^m*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^m + m*AppellF1[1, m
, -m, 2, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f*x)/2]^4*(-(Cos
[e + f*x]*Csc[(e + f*x)/2]^2))^(-1 + m)*(Sec[(e + f*x)/2]^2)^m*(Cos[e + f*x
]*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^2 + Csc[(e + f*x)/2]^2*Sin[e + f*x]) +
m*(-8*AppellF1[1, m, 1 - m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + A
ppellF1[1, m, -m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*(Csc[(e + f*
x)/2]^2)^m*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(-1 + m)*(-(Sec[(e + f*x)/2]^2
*Sin[e + f*x]) + Cos[e + f*x]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + (Csc[(
e + f*x)/2]^2)^m*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^m*((m*AppellF1[2, m, 1 -
 m, 3, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e +
 f*x)/2])/2 + (m*AppellF1[2, 1 + m, -m, 3, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/2 - 8*(-((1 - m)*AppellF1[2,
m, 2 - m, 3, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Ta
n[(e + f*x)/2])/2 + (m*AppellF1[2, 1 + m, 1 - m, 3, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/2)))))/(8*(-1 + m)*(
Csc[(e + f*x)/2]^2)^m) - (m*(2^(2 + m)*AppellF1[1 - m, -m, 1, 2 - m, (Cos[e
 + f*x]*Sec[(e + f*x)/2]^2)/2, Cos[e + f*x]*Sec[(e + f*x)/2]^2]*Cos[e + f*x
]*(Csc[(e + f*x)/2]^2)^(1 + m) + (-1 + m)*(AppellF1[1, m, -m, 2, Cot[(e + f
*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f*x)/2]^4*(-(Cos[e + f*x]*Csc[(e +
f*x)/2]^2))^m*(Sec[(e + f*x)/2]^2)^m + (-8*AppellF1[1, m, 1 - m, 2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[1, m, -m, 2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2])*(Csc[(e + f*x)/2]^2)^m*(Cos[e + f*x]*Sec[(e + f*x)
/2]^2)^m))*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + m)*Tan[(e + f*x)/2]^2*(-
(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e
```

```
+ f*x]*Tan[e + f*x]))/(8*(-1 + m)*(Csc[(e + f*x)/2]^2)^m)))
```

Maple [F] time = 0.386, size = 0, normalized size = 0.

$$\int \left(\cot \left(fx+e\right)\right)^{3} \left(b\sec \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(b*sec(f*x+e))^m,x)
```

```
[Out] int(cot(f*x+e)^3*(b*sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec \left(fx+e\right)\right)^{m} \cot \left(fx+e\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(b*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec \left(fx+e\right)\right)^{m} \cot \left(fx+e\right)^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(b*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^m*cot(f*x + e)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(e+fx\right)\right)^m \cot^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(b*sec(f*x+e))**m,x)
```

```
[Out] Integral((b*sec(e + f*x))**m*cot(e + f*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^m \cot\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(b*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e)^3, x)
```

3.356 $\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx$

Optimal. Leaf size=40

$$-\frac{(b\sec(e+fx))^m\,{}_2F_1\left(3,\frac{m}{2};\frac{m+2}{2};\sec^2(e+fx)\right)}{fm}$$

```
[Out] -((Hypergeometric2F1[3, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^m)
/(f*m))
```

Rubi [A] time = 0.0439608, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2606, 364}

$$-\frac{(b\sec(e+fx))^m\,{}_2F_1\left(3,\frac{m}{2};\frac{m+2}{2};\sec^2(e+fx)\right)}{fm}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*(b*Sec[e + f*x])^m,x]
```

```
[Out] -((Hypergeometric2F1[3, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^m)
/(f*m))
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx = \frac{b\,\text{Subst}\left(\int \frac{(bx)^{-1+m}}{\left(-1+x^2\right)^3}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= -\frac{{}_2F_1\left(3, \frac{m}{2}; \frac{2+m}{2}; \sec^2(e+fx)\right)(b\sec(e+fx))^m}{fm}$$

Mathematica [C] time = 21.7939, size = 4177, normalized size = 104.42

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^5*(b*Sec[e + f*x])^m,x]
```

```
[Out] -(Cot[e + f*x]^5*(b*Sec[e + f*x])^m*((1 - Tan[(e + f*x)/2]^2)^(-1))^(5 + m)
*(-1 + Tan[(e + f*x)/2]^2)^5*(-64*AppellF1[1, m, 1 - m, 2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2*(1 - Tan[(e + f*x)/2]^2)^m + 12
*AppellF1[1, m, -m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*
x)/2]^2*(1 - Tan[(e + f*x)/2]^2)^m - AppellF1[2, m, -m, 3, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^4*(1 - Tan[(e + f*x)/2]^2)^m - (2
^(5 + m)*AppellF1[1 - m, -m, 1, 2 - m, (1 - Tan[(e + f*x)/2]^2)/2, 1 - Tan[
(e + f*x)/2]^2]*(-1 + Tan[(e + f*x)/2]^2))/(-1 + m) + (12*AppellF1[1, m, -m
, 2, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f*x)/2]^2*(1 - Cot[(
e + f*x)/2]^2)^m*(1 + Tan[(e + f*x)/2]^2)^m)/(1 + Cot[(e + f*x)/2]^2)^m - (
AppellF1[2, m, -m, 3, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f*x
)/2]^4*(1 - Cot[(e + f*x)/2]^2)^m*(1 + Tan[(e + f*x)/2]^2)^m)/(1 + Cot[(e +
 f*x)/2]^2)^m))/(64*f*((-5*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]*((1 - Tan[(e
 + f*x)/2]^2)^(-1))^(5 + m)*(-1 + Tan[(e + f*x)/2]^2)^4*(-64*AppellF1[1, m,
 1 - m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2*(1 -
 Tan[(e + f*x)/2]^2)^m + 12*AppellF1[1, m, -m, 2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2*(1 - Tan[(e + f*x)/2]^2)^m - AppellF1[2,
 m, -m, 3, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^4*(1 -
 Tan[(e + f*x)/2]^2)^m - (2^(5 + m)*AppellF1[1 - m, -m, 1, 2 - m, (1 - Tan[
(e + f*x)/2]^2)/2, 1 - Tan[(e + f*x)/2]^2]*(-1 + Tan[(e + f*x)/2]^2))/(-1 +
 m) + (12*AppellF1[1, m, -m, 2, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Co
t[(e + f*x)/2]^2*(1 - Cot[(e + f*x)/2]^2)^m*(1 + Tan[(e + f*x)/2]^2)^m)/(1
+ Cot[(e + f*x)/2]^2)^m - (AppellF1[2, m, -m, 3, Cot[(e + f*x)/2]^2, -Cot[(
e + f*x)/2]^2]*Cot[(e + f*x)/2]^4*(1 - Cot[(e + f*x)/2]^2)^m*(1 + Tan[(e +
f*x)/2]^2)^m)/(1 + Cot[(e + f*x)/2]^2)^m))/64 - ((5 + m)*Sec[(e + f*x)/2]^2
*Tan[(e + f*x)/2]*((1 - Tan[(e + f*x)/2]^2)^(-1))^(6 + m)*(-1 + Tan[(e + f*
```

x)/2]^2)^5*(-64*AppellF1[1, m, 1 - m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2*(1 - Tan[(e + f*x)/2]^2)^m + 12*AppellF1[1, m, -m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2*(1 - Tan[(e + f*x)/2]^2)^m - AppellF1[2, m, -m, 3, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^4*(1 - Tan[(e + f*x)/2]^2)^m - (2^(5 + m)*AppellF1[1 - m, -m, 1, 2 - m, (1 - Tan[(e + f*x)/2]^2)/2, 1 - Tan[(e + f*x)/2]^2]*(-1 + Tan[(e + f*x)/2]^2))/(-1 + m) + (12*AppellF1[1, m, -m, 2, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f*x)/2]^2*(1 - Cot[(e + f*x)/2]^2)^m*(1 + Tan[(e + f*x)/2]^2)^m)/(1 + Cot[(e + f*x)/2]^2)^m - (AppellF1[2, m, -m, 3, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f*x)/2]^4*(1 - Cot[(e + f*x)/2]^2)^m*(1 + Tan[(e + f*x)/2]^2)^m)/(1 + Cot[(e + f*x)/2]^2)^m))/64 - (((1 - Tan[(e + f*x)/2]^2)^(-1))^(5 + m)*(-1 + Tan[(e + f*x)/2]^2)^5*(-((2^(5 + m)*AppellF1[1 - m, -m, 1, 2 - m, (1 - Tan[(e + f*x)/2]^2)/2, 1 - Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(-1 + m)) + 64*m*AppellF1[1, m, 1 - m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]^3*(1 - Tan[(e + f*x)/2]^2)^(-1 + m) - 12*m*AppellF1[1, m, -m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]^3*(1 - Tan[(e + f*x)/2]^2)^(-1 + m) + m*AppellF1[2, m, -m, 3, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]^5*(1 - Tan[(e + f*x)/2]^2)^(-1 + m) - 64*AppellF1[1, m, 1 - m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]*(1 - Tan[(e + f*x)/2]^2)^m + 12*AppellF1[1, m, -m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]*(1 - Tan[(e + f*x)/2]^2)^m - 2*AppellF1[2, m, -m, 3, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]^3*(1 - Tan[(e + f*x)/2]^2)^m - 64*Tan[(e + f*x)/2]^2*(-((1 - m)*AppellF1[2, m, 2 - m, 3, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/2 + (m*AppellF1[2, 1 + m, 1 - m, 3, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/2)*(1 - Tan[(e + f*x)/2]^2)^m + 12*Tan[(e + f*x)/2]^2*((m*AppellF1[2, m, 1 - m, 3, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/2 + (m*AppellF1[2, 1 + m, -m, 3, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/2)*(1 - Tan[(e + f*x)/2]^2)^m - Tan[(e + f*x)/2]^4*((2*m*AppellF1[3, m, 1 - m, 4, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + (2*m*AppellF1[3, 1 + m, -m, 4, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3)*(1 - Tan[(e + f*x)/2]^2)^m - (2^(5 + m)*(((1 - m)*m*AppellF1[2 - m, 1 - m, 1, 3 - m, (1 - Tan[(e + f*x)/2]^2)/2, 1 - Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2*(2 - m)) - ((1 - m)*AppellF1[2 - m, -m, 2, 3 - m, (1 - Tan[(e + f*x)/2]^2)/2, 1 - Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2 - m))*(-1 + Tan[(e + f*x)/2]^2))/(-1 + m) - (m*AppellF1[2, m, -m, 3, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f*x)/2]*(1 - Cot[(e + f*x)/2]^2)^m*Csc[(e + f*x)/2]^2*(1 + Tan[(e + f*x)/2]^2)^(-1 + m))/(1 + Cot[(e + f*x)/2]^2)^m + (12*m*AppellF1[1, m, -m, 2, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*(1 - Cot[(e + f*x)/2]^2)^m*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*(1 + Tan[(e + f*x)/2]^2)^(

```
-1 + m))/(1 + Cot[(e + f*x)/2]^2)^m + 12*m*AppellF1[1, m, -m, 2, Cot[(e + f
*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f*x)/2]^3*(1 - Cot[(e + f*x)/2]^2)^
m*(1 + Cot[(e + f*x)/2]^2)^(-1 - m)*Csc[(e + f*x)/2]^2*(1 + Tan[(e + f*x)/2
]^2)^m - m*AppellF1[2, m, -m, 3, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*C
ot[(e + f*x)/2]^5*(1 - Cot[(e + f*x)/2]^2)^m*(1 + Cot[(e + f*x)/2]^2)^(-1 -
 m)*Csc[(e + f*x)/2]^2*(1 + Tan[(e + f*x)/2]^2)^m + (12*m*AppellF1[1, m, -m
, 2, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f*x)/2]^3*(1 - Cot[(
e + f*x)/2]^2)^(-1 + m)*Csc[(e + f*x)/2]^2*(1 + Tan[(e + f*x)/2]^2)^m)/(1 +
 Cot[(e + f*x)/2]^2)^m - (m*AppellF1[2, m, -m, 3, Cot[(e + f*x)/2]^2, -Cot[
(e + f*x)/2]^2]*Cot[(e + f*x)/2]^5*(1 - Cot[(e + f*x)/2]^2)^(-1 + m)*Csc[(e
 + f*x)/2]^2*(1 + Tan[(e + f*x)/2]^2)^m)/(1 + Cot[(e + f*x)/2]^2)^m - (12*A
ppellF1[1, m, -m, 2, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f*x)
/2]*(1 - Cot[(e + f*x)/2]^2)^m*Csc[(e + f*x)/2]^2*(1 + Tan[(e + f*x)/2]^2)^
m)/(1 + Cot[(e + f*x)/2]^2)^m + (2*AppellF1[2, m, -m, 3, Cot[(e + f*x)/2]^2
, -Cot[(e + f*x)/2]^2]*Cot[(e + f*x)/2]^3*(1 - Cot[(e + f*x)/2]^2)^m*Csc[(e
 + f*x)/2]^2*(1 + Tan[(e + f*x)/2]^2)^m)/(1 + Cot[(e + f*x)/2]^2)^m + (12*C
ot[(e + f*x)/2]^2*(1 - Cot[(e + f*x)/2]^2)^m*(-(m*AppellF1[2, m, 1 - m, 3,
Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^
2)/2 - (m*AppellF1[2, 1 + m, -m, 3, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2
]*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^2)/2)*(1 + Tan[(e + f*x)/2]^2)^m)/(1 +
Cot[(e + f*x)/2]^2)^m - (Cot[(e + f*x)/2]^4*(1 - Cot[(e + f*x)/2]^2)^m*((-2
*m*AppellF1[3, m, 1 - m, 4, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e
 + f*x)/2]*Csc[(e + f*x)/2]^2)/3 - (2*m*AppellF1[3, 1 + m, -m, 4, Cot[(e +
f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^2)/3)*(1
+ Tan[(e + f*x)/2]^2)^m)/(1 + Cot[(e + f*x)/2]^2)^m))/64))
```

Maple [F] time = 0.191, size = 0, normalized size = 0.

$$\int \left(\cot \left(fx+e\right)\right)^{5} \left(b\sec \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(b*sec(f*x+e))^m,x)
```

```
[Out] int(cot(f*x+e)^5*(b*sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec \left(fx+e\right)\right)^{m} \cot \left(fx+e\right)^{5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(b*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sec\left(fx+e\right)\right)^{m}\cot\left(fx+e\right)^{5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(b*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^m*cot(f*x + e)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(b*sec(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(fx+e\right)\right)^{m}\cot\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(b*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e)^5, x)
```

3.357 $\int (b\sec(e+fx))^m \tan^4(e+fx)\,dx$

Optimal. Leaf size=63

$$\frac{\tan^5(e+fx)\cos^2(e+fx)^{\frac{m+5}{2}}(b\sec(e+fx))^m\,{}_2F_1\left(\frac{5}{2},\frac{m+5}{2};\frac{7}{2};\sin^2(e+fx)\right)}{5f}$$

```
[Out] ((Cos[e + f*x]^2)^((5 + m)/2)*Hypergeometric2F1[5/2, (5 + m)/2, 7/2, Sin[e
+ f*x]^2]*(b*Sec[e + f*x])^m*Tan[e + f*x]^5)/(5*f)
```

Rubi [A] time = 0.0395067, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2617}

$$\frac{\tan^5(e+fx)\cos^2(e+fx)^{\frac{m+5}{2}}(b\sec(e+fx))^m\,{}_2F_1\left(\frac{5}{2},\frac{m+5}{2};\frac{7}{2};\sin^2(e+fx)\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^m*Tan[e + f*x]^4,x]
```

```
[Out] ((Cos[e + f*x]^2)^((5 + m)/2)*Hypergeometric2F1[5/2, (5 + m)/2, 7/2, Sin[e
+ f*x]^2]*(b*Sec[e + f*x])^m*Tan[e + f*x]^5)/(5*f)
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (b\sec(e+fx))^m \tan^4(e+fx)\,dx = \frac{\cos^2(e+fx)^{\frac{5+m}{2}}\,{}_2F_1\left(\frac{5}{2},\frac{5+m}{2};\frac{7}{2};\sin^2(e+fx)\right)(b\sec(e+fx))^m\tan^5(e+fx)}{5f}$$

Mathematica [C] time = 29.413, size = 10908, normalized size = 173.14

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Sec[e + f*x])^m*Tan[e + f*x]^4,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.181, size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{m}\left(\tan\left(fx+e\right)\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^m*tan(f*x+e)^4,x)
```

```
[Out] int((b*sec(f*x+e))^m*tan(f*x+e)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{m}\tan\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^m*tan(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)\right)^{m}\tan\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^m*tan(f*x + e)^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(e+fx\right)\right)^{m}\tan^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**m*tan(f*x+e)**4,x)
```

```
[Out] Integral((b*sec(e + f*x))**m*tan(e + f*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{m}\tan\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^m*tan(f*x + e)^4, x)
```

3.358 $\int (b\sec(e+fx))^m \tan^2(e+fx)\,dx$

Optimal. Leaf size=63

$$\frac{\tan^3(e+fx)\cos^2(e+fx)^{\frac{m+3}{2}}(b\sec(e+fx))^m\,{}_2F_1\left(\frac{3}{2},\frac{m+3}{2};\frac{5}{2};\sin^2(e+fx)\right)}{3f}$$

```
[Out] ((Cos[e + f*x]^2)^((3 + m)/2)*Hypergeometric2F1[3/2, (3 + m)/2, 5/2, Sin[e
+ f*x]^2]*(b*Sec[e + f*x])^m*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.0376106, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2617}

$$\frac{\tan^3(e+fx)\cos^2(e+fx)^{\frac{m+3}{2}}(b\sec(e+fx))^m\,{}_2F_1\left(\frac{3}{2},\frac{m+3}{2};\frac{5}{2};\sin^2(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^m*Tan[e + f*x]^2,x]
```

```
[Out] ((Cos[e + f*x]^2)^((3 + m)/2)*Hypergeometric2F1[3/2, (3 + m)/2, 5/2, Sin[e
+ f*x]^2]*(b*Sec[e + f*x])^m*Tan[e + f*x]^3)/(3*f)
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

Rubi steps

$$\int (b\sec(e+fx))^m \tan^2(e+fx)\,dx = \frac{\cos^2(e+fx)^{\frac{3+m}{2}}\,{}_2F_1\left(\frac{3}{2},\frac{3+m}{2};\frac{5}{2};\sin^2(e+fx)\right)(b\sec(e+fx))^m\tan^3(e+fx)}{3f}$$

Mathematica [C] time = 24.9202, size = 6612, normalized size = 104.95

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Sec[e + f*x])^m*Tan[e + f*x]^2,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.25, size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{m}\left(\tan\left(fx+e\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^m*tan(f*x+e)^2,x)
```

```
[Out] int((b*sec(f*x+e))^m*tan(f*x+e)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{m}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^m*tan(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)\right)^{m}\tan\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^m*tan(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(e+fx\right)\right)^{m}\tan^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**m*tan(f*x+e)**2,x)
```

```
[Out] Integral((b*sec(e + f*x))**m*tan(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{m}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^m*tan(f*x + e)^2, x)
```

3.359 $\int \cot^2(e+fx)(b\sec(e+fx))^m\,dx$

Optimal. Leaf size=59

$$-\frac{\cot(e+fx)\cos^2(e+fx)^{\frac{m-1}{2}}(b\sec(e+fx))^m\,{}_2F_1\left(-\frac{1}{2},\frac{m-1}{2};\frac{1}{2};\sin^2(e+fx)\right)}{f}$$

```
[Out] -(((Cos[e + f*x]^2)^((-1 + m)/2)*Cot[e + f*x]*Hypergeometric2F1[-1/2, (-1 +
 m)/2, 1/2, Sin[e + f*x]^2]*(b*Sec[e + f*x])^m)/f)
```

Rubi [A] time = 0.0370596, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2617}

$$-\frac{\cot(e+fx)\cos^2(e+fx)^{\frac{m-1}{2}}(b\sec(e+fx))^m\,{}_2F_1\left(-\frac{1}{2},\frac{m-1}{2};\frac{1}{2};\sin^2(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(b*Sec[e + f*x])^m,x]
```

```
[Out] -(((Cos[e + f*x]^2)^((-1 + m)/2)*Cot[e + f*x]*Hypergeometric2F1[-1/2, (-1 +
 m)/2, 1/2, Sin[e + f*x]^2]*(b*Sec[e + f*x])^m)/f)
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \cot^2(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\cos^2(e+fx)^{\frac{1}{2}(-1+m)}\cot(e+fx)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(-1+m);\frac{1}{2};\sin^2(e+fx)\right)(b\sec(e+fx))^m}{f}$$

Mathematica [C] time = 21.6067, size = 4872, normalized size = 82.58

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^2*(b*Sec[e + f*x])^m,x]
```

```
[Out] (Cot[(e + f*x)/2]*Cot[e + f*x]^2*(b*Sec[e + f*x])^m*(Cos[(e + f*x)/2]^2*Sec
[e + f*x])^m*(-(AppellF1[-1/2, m, -m, 1/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^m) + 3*(Sec[(e + f*x)/2]^2)^m*Ta
n[(e + f*x)/2]^2*((-4*AppellF1[1/2, m, 1 - m, 3/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2)/(3*AppellF1[1/2, m, 1 - m, 3/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + m)*AppellF1[3/2, m, 2 - m, 5/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3/2, 1 + m, 1 - m,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + Appel
lF1[1/2, m, -m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]/(3*AppellF1[1
/2, m, -m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*m*(AppellF1[3/
2, m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2,
1 + m, -m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^
2))))/(2*f*(-(Csc[(e + f*x)/2]^2*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^m*(-(App
ellF1[-1/2, m, -m, 1/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f
*x]*Sec[(e + f*x)/2]^2)^m) + 3*(Sec[(e + f*x)/2]^2)^m*Tan[(e + f*x)/2]^2*((
-4*AppellF1[1/2, m, 1 - m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Co
s[(e + f*x)/2]^2)/(3*AppellF1[1/2, m, 1 - m, 3/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] + 2*((-1 + m)*AppellF1[3/2, m, 2 - m, 5/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + AppellF1[1/2, m, -m, 3/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]/(3*AppellF1[1/2, m, -m, 3/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*m*(AppellF1[3/2, m, 1 - m, 5/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + m, -m, 5/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))))/4 + (Cot[(e +
 f*x)/2]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^m*(-((Cos[e + f*x]*Sec[(e + f*x)
/2]^2)^m*(-(m*AppellF1[1/2, m, 1 - m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) - m*AppellF1[1/2, 1 + m, -m,
3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f
*x)/2])) - m*AppellF1[-1/2, m, -m, 1/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(-1 + m)*(-(Sec[(e + f*x)/2]^2*Sin[
e + f*x]) + Cos[e + f*x]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + 3*(Sec[(e +
 f*x)/2]^2)^(1 + m)*Tan[(e + f*x)/2]*((-4*AppellF1[1/2, m, 1 - m, 3/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2)/(3*AppellF1[1/2, m
, 1 - m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + m)*Appell
F1[3/2, m, 2 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF
1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e
```

```
+ f*x)/2]^2) + AppellF1[1/2, m, -m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]/(3*AppellF1[1/2, m, -m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
] + 2*m*(AppellF1[3/2, m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + AppellF1[3/2, 1 + m, -m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
])*Tan[(e + f*x)/2]^2)) + 3*m*(Sec[(e + f*x)/2]^2)^m*Tan[(e + f*x)/2]^3*((-
4*AppellF1[1/2, m, 1 - m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos
[(e + f*x)/2]^2)/(3*AppellF1[1/2, m, 1 - m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2] + 2*((-1 + m)*AppellF1[3/2, m, 2 - m, 5/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + AppellF1[1/2, m, -m, 3/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]/(3*AppellF1[1/2, m, -m, 3/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*m*(AppellF1[3/2, m, 1 - m, 5/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + m, -m, 5/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + 3*(Sec[(e + f*
x)/2]^2)^m*Tan[(e + f*x)/2]^2*((4*AppellF1[1/2, m, 1 - m, 3/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]*Sin[(e + f*x)/2])/(3*AppellF1
[1/2, m, 1 - m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + m)
*AppellF1[3/2, m, 2 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*
AppellF1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*
Tan[(e + f*x)/2]^2) - (4*Cos[(e + f*x)/2]^2*(-((1 - m)*AppellF1[3/2, m, 2 -
 m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e
 + f*x)/2])/3 + (m*AppellF1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/(3*AppellF1[1/2,
 m, 1 - m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + m)*Appe
llF1[3/2, m, 2 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*Appel
lF1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(
e + f*x)/2]^2) + ((m*AppellF1[3/2, m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + (m*AppellF1[3/2, 1
 + m, -m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*
Tan[(e + f*x)/2])/3)/(3*AppellF1[1/2, m, -m, 3/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] + 2*m*(AppellF1[3/2, m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2] + AppellF1[3/2, 1 + m, -m, 5/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (AppellF1[1/2, m, -m, 3/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(2*m*(AppellF1[3/2, m, 1 - m, 5/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + m, -m, 5/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*((m
*AppellF1[3/2, m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[
(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + (m*AppellF1[3/2, 1 + m, -m, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3)
 + 2*m*Tan[(e + f*x)/2]^2*((-3*(1 - m)*AppellF1[5/2, m, 2 - m, 7/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 +
(6*m*AppellF1[5/2, 1 + m, 1 - m, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1 + m)*AppellF1[5/2, 2 + m
, -m, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[
(e + f*x)/2])/5)))/(3*AppellF1[1/2, m, -m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
```

```
 + f*x)/2]^2] + 2*m*(AppellF1[3/2, m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] + AppellF1[3/2, 1 + m, -m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (4*AppellF1[1/2, m, 1 - m, 3/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2*(2*((-1 + m)*Appell
F1[3/2, m, 2 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF
1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e
+ f*x)/2]^2*Tan[(e + f*x)/2] + 3*(-((1 - m)*AppellF1[3/2, m, 2 - m, 5/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])
/3 + (m*AppellF1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + 2*Tan[(e + f*x)/2]^2*((-1
+ m)*((-3*(2 - m)*AppellF1[5/2, m, 3 - m, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*m*AppellF1[5/2, 1
+ m, 2 - m, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^
2*Tan[(e + f*x)/2])/5) + m*((-3*(1 - m)*AppellF1[5/2, 1 + m, 2 - m, 7/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])
/5 + (3*(1 + m)*AppellF1[5/2, 2 + m, 1 - m, 7/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2,
m, 1 - m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + m)*Appel
lF1[3/2, m, 2 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*Appell
F1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e
 + f*x)/2]^2)^2)))/2 + (m*Cot[(e + f*x)/2]*(Cos[(e + f*x)/2]^2*Sec[e + f*x]
)^(-1 + m)*(-(AppellF1[-1/2, m, -m, 1/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^m) + 3*(Sec[(e + f*x)/2]^2)^m*Tan[
(e + f*x)/2]^2*((-4*AppellF1[1/2, m, 1 - m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2]*Cos[(e + f*x)/2]^2)/(3*AppellF1[1/2, m, 1 - m, 3/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + m)*AppellF1[3/2, m, 2 - m, 5/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3/2, 1 + m, 1 - m, 5
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + AppellF
1[1/2, m, -m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]/(3*AppellF1[1/2
, m, -m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*m*(AppellF1[3/2,
 m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1
+ m, -m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)
))*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*
Sec[e + f*x]*Tan[e + f*x]))/2))
```

Maple [F] time = 0.271, size = 0, normalized size = 0.

$$\int \left(\cot\left(fx+e\right)\right)^{2} \left(b\sec\left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(b*sec(f*x+e))^m,x)
```

```
[Out] int(cot(f*x+e)^2*(b*sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(fx+e))^m \cot(fx+e)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(b*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(fx+e))^m \cot(fx+e)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(b*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^m*cot(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(e+fx))^m \cot^2(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(b*sec(f*x+e))**m,x)
```

```
[Out] Integral((b*sec(e + f*x))**m*cot(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{m}\cot\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(b*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e)^2, x)
```

3.360 $\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx$

Optimal. Leaf size=63

$$-\frac{\cot^3(e+fx)\cos^2(e+fx)^{\frac{m-3}{2}}(b\sec(e+fx))^m\,{}_2F_1\left(-\frac{3}{2},\frac{m-3}{2};-\frac{1}{2};\sin^2(e+fx)\right)}{3f}$$

```
[Out] -((Cos[e + f*x]^2)^((-3 + m)/2)*Cot[e + f*x]^3*Hypergeometric2F1[-3/2, (-3
+ m)/2, -1/2, Sin[e + f*x]^2]*(b*Sec[e + f*x])^m)/(3*f)
```

Rubi [A] time = 0.0373571, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2617}

$$-\frac{\cot^3(e+fx)\cos^2(e+fx)^{\frac{m-3}{2}}(b\sec(e+fx))^m\,{}_2F_1\left(-\frac{3}{2},\frac{m-3}{2};-\frac{1}{2};\sin^2(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(b*Sec[e + f*x])^m,x]
```

```
[Out] -((Cos[e + f*x]^2)^((-3 + m)/2)*Cot[e + f*x]^3*Hypergeometric2F1[-3/2, (-3
+ m)/2, -1/2, Sin[e + f*x]^2]*(b*Sec[e + f*x])^m)/(3*f)
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\cos^2(e+fx)^{\frac{1}{2}(-3+m)}\cot^3(e+fx)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}(-3+m);-\frac{1}{2};\sin^2(e+fx)\right)(b\sec(e}{3f}$$

Mathematica [C] time = 25.1862, size = 6671, normalized size = 105.89

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4*(b*Sec[e + f*x])^m,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.179, size = 0, normalized size = 0.

$$\int \left(\cot\left(fx+e\right)\right)^{4}\left(b\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(b*sec(f*x+e))^m,x)
```

```
[Out] int(cot(f*x+e)^4*(b*sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{m}\cot\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(b*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)\right)^{m}\cot\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(b*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^m*cot(f*x + e)^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(e+fx\right)\right)^{m}\cot^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(b*sec(f*x+e))**m,x)
```

```
[Out] Integral((b*sec(e + f*x))**m*cot(e + f*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{m}\cot\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(b*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e)^4, x)
```

3.361 $\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx$

Optimal. Leaf size=63

$$-\frac{\cot^5(e+fx)\cos^2(e+fx)^{\frac{m-5}{2}}(b\sec(e+fx))^m\,{}_2F_1\left(-\frac{5}{2},\frac{m-5}{2};-\frac{3}{2};\sin^2(e+fx)\right)}{5f}$$

```
[Out] -((Cos[e + f*x]^2)^((-5 + m)/2)*Cot[e + f*x]^5*Hypergeometric2F1[-5/2, (-5
+ m)/2, -3/2, Sin[e + f*x]^2]*(b*Sec[e + f*x])^m)/(5*f)
```

Rubi [A] time = 0.0372654, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2617}

$$-\frac{\cot^5(e+fx)\cos^2(e+fx)^{\frac{m-5}{2}}(b\sec(e+fx))^m\,{}_2F_1\left(-\frac{5}{2},\frac{m-5}{2};-\frac{3}{2};\sin^2(e+fx)\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*(b*Sec[e + f*x])^m,x]
```

```
[Out] -((Cos[e + f*x]^2)^((-5 + m)/2)*Cot[e + f*x]^5*Hypergeometric2F1[-5/2, (-5
+ m)/2, -3/2, Sin[e + f*x]^2]*(b*Sec[e + f*x])^m)/(5*f)
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

Rubi steps

$$\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\cos^2(e+fx)^{\frac{1}{2}(-5+m)}\cot^5(e+fx)\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}(-5+m);-\frac{3}{2};\sin^2(e+fx)\right)(b\sec(e+}{5f}$$

Mathematica [F] time = 0.570762, size = 0, normalized size = 0.

$$\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^6*(b*Sec[e + f*x])^m,x]
```

```
[Out] Integrate[Cot[e + f*x]^6*(b*Sec[e + f*x])^m, x]
```

Maple [F] time = 0.192, size = 0, normalized size = 0.

$$\int \left(\cot\left(fx+e\right)\right)^{6}\left(b\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(b*sec(f*x+e))^m,x)
```

```
[Out] int(cot(f*x+e)^6*(b*sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{m}\cot\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(b*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e)^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)\right)^{m}\cot\left(fx+e\right)^{6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(b*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^m*cot(f*x + e)^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(b*sec(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)\right)^{m}\cot\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(b*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e)^6, x)
```

3.362 $\int (a\sec(e+fx))^m (b\tan(e+fx))^n\,dx$

Optimal. Leaf size=82

$$\frac{(a\sec(e+fx))^m (b\tan(e+fx))^{n+1} \cos^2(e+fx)^{\frac{1}{2}(m+n+1)}\,{}_2F_1\left(\frac{n+1}{2},\frac{1}{2}(m+n+1);\frac{n+3}{2};\sin^2(e+fx)\right)}{bf(n+1)}$$

```
[Out] ((Cos[e + f*x]^2)^((1 + m + n)/2)*Hypergeometric2F1[(1 + n)/2, (1 + m + n)/
2, (3 + n)/2, Sin[e + f*x]^2]*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(1 + n))/
(b*f*(1 + n))
```

Rubi [A] time = 0.045036, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2617}

$$\frac{(a\sec(e+fx))^m (b\tan(e+fx))^{n+1} \cos^2(e+fx)^{\frac{1}{2}(m+n+1)}\,{}_2F_1\left(\frac{n+1}{2},\frac{1}{2}(m+n+1);\frac{n+3}{2};\sin^2(e+fx)\right)}{bf(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```


```
[Out] ((Cos[e + f*x]^2)^((1 + m + n)/2)*Hypergeometric2F1[(1 + n)/2, (1 + m + n)/
2, (3 + n)/2, Sin[e + f*x]^2]*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(1 + n))/
(b*f*(1 + n))
```


Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (a\sec(e+fx))^m (b\tan(e+fx))^n\,dx = \frac{\cos^2(e+fx)^{\frac{1}{2}(1+m+n)}\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{2}(1+m+n);\frac{3+n}{2};\sin^2(e+fx)\right)(a\sec(e+fx))}{bf(1+n)}$$

Mathematica [A] time = 0.144983, size = 80, normalized size = 0.98

$$\frac{b\left(-\tan^2(e+fx)\right)^{\frac{1-n}{2}}(a\sec(e+fx))^m(b\tan(e+fx))^{n-1}\,{}_2F_1\left(\frac{m}{2},\frac{1-n}{2};\frac{m+2}{2};\sec^2(e+fx)\right)}{fm}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] (b*Hypergeometric2F1[m/2, (1 - n)/2, (2 + m)/2, Sec[e + f*x]^2]*(a*Sec[e +
f*x])^m*(b*Tan[e + f*x])^(-1 + n)*(-Tan[e + f*x]^2)^((1 - n)/2))/(f*m)
```

Maple [F] time = 0.748, size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)\right)^m\left(b\tan\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

```
[Out] int((a*sec(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)\right)^m\left(b\tan\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sec\left(fx+e\right)\right)^m\left(b\tan\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec \left(e+fx\right)\right)^{m}\left(b\tan \left(e+fx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(f*x+e))**m*(b*tan(f*x+e))**n,x)
```

```
[Out] Integral((a*sec(e + f*x))**m*(b*tan(e + f*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec \left(fx+e\right)\right)^{m}\left(b\tan \left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

3.363 $\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx$

Optimal. Leaf size=74

$$\frac{2(d\tan(a+bx))^{n+3}}{bd^3(n+3)}+\frac{(d\tan(a+bx))^{n+5}}{bd^5(n+5)}+\frac{(d\tan(a+bx))^{n+1}}{bd(n+1)}$$

```
[Out] (d*Tan[a + b*x])^(1 + n)/(b*d*(1 + n)) + (2*(d*Tan[a + b*x])^(3 + n))/(b*d^
3*(3 + n)) + (d*Tan[a + b*x])^(5 + n)/(b*d^5*(5 + n))
```

Rubi [A] time = 0.0660517, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2607, 270}

$$\frac{2(d\tan(a+bx))^{n+3}}{bd^3(n+3)}+\frac{(d\tan(a+bx))^{n+5}}{bd^5(n+5)}+\frac{(d\tan(a+bx))^{n+1}}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^6*(d*Tan[a + b*x])^n,x]
```

```
[Out] (d*Tan[a + b*x])^(1 + n)/(b*d*(1 + n)) + (2*(d*Tan[a + b*x])^(3 + n))/(b*d^
3*(3 + n)) + (d*Tan[a + b*x])^(5 + n)/(b*d^5*(5 + n))
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx = \frac{\text{Subst}\left(\int (dx)^n\left(1+x^2\right)^2\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left((dx)^n+\frac{2(dx)^{2+n}}{d^2}+\frac{(dx)^{4+n}}{d^4}\right)dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{(d\tan(a+bx))^{1+n}}{bd(1+n)}+\frac{2(d\tan(a+bx))^{3+n}}{bd^3(3+n)}+\frac{(d\tan(a+bx))^{5+n}}{bd^5(5+n)}$$

Mathematica [A] time = 2.09949, size = 101, normalized size = 1.36

$$\frac{d(d\tan(a+bx))^{n-1}\left(\tan^2(a+bx)\sec^4(a+bx)\left(2(n+3)\cos(2(a+bx))+\cos(4(a+bx))+n^2+6n+8\right)+8\left(-\tan^2(a\right.\right.}{b(n+1)(n+3)(n+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^6*(d*Tan[a + b*x])^n,x]
```

```
[Out] (d*(d*Tan[a + b*x])^(-1 + n)*((8 + 6*n + n^2 + 2*(3 + n)*Cos[2*(a + b*x)] +
 Cos[4*(a + b*x)])*Sec[a + b*x]^4*Tan[a + b*x]^2 + 8*(-Tan[a + b*x]^2)^((1
- n)/2)))/(b*(1 + n)*(3 + n)*(5 + n))
```

Maple [F] time = 0.196, size = 0, normalized size = 0.

$$\int (\sec(bx+a))^6 (d\tan(bx+a))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^6*(d*tan(b*x+a))^n,x)
```

```
[Out] int(sec(b*x+a)^6*(d*tan(b*x+a))^n,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66651, size = 215, normalized size = 2.91

$$\frac{\left(8\,\cos\left(bx+a\right)^{4}+4\,(n+1)\cos\left(bx+a\right)^{2}+n^{2}+4\,n+3\right)\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{n}\sin\left(bx+a\right)}{\left(bn^{3}+9\,bn^{2}+23\,bn+15\,b\right)\cos\left(bx+a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] (8*cos(b*x + a)^4 + 4*(n + 1)*cos(b*x + a)^2 + n^2 + 4*n + 3)*(d*sin(b*x +
a)/cos(b*x + a))^n*sin(b*x + a)/((b*n^3 + 9*b*n^2 + 23*b*n + 15*b)*cos(b*x
+ a)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**6*(d*tan(b*x+a))**n,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*(d*tan(b*x+a))^n,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.364 $\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx$

Optimal. Leaf size=49

$$\frac{(d\tan(a+bx))^{n+3}}{bd^3(n+3)}+\frac{(d\tan(a+bx))^{n+1}}{bd(n+1)}$$

```
[Out] (d*Tan[a + b*x])^(1 + n)/(b*d*(1 + n)) + (d*Tan[a + b*x])^(3 + n)/(b*d^3*(3
 + n))
```

Rubi [A] time = 0.0511222, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2607, 14}

$$\frac{(d\tan(a+bx))^{n+3}}{bd^3(n+3)}+\frac{(d\tan(a+bx))^{n+1}}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^4*(d*Tan[a + b*x])^n,x]
```

```
[Out] (d*Tan[a + b*x])^(1 + n)/(b*d*(1 + n)) + (d*Tan[a + b*x])^(3 + n)/(b*d^3*(3
 + n))
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx = \frac{\text{Subst}\left(\int (dx)^n\left(1+x^2\right)dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left((dx)^n + \frac{(dx)^{2+n}}{d^2}\right)dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{(d\tan(a+bx))^{1+n}}{bd(1+n)} + \frac{(d\tan(a+bx))^{3+n}}{bd^3(3+n)}$$

Mathematica [A] time = 1.14592, size = 78, normalized size = 1.59

$$\frac{d(d\tan(a+bx))^{n-1}\left(2\left(-\tan^2(a+bx)\right)^{\frac{1-n}{2}} + \tan^2(a+bx)\sec^2(a+bx)(\cos(2(a+bx))+n+2)\right)}{b(n+1)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^4*(d*Tan[a + b*x])^n,x]
```

```
[Out] (d*(d*Tan[a + b*x])^(-1 + n)*((2 + n + Cos[2*(a + b*x)])*Sec[a + b*x]^2*Tan
[a + b*x]^2 + 2*(-Tan[a + b*x]^2)^((1 - n)/2)))/(b*(1 + n)*(3 + n))
```

Maple [F] time = 0.139, size = 0, normalized size = 0.

$$\int (\sec(bx+a))^4 (d\tan(bx+a))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4*(d*tan(b*x+a))^n,x)
```

```
[Out] int(sec(b*x+a)^4*(d*tan(b*x+a))^n,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71338, size = 151, normalized size = 3.08

$$\frac{\left(2\,\cos\left(bx+a\right)^{2}+n+1\right)\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{n}\sin\left(bx+a\right)}{\left(bn^{2}+4\,bn+3\,b\right)\cos\left(bx+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] (2*cos(b*x + a)^2 + n + 1)*(d*sin(b*x + a)/cos(b*x + a))^n*sin(b*x + a)/((b
*n^2 + 4*b*n + 3*b)*cos(b*x + a)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\tan\left(a+bx\right)\right)^{n}\sec^{4}\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4*(d*tan(b*x+a))**n,x)
```

```
[Out] Integral((d*tan(a + b*x))**n*sec(a + b*x)**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*(d*tan(b*x+a))^n,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.365 $\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx$

Optimal. Leaf size=24

$$\frac{(d\tan(a+bx))^{n+1}}{bd(n+1)}$$

```
[Out] (d*Tan[a + b*x])^(1 + n)/(b*d*(1 + n))
```

Rubi [A] time = 0.0385258, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2607, 32}

$$\frac{(d\tan(a+bx))^{n+1}}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^2*(d*Tan[a + b*x])^n,x]
```

```
[Out] (d*Tan[a + b*x])^(1 + n)/(b*d*(1 + n))
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx &= \frac{\text{Subst}\left(\int (dx)^n\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{(d\tan(a+bx))^{1+n}}{bd(1+n)}\end{aligned}$$

Mathematica [A] time = 0.020364, size = 25, normalized size = 1.04

$$\frac{\tan(a+bx)(d\tan(a+bx))^n}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^2*(d*Tan[a + b*x])^n,x]
```

```
[Out] (Tan[a + b*x]*(d*Tan[a + b*x])^n)/(b*(1 + n))
```

Maple [A] time = 0.017, size = 25, normalized size = 1.

$$\frac{(d\tan(bx+a))^{1+n}}{bd(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*(d*tan(b*x+a))^n,x)
```

```
[Out] (d*tan(b*x+a))^(1+n)/b/d/(1+n)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65202, size = 96, normalized size = 4.

$$\frac{\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^n\sin(bx+a)}{(bn+b)\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] (d*sin(b*x + a)/cos(b*x + a))^n*sin(b*x + a)/((b*n + b)*cos(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(a+bx))^n \sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*(d*tan(b*x+a))**n,x)
```

```
[Out] Integral((d*tan(a + b*x))**n*sec(a + b*x)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*(d*tan(b*x+a))^n,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.366 $\int (d\tan(a+bx))^n\,dx$

Optimal. Leaf size=50

$$\frac{(d\tan(a+bx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(a+bx)\right)}{bd(n+1)}$$

```
[Out] (Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[a + b*x]^2]*(d*Tan[a + b*x
])^(1 + n))/(b*d*(1 + n))
```

Rubi [A] time = 0.0288274, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3476, 364}

$$\frac{(d\tan(a+bx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(a+bx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[a + b*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[a + b*x]^2]*(d*Tan[a + b*x
])^(1 + n))/(b*d*(1 + n))
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (d\tan(a+bx))^n\,dx = \frac{d\,\mathrm{Subst}\left(\int \frac{x^n}{d^2+x^2}\,dx, x, d\tan(a+bx)\right)}{b}$$
$$= \frac{{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(a+bx)\right)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

Mathematica [A] time = 0.0475396, size = 53, normalized size = 1.06

$$\frac{\tan(a+bx)(d\tan(a+bx))^n\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+1}{2}+1;-\tan^2(a+bx)\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[a + b*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n)/2, 1 + (1 + n)/2, -Tan[a + b*x]^2]*Tan[a + b*
x]*(d*Tan[a + b*x])^n)/(b*(1 + n))
```

Maple [F] time = 0.347, size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(b*x+a))^n,x)
```

```
[Out] int((d*tan(b*x+a))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((d\tan(bx+a))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] integral((d*tan(b*x + a))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(a+bx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(b*x+a))**n,x)
```

```
[Out] Integral((d*tan(a + b*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(b*x+a))^n,x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^n, x)
```

3.367 $\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx$

Optimal. Leaf size=50

$$\frac{(d\tan(a+bx))^{n+1}\,{}_2F_1\left(2,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(a+bx)\right)}{bd(n+1)}$$

```
[Out] (Hypergeometric2F1[2, (1 + n)/2, (3 + n)/2, -Tan[a + b*x]^2]*(d*Tan[a + b*x
])^(1 + n))/(b*d*(1 + n))
```

Rubi [A] time = 0.0466357, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2607, 364}

$$\frac{(d\tan(a+bx))^{n+1}\,{}_2F_1\left(2,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(a+bx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*(d*Tan[a + b*x])^n,x]
```

```
[Out] (Hypergeometric2F1[2, (1 + n)/2, (3 + n)/2, -Tan[a + b*x]^2]*(d*Tan[a + b*x
])^(1 + n))/(b*d*(1 + n))
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx = \frac{\operatorname{Subst}\left(\int \frac{(dx)^n}{\left(1+x^2\right)^2}\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{{}_2F_1\left(2, \frac{1+n}{2}; \frac{3+n}{2}; -\tan^2(a+bx)\right)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

<u>Mathematica</u> [C] time = 4.37625, size = 939, normalized size = 18.78

$$\overline{b\left(\frac{2(n+1)\left(-F_1\left(\frac{n+3}{2};n,2;\frac{n+5}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)+8F_1\left(\frac{n+3}{2};n,3;\frac{n+5}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)-12F_1\left(\frac{n+3}{2};n,4;\frac{n+5}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan\right.}{}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[a + b*x]^2*(d*Tan[a + b*x])^n,x]
```

```
[Out] (2*(AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)
/2]^2] - 4*AppellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a
 + b*x)/2]^2] + 4*AppellF1[(1 + n)/2, n, 3, (3 + n)/2, Tan[(a + b*x)/2]^2,
-Tan[(a + b*x)/2]^2])*Cos[a + b*x]^2*Tan[(a + b*x)/2]*(d*Tan[a + b*x])^n)/(
b*((AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)
/2]^2] - 4*AppellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a
 + b*x)/2]^2] + 4*AppellF1[(1 + n)/2, n, 3, (3 + n)/2, Tan[(a + b*x)/2]^2,
-Tan[(a + b*x)/2]^2])*Sec[(a + b*x)/2]^2 + n*(AppellF1[(1 + n)/2, n, 1, (3
+ n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 4*AppellF1[(1 + n)/2, n,
 2, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 4*AppellF1[(1 + n
)/2, n, 3, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*Sec[(a + b*
x)/2]^2*Sec[a + b*x] + (2*(1 + n)*(-AppellF1[(3 + n)/2, n, 2, (5 + n)/2, Ta
n[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 8*AppellF1[(3 + n)/2, n, 3, (5 + n
)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 12*AppellF1[(3 + n)/2, n, 4
, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + n*AppellF1[(3 + n)/
2, 1 + n, 1, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 4*n*Appe
llF1[(3 + n)/2, 1 + n, 2, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^
2] + 4*n*AppellF1[(3 + n)/2, 1 + n, 3, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[
(a + b*x)/2]^2])*Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2]^2)/(3 + n) - 2*n*(Appe
llF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] -
 4*AppellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/
2]^2] + 4*AppellF1[(1 + n)/2, n, 3, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a
+ b*x)/2]^2])*Sec[a + b*x]*Tan[(a + b*x)/2]^2))
```

Maple [F] time = 0.651, size = 0, normalized size = 0.

$$\int (\cos (bx+a))^2 (d \tan (bx+a))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*(d*tan(b*x+a))^n,x)
```

```
[Out] int(cos(b*x+a)^2*(d*tan(b*x+a))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d \tan (bx+a))^n \cos (bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^n*cos(b*x + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((d \tan (bx+a))^n \cos (bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] integral((d*tan(b*x + a))^n*cos(b*x + a)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d \tan (a+bx))^n \cos^2 (a+bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*(d*tan(b*x+a))**n,x)
```

```
[Out] Integral((d*tan(a + b*x))**n*cos(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^n \cos(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*(d*tan(b*x+a))^n,x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^n*cos(b*x + a)^2, x)
```

3.368 $\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx$

Optimal. Leaf size=50

$$\frac{(d\tan(a+bx))^{n+1}\,{}_2F_1\left(3,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(a+bx)\right)}{bd(n+1)}$$

```
[Out] (Hypergeometric2F1[3, (1 + n)/2, (3 + n)/2, -Tan[a + b*x]^2]*(d*Tan[a + b*x
])^(1 + n))/(b*d*(1 + n))
```

Rubi [A] time = 0.0445644, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2607, 364}

$$\frac{(d\tan(a+bx))^{n+1}\,{}_2F_1\left(3,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(a+bx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4*(d*Tan[a + b*x])^n,x]
```

```
[Out] (Hypergeometric2F1[3, (1 + n)/2, (3 + n)/2, -Tan[a + b*x]^2]*(d*Tan[a + b*x
])^(1 + n))/(b*d*(1 + n))
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx = \frac{\text{Subst}\left(\int \frac{(dx)^n}{(1+x^2)^3}\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{{}_2F_1\left(3, \frac{1+n}{2}; \frac{3+n}{2}; -\tan^2(a+bx)\right)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

Mathematica [C] time = 13.2093, size = 1712, normalized size = 34.24

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[a + b*x]^4*(d*Tan[a + b*x])^n,x]
```

```
[Out] (-8*(3 + n)*(AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[
(a + b*x)/2]^2] - 8*(AppellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(a + b*x)/2]^
2, -Tan[(a + b*x)/2]^2] - 3*AppellF1[(1 + n)/2, n, 3, (3 + n)/2, Tan[(a + b
*x)/2]^2, -Tan[(a + b*x)/2]^2] + 4*AppellF1[(1 + n)/2, n, 4, (3 + n)/2, Tan
[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 2*AppellF1[(1 + n)/2, n, 5, (3 + n)
/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]))*Cos[(a + b*x)/2]^3*Cos[a + b
*x]^5*Sin[(a + b*x)/2]^2*(d*Tan[a + b*x])^n)/(b*(1 + n)*((3 + n)*AppellF1[(
1 + n)/2, n, 1, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(1 + Co
s[a + b*x]) + 2*(16*AppellF1[(3 + n)/2, n, 3, (5 + n)/2, Tan[(a + b*x)/2]^2
, -Tan[(a + b*x)/2]^2] - 72*AppellF1[(3 + n)/2, n, 4, (5 + n)/2, Tan[(a + b
*x)/2]^2, -Tan[(a + b*x)/2]^2] + 128*AppellF1[(3 + n)/2, n, 5, (5 + n)/2, T
an[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 80*AppellF1[(3 + n)/2, n, 6, (5 +
 n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + n*AppellF1[(3 + n)/2, 1 +
 n, 1, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 8*n*AppellF1[(
3 + n)/2, 1 + n, 2, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 2
4*n*AppellF1[(3 + n)/2, 1 + n, 3, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a +
b*x)/2]^2] - 32*n*AppellF1[(3 + n)/2, 1 + n, 4, (5 + n)/2, Tan[(a + b*x)/2]
^2, -Tan[(a + b*x)/2]^2] + 16*n*AppellF1[(3 + n)/2, 1 + n, 5, (5 + n)/2, Ta
n[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 24*AppellF1[(1 + n)/2, n, 2, (3 +
n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[(a + b*x)/2]^2 - 8*n*App
ellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*
Cos[(a + b*x)/2]^2 + 72*AppellF1[(1 + n)/2, n, 3, (3 + n)/2, Tan[(a + b*x)/
2]^2, -Tan[(a + b*x)/2]^2]*Cos[(a + b*x)/2]^2 + 24*n*AppellF1[(1 + n)/2, n,
 3, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[(a + b*x)/2]^2
- 96*AppellF1[(1 + n)/2, n, 4, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x
)/2]^2]*Cos[(a + b*x)/2]^2 - 32*n*AppellF1[(1 + n)/2, n, 4, (3 + n)/2, Tan[
```

```
(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[(a + b*x)/2]^2 + AppellF1[(3 + n)/
2, n, 2, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(-1 + Cos[a +
b*x]) - 16*AppellF1[(3 + n)/2, n, 3, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a
 + b*x)/2]^2]*Cos[a + b*x] + 72*AppellF1[(3 + n)/2, n, 4, (5 + n)/2, Tan[(a
 + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[a + b*x] - 128*AppellF1[(3 + n)/2, n
, 5, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[a + b*x] + 80*
AppellF1[(3 + n)/2, n, 6, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^
2]*Cos[a + b*x] - n*AppellF1[(3 + n)/2, 1 + n, 1, (5 + n)/2, Tan[(a + b*x)/
2]^2, -Tan[(a + b*x)/2]^2]*Cos[a + b*x] + 8*n*AppellF1[(3 + n)/2, 1 + n, 2,
 (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[a + b*x] - 24*n*Ap
pellF1[(3 + n)/2, 1 + n, 3, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2
]^2]*Cos[a + b*x] + 32*n*AppellF1[(3 + n)/2, 1 + n, 4, (5 + n)/2, Tan[(a +
b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[a + b*x] - 16*n*AppellF1[(3 + n)/2, 1 +
 n, 5, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[a + b*x] + 8
*(3 + n)*AppellF1[(1 + n)/2, n, 5, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a +
 b*x)/2]^2]*(1 + Cos[a + b*x])))*(Sin[(a + b*x)/2] - Sin[(3*(a + b*x))/2]))
```

Maple [F] time = 180., size = 0, normalized size = 0.

hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4*(d*tan(b*x+a))^n,x)
```

```
[Out] int(cos(b*x+a)^4*(d*tan(b*x+a))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^n \cos(bx+a)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^n*cos(b*x + a)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((d\tan(bx+a))^n\cos(bx+a)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] integral((d*tan(b*x + a))^n*cos(b*x + a)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4*(d*tan(b*x+a))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^n\cos(bx+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*(d*tan(b*x+a))^n,x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^n*cos(b*x + a)^4, x)
```

3.369 $\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx$

Optimal. Leaf size=78

$$\frac{\sec^5(a+bx)\cos^2(a+bx)^{\frac{n+6}{2}}(d\tan(a+bx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{n+6}{2};\frac{n+3}{2};\sin^2(a+bx)\right)}{bd(n+1)}$$

```
[Out] ((Cos[a + b*x]^2)^((6 + n)/2)*Hypergeometric2F1[(1 + n)/2, (6 + n)/2, (3 +
n)/2, Sin[a + b*x]^2]*Sec[a + b*x]^5*(d*Tan[a + b*x])^(1 + n))/(b*d*(1 + n)
)
```

Rubi [A] time = 0.0384604, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2617}

$$\frac{\sec^5(a+bx)\cos^2(a+bx)^{\frac{n+6}{2}}(d\tan(a+bx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{n+6}{2};\frac{n+3}{2};\sin^2(a+bx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^5*(d*Tan[a + b*x])^n,x]
```

```
[Out] ((Cos[a + b*x]^2)^((6 + n)/2)*Hypergeometric2F1[(1 + n)/2, (6 + n)/2, (3 +
n)/2, Sin[a + b*x]^2]*Sec[a + b*x]^5*(d*Tan[a + b*x])^(1 + n))/(b*d*(1 + n)
)
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx=\frac{\cos^2(a+bx)^{\frac{6+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{6+n}{2};\frac{3+n}{2};\sin^2(a+bx)\right)\sec^5(a+bx)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

Mathematica [A] time = 0.107296, size = 72, normalized size = 0.92

$$\frac{d\sec^5(a+bx)\left(-\tan^2(a+bx)\right)^{\frac{1-n}{2}}(d\tan(a+bx))^{n-1}\,{}_2F_1\left(\frac{5}{2},\frac{1-n}{2};\frac{7}{2};\sec^2(a+bx)\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^5*(d*Tan[a + b*x])^n,x]
```

```
[Out] (d*Hypergeometric2F1[5/2, (1 - n)/2, 7/2, Sec[a + b*x]^2]*Sec[a + b*x]^5*(d
*Tan[a + b*x])^(-1 + n)*(-Tan[a + b*x]^2)^((1 - n)/2))/(5*b)
```

Maple [F] time = 0.142, size = 0, normalized size = 0.

$$\int (\sec(bx+a))^5 (d\tan(bx+a))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^5*(d*tan(b*x+a))^n,x)
```

```
[Out] int(sec(b*x+a)^5*(d*tan(b*x+a))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^n \sec(bx+a)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^n*sec(b*x + a)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((d\tan(bx+a))^n \sec(bx+a)^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] integral((d*tan(b*x + a))^n*sec(b*x + a)^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(a+bx))^n \sec^5(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**5*(d*tan(b*x+a))**n,x)
```

```
[Out] Integral((d*tan(a + b*x))**n*sec(a + b*x)**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^n \sec(bx+a)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*(d*tan(b*x+a))^n,x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^n*sec(b*x + a)^5, x)
```

3.370 $\int \sec^3(a+bx)(d\tan(a+bx))^n\,dx$

Optimal. Leaf size=78

$$\frac{\sec^3(a+bx)\cos^2(a+bx)^{\frac{n+4}{2}}(d\tan(a+bx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{n+4}{2};\frac{n+3}{2};\sin^2(a+bx)\right)}{bd(n+1)}$$

```
[Out] ((Cos[a + b*x]^2)^((4 + n)/2)*Hypergeometric2F1[(1 + n)/2, (4 + n)/2, (3 +
n)/2, Sin[a + b*x]^2]*Sec[a + b*x]^3*(d*Tan[a + b*x])^(1 + n))/(b*d*(1 + n)
)
```

Rubi [A] time = 0.0384939, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2617}

$$\frac{\sec^3(a+bx)\cos^2(a+bx)^{\frac{n+4}{2}}(d\tan(a+bx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{n+4}{2};\frac{n+3}{2};\sin^2(a+bx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^3*(d*Tan[a + b*x])^n,x]
```

```
[Out] ((Cos[a + b*x]^2)^((4 + n)/2)*Hypergeometric2F1[(1 + n)/2, (4 + n)/2, (3 +
n)/2, Sin[a + b*x]^2]*Sec[a + b*x]^3*(d*Tan[a + b*x])^(1 + n))/(b*d*(1 + n)
)
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

Rubi steps

$$\int \sec^3(a+bx)(d\tan(a+bx))^n\,dx = \frac{\cos^2(a+bx)^{\frac{4+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{4+n}{2};\frac{3+n}{2};\sin^2(a+bx)\right)\sec^3(a+bx)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

Mathematica [A] time = 0.0945355, size = 72, normalized size = 0.92

$$\frac{d\sec^3(a+bx)\left(-\tan^2(a+bx)\right)^{\frac{1-n}{2}}(d\tan(a+bx))^{n-1}{}_2F_1\left(\frac{3}{2},\frac{1-n}{2};\frac{5}{2};\sec^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^3*(d*Tan[a + b*x])^n,x]
```

```
[Out] (d*Hypergeometric2F1[3/2, (1 - n)/2, 5/2, Sec[a + b*x]^2]*Sec[a + b*x]^3*(d
*Tan[a + b*x])^(-1 + n)*(-Tan[a + b*x]^2)^((1 - n)/2))/(3*b)
```

Maple [F] time = 0.13, size = 0, normalized size = 0.

$$\int (\sec(bx+a))^3 (d\tan(bx+a))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3*(d*tan(b*x+a))^n,x)
```

```
[Out] int(sec(b*x+a)^3*(d*tan(b*x+a))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^n \sec(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^n*sec(b*x + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((d\tan(bx+a))^n \sec(bx+a)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] integral((d*tan(b*x + a))^n*sec(b*x + a)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(a+bx))^n \sec^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3*(d*tan(b*x+a))**n,x)
```

```
[Out] Integral((d*tan(a + b*x))**n*sec(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^n \sec(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*(d*tan(b*x+a))^n,x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^n*sec(b*x + a)^3, x)
```

3.371 $\int \sec(a+bx)(d\tan(a+bx))^n\,dx$

Optimal. Leaf size=76

$$\frac{\sec(a+bx)\cos^2(a+bx)^{\frac{n+2}{2}}(d\tan(a+bx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{n+2}{2};\frac{n+3}{2};\sin^2(a+bx)\right)}{bd(n+1)}$$

```
[Out] ((Cos[a + b*x]^2)^((2 + n)/2)*Hypergeometric2F1[(1 + n)/2, (2 + n)/2, (3 +
n)/2, Sin[a + b*x]^2]*Sec[a + b*x]*(d*Tan[a + b*x])^(1 + n))/(b*d*(1 + n))
```

Rubi [A] time = 0.0239285, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2617}

$$\frac{\sec(a+bx)\cos^2(a+bx)^{\frac{n+2}{2}}(d\tan(a+bx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{n+2}{2};\frac{n+3}{2};\sin^2(a+bx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]*(d*Tan[a + b*x])^n,x]
```

```
[Out] ((Cos[a + b*x]^2)^((2 + n)/2)*Hypergeometric2F1[(1 + n)/2, (2 + n)/2, (3 +
n)/2, Sin[a + b*x]^2]*Sec[a + b*x]*(d*Tan[a + b*x])^(1 + n))/(b*d*(1 + n))
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \sec(a+bx)(d\tan(a+bx))^n\,dx = \frac{\cos^2(a+bx)^{\frac{2+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{2+n}{2};\frac{3+n}{2};\sin^2(a+bx)\right)\sec(a+bx)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

Mathematica [A] time = 0.0665986, size = 64, normalized size = 0.84

$$\frac{\csc(a+bx)\left(-\tan^2(a+bx)\right)^{\frac{1-n}{2}}(d\tan(a+bx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3}{2};\sec^2(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]*(d*Tan[a + b*x])^n,x]
```

```
[Out] (Csc[a + b*x]*Hypergeometric2F1[1/2, (1 - n)/2, 3/2, Sec[a + b*x]^2]*(d*Tan
[a + b*x])^n*(-Tan[a + b*x]^2)^((1 - n)/2))/b
```

Maple [F] time = 0.223, size = 0, normalized size = 0.

$$\int \sec(bx+a)(d\tan(bx+a))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*(d*tan(b*x+a))^n,x)
```

```
[Out] int(sec(b*x+a)*(d*tan(b*x+a))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^n\sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^n*sec(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((d\tan(bx+a))^n\sec(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] integral((d*tan(b*x + a))^n*sec(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(a+bx))^n \sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*(d*tan(b*x+a))**n,x)
```

```
[Out] Integral((d*tan(a + b*x))**n*sec(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^n \sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*(d*tan(b*x+a))^n,x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^n*sec(b*x + a), x)
```

3.372 $\int \cos(a+bx)(d\tan(a+bx))^n\,dx$

Optimal. Leaf size=72

$$\frac{\cos(a+bx)\cos^2(a+bx)^{n/2}(d\tan(a+bx))^{n+1}\,{}_2F_1\left(\frac{n}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(a+bx)\right)}{bd(n+1)}$$

```
[Out] (Cos[a + b*x]*(Cos[a + b*x]^2)^(n/2)*Hypergeometric2F1[n/2, (1 + n)/2, (3 +
 n)/2, Sin[a + b*x]^2]*(d*Tan[a + b*x])^(1 + n))/(b*d*(1 + n))
```

Rubi [A] time = 0.03143, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2617}

$$\frac{\cos(a+bx)\cos^2(a+bx)^{n/2}(d\tan(a+bx))^{n+1}\,{}_2F_1\left(\frac{n}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(a+bx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*(d*Tan[a + b*x])^n,x]
```

```
[Out] (Cos[a + b*x]*(Cos[a + b*x]^2)^(n/2)*Hypergeometric2F1[n/2, (1 + n)/2, (3 +
 n)/2, Sin[a + b*x]^2]*(d*Tan[a + b*x])^(1 + n))/(b*d*(1 + n))
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \cos(a+bx)(d\tan(a+bx))^n\,dx=\frac{\cos(a+bx)\cos^2(a+bx)^{n/2}\,{}_2F_1\left(\frac{n}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(a+bx)\right)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

Mathematica [C] time = 2.3333, size = 452, normalized size = 6.28

$$-\frac{2\sin\left(\frac{1}{2}(a+bx)\right)\cos\left(\frac{1}{2}(a+bx)\right)\cos(a+bx)\left(F_1\left(\right.\right.}{b(n+1)\left(\frac{\sec^2\left(\frac{1}{2}(a+bx)\right)\left((n+3)(\cos(a+bx)+1)F_1\left(\frac{n+1}{2};n,2;\frac{n+3}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)-(\cos(a+bx)-1)\left(F_1\left(\frac{n+3}{2};n,2;\frac{n+5}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan\right.\right.\right.}{}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[a + b*x]*(d*Tan[a + b*x])^n,x]
```

```
[Out] (-2*(AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x
)/2]^2] - 2*AppellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(
a + b*x)/2]^2])*Cos[(a + b*x)/2]*Cos[a + b*x]*Sin[(a + b*x)/2]*(d*Tan[a + b
*x])^n)/(b*(1 + n)*(-AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(a + b*x)/2]^
2, -Tan[(a + b*x)/2]^2] + ((-((AppellF1[(3 + n)/2, n, 2, (5 + n)/2, Tan[(a
+ b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 4*AppellF1[(3 + n)/2, n, 3, (5 + n)/2,
Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - n*AppellF1[(3 + n)/2, 1 + n, 1,
(5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 2*n*AppellF1[(3 + n)/
2, 1 + n, 2, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*(-1 + Cos
[a + b*x])) + (3 + n)*AppellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(a + b*x)/2]
^2, -Tan[(a + b*x)/2]^2]*(1 + Cos[a + b*x]))*Sec[(a + b*x)/2]^2)/(3 + n)))
```

Maple [F] time = 0.767, size = 0, normalized size = 0.

$$\int \cos(bx+a)\,(d\tan(bx+a))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*(d*tan(b*x+a))^n,x)
```

```
[Out] int(cos(b*x+a)*(d*tan(b*x+a))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^n\cos(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^n*cos(b*x + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((d\tan(bx+a))^n\cos(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] integral((d*tan(b*x + a))^n*cos(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(a+bx))^n\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*(d*tan(b*x+a))**n,x)
```

```
[Out] Integral((d*tan(a + b*x))**n*cos(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^n\cos(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*(d*tan(b*x+a))^n,x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^n*cos(b*x + a), x)
```

3.373 $\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx$

Optimal. Leaf size=78

$$\frac{\cos^3(a+bx)\cos^2(a+bx)^{\frac{n-2}{2}}(d\tan(a+bx))^{n+1}\,{}_2F_1\left(\frac{n-2}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(a+bx)\right)}{bd(n+1)}$$

```
[Out] (Cos[a + b*x]^3*(Cos[a + b*x]^2)^((-2 + n)/2)*Hypergeometric2F1[(-2 + n)/2,
 (1 + n)/2, (3 + n)/2, Sin[a + b*x]^2]*(d*Tan[a + b*x])^(1 + n))/(b*d*(1 +
n))
```

Rubi [A] time = 0.037228, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2617}

$$\frac{\cos^3(a+bx)\cos^2(a+bx)^{\frac{n-2}{2}}(d\tan(a+bx))^{n+1}\,{}_2F_1\left(\frac{n-2}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(a+bx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*(d*Tan[a + b*x])^n,x]
```

```
[Out] (Cos[a + b*x]^3*(Cos[a + b*x]^2)^((-2 + n)/2)*Hypergeometric2F1[(-2 + n)/2,
 (1 + n)/2, (3 + n)/2, Sin[a + b*x]^2]*(d*Tan[a + b*x])^(1 + n))/(b*d*(1 +
n))
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx = \frac{\cos^3(a+bx)\cos^2(a+bx)^{\frac{1}{2}(-2+n)}\,{}_2F_1\left(\frac{1}{2}(-2+n),\frac{1+n}{2};\frac{3+n}{2};\sin^2(a+bx)\right)(d\tan(a}{bd(1+n)}$$

Mathematica [C] time = 6.27976, size = 1313, normalized size = 16.83

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[a + b*x]^3*(d*Tan[a + b*x])^n,x]
```

```
[Out] (4*(3 + n)*(AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(
a + b*x)/2]^2] - 6*AppellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(a + b*x)/2]^2,
 -Tan[(a + b*x)/2]^2] + 12*AppellF1[(1 + n)/2, n, 3, (3 + n)/2, Tan[(a + b*
x)/2]^2, -Tan[(a + b*x)/2]^2] - 8*AppellF1[(1 + n)/2, n, 4, (3 + n)/2, Tan[
(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*Cos[(a + b*x)/2]^3*Cos[a + b*x]^3*Sin
[(a + b*x)/2]*(d*Tan[a + b*x])^n)/(b*(1 + n)*((3 + n)*AppellF1[(1 + n)/2, n
, 1, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(1 + Cos[a + b*x])
 - 2*(AppellF1[(3 + n)/2, n, 2, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*
x)/2]^2] - 12*AppellF1[(3 + n)/2, n, 3, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan
[(a + b*x)/2]^2] + 36*AppellF1[(3 + n)/2, n, 4, (5 + n)/2, Tan[(a + b*x)/2]
^2, -Tan[(a + b*x)/2]^2] - 32*AppellF1[(3 + n)/2, n, 5, (5 + n)/2, Tan[(a +
 b*x)/2]^2, -Tan[(a + b*x)/2]^2] - n*AppellF1[(3 + n)/2, 1 + n, 1, (5 + n)/
2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 6*n*AppellF1[(3 + n)/2, 1 + n
, 2, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 12*n*AppellF1[(3
 + n)/2, 1 + n, 3, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 8*
n*AppellF1[(3 + n)/2, 1 + n, 4, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*
x)/2]^2] + 18*AppellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan
[(a + b*x)/2]^2]*Cos[(a + b*x)/2]^2 + 6*n*AppellF1[(1 + n)/2, n, 2, (3 + n)
/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[(a + b*x)/2]^2 + 8*(3 + n)
*AppellF1[(1 + n)/2, n, 4, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]
^2]*Cos[(a + b*x)/2]^2 - AppellF1[(3 + n)/2, n, 2, (5 + n)/2, Tan[(a + b*x)
/2]^2, -Tan[(a + b*x)/2]^2]*Cos[a + b*x] + 12*AppellF1[(3 + n)/2, n, 3, (5
+ n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[a + b*x] - 36*AppellF1
[(3 + n)/2, n, 4, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[a
 + b*x] + 32*AppellF1[(3 + n)/2, n, 5, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[
(a + b*x)/2]^2]*Cos[a + b*x] + n*AppellF1[(3 + n)/2, 1 + n, 1, (5 + n)/2, T
an[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[a + b*x] - 6*n*AppellF1[(3 + n)
/2, 1 + n, 2, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[a + b
*x] + 12*n*AppellF1[(3 + n)/2, 1 + n, 3, (5 + n)/2, Tan[(a + b*x)/2]^2, -Ta
n[(a + b*x)/2]^2]*Cos[a + b*x] - 8*n*AppellF1[(3 + n)/2, 1 + n, 4, (5 + n)/
2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[a + b*x] - 6*(3 + n)*Appell
F1[(1 + n)/2, n, 3, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(1
+ Cos[a + b*x]))))
```

Maple [F] time = 0.826, size = 0, normalized size = 0.

$$\int \left(\cos \left(bx + a\right)\right)^{3} \left(d \tan \left(bx + a\right)\right)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*(d*tan(b*x+a))^n,x)
```

```
[Out] int(cos(b*x+a)^3*(d*tan(b*x+a))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d \tan \left(bx + a\right)\right)^{n} \cos \left(bx + a\right)^{3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^n*cos(b*x + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d \tan \left(bx + a\right)\right)^{n} \cos \left(bx + a\right)^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] integral((d*tan(b*x + a))^n*cos(b*x + a)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*(d*tan(b*x+a))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\tan(bx+a))^n \cos(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*(d*tan(b*x+a))^n,x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^n*cos(b*x + a)^3, x)
```

3.374 $\int (b\csc(e+fx))^m \tan^3(e+fx)\,dx$

Optimal. Leaf size=40

$$-\frac{(b\csc(e+fx))^m\,{}_2F_1\left(2,\frac{m}{2};\frac{m+2}{2};\csc^2(e+fx)\right)}{fm}$$

```
[Out] -(((b*Csc[e + f*x])^m*Hypergeometric2F1[2, m/2, (2 + m)/2, Csc[e + f*x]^2])
/(f*m))
```

Rubi [A] time = 0.046136, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2606, 364}

$$-\frac{(b\csc(e+fx))^m\,{}_2F_1\left(2,\frac{m}{2};\frac{m+2}{2};\csc^2(e+fx)\right)}{fm}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^m*Tan[e + f*x]^3,x]
```


```
[Out] -(((b*Csc[e + f*x])^m*Hypergeometric2F1[2, m/2, (2 + m)/2, Csc[e + f*x]^2])
/(f*m))
```


Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (b\csc(e+fx))^m\tan^3(e+fx)\,dx=-\frac{b\,\text{Subst}\left(\int\frac{(bx)^{-1+m}}{\left(-1+x^2\right)^2}\,dx,x,\csc(e+fx)\right)}{f}$$
$$=-\frac{(b\csc(e+fx))^m\,{}_2F_1\left(2,\frac{m}{2};\frac{2+m}{2};\csc^2(e+fx)\right)}{fm}$$

Mathematica [A] time = 0.0838147, size = 52, normalized size = 1.3

$$-\frac{\sin^4(e+fx)(b\csc(e+fx))^m\,{}_2F_1\left(2,2-\frac{m}{2};3-\frac{m}{2};\sin^2(e+fx)\right)}{f(m-4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^m*Tan[e + f*x]^3,x]
```

```
[Out] -(((b*Csc[e + f*x])^m*Hypergeometric2F1[2, 2 - m/2, 3 - m/2, Sin[e + f*x]^2
]*Sin[e + f*x]^4)/(f*(-4 + m)))
```

Maple [F] time = 0.287, size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^m\left(\tan\left(fx+e\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^m*tan(f*x+e)^3,x)
```

```
[Out] int((b*csc(f*x+e))^m*tan(f*x+e)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^m\tan\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^m*tan(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\csc\left(fx+e\right)\right)^{m}\tan\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^m*tan(f*x + e)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\csc\left(e+fx\right)\right)^{m}\tan^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**m*tan(f*x+e)**3,x)
```

```
[Out] Integral((b*csc(e + f*x))**m*tan(e + f*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\csc\left(fx+e\right)\right)^{m}\tan\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^m*tan(f*x + e)^3, x)
```

3.375 $\int (b\csc(e+fx))^m \tan(e+fx)\,dx$

Optimal. Leaf size=39

$$\frac{(b\csc(e+fx))^m\,{}_2F_1\left(1,\frac{m}{2};\frac{m+2}{2};\csc^2(e+fx)\right)}{fm}$$

```
[Out] ((b*Csc[e + f*x])^m*Hypergeometric2F1[1, m/2, (2 + m)/2, Csc[e + f*x]^2])/(
f*m)
```

Rubi [A] time = 0.0357686, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2606, 364}

$$\frac{(b\csc(e+fx))^m\,{}_2F_1\left(1,\frac{m}{2};\frac{m+2}{2};\csc^2(e+fx)\right)}{fm}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^m*Tan[e + f*x],x]
```

```
[Out] ((b*Csc[e + f*x])^m*Hypergeometric2F1[1, m/2, (2 + m)/2, Csc[e + f*x]^2])/(
f*m)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (b\csc(e+fx))^m\tan(e+fx)\,dx = -\frac{b\,\text{Subst}\left(\int \frac{(bx)^{-1+m}}{-1+x^2}\,dx, x, \csc(e+fx)\right)}{f}$$
$$= \frac{(b\csc(e+fx))^m\,{}_2F_1\left(1,\frac{m}{2};\frac{2+m}{2};\csc^2(e+fx)\right)}{fm}$$

Mathematica [A] time = 0.0491627, size = 52, normalized size = 1.33

$$-\frac{\sin^2(e+fx)(b\csc(e+fx))^m\,{}_2F_1\left(1,1-\frac{m}{2};2-\frac{m}{2};\sin^2(e+fx)\right)}{f(m-2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^m*Tan[e + f*x],x]
```

```
[Out] -(((b*Csc[e + f*x])^m*Hypergeometric2F1[1, 1 - m/2, 2 - m/2, Sin[e + f*x]^2
]*Sin[e + f*x]^2)/(f*(-2 + m)))
```

Maple [F] time = 0.441, size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^m\tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^m*tan(f*x+e),x)
```

```
[Out] int((b*csc(f*x+e))^m*tan(f*x+e),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^m\tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^m*tan(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\csc\left(fx+e\right)\right)^{m}\tan\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^m*tan(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\csc\left(e+fx\right)\right)^{m}\tan\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**m*tan(f*x+e),x)
```

```
[Out] Integral((b*csc(e + f*x))**m*tan(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\csc\left(fx+e\right)\right)^{m}\tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^m*tan(f*x + e), x)
```

3.376 $\int \cot(e+fx)(b\csc(e+fx))^m \, dx$

Optimal. Leaf size=18

$$-\frac{(b\csc(e+fx))^m}{fm}$$

```
[Out] -((b*Csc[e + f*x])^m/(f*m))
```

Rubi [A] time = 0.0214582, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2606, 32}

$$-\frac{(b\csc(e+fx))^m}{fm}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(b*Csc[e + f*x])^m,x]
```

```
[Out] -((b*Csc[e + f*x])^m/(f*m))
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cot(e+fx)(b\csc(e+fx))^m \, dx &= -\frac{b\,\text{Subst}\left(\int (bx)^{-1+m} \, dx, x, \csc(e+fx)\right)}{f}\\ &= -\frac{(b\csc(e+fx))^m}{fm}\end{aligned}$$

Mathematica [A] time = 0.0174604, size = 18, normalized size = 1.

$$-\frac{(b\csc(e+fx))^m}{fm}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(b*Csc[e + f*x])^m,x]
```

```
[Out] -((b*Csc[e + f*x])^m/(f*m))
```

Maple [A] time = 0.01, size = 19, normalized size = 1.1

$$-\frac{\left(b\csc\left(fx+e\right)\right)^m}{fm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(b*csc(f*x+e))^m,x)
```

```
[Out] -(b*csc(f*x+e))^m/f/m
```

Maxima [A] time = 0.969372, size = 28, normalized size = 1.56

$$-\frac{b^m\sin\left(fx+e\right)^{-m}}{fm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(b*csc(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] -b^m*sin(f*x + e)^(-m)/(f*m)
```

Fricas [A] time = 1.60248, size = 36, normalized size = 2.

$$-\frac{\left(\frac{b}{\sin\left(fx+e\right)}\right)^m}{fm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(b*csc(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] -(b/sin(f*x + e))^m/(f*m)
```

Sympy [A] time = 0.523902, size = 56, normalized size = 3.11

$$\begin{cases} x\cot(e) & \text{for}\ f = 0 \wedge m = 0 \\ x\left(b\csc(e)\right)^m \cot(e) & \text{for}\ f = 0 \\ -\frac{\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} + \frac{\log\left(\tan\left(e+fx\right)\right)}{f} & \text{for}\ m = 0 \\ -\frac{b^m \csc^m\left(e+fx\right)}{fm} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(b*csc(f*x+e))**m,x)
```

```
[Out] Piecewise((x*cot(e), Eq(f, 0) & Eq(m, 0)), (x*(b*csc(e))**m*cot(e), Eq(f, 0
)), (-log(tan(e + f*x)**2 + 1)/(2*f) + log(tan(e + f*x))/f, Eq(m, 0)), (-b*
*m*csc(e + f*x)**m/(f*m), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^m \cot\left(fx+e\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(b*csc(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^m*cot(f*x + e), x)
```

3.377 $\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx$

Optimal. Leaf size=43

$$\frac{(b\csc(e+fx))^m}{fm}-\frac{(b\csc(e+fx))^{m+2}}{b^2f(m+2)}$$

```
[Out] (b*Csc[e + f*x])^m/(f*m) - (b*Csc[e + f*x])^(2 + m)/(b^2*f*(2 + m))
```

Rubi [A] time = 0.0472561, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2606, 14}

$$\frac{(b\csc(e+fx))^m}{fm}-\frac{(b\csc(e+fx))^{m+2}}{b^2f(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(b*Csc[e + f*x])^m,x]
```

```
[Out] (b*Csc[e + f*x])^m/(f*m) - (b*Csc[e + f*x])^(2 + m)/(b^2*f*(2 + m))
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx = -\frac{b\,\text{Subst}\left(\int (bx)^{-1+m}\left(-1+x^2\right)\,dx, x, \csc(e+fx)\right)}{f}$$
$$= -\frac{b\,\text{Subst}\left(\int \left(-(bx)^{-1+m}+\frac{(bx)^{1+m}}{b^2}\right)\,dx, x, \csc(e+fx)\right)}{f}$$
$$= \frac{(b\csc(e+fx))^m}{fm} - \frac{(b\csc(e+fx))^{2+m}}{b^2f(2+m)}$$

Mathematica [A] time = 0.0819227, size = 36, normalized size = 0.84

$$\frac{\left(-m\csc^2(e+fx)+m+2\right)(b\csc(e+fx))^m}{fm(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(b*Csc[e + f*x])^m,x]
```

```
[Out] ((b*Csc[e + f*x])^m*(2 + m - m*Csc[e + f*x]^2))/(f*m*(2 + m))
```

Maple [C] time = 0.601, size = 6612, normalized size = 153.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(b*csc(f*x+e))^m,x)
```

```
[Out] result too large to display
```

Maxima [A] time = 0.968742, size = 68, normalized size = 1.58

$$\frac{\frac{b^m\sin\left(fx+e\right)^{-m}}{m} - \frac{b^m\sin\left(fx+e\right)^{-m}}{(m+2)\sin\left(fx+e\right)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(b*csc(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] (b^m*sin(f*x + e)^(-m)/m - b^m*sin(f*x + e)^(-m)/((m + 2)*sin(f*x + e)^2))/
f
```

Fricas [A] time = 1.66319, size = 134, normalized size = 3.12

$$-\frac{\left((m+2)\cos\left(fx+e\right)^{2}-2\right)\left(\frac{b}{\sin\left(fx+e\right)}\right)^{m}}{fm^{2}-\left(fm^{2}+2\,fm\right)\cos\left(fx+e\right)^{2}+2\,fm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(b*csc(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] -((m + 2)*cos(f*x + e)^2 - 2)*(b/sin(f*x + e))^m/(f*m^2 - (f*m^2 + 2*f*m)*c
os(f*x + e)^2 + 2*f*m)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(b*csc(f*x+e))**m,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\csc\left(fx+e\right)\right)^{m}\cot\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(b*csc(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^m*cot(f*x + e)^3, x)
```

3.378 $\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx$

Optimal. Leaf size=69

$$\frac{2(b\csc(e+fx))^{m+2}}{b^2f(m+2)}-\frac{(b\csc(e+fx))^{m+4}}{b^4f(m+4)}-\frac{(b\csc(e+fx))^m}{fm}$$

```
[Out] -((b*Csc[e + f*x])^m/(f*m)) + (2*(b*Csc[e + f*x])^(2 + m))/(b^2*f*(2 + m))
- (b*Csc[e + f*x])^(4 + m)/(b^4*f*(4 + m))
```

Rubi [A] time = 0.0604031, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2606, 270}

$$\frac{2(b\csc(e+fx))^{m+2}}{b^2f(m+2)}-\frac{(b\csc(e+fx))^{m+4}}{b^4f(m+4)}-\frac{(b\csc(e+fx))^m}{fm}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*(b*Csc[e + f*x])^m,x]
```

```
[Out] -((b*Csc[e + f*x])^m/(f*m)) + (2*(b*Csc[e + f*x])^(2 + m))/(b^2*f*(2 + m))
- (b*Csc[e + f*x])^(4 + m)/(b^4*f*(4 + m))
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx = -\frac{b\,\text{Subst}\left(\int (bx)^{-1+m}\left(-1+x^2\right)^2\,dx, x, \csc(e+fx)\right)}{f}$$

$$= -\frac{b\,\text{Subst}\left(\int \left((bx)^{-1+m} - \frac{2(bx)^{1+m}}{b^2} + \frac{(bx)^{3+m}}{b^4}\right)dx, x, \csc(e+fx)\right)}{f}$$

$$= -\frac{(b\csc(e+fx))^m}{fm} + \frac{2(b\csc(e+fx))^{2+m}}{b^2f(2+m)} - \frac{(b\csc(e+fx))^{4+m}}{b^4f(4+m)}$$

Mathematica [A] time = 0.293394, size = 63, normalized size = 0.91

$$-\frac{\left(m(m+2)\csc^4(e+fx) - 2m(m+4)\csc^2(e+fx) + m^2 + 6m + 8\right)(b\csc(e+fx))^m}{fm(m+2)(m+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*(b*Csc[e + f*x])^m,x]
```

```
[Out] -(((b*Csc[e + f*x])^m*(8 + 6*m + m^2 - 2*m*(4 + m)*Csc[e + f*x]^2 + m*(2 +
m)*Csc[e + f*x]^4))/(f*m*(2 + m)*(4 + m)))
```

Maple [C] time = 0.553, size = 16599, normalized size = 240.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(b*csc(f*x+e))^m,x)
```

```
[Out] result too large to display
```

Maxima [A] time = 0.995602, size = 105, normalized size = 1.52

$$-\frac{\frac{b^m \sin\left(fx+e\right)^{-m}}{m} - \frac{2\,b^m \sin\left(fx+e\right)^{-m}}{(m+2)\sin\left(fx+e\right)^2} + \frac{b^m \sin\left(fx+e\right)^{-m}}{(m+4)\sin\left(fx+e\right)^4}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(b*csc(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] -(b^m*sin(f*x + e)^(-m)/m - 2*b^m*sin(f*x + e)^(-m)/((m + 2)*sin(f*x + e)^2
) + b^m*sin(f*x + e)^(-m)/((m + 4)*sin(f*x + e)^4))/f
```

Fricas [A] time = 1.76921, size = 269, normalized size = 3.9

$$-\frac{\left(\left(m^2+6\,m+8\right)\cos\left(fx+e\right)^4-4\,(m+4)\cos\left(fx+e\right)^2+8\right)\left(\frac{b}{\sin\left(fx+e\right)}\right)^m}{\left(fm^3+6\,fm^2+8\,fm\right)\cos\left(fx+e\right)^4+fm^3+6\,fm^2-2\left(fm^3+6\,fm^2+8\,fm\right)\cos\left(fx+e\right)^2+8\,fm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(b*csc(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] -((m^2 + 6*m + 8)*cos(f*x + e)^4 - 4*(m + 4)*cos(f*x + e)^2 + 8)*(b/sin(f*x
 + e))^m/((f*m^3 + 6*f*m^2 + 8*f*m)*cos(f*x + e)^4 + f*m^3 + 6*f*m^2 - 2*(f
*m^3 + 6*f*m^2 + 8*f*m)*cos(f*x + e)^2 + 8*f*m)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(b*csc(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\csc\left(fx+e\right)\right)^m\cot\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(b*csc(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^m*cot(f*x + e)^5, x)
```

3.379 $\int (b\csc(e+fx))^m \tan^4(e+fx)\,dx$

Optimal. Leaf size=63

$$\frac{\tan^3(e+fx)\sin^2(e+fx)^{\frac{m-3}{2}}(b\csc(e+fx))^m\,{}_2F_1\left(-\frac{3}{2},\frac{m-3}{2};-\frac{1}{2};\cos^2(e+fx)\right)}{3f}$$

```
[Out] ((b*Csc[e + f*x])^m*Hypergeometric2F1[-3/2, (-3 + m)/2, -1/2, Cos[e + f*x]^
2]*(Sin[e + f*x]^2)^((-3 + m)/2)*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.0361783, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2617}

$$\frac{\tan^3(e+fx)\sin^2(e+fx)^{\frac{m-3}{2}}(b\csc(e+fx))^m\,{}_2F_1\left(-\frac{3}{2},\frac{m-3}{2};-\frac{1}{2};\cos^2(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^m*Tan[e + f*x]^4,x]
```

```
[Out] ((b*Csc[e + f*x])^m*Hypergeometric2F1[-3/2, (-3 + m)/2, -1/2, Cos[e + f*x]^
2]*(Sin[e + f*x]^2)^((-3 + m)/2)*Tan[e + f*x]^3)/(3*f)
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (b\csc(e+fx))^m \tan^4(e+fx)\,dx = \frac{(b\csc(e+fx))^m\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}(-3+m);-\frac{1}{2};\cos^2(e+fx)\right)\sin^2(e+fx)^{\frac{1}{2}(-3+m)}\tan^3(e}{3f}$$

Mathematica [A] time = 0.699332, size = 79, normalized size = 1.25

$$\frac{\tan^5(e+fx)\sec^2(e+fx)^{-m/2}(b\csc(e+fx))^m\,{}_2F_1\left(1-\frac{m}{2},\frac{5}{2}-\frac{m}{2};\frac{7}{2}-\frac{m}{2};-\tan^2(e+fx)\right)}{f(5-m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^m*Tan[e + f*x]^4,x]
```

```
[Out] ((b*Csc[e + f*x])^m*Hypergeometric2F1[1 - m/2, 5/2 - m/2, 7/2 - m/2, -Tan[e
 + f*x]^2]*Tan[e + f*x]^5)/(f*(5 - m)*(Sec[e + f*x]^2)^(m/2))
```

Maple [F] time = 0.221, size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^m \left(\tan\left(fx+e\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^m*tan(f*x+e)^4,x)
```

```
[Out] int((b*csc(f*x+e))^m*tan(f*x+e)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^m \tan\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^m*tan(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\csc\left(fx+e\right)\right)^m \tan\left(fx+e\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^m*tan(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**m*tan(f*x+e)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \csc \left(fx+e\right)\right)^{m} \tan \left(fx+e\right)^{4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^m*tan(f*x + e)^4, x)
```

3.380 $\int (b\csc(e+fx))^m \tan^2(e+fx)\,dx$

Optimal. Leaf size=58

$$\frac{\tan(e+fx)\sin^2(e+fx)^{\frac{m-1}{2}}(b\csc(e+fx))^m\,{}_2F_1\left(-\frac{1}{2},\frac{m-1}{2};\frac{1}{2};\cos^2(e+fx)\right)}{f}$$

```
[Out] ((b*Csc[e + f*x])^m*Hypergeometric2F1[-1/2, (-1 + m)/2, 1/2, Cos[e + f*x]^2
]*(Sin[e + f*x]^2)^((-1 + m)/2)*Tan[e + f*x])/f
```

Rubi [A] time = 0.0357518, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2617}

$$\frac{\tan(e+fx)\sin^2(e+fx)^{\frac{m-1}{2}}(b\csc(e+fx))^m\,{}_2F_1\left(-\frac{1}{2},\frac{m-1}{2};\frac{1}{2};\cos^2(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^m*Tan[e + f*x]^2,x]
```

```
[Out] ((b*Csc[e + f*x])^m*Hypergeometric2F1[-1/2, (-1 + m)/2, 1/2, Cos[e + f*x]^2
]*(Sin[e + f*x]^2)^((-1 + m)/2)*Tan[e + f*x])/f
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (b\csc(e+fx))^m\tan^2(e+fx)\,dx = \frac{(b\csc(e+fx))^m\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(-1+m);\frac{1}{2};\cos^2(e+fx)\right)\sin^2(e+fx)^{\frac{1}{2}(-1+m)}\tan(e+f}{f}$$

Mathematica [A] time = 0.56063, size = 79, normalized size = 1.36

$$\frac{\tan^3(e+fx)\sec^2(e+fx)^{-m/2}(b\csc(e+fx))^m\,{}_2F_1\left(1-\frac{m}{2},\frac{3}{2}-\frac{m}{2};\frac{5}{2}-\frac{m}{2};-\tan^2(e+fx)\right)}{f(3-m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^m*Tan[e + f*x]^2,x]
```

```
[Out] ((b*Csc[e + f*x])^m*Hypergeometric2F1[1 - m/2, 3/2 - m/2, 5/2 - m/2, -Tan[e
 + f*x]^2]*Tan[e + f*x]^3)/(f*(3 - m)*(Sec[e + f*x]^2)^(m/2))
```

Maple [F] time = 0.199, size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{m}\left(\tan\left(fx+e\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^m*tan(f*x+e)^2,x)
```

```
[Out] int((b*csc(f*x+e))^m*tan(f*x+e)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{m}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^m*tan(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\csc\left(fx+e\right)\right)^{m}\tan\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^m*tan(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(e+fx\right)\right)^m\tan^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**m*tan(f*x+e)**2,x)
```

```
[Out] Integral((b*csc(e + f*x))**m*tan(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^m\tan\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^m*tan(f*x + e)^2, x)
```

3.381 $\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx$

Optimal. Leaf size=63

$$-\frac{\cot^3(e+fx)\sin^2(e+fx)^{\frac{m+3}{2}}(b\csc(e+fx))^m\,{}_2F_1\left(\frac{3}{2},\frac{m+3}{2};\frac{5}{2};\cos^2(e+fx)\right)}{3f}$$

```
[Out] -(Cot[e + f*x]^3*(b*Csc[e + f*x])^m*Hypergeometric2F1[3/2, (3 + m)/2, 5/2,
Cos[e + f*x]^2]*(Sin[e + f*x]^2)^((3 + m)/2))/(3*f)
```

Rubi [A] time = 0.0356907, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2617}

$$-\frac{\cot^3(e+fx)\sin^2(e+fx)^{\frac{m+3}{2}}(b\csc(e+fx))^m\,{}_2F_1\left(\frac{3}{2},\frac{m+3}{2};\frac{5}{2};\cos^2(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(b*Csc[e + f*x])^m,x]
```

```
[Out] -(Cot[e + f*x]^3*(b*Csc[e + f*x])^m*Hypergeometric2F1[3/2, (3 + m)/2, 5/2,
Cos[e + f*x]^2]*(Sin[e + f*x]^2)^((3 + m)/2))/(3*f)
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx = -\frac{\cot^3(e+fx)(b\csc(e+fx))^m\,{}_2F_1\left(\frac{3}{2},\frac{3+m}{2};\frac{5}{2};\cos^2(e+fx)\right)\sin^2(e+fx)^{\frac{3+m}{2}}}{3f}$$

Mathematica [B] time = 1.05563, size = 186, normalized size = 2.95

$$-\frac{\tan\left(\frac{1}{2}(e+fx)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)^{-m}(b\csc(e+fx))^m\left(-4(m+1)\,{}_2F_1\left(1-m,\frac{1}{2}-\frac{m}{2};\frac{3}{2}-\frac{m}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+(m+1\right.}{2f\left(m\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(b*Csc[e + f*x])^m,x]
```

```
[Out] -((b*Csc[e + f*x])^m*(-4*(1 + m)*Hypergeometric2F1[1 - m, 1/2 - m/2, 3/2 -
m/2, -Tan[(e + f*x)/2]^2] + (-1 + m)*Cot[(e + f*x)/2]^2*Hypergeometric2F1[-
1/2 - m/2, -m, 1/2 - m/2, -Tan[(e + f*x)/2]^2] + (1 + m)*Hypergeometric2F1[
1/2 - m/2, -m, 3/2 - m/2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2])/(2*f*(-1
+ m^2)*(Sec[(e + f*x)/2]^2)^m)
```

Maple [F] time = 0.201, size = 0, normalized size = 0.

$$\int \left(\cot\left(fx+e\right)\right)^2 \left(b\csc\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(b*csc(f*x+e))^m,x)
```

```
[Out] int(cot(f*x+e)^2*(b*csc(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^m \cot\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(b*csc(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^m*cot(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\csc\left(fx+e\right)\right)^{m}\cot\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(b*csc(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^m*cot(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\csc\left(e+fx\right)\right)^{m}\cot^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(b*csc(f*x+e))**m,x)
```

```
[Out] Integral((b*csc(e + f*x))**m*cot(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\csc\left(fx+e\right)\right)^{m}\cot\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(b*csc(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^m*cot(f*x + e)^2, x)
```

3.382 $\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx$

Optimal. Leaf size=63

$$-\frac{\cot^5(e+fx)\sin^2(e+fx)^{\frac{m+5}{2}}(b\csc(e+fx))^m\,{}_2F_1\left(\frac{5}{2},\frac{m+5}{2};\frac{7}{2};\cos^2(e+fx)\right)}{5f}$$

```
[Out] -(Cot[e + f*x]^5*(b*Csc[e + f*x])^m*Hypergeometric2F1[5/2, (5 + m)/2, 7/2,
Cos[e + f*x]^2]*(Sin[e + f*x]^2)^((5 + m)/2))/(5*f)
```

Rubi [A] time = 0.0364904, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2617}

$$-\frac{\cot^5(e+fx)\sin^2(e+fx)^{\frac{m+5}{2}}(b\csc(e+fx))^m\,{}_2F_1\left(\frac{5}{2},\frac{m+5}{2};\frac{7}{2};\cos^2(e+fx)\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(b*Csc[e + f*x])^m,x]
```

```
[Out] -(Cot[e + f*x]^5*(b*Csc[e + f*x])^m*Hypergeometric2F1[5/2, (5 + m)/2, 7/2,
Cos[e + f*x]^2]*(Sin[e + f*x]^2)^((5 + m)/2))/(5*f)
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx=-\frac{\cot^5(e+fx)(b\csc(e+fx))^m\,{}_2F_1\left(\frac{5}{2},\frac{5+m}{2};\frac{7}{2};\cos^2(e+fx)\right)\sin^2(e+fx)^{\frac{5+m}{2}}}{5f}$$

Mathematica [B] time = 4.85394, size = 302, normalized size = 4.79

$$\frac{\cot^3\left(\frac{1}{2}(e+fx)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)^{-m}(b\csc(e+fx))^m\left(\tan^4\left(\frac{1}{2}(e+fx)\right)\left(\frac{\tan^2\left(\frac{1}{2}(e+fx)\right){}_2F_1\left(\frac{3}{2}-\frac{m}{2},-m;\frac{5}{2}-\frac{m}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}{3-m}-\frac{16}{\quad}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(b*Csc[e + f*x])^m,x]
```

```
[Out] (Cot[(e + f*x)/2]^3*(b*Csc[e + f*x])^m*(-(Hypergeometric2F1[-3/2 - m/2, -m,
 -1/2 - m/2, -Tan[(e + f*x)/2]^2]/(3 + m)) + (5*Hypergeometric2F1[-1/2 - m/
2, -m, 1/2 - m/2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)/(1 + m) + Tan[(e
 + f*x)/2]^4*((-16*Hypergeometric2F1[1 - m, 1/2 - m/2, 3/2 - m/2, -Tan[(e +
 f*x)/2]^2])/(-1 + m) + (5*Hypergeometric2F1[1/2 - m/2, -m, 3/2 - m/2, -Tan
[(e + f*x)/2]^2])/(-1 + m) + (Hypergeometric2F1[3/2 - m/2, -m, 5/2 - m/2, -
Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)/(3 - m))))/(8*f*(Sec[(e + f*x)/2]^2
)^m)
```

Maple [F] time = 0.234, size = 0, normalized size = 0.

$$\int \left(\cot\left(fx+e\right)\right)^4 \left(b\csc\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(b*csc(f*x+e))^m,x)
```

```
[Out] int(cot(f*x+e)^4*(b*csc(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^m \cot\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(b*csc(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^m*cot(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\csc\left(fx+e\right)\right)^{m}\cot\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(b*csc(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^m*cot(f*x + e)^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\csc\left(e+fx\right)\right)^{m}\cot^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(b*csc(f*x+e))**m,x)
```

```
[Out] Integral((b*csc(e + f*x))**m*cot(e + f*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\csc\left(fx+e\right)\right)^{m}\cot\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(b*csc(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^m*cot(f*x + e)^4, x)
```

3.383 $\int (b\csc(e+fx))^m (d\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=79

$$\frac{2\cos^2(e+fx)^{5/4}(d\tan(e+fx))^{5/2}(b\csc(e+fx))^m\,{}_2F_1\left(\frac{5}{4},\frac{1}{4}(5-2m);\frac{1}{4}(9-2m);\sin^2(e+fx)\right)}{df(5-2m)}$$

```
[Out] (2*(Cos[e + f*x]^2)^(5/4)*(b*Csc[e + f*x])^m*Hypergeometric2F1[5/4, (5 - 2*
m)/4, (9 - 2*m)/4, Sin[e + f*x]^2]*(d*Tan[e + f*x])^(5/2))/(d*f*(5 - 2*m))
```

Rubi [A] time = 0.166997, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2618, 2602, 2577}

$$\frac{2\cos^2(e+fx)^{5/4}(d\tan(e+fx))^{5/2}(b\csc(e+fx))^m\,{}_2F_1\left(\frac{5}{4},\frac{1}{4}(5-2m);\frac{1}{4}(9-2m);\sin^2(e+fx)\right)}{df(5-2m)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^m*(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(5/4)*(b*Csc[e + f*x])^m*Hypergeometric2F1[5/4, (5 - 2*
m)/4, (9 - 2*m)/4, Sin[e + f*x]^2]*(d*Tan[e + f*x])^(5/2))/(d*f*(5 - 2*m))
```

Rule 2618

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m], Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (b\csc(e+fx))^m (d\tan(e+fx))^{3/2}\,dx = \left((b\csc(e+fx))^m \left(\frac{\sin(e+fx)}{b}\right)^m\right)\int \left(\frac{\sin(e+fx)}{b}\right)^{-m} (d\tan(e+fx))^{3/2}\,dx$$

$$= \frac{\left(\cos^{\frac{5}{2}}(e+fx)(b\csc(e+fx))^{3+m}\left(\frac{\sin(e+fx)}{b}\right)^{\frac{1}{2}+m}(d\tan(e+fx))^{5/2}\right)\int \frac{\left(\frac{\sin(e+fx)}{b}\right)^{\frac{3}{2}}}{\cos^{\frac{3}{2}}(e+f}}{bd}$$

$$= \frac{2\cos^2(e+fx)^{5/4}(b\csc(e+fx))^{3+m}\,{}_2F_1\left(\frac{5}{4},\frac{1}{4}(5-2m);\frac{1}{4}(9-2m);\sin^2(e+fx)\right)}{b^3df(5-2m)}$$

Mathematica [A] time = 5.68104, size = 87, normalized size = 1.1

$$-\frac{2(d\tan(e+fx))^{5/2}\sec^2(e+fx)^{-m/2}(b\csc(e+fx))^m\,{}_2F_1\left(\frac{1}{4}(5-2m),1-\frac{m}{2};\frac{1}{4}(9-2m);-\tan^2(e+fx)\right)}{df(2m-5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^m*(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*(b*Csc[e + f*x])^m*Hypergeometric2F1[(5 - 2*m)/4, 1 - m/2, (9 - 2*m)/4,
 -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(5/2))/(d*f*(-5 + 2*m)*(Sec[e + f*x]^2)^
(m/2))
```

Maple [F] time = 0.174, size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^m\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^m*(d*tan(f*x+e))^(3/2),x)
```

```
[Out] int((b*csc(f*x+e))^m*(d*tan(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\csc\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(f*x + e))^(3/2)*(b*csc(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{d\tan\left(fx+e\right)}\left(b\csc\left(fx+e\right)\right)^{m}d\tan\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(f*x + e))*(b*csc(f*x + e))^m*d*tan(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**m*(d*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\csc\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*(d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(f*x + e))^(3/2)*(b*csc(f*x + e))^m, x)
```

3.384 $\int (b\csc(e+fx))^m \sqrt{d\tan(e+fx)}\,dx$

Optimal. Leaf size=79

$$\frac{2\cos^2(e+fx)^{3/4}(d\tan(e+fx))^{3/2}(b\csc(e+fx))^m\,{}_2F_1\left(\frac{3}{4},\frac{1}{4}(3-2m);\frac{1}{4}(7-2m);\sin^2(e+fx)\right)}{df(3-2m)}$$

```
[Out] (2*(Cos[e + f*x]^2)^(3/4)*(b*Csc[e + f*x])^m*Hypergeometric2F1[3/4, (3 - 2*
m)/4, (7 - 2*m)/4, Sin[e + f*x]^2]*(d*Tan[e + f*x])^(3/2))/(d*f*(3 - 2*m))
```

Rubi [A] time = 0.145495, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2618, 2602, 2577}

$$\frac{2\cos^2(e+fx)^{3/4}(d\tan(e+fx))^{3/2}(b\csc(e+fx))^m\,{}_2F_1\left(\frac{3}{4},\frac{1}{4}(3-2m);\frac{1}{4}(7-2m);\sin^2(e+fx)\right)}{df(3-2m)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^m*Sqrt[d*Tan[e + f*x]],x]
```


```
[Out] (2*(Cos[e + f*x]^2)^(3/4)*(b*Csc[e + f*x])^m*Hypergeometric2F1[3/4, (3 - 2*
m)/4, (7 - 2*m)/4, Sin[e + f*x]^2]*(d*Tan[e + f*x])^(3/2))/(d*f*(3 - 2*m))
```


Rule 2618

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m], Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (b\csc(e+fx))^m\sqrt{d\tan(e+fx)}\,dx=\left((b\csc(e+fx))^m\left(\frac{\sin(e+fx)}{b}\right)^m\right)\int\left(\frac{\sin(e+fx)}{b}\right)^{-m}\sqrt{d\tan(e+fx)}\,dx$$

$$=\frac{\left(\cos^{\frac{3}{2}}(e+fx)(b\csc(e+fx))^{2+m}\left(\frac{\sin(e+fx)}{b}\right)^{\frac{1}{2}+m}(d\tan(e+fx))^{3/2}\right)\int\frac{\left(\frac{\sin(e+fx)}{b}\right)^{\frac{1}{2}-m}}{\sqrt{\cos(e+fx)}}}{bd}$$

$$=\frac{2\cos^2(e+fx)^{3/4}(b\csc(e+fx))^{2+m}\,{}_2F_1\left(\frac{3}{4},\frac{1}{4}(3-2m);\frac{1}{4}(7-2m);\sin^2(e+fx)\right)\sin}{b^2df(3-2m)}$$

Mathematica [A] time = 3.10688, size = 87, normalized size = 1.1

$$-\frac{2(d\tan(e+fx))^{3/2}\sec^2(e+fx)^{-m/2}(b\csc(e+fx))^m\,{}_2F_1\left(\frac{1}{4}(3-2m),1-\frac{m}{2};\frac{1}{4}(7-2m);-\tan^2(e+fx)\right)}{df(2m-3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^m*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (-2*(b*Csc[e + f*x])^m*Hypergeometric2F1[(3 - 2*m)/4, 1 - m/2, (7 - 2*m)/4,
 -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(3/2))/(d*f*(-3 + 2*m)*(Sec[e + f*x]^2)^
(m/2))
```

Maple [F] time = 0.171, size = 0, normalized size = 0.

$$\int\left(b\csc\left(fx+e\right)\right)^m\sqrt{d\tan\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^m*(d*tan(f*x+e))^(1/2),x)
```

```
[Out] int((b*csc(f*x+e))^m*(d*tan(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\tan\left(fx+e\right)}\left(b\csc\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*tan(f*x + e))*(b*csc(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\tan\left(fx+e\right)}\left(b\csc\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(f*x + e))*(b*csc(f*x + e))^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(e+fx\right)\right)^{m}\sqrt{d\tan\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**m*(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral((b*csc(e + f*x))**m*sqrt(d*tan(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\tan\left(fx+e\right)}\left(b\csc\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*tan(f*x + e))*(b*csc(f*x + e))^m, x)
```

3.385 $\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx$

Optimal. Leaf size=79

$$\frac{2\sqrt[4]{\cos^2(e+fx)}\sqrt{d\tan(e+fx)}(b\csc(e+fx))^m\,{}_2F_1\left(\frac{1}{4},\frac{1}{4}(1-2m);\frac{1}{4}(5-2m);\sin^2(e+fx)\right)}{df(1-2m)}$$

```
[Out] (2*(Cos[e + f*x]^2)^(1/4)*(b*Csc[e + f*x])^m*Hypergeometric2F1[1/4, (1 - 2*
m)/4, (5 - 2*m)/4, Sin[e + f*x]^2]*Sqrt[d*Tan[e + f*x]])/(d*f*(1 - 2*m))
```

Rubi [A] time = 0.150533, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2618, 2602, 2577}

$$\frac{2\sqrt[4]{\cos^2(e+fx)}\sqrt{d\tan(e+fx)}(b\csc(e+fx))^m\,{}_2F_1\left(\frac{1}{4},\frac{1}{4}(1-2m);\frac{1}{4}(5-2m);\sin^2(e+fx)\right)}{df(1-2m)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^m/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(1/4)*(b*Csc[e + f*x])^m*Hypergeometric2F1[1/4, (1 - 2*
m)/4, (5 - 2*m)/4, Sin[e + f*x]^2]*Sqrt[d*Tan[e + f*x]])/(d*f*(1 - 2*m))
```

Rule 2618

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m], Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx = \left((b\csc(e+fx))^m\left(\frac{\sin(e+fx)}{b}\right)^m\right)\int \frac{\left(\frac{\sin(e+fx)}{b}\right)^{-m}}{\sqrt{d\tan(e+fx)}}\,dx$$

$$= \frac{\left(\sqrt{\cos(e+fx)}(b\csc(e+fx))^{1+m}\left(\frac{\sin(e+fx)}{b}\right)^{\frac{1}{2}+m}\sqrt{d\tan(e+fx)}\right)\int\sqrt{\cos(e+fx)}\left(\frac{\sin(e+fx)}{b}\right)^{-\frac{1}{2}-}}{bd}$$

$$= \frac{2\sqrt[4]{\cos^2(e+fx)}(b\csc(e+fx))^{1+m}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4}(1-2m);\frac{1}{4}(5-2m);\sin^2(e+fx)\right)\sin(e+fx)\sqrt{d\tan}}{bdf(1-2m)}$$

Mathematica [A] time = 1.22483, size = 87, normalized size = 1.1

$$-\frac{2\sqrt{d\tan(e+fx)}\sec^2(e+fx)^{-m/2}(b\csc(e+fx))^m\,{}_2F_1\left(\frac{1}{4}(1-2m),1-\frac{m}{2};\frac{1}{4}(5-2m);-\tan^2(e+fx)\right)}{df(2m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^m/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (-2*(b*Csc[e + f*x])^m*Hypergeometric2F1[(1 - 2*m)/4, 1 - m/2, (5 - 2*m)/4,
 -Tan[e + f*x]^2]*Sqrt[d*Tan[e + f*x]])/(d*f*(-1 + 2*m)*(Sec[e + f*x]^2)^(m
/2))
```

Maple [F] time = 0.168, size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^m \frac{1}{\sqrt{d\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^m/(d*tan(f*x+e))^(1/2),x)
```

```
[Out] int((b*csc(f*x+e))^m/(d*tan(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\csc(fx+e))^m}{\sqrt{d\tan(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^m/sqrt(d*tan(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{d\tan(fx+e)}\,(b\csc(fx+e))^m}{d\tan(fx+e)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(f*x + e))*(b*csc(f*x + e))^m/(d*tan(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**m/(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral((b*csc(e + f*x))**m/sqrt(d*tan(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\csc\left(fx+e\right)\right)^{m}}{\sqrt{d\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^m/sqrt(d*tan(f*x + e)), x)
```

3.386 $\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=79

$$-\frac{2(b\csc(e+fx))^m\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4}(-2m-1);\frac{1}{4}(3-2m);\sin^2(e+fx)\right)}{df(2m+1)\sqrt[4]{\cos^2(e+fx)}\sqrt{d\tan(e+fx)}}$$

```
[Out] (-2*(b*Csc[e + f*x])^m*Hypergeometric2F1[-1/4, (-1 - 2*m)/4, (3 - 2*m)/4, S
in[e + f*x]^2])/(d*f*(1 + 2*m)*(Cos[e + f*x]^2)^(1/4)*Sqrt[d*Tan[e + f*x]])
```

Rubi [A] time = 0.173675, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2618, 2602, 2577}

$$-\frac{2(b\csc(e+fx))^m\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4}(-2m-1);\frac{1}{4}(3-2m);\sin^2(e+fx)\right)}{df(2m+1)\sqrt[4]{\cos^2(e+fx)}\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^m/(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*(b*Csc[e + f*x])^m*Hypergeometric2F1[-1/4, (-1 - 2*m)/4, (3 - 2*m)/4, S
in[e + f*x]^2])/(d*f*(1 + 2*m)*(Cos[e + f*x]^2)^(1/4)*Sqrt[d*Tan[e + f*x]])
```

Rule 2618

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m], Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx &= \left((b\csc(e+fx))^m\left(\frac{\sin(e+fx)}{b}\right)^m\right)\int \frac{\left(\frac{\sin(e+fx)}{b}\right)^{-m}}{(d\tan(e+fx))^{3/2}}\,dx\\
&= \frac{\left((b\csc(e+fx))^m\left(\frac{\sin(e+fx)}{b}\right)^{\frac{1}{2}+m}\right)\int \cos^{\frac{3}{2}}(e+fx)\left(\frac{\sin(e+fx)}{b}\right)^{-\frac{3}{2}-m}\,dx}{bd\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}\\
&= -\frac{2(b\csc(e+fx))^m\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4}(-1-2m);\frac{1}{4}(3-2m);\sin^2(e+fx)\right)}{df(1+2m)\sqrt[4]{\cos^2(e+fx)}\sqrt{d\tan(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 3.5421, size = 87, normalized size = 1.1

$$-\frac{2\sec^2(e+fx)^{-m/2}(b\csc(e+fx))^m\,{}_2F_1\left(\frac{1}{4}(-2m-1),1-\frac{m}{2};\frac{1}{4}(3-2m);-\tan^2(e+fx)\right)}{df(2m+1)\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^m/(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*(b*Csc[e + f*x])^m*Hypergeometric2F1[(-1 - 2*m)/4, 1 - m/2, (3 - 2*m)/4
, -Tan[e + f*x]^2])/(d*f*(1 + 2*m)*(Sec[e + f*x]^2)^(m/2)*Sqrt[d*Tan[e + f*
x]])
```

Maple [F] time = 0.146, size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^m\left(d\tan\left(fx+e\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^m/(d*tan(f*x+e))^(3/2),x)
```

```
[Out] int((b*csc(f*x+e))^m/(d*tan(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\csc\left(fx+e\right)\right)^{m}}{\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m/(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^m/(d*tan(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{d\tan\left(fx+e\right)}\left(b\csc\left(fx+e\right)\right)^{m}}{d^{2}\tan\left(fx+e\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m/(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(f*x + e))*(b*csc(f*x + e))^m/(d^2*tan(f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**m/(d*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\csc(fx+e))^m}{(d\tan(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m/(d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^m/(d*tan(f*x + e))^(3/2), x)
```

3.387 $\int (a\csc(e+fx))^m (b\tan(e+fx))^n\,dx$

Optimal. Leaf size=89

$$\frac{\cos^2(e+fx)^{\frac{n+1}{2}}(a\csc(e+fx))^m(b\tan(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{1}{2}(-m+n+1);\frac{1}{2}(-m+n+3);\sin^2(e+fx)\right)}{bf(-m+n+1)}$$

```
[Out] ((Cos[e + f*x]^2)^((1 + n)/2)*(a*Csc[e + f*x])^m*Hypergeometric2F1[(1 + n)/
2, (1 - m + n)/2, (3 - m + n)/2, Sin[e + f*x]^2]*(b*Tan[e + f*x])^(1 + n))/
(b*f*(1 - m + n))
```

Rubi [A] time = 0.149748, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2618, 2602, 2577}

$$\frac{\cos^2(e+fx)^{\frac{n+1}{2}}(a\csc(e+fx))^m(b\tan(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{1}{2}(-m+n+1);\frac{1}{2}(-m+n+3);\sin^2(e+fx)\right)}{bf(-m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csc[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + n)/2)*(a*Csc[e + f*x])^m*Hypergeometric2F1[(1 + n)/
2, (1 - m + n)/2, (3 - m + n)/2, Sin[e + f*x]^2]*(b*Tan[e + f*x])^(1 + n))/
(b*f*(1 - m + n))
```

Rule 2618

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m], Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (a\csc(e+fx))^m (b\tan(e+fx))^n\,dx = \left((a\csc(e+fx))^m \left(\frac{\sin(e+fx)}{a}\right)^m\right)\int \left(\frac{\sin(e+fx)}{a}\right)^{-m} (b\tan(e+fx))^n\,dx$$

$$= \frac{\left(\cos^{1+n}(e+fx)(a\csc(e+fx))^{1+m}\left(\frac{\sin(e+fx)}{a}\right)^{m-n}(b\tan(e+fx))^{1+n}\right)\int \cos^{-n}(e\cdots}{ab}$$

$$= \frac{\cos^2(e+fx)^{\frac{1+n}{2}}(a\csc(e+fx))^{1+m}\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{2}(1-m+n);\frac{1}{2}(3-m+n);\sin^2(e\cdots\right.}{abf(1-m+n)}$$

Mathematica [C] time = 1.96385, size = 287, normalized size = 3.22

$$-\frac{a(m-n-3)(a\csc(e+fx))\cdots}{f(m-n-1)\left((m-n-3)F_1\left(\frac{1}{2}(-m+n+1);n,1-m;\frac{1}{2}(-m+n+3);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2\tan^2\left(\cdots\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*Csc[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] -((a*(-3 + m - n)*AppellF1[(1 - m + n)/2, n, 1 - m, (3 - m + n)/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(a*Csc[e + f*x])^(-1 + m)*(b*Tan[e + f*x])
^n)/(f*(-1 + m - n)*((-3 + m - n)*AppellF1[(1 - m + n)/2, n, 1 - m, (3 - m
+ n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*((-1 + m)*AppellF1[(3
- m + n)/2, n, 2 - m, (5 - m + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2] + n*AppellF1[(3 - m + n)/2, 1 + n, 1 - m, (5 - m + n)/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))
```

Maple [F] time = 0.789, size = 0, normalized size = 0.

$$\int \left(a\csc\left(fx+e\right)\right)^m \left(b\tan\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csc(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

```
[Out] int((a*csc(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\csc\left(fx+e\right)\right)^{m}\left(b\tan\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csc(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*csc(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\csc\left(fx+e\right)\right)^{m}\left(b\tan\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csc(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*csc(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\csc\left(e+fx\right)\right)^{m}\left(b\tan\left(e+fx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csc(f*x+e))**m*(b*tan(f*x+e))**n,x)
```

```
[Out] Integral((a*csc(e + f*x))**m*(b*tan(e + f*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\csc\left(fx+e\right)\right)^{m}\left(b\tan\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csc(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*csc(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType, Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: # isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```